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A Fault-Tolerant Dynamic Scheduling Algorithm for Real-Time Systems on Heterogeneous Multiprocessor

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在異質多處理器上針對即時系統具有容錯能力之動態排程演算法 A Fault-Tolerant Dynamic Scheduling Algorithm for Real-Time Systems on Heterogeneous Multiprocessor

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摘要

即時系統已經廣泛地被應用在許多需要嚴格地符合時間要求的環境 中。在即時系統中的工作必須在時間限制內完成,否則可能造成嚴重的後 果。由於對穩定性的高度要求,容錯能力也是即時系統所必須具備的。由 於工作在進入系統後才能開始被排程,因此需要的是動態的排程演算法。 本論文即是提出一個在異質多處理器上針對即時系統具有容錯能力的動態 排程演算法。我們將會提出一個以工作可排程的時間與所需要的執行時間 作為考量的 heuristic 函式,來決定工作排程的優先順序。針對為達到容 錯目的所用的 backup,我們也提出新的排程策略,稱為 MNO。經由動態地 模擬一個即時系統,結果顯示我們提出的方法能夠決定出更恰當的排程順 序,而且挪出更多的可排程時間給後來的工作,使得較多的工作能夠在時 間限制前完成執行。並且在不同的環境中,不需要搭配任何參數也能得到 較好的結果。

A Fault-Tolerant Dynamic Scheduling Algorithm for Real-Time Systems on Heterogeneous Multiprocessor

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Abstract

Real-time systems are being increasingly used in several applications which are time critical. Tasks corresponding to these applications have deadlines to be met. Fault-tolerance is an important requirement of such systems, due to the catastrophic consequences of not tolerating faults. In this thesis, we propose an algorithm do dynamically schedule arriving real-time tasks with PB fault-tolerant requirement on to a set of heterogeneous multiprocessor. Our algorithm, named density first with minimum non-overlap scheduling algorithm (DNA), proposes two performance improving techniques. First, a new heuristic function, called density, takes account of the needed computation time and schedulable time of a task. The task with the maximum density value will be given the highest priority. Second, the MNO strategy for backup scheduling will minimize the time reserved for backups. In the result of dynamic simulation, we can find that our algorithm has fewer rejected tasks and more general and suitable for any kind of environment.

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Chapter 1. Introduction

A heterogeneous system is a computing platform consisting of different kinds of processors which are interconnected with some topology. These processors vary in computation power and dedicated purposes. Because of the multiple and various processors, the heterogeneous systems can support parallel and distributed applications. It is important to distribute the tasks to the appropriate processors for achieving high performance [1-14, 19-28].

In recent years, computing systems have been used in several applications which have stringent timing constraints, such as autopilot systems, satellite and nuclear plant control. *Real-time systems* are defined as those systems in which the correctness of the system depends on not only the logical result of computation, but also on the time at which the results are produced [14]. Real-time systems are broadly classified into three categories as follows: [16] (1) *hard real-time* systems, in which the consequences of not executing a task before its deadline may be catastrophic, (2) *firm real-time* systems, in which the result produced by the corresponding task ceases to be useful as soon as the deadline expires, but the consequences of not meeting the deadlines are not very severe, (3) *soft real-time* systems, in which the expires. We will use the hard real-time system as our system model.

The problem of scheduling real-time tasks in heterogeneous multiprocessors is to determine when and on which processor a given task executes [14, 16]. This can be done either statically or dynamically. If the characteristics of tasks such as arrival time and deadline could be determined a priori, the scheduling process could be done by a static algorithm. Static algorithms are often used to schedule periodic tasks. However, this approach is not applicable to aperiodic tasks whose characteristics are not known a priori. Scheduling such tasks require a dynamic scheduling algorithm [12]. In dynamic scheduling, when new tasks

arrive at the system, the scheduler dynamically determines whether these tasks could be scheduled successfully in the current schedule. A task is feasible if it can be scheduled successfully. In hard real-time systems, if a task is found to be unfeasible, it should be rejected as soon as possible.

Real-time systems usually need high reliability. Thus, fault-tolerance is an important issue in many real-time applications. A system is fault-tolerant if it produces correct results even in the presence of faults [29]. In real-time multiprocessor systems, fault tolerance can be provided by scheduling multiple copies of a task on different processors. Three different models have evolved for fault-tolerant scheduling of real-time tasks. Firstly, the *Primary Backup (PB)* model duplicates an additional copy, called *backup*, except the original one, called *primary* [3]. The backup is executed only if the primary fails to produce the correct results. Secondly, in the *Triple Modular Redundancy (TMR)* model, three copies of a task are executed concurrently to achieve error checking by comparing results after completion [15, 16]. Thirdly, in the *Imprecise Computational (IC)* model, a task is divided into mandatory and optional parts [5]. The mandatory part must be completed before the deadline of a task for acceptable quality of result, and the optional part refines the result.

In this thesis, we address the problem of dynamically scheduling hard real-time tasks with PB fault-tolerant requirement on to a set of heterogeneous multiprocessor. The objective of any dynamic real-time scheduling algorithm is to improve the number of tasks whose deadlines are met. Most dynamic scheduling algorithms for real-time tasks take the steps similar to the list scheduling algorithms. First, a heuristic function determines the scheduling priority of tasks, and then, tasks are scheduled on appropriate processors in the order of nonincreasing priority. The integrated heuristic function used in existing algorithms emphasizes whether a task could be executed earlier. However, the needed computation time and the time which is schedulable for a task will determine whether it is flexible to be scheduled. For this observation, we will propose an advanced heuristic function, name *density* function. The density function takes account of the relation between computation time and schedulable time, and will select the least flexible to be scheduled first. We also propose a new strategy for scheduling backup copy, named *MNO*, to minimize the reserved processor time for the backups. The MNO strategy takes advantage of backup overlapping and will find a processor where the least extra time, i.e. non-overlapped time, is needed for a backup. New arriving tasks will benefit from MNO because the schedulable time for them is getting more.

For evaluating the performance, we construct a dynamic simulation and compare our algorithm with *FTMA* [8] and *distance myopic algorithm* [2]. Unlike these two algorithms, one feature of our algorithm is that it doesn't need any input parameter for any kind of system environments. In the simulation result, we can see the density function selects more appropriate tasks, and MNO saves more time for new tasks. Moreover, DNA outperforms FTMA and distance myopic with any combination of parameters.

The remainder of this thesis is organized as follows. In chapter 2, we define the system model and describe representative previous works done in the area of real-time fault-tolerant scheduling. In chapter 3, we propose an algorithm for dynamically scheduling real-time tasks with fault-tolerant requirement. In chapter 4, the performance of the proposed algorithm is evaluated through dynamic simulation and compared with the algorithms described in chapter 2. Finally, in chapter 5, we make some conclusions and future work.

Chapter 2. System Model and Fundamental Background

In this chapter, we first present our assumptions and system model, including task model, scheduler model, and fault tolerance model. Then we discuss the existing work on fault-tolerant scheduling algorithm. Finally, the limitations of those previous algorithms will be highlighted to induce the motivation for our research work.

2.1 System Model and Assumptions

The target environment is a multiprocessor system consisting of multiple heterogeneous processing units. The heterogeneity of processors means that the computation time of a task varies from processor to processor, such as Application-Specific Integrated Circuit (ASIC), Field Programmable Gate Array (FPGA), and Digital Signal Processor (DSP) which are applied widely on real-time embedded systems [11]. Basically, we will give some basic concepts in the following.

2.1.1 Task Model

The real-time systems discussed in this thesis are hard real-time [16]. Every real-time task T_i has the following attributes:

- (i) arrival time (a_i) , at which T_i enters the task queue.
- (ii) ready time (r_i) , at which T_i is really ready to execute.
- (iii) deadline (d_i) , by which T_i must finish execution.
- (iv) computation time (c_{ip}) , the c_{ip} represents the computation time of task T_i on processor p.

Generally, a task is ready to execute at arrival time, i.e., $a_i = r_i$.

We assume that tasks are aperiodic [2], i.e., the task arrivals are not known a priori. Attributes of a task is unknown until it is generated and enters the system. Periodic task model is a special case of aperiodic task model, so that the algorithm proposed in this thesis is also applied to periodic tasks.

We also assume that tasks are independent, i.e., there are no precedence constraints between tasks. Nevertheless, dealing with precedence constraints is equivalent to working with the modified ready times and deadlines [5]. Therefore, the proposed algorithm can also be applied to tasks with precedence constraints among them. There are also no communications between tasks.

Finally, tasks are nonpreemptable, i.e., when a task starts execution on a processor, it finishes to its completion.

2.1.2 Scheduler Model



Single scheduler in this model may make a point of failure. The scheduler model may be fault-tolerant by employing modular redundancy technique in which another backup scheduler runs scheduling in parallel with the primary scheduler [2]. Final schedule is chosen from the results of the two schedulers by an acceptance test. A simple acceptance test is to check whether all the tasks in the schedule finish before their deadlines.

Due to the assumption of the hard real-time task model, we assume each task is scheduled as soon as possible. That is, the scheduling action of the scheduler may be periodic

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Fig. 2.1 Scheduler model

with a small time quantum or may be starting as a task arrives. Because the scheduler is a dedicated processor, the high frequency of the scheduling would not affect the other general processors.



In this thesis, we uses *Primary-Backup (PB)* model for fault tolerance [3]. In the PB model, each task has two copies, namely, primary copy and backup copy. The backup copy is redundant for the purpose of fault tolerance, and starts execution only when the primary copy fails. Using PB model for fault tolerance, the following necessary conditions are required.

- (1) *Mutually exclusive in space*: primary and backup copies must be scheduled on different processors (to tolerate permanent processor faults).
- (2) *Mutually exclusive in time*: the start time of a backup copy must be latter than the finish time of its primary copy. This condition also implies two facts. First, the two versions of a task are not parallelizable. Second, the sum of computation times of primary and backup copies should be less than or equal to $(d_i r_i)$ so that the both copies can be schedulable within this interval.

Because backup is the only redundant copy of a task, we assume that each task encounters at most one failure either due to hardware failure or due to software failure. This also implies that there is at most one failure in the system at any instant of time.

2.2 Related Work

Many scheduling problems have been proved to be NP-complete [6], i.e., it was believed that there is no optimal polynomial-time algorithm for them. It was also shown that an algorithm does not exist for optimally scheduling dynamically arriving tasks on a multiprocessor system [7]. These negative results motivated the need for heuristic approaches to solve the scheduling problems. Generally, the heuristic scheduling algorithms try to find a feasible schedule by repeating two steps. The first step is to select one task with the highest priority which is determined by a heuristic function. The heuristic function synthesizes various characteristics of a task to form the priority of scheduling order. The next step is to decide one processor on which the selected task is assigned. The two steps repeat until all tasks in the queue are either scheduled or rejected.

In the past decade, many heuristic scheduling algorithms have been proposed to dynamically schedule a set of tasks whose deadlines and computation times are unknown until arriving into the system. For homogenous multiprocessor systems with resource constraint, an algorithm called *myopic algorithm* was proposed [1]. Initially, myopic sorts all tasks in the nondecreasing order of deadlines. In the task selection step, it checks whether the current partial schedule is *strongly feasible*. If so, a given heuristic is applied to all tasks in *feasibility check window*. The task with the lowest heuristic is chosen to extend the current schedule. If the current schedule is not strongly feasible, backtracking is done by discarding the current schedule, and extending the previous schedule by a different task. A partial schedule is said *strongly feasible* if all the schedules obtained by extending this schedule with

any one task in the feasibility check window are also feasible. *Feasibility check window* contains the first k tasks in the sorted task queue. In order to reduce the cost of looking ahead, [1] considers only the tasks in the feasibility check window both for strong-feasibility checking and for heuristic calculating. It has been shown that an integrated heuristic, which is a function of the deadline and the earliest start time of a task, outperforms other simple heuristics such as minimum deadline first, minimum computation time first, least laxity first, etc.

Myopic is efficient and effective. However, it is limited on homogeneous systems, and does not have the fault-tolerant mechanism which is more and more important for real-time systems. Another issue is about the heuristic function. Myopic sorts all the tasks initially in nondecreasing order of deadlines. When scheduling is in progress, the tasks under consideration are limited in the feasibility check window. Because the size of feasibility check window is small, the order of tasks to be scheduled is close to the order of nondecreasing deadlines. A more realistic heuristic will be defined in the next chapter.

In the PB model, backups are redundant if their primaries finish successfully. In order to utilize the redundant time, the concept of overlapping backups is proposed by *backup overloading algorithm* [3]. Overlapping backups means that more than one backup copy can be overlapped with each other on the same processor. The necessary condition of backup overloading is as follows. If the primary copies of two tasks are scheduled on two different processors, then their backups can be overlapped with each other on a processor. Fig. 2.2 illustrates the backup overloading. *Backup1* and *Backup3* can be overlapped with each other whose primaries, *Primary1* and *Primary3*, are scheduled on different processors respectively. Similarly, the *Backup2* and *Backup4* could also be overlapped with each other. Another concept proposed by [3] is *backup deallocation*. Backup deallocation means the reclamation of resources reserved for backup tasks when the corresponding primaries complete

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Fig. 2.2 Backup overloading

successfully. For example, the time interval used by the backups can reutilized if no faults occur. Both of these two techniques help improve acceptance ratio of arriving tasks.

For dynamically scheduling tasks with fault tolerance, *distance myopic algorithm* which is extended from myopic algorithm is proposed in [2]. The main difference between distance myopic and the original myopic algorithm is the construction of task queue. Because PB based fault tolerance is included in this method, all primaries in the task queue are sorted in nondecreasing order of deadlines. Then, the backups are inserted into the task queue at the distance, called *distance concept*, from their primary copies. The primary always precedes its backup in the task queue. The *distance* is an input parameter to the scheduling algorithm to determine the relative positions of the two copies of a task in the task queue. The primary and backup of a task are scheduled separately except that the backup could be scheduled until the primary has been scheduled. The other enhancement of distance myopic is *flexible backup overloading*, which introduces a trade-off between degree of fault tolerance and performance. The flexible overloading scheme permits more than one fault occur at any instant of time by forming the processors into different groups. In each group, there is at most one fault at a time.

Putting the primaries and backups in the single task queue is one of main disadvantages in this algorithm. The reason is that a backup may appear in the feasibility check window when its primary is also in the window. The strong feasibility checking is false because the backup is unschedulable. The situation will result in backtracking and eventually rejecting this task. Another disadvantage is that it is difficult to choose the right combination of the two parameters, distance and the size of feasibility check window. With varying situations of system environment, the same values of parameters will not always produce good result. Thus, adjustments to the parameters are needed in different situations.

Both of myopic and distance myopic are scheduling real-time tasks on homogeneous multiprocessor systems. *Fault-tolerant myopic algorithm (FTMA)* [8], which is extended from distance myopic, is used for heterogeneous multiprocessor systems. In addition, the enhancements of FTMA include task queue construction and feasibility check window movement. The main motivation of FTMA is to overcome the disadvantage of single task queue in distance myopic algorithm. FTMA puts primary and backup copies separately in two different task queues, namely primary task queue and backup task queue. Both primaries and backups are sorted in nondecreasing order of deadlines. FTMA selects the task with smaller heuristic value from the first primary and backup in the queues into the check window. The heuristic of a backup is defined as infinite when its primary has not been scheduled. In this way, the two versions of a task will not appear in the window simultaneously. The two-task-queue method also eliminates the need of the distance parameter which is difficult to be chosen. However, the other parameter, size of feasibility check window, is still needed in FTMA. As distance myopic, adjustment to this parameter is still needed in different situations of the system.

eFRCD is a static scheduling algorithm on heterogeneous systems [4]. It takes account of the heterogeneities of computation, communication and reliability. Tasks are judiciously allocated to processors so as to reduce the reliability cost, defined to be the product of processor failure rate and task execution time. Like the static algorithms proposed in [19, 30], eFRCD schedules all tasks with known attributes a priori. These static algorithms can not be applied to a dynamic system due to their high complexity. The assumption with a priori known attributes is also not realistic for the systems of aperiodic tasks.

In the next chapter, we will describe the proposed algorithm which schedules real-time tasks dynamically with fault tolerance on heterogeneous systems without any parameters.



Chapter 3. Density First with Minimum Non-overlap Scheduling Algorithm

We have described the relative background about dynamically scheduling real-time tasks with fault tolerance in the previous chapter. In this chapter, we will propose the *density first with minimum non-overlap scheduling algorithm (DNA)*. As those algorithms introduced in chapter 2, DNA repeats two phases continuously until all tasks in the queue are scheduled or rejected. The first one is the task selection phase followed by the second one, processor assignment phase. In the task selection phase, we define a new heuristic, namely *density*, to prioritize tasks in the task queue. In the processor assignment phase, the primary copy is scheduled first, and then the next step is backup scheduling. A new strategy, called *minimum non-overlap (MNO)*, is proposed for scheduling backups.

In section 3.1, we describe the definition of the density heuristic. In section 3.2, we state the minimum non-overlap strategy for backup scheduling. Finally, the complete algorithm will be presented in section 3.3 followed by the discussion of complexity.

3.1 A New Heuristic Function

In the first phase, a heuristic function is used to decide the priorities of tasks. A task with the highest priority will be selected to be scheduled first. Most scheduling algorithms described in section 2.2 use deadlines and earliest finish times as the integrated heuristic function. This implies that tasks which may or have to finish earlier are given higher priorities. Nevertheless, real-time tasks are not concerned about when to start computation but rather about meeting their deadlines. For example, tasks with earlier deadlines or finish times may have small computation times so that they are still schedulable even with lower scheduling order. On the other hand, tasks with later deadlines or finish times should be given higher priorities if their computation times are large with respect to their schedulable intervals. The question of whether or not a task T_i could be scheduled by its deadline depends on the relationship between the schedulable intervals of T_i and its computation times on each processor. Thus, we introduce a concept of *density* as the heuristic function.

The time of all schedulable intervals for a task is called schedulable time generally. The density of a task is a ratio of the computation time to the schedulable time. When the computation time needed by T_i is getting closer to its schedulable time, we say that T_i has large density. With large density, it is less flexible to schedule T_i . That is, if T_i does not have higher priority of scheduling order, it may be rejected because the schedulable time is getting less after other tasks are scheduled. Conversely, if the computation time needed by T_i is small with respect to its schedulable time, i.e. small density, T_i may be still schedulable with lower priority. This is because it is easier to find schedulable intervals for tasks with small density than tasks with large density. Before we give the formal definition of density, some terminologies will be defined first.

Let Pr_i and Bk_i be the primary and backup copies of task T_i respectively, and Proc is the set of all processors in the system. Because of the time exclusion between primary and backup copies of a task, the scheduled start time of Bk_i must be greater than or equal to the scheduled finish time of Pr_i . It implies that the deadline of T_i , d_i , is impossible to be the latest finish time of Pr_i , and the ready time of T_i , r_i , is also impossible to the earliest start time of Bk_i . In the following, the start or finish time for primary and backup will be redefined. First, Bk_i is impossible to be scheduled if the interval between the scheduled finish time of Pr_i and d_i is less than the minimum computation time of T_i . Thus, Pr_i must be finished before its latest finish time, $LFP(T_i)$. Actually, $LFP(T_i)$ could be thought of as the deadline of Pr_i . Finally, we give the following definitions.

Definition 3.1 The Latest Finish time of Primary (LFP) is defined as:

$$LFP(T_i) = d_i - MIN\{c_{in}\} \text{, for all } p \in Proc$$
(1)

, where d_i is the deadline of T_i , and c_{ip} is the computation time of T_i on processor p.

In additional to the LFP, T_i has the earliest finish time for its primary.

Definition 3.2 The *Earliest Finish time of Primary (EFP)* is defined as:

$$EFP(T_i) = MIN\{EST(Pr_i)_p + c_{in}\}, \text{ for all } p \in Proc$$
(2)

, where $EST(Pr_i)_p$ is the earliest start time of Pr_i on processor p.

The scheduled start time of Bk_i could be as early as the earliest finish time of Pr_i . Thus, we define the earliest start time of backup (*ESB*) for T_i as:

$$ESB(T_i) = EFP(T_i) \tag{3}$$

 $ESB(T_i)$ could also be thought of as the ready time of Bk_i . With the definitions of *LFP* and *ESB*, we could claim that Pr_i should be scheduled between r_i and $LFP(T_i)$, and Bk_i should be scheduled between $ESB(T_i)$ and d_i .

Neither primary nor backup of a task could be scheduled on the processors without sufficient schedulable time for it. The processor on which the primary or backup could be scheduled may be different. Thus, we define a set of available processors for the primary and backup respectively. Each set contains processors on which the primary (backup) can be scheduled.

Definition 3.3 We define $availP(Pr_i)$, the set of available processor for Pr_i , and $availP(Bk_i)$, the set of available processor for Bk_i , as follows:

(1) $availP(Pr_i) = \{ processor j \}, where there is at least one time slot greater than or$

equal to c_{ij} between r_i and $LFP(T_i)$ on *processor j*. The slot cannot be overlapped with any scheduled tasks, neither primaries nor backups.

(2) $availP(Bk_i) = \{ processor j \}$, where there is at least one time slot greater than or equal to c_{ij} between $ESB(T_i)$ and d_i on processor j. The slot can be overlapped with any scheduled backups.

Here, we give an example to demonstrate the above definitions. We assume there are four processors in the system, the current time is 10, and there are three tasks have arrived into the system. Fig. 3.1 shows the current schedule, and Table 3.1 lists the attributes of the tasks in the queue. This example will be used for illustration throughout this chapter. In this example, Pr_9 must finish before the time = 40 because the Br_9 may be scheduled on processor 3 where T_9 has the maximum computation time. Thus, the $LFP(T_9)$ is 40. Pr_9 could start at time = 17, and has the earliest finish time, 26, on processor 2. This implies Bk_9 could start as early as time = 26. Thus the $ESB(T_9)$ is 26. Pr_9 can be scheduled in the range from the ready time, $r_9 = 10$, to the LFP(T_9), 40. On processor 3, there are two time slots, (10, 15) and (28, 40), which are not reserved for any task yet between r_9 and $LFP(T_9)$. But both of them are less than c_{93} so that Pr_9 could not be scheduled on processor 3. Similarly, Pr_9 could not be scheduled on processor 4 because the only unreserved time slot between r_9 and $LFP(T_9)$, (21, 30), is less than c_{94} . Thus, the set of processors which Pr_9 can be scheduled, i.e. $availP(Pr_9)$, is {processor 1, processor 2}. Bk_9 can be scheduled in the range from the $ESB(T_9)$, 26, to the deadline, $d_9 = 45$. Between $ESB(T_9)$ and d_9 , we assume all scheduled backups may be overlapped with Bk_9 . On processor 1, 2, 3, there are time slots large enough to execute Bk_9 . However, time slots (26, 30) and (40, 45) are too small to execute Bk_9 . Thus, the avail $P(Bk_9)$ is {processor 1, processor 2, processor 3}.

As we mentioned early in this section, the density of a task is a ratio of the computation

	r _i	d_i	C _{il}	c_{i2}	c _{i3}	<i>C</i> _{<i>i</i>4}
T_9	10	45	5	11	15	10
T_{10}	10	50	15	14	11	10
T_{11}	10	37	5	7	5	8

Table 3.1. Attributes of tasks in the task queue.



time to its schedulable time. First, because of the two copies of a task, the computation time needed by T_i includes the time for Pr_i and Bk_i . Actually, we define the time needed by primary and backup as the average computation time on those processors in the *availP*(Pr_i) and *availP*(Bk_i) respectively.

Definition 3.5 For task T_i , we define Pr_mean , the average computation time of Pr_i on $availP(Pr_i)$, and Bk_mean , the average computation time of Bk_i on $availP(Bk_i)$ as follows:

$$Pr_mean(T_i) = \sum_{p \in availP(Pr_i)} / |availP(Pr_i)|$$
(4)

$$Bk_mean(T_i) = \sum_{p \in availP(Bk_i)} / |availP(Bk_i)|$$
(5)

, where $|availP(Pr_i)|$ and $|availP(Bk_i)|$ are the number of processor in $availP(Pr_i)$ and

 $availP(Bk_i)$ respectively.

Next, the schedulable time for T_i is the sum of time slots which are schedulable for Pr_i or Bk_i . A time slot, represented as (start, end), which is schedulable for Pr_i on processor p is defined as $prslot_{ip}$. $prslot_{ip}$ must satisfies the following conditions: (1) $start \ge r_i$, (2) $end \le LFP(T_i)$, (3) $end - start \ge c_{ip}$, (4) $prslot_{ip}$ cannot be overlapped with any scheduled primaries or backups on processor p. A time slot which is schedulable for Bk_i on processor p is defined as $bkslot_{ip}$. Similarly, $bkslot_{ip}$ must satisfy: (1) $start \ge ESB(T_i)$, (2) $end \le d_i$, (3) $end - start \ge c_{ip}$, (4) $bkslot_{ip}$ cannot be overlapped with scheduled primaries on processor p. For T_i , the total schedulable time is the sum of all $prslot_{ip}$ and $bkslot_{ip}$.



Definition 3.6 The Sum of Schedulable Time of T_i , named SST, is defined as:

$$SST(T_i) = \sum_{p \in availP(Pr_i)} prslot_{ip} + \sum_{p \in availP(Bk_i)} bkslot_{ip},$$
(6)

After introducing the above definitions, we could define the new heuristic function, named *density* function, as follows.

Definition 3.4 For each task T_i in the task queue, we define the *density* heuristic function as:

$$density(T_i) = \frac{Pr_mean(T_i) + Bk_mean(T_i)}{SST(T_i)}$$
(7)

The density of a task indicates the tightness of the interval between the ready time and the deadline with respect to its computation time. The highest density means that it is least flexible to schedule a task. On the other hand, low density means that it is easy to find schedulable intervals for primary and backup. Thus, we will select the task with the maximum density in task selection phase. It is to be noted that the density is defined for a task T_i , rather than Pr_i or Bk_i separately. The reason is that T_i will be rejected if either Pr_i or Bk_i fails to be scheduled. Getting less flexible to schedule either Pr_i or Bk_i implies that it is also inflexible to schedule T_i .

Table 3.2 gives an example of density calculation for the tasks in table 3.1. T_{10} are given the highest priority even though both of the $EFT(Pr_{10})$ and d_{10} are larger than the other tasks.

	LFP	ESB	availP(Pr)	availP(Bk)	Pr_mean	Bk_mean	$\sum prslot$	$\sum bkslot$	density
<i>T</i> 9	40	26	proc1,2	proc1,2,3	8	10.333	28	55	0.220
T_{10}	40	36	proc1,3	proc2,3,4	13	11.667	29	38	0.368
\overline{T}_{11}	32	15	proc1,2,3,4	proc1,2,3,4	6.25	6.25	36	54	0.139

Table 3.2. Density calculation for T_9 , T_{10} , T_{11} in Table 3.1.

3.2 Minimum Non-Overlap (MNO) for Backup

After selecting the task with the maximum density in the first phase, DNA starts to schedule the primary and backup in the second phase. As many scheduling algorithms for heterogeneous systems, we will schedules the primary on the processor where it could finish as early as possible. In the next step for scheduling backup, most algorithms like those described in section 2.2.1 also assign a backup to a processor according to the earliest finish time. The processor time reserved for backups, however, is redundant and will be deallocated if their primaries finish successfully. If the processor time reserved for backups could be minimized, there is more schedulable time for other new tasks. An intuitive method for this idea is to overlap backups as much as possible. It works for homogeneous systems, but doesn't work for heterogeneous systems. This is because the computation time of a task varies from processor to processor in heterogeneous systems. For a task T_i , having the maximum



Fig. 3.2. The maximum overlapped time of Bk_i is 10 on processor 1. The minimum non-overlapped time of Bk_i is 2 on processor 3.

overlapped time on processor p does not necessarily mean that T_i also has the minimum non-overlapped time on processor p. The non-overlapped time is actually the extra processor time being reserved for Bk_i . For example, in Fig. 3.2, lines with double arrows represent the computation time and the possible schedule of Bk_i where it has the minimum non-overlapped time on that processor. It is shown that Bk_i has the maximum overlapped time on processor 1, however 6 extra time units, i.e. interval (23, 29), are required. Similarly, 6 extra time units will be required for processor 2. Bk_i has the minimum non-overlapped time, 2, on processor 3 means that only 2 extra time units will be reserved for it. Thus, we will intend to minimize the extra time being reserved for backups, i.e. non-overlapped time. That is, a backup will be scheduled on the processor where it has the minimum non-overlapped time. We give the definition of minimum non-overlap of a backup as follows.

Definition 3.7 The *Minimum Non_Overlap* (*MNO*) of *Bk_i* is defined as:

$$MNO(Bk_i) = MIN\{c_{in} - MAXoverlap_{in}\}, \text{ for each } p \in Proc$$
(8)

, where $Maxoverlap_{ip}$ is the maximum overlapped time of Bk_i on processor p.



Fig. 3.3. $MNO(Bk_{10})$ is 3 on processor 4.

On each processor, we try to find the maximum overlapped time for Bk_i in order to minimize the non-overlapped time. Finally, Bk_i will be scheduled on the processor where $MNO(Bk_i)$ is obtained.

For the example in section 3.1, T_{10} has the maximum density and Pr_{10} is scheduled on processor 1 by its EFT shown in Fig.3.3. The schedule of Bk_{10} on processor 2 is exactly the interval ($ESB(T_{10})$, d_{10}), and the non-overlapped times is 7. On processor 3, there is no any backup to be overlapped with so that the non-overlapped time equals the $c_{10,3}$, 11. On processor 4, Bk_4 could be overlapped with Bk_{10} completely, and the non-overlapped time is only 3. Obviously, if Bk10 is scheduled on processor 4, the extra processor time reserved for it is only 3. Thus, $MNO(Bk_{10})$ is 3 and Bk_{10} will be scheduled on processor 4 even though the maximum overlapped time is on processor 2, and the earliest finish time is on processor 3.

3.3 The DNA Algorithm

After introducing the concepts of density and MNO, the complete algorithm of DNA is described in this section. Fig. 3.4 shows the flow of our DNA algorithm. Similar to most



Fig. 3.4 Flow chart of DNA algorithm

heuristic scheduling algorithms, the steps involved in DNA divided into two phases. The two phases repeats continuously until the task queue is empty. The first phase is to select the task with the maximum density as the candidate to be scheduled. In the second phase, we try to schedule the primary copy of the selected task first. If successfully, the next step is trying to schedule the backup copy. A task is said to be scheduled successfully only if both copies are scheduled successfully. Conversely, the selected task will be rejected if either the primary or



Fig. 3.5. Final schedule.

the backup fails to be scheduled. For the previous example, after T_{10} is scheduled successfully, we recalculate the density of the remaining tasks, T_9 , T_{11} . The next one to be scheduled is T_9 with density = 0.395, and T_{11} is the last one. The final schedule is shown in Fig. 3.5.

It is to be noted that after a task with the maximum density has been selected, we schedule the primary first, and then, schedule the backup immediately. This is different from many fault-tolerant scheduling algorithms, such as distance myopic and FTMA which schedule primaries and their backups as distinct tasks [2, 8]. The reason is that the density heuristic function determines the priorities among all original tasks, neither primaries nor backups. The advantage is that there is no need for task queue construction any more. That is, the distance parameter in distance myopic algorithm and the separated task queues in FTMA are eliminated.

It is also to be noted that we have no checking strong feasibility and backtracking which are used in distance myopic and FTMA [2, 8]. The purpose of checking strong feasibility is for looking ahead. Although the feasibility check window decreases the number of tasks to be checked, the overhead of checking each task exists at each selection phase. We just apply the density heuristic function to all the remaining tasks in the task queue. Because of the assumption of dynamic systems and scheduler model defined in chapter 2, the number of tasks in the task queue should not become so large while the scheduler starts scheduling. Thus, the overhead of the task selection shall not be very heavy. We need no backtracking in our algorithm. If either primary or backup fails to be scheduled, this task is just rejected. Both steps of checking and backtracking will increase the running time of algorithms in realistic systems. Thus, we neither check the strong feasibility of the current partial schedule, nor backtrack to the previous schedule if the selected task is not schedulable. It is just scheduling all the tasks one by one.

Because we take all tasks into consideration in the task selection phase, the complexity of worst case is $O(n^2)$. *n* is the number of tasks to be scheduled. The complexity is higher than that of FTMA and distance myopic, O(n). Nevertheless, *n* shall not be very large in the assumption of our scheduler model. In addition, a trick could also reduce the run time cost. The density heuristic function is applied to all tasks only before the repeat of the two scheduling phases. After a task is scheduled successfully, the heuristic function may not be applied to all the remaining tasks in the next task selection phase. Only those tasks, which have schedulable time slots overlapped with the just scheduled primary or backup, will be given the recalculated density.

In the next chapter, we will evaluate our DNA algorithm using simulation. The simulation result of DNA will compare with distance myopic and FTMA.

Chapter 4. Simulation and Performance Evaluations

In this chapter, we will evaluate the performance of *density first with minimum non-overlap scheduling algorithm* (*DNA*) through simulation. In section 4.1, we will describe the architecture of the simulator and some simulation parameters. Next, we will give the performance evaluations in section 4.2.

4.1 Simulation Construction

Because DNA is a dynamic scheduling algorithm, we will construct a dynamic simulation instead of a static simulation. The flow of dynamic simulation is divided into two parts. The first one is the task generator which generates a set of real-time tasks as the input of the second part, simulator. The simulator simulates the events in the systems and the actions of the scheduler. In the following, we will describe how to construct these two parts.

4.1.1 Task Generator

The task generator generates a set of real-time tasks in the non-decreasing order of arriving times. Each task has the attributes as described in section 2.1.1. The parameters which affect these attributes in the task generation are summarized in Table 4.1. In the following, we will describe how to decide the attributes of a task.

The computation time of a task varies from processor to processor, and is bounded by the minimum and maximum computation time, *MIN_C* and *MAX_C*. The *heterogeneity* variable, which is chosen uniformly between 0 and 1, represents the heterogeneity of computation times of a task. It determines the range of possible computation times for a task. Thus,

parameter	explanation	range of possible values
MIN_C	minimum computation time	10
MAX_C	maximum computation time	80
λ	task arrival rate	[0.3, 0.9] (real)
R	laxity	[2, 7] (real)
Р	number of processors	[3, 10] (integer)
BurstP	probability of a burst	$\lambda/100$
MAX_Burst	maximum task number for a burst	10
MIN_Burst	minimum task number for a burst	30

Table 4.1. Parameters for task generator.

computation times of a task are chosen uniformly in the range of MIN_C and $MIN_C + (MAX_C - MIN_C) \times$ heterogeneity. The lower bound, however, is not always MIN_C , so the range will be shifted by the variable *shift*. Because the upper bound cannot larger than MAX_C , the *shift* is chosen uniformly between 0 and $MAX_C - (MAX_C - MIN_C) \times$ heterogeneity. Finally, computation times of a task are chosen uniformly between $(MIN_C + shift)$ and $(MIN_C + ((MAX_C - MIN_C) \times heterogeneity) + shift)$. Fig. 4.1 shows the range of computation time.

The arrival times of tasks depend on the interarrival time between each task. The interarrival time is exponentially distributed with mean [2]:

$$\frac{1}{\lambda \times P} \times \frac{MIN_C + MAX_C}{2}$$

, where λ is the task arrival rate, and *P* is the number of processors. We also assume there is a possibility of bursts of tasks. We define the mean of interarrival time for bursting is

$$\frac{1}{\lambda \times P} \times \frac{MIN_C}{10}$$

The probability of burst, BurstP, varies with λ and is defined as $\lambda/100$. When a burst



Fig. 4.1. The solid line with double arrows is the range of possible computation time.



Fig. 4.2. d_i is chosen uniformly in the range of the solid line with double arrows when laxity = 3.

happens, there are at least *MIN_Burst* tasks and at most *MAX_Burst* tasks arriving at the systems in a very short interval.

Because both copies may be scheduled with the first two maximum computation times, the deadline of a task must be late enough to satisfy this possible scheduling. Thus, the deadline of a task T_i is uniformly chosen between $(a_i + \max c_{ip} + \text{second max } c_{ip})$ and $(a_i + R \times \max c_{ip})$. The laxity parameter, R, indicates the tightness of the deadline, and is at least 2. Fig. 4.2 depicts the lower and upper bound of deadline when laxity = 3.

4.1.2 Simulator

Our evaluation was done by implementing a discrete-event dynamic simulator [17]. The dynamic simulator simulates all events which may happen in a realistic system. The possible events include the arrival, start and completion of a task, start of the scheduling, and backup



number of rejected tasks

Fig. 4.3. Flow of the dynamic simulator.

deallocation as well as occurrence of faults. Fig. 4.3 depicts the flow of the dynamic simulator. After reading the task set generated by the task generator, the Timer function decides the time of next event, and calls the corresponding operation. After dealing with the events, the update function updates the status of the system. The flow will repeat until all tasks in the set have arrived into the system and completed or been rejected. At the end time, the simulator terminates and reports the number of rejected tasks.

For reality, we simulate the failure events. The failures may be due to hardware fault or software fault [2]. Because the backup copy is the only one redundancy, we assume that each task encounters at most one failure. That is, the backup always succeeds if its primary fails. A software fault will terminate the task immediately. The hardware faults are the faults happened to processors. All tasks on the failed processor will be terminated and deallocated, whatever they are running or ready to run. The hardware faults could be transient or permanent. If a transient fault happens, the failed processor will be available again in some recovery time. The recovery time is distributed normally between 0 and *MAX_Recovery*. If a

parameter	explanation	range of possible value
FaultP	probability that a primary fails	[0, 0.5] (real)
Soft_FP	probability that a primary fails due to software fault	0.2
Hard_FP	probability that a primary fails due to hardware fault	0.8
PermHard_FP	probability that a hardware fault is permanent	0.000001
MAX_Recovery	maximum recovery time after a transient hardware fault happened	50

Table 4.2. Parameters for fault probability.

permanent fault happens, the failed processor would never be available to the end of simulation. We define the relative probabilities and parameters for failure events in Table 4.2.

4.2 Performance Evaluations

In this section, we will evaluate the performance of the DNA algorithm by comparison with FTMA and a modified distance myopic algorithm for heterogeneous systems, called HDMA. HDMA is proposed in [8]. The total number of tasks arrived into the system is 20,000. For each set of parameters of the task generator, 20 task sets are generated as the inputs of the three algorithms. We take the average of the 20 rejection number as the final result. HDMA needs two parameters, size of feasibility check window (K) and *distance*, and FTMA needs one, i.e. K. Because the better results depend on the combination of these parameters, we will run HDMA with various combinations of K and *distance*, and run FTMA with various K for each task set. We will choose the best result among the various combinations of parameters as the final result of HDMA and FTMA for each task set.

Next, we define the metric for the performance evaluations. The objective of any dynamic real-time scheduling algorithm is to improve the *guarantee ratio*. The guarantee ratio is defined as the percentage of tasks whose deadlines are met [1]. The formal definition is

given below:

$$Guarantee Ratio = \frac{number of tasks whose deadlines are met}{total number of tasks arrived in the system} \times 100\%$$
(9)

In the next subsections, we will evaluate the performance of DNA and the other algorithms with four simulation parameters. These parameters are task arrival rate (λ), laxity (*R*), processor number (*P*), and fault probability (*FaultP*).

4.2.1 The Effect of Task Load

The task arrival rate () has been varied in Fig. 4.4. The size of feasibility check window, *K*, ranges from 6 to 10 for FTMA. For HDMA, *K* ranges from 3 to 8 and the *distance* ranges from 5 to 8. Higher λ means lower interarrival time and, thus, higher task load. As task load increases, the guarantee ratio decreases for all algorithms. Obviously, HDMA has poor performance because of the single task queue for primaries and backups. Appearances of a primary and its backup in the feasibility check window results in continuously backtracking and rejecting eventually. FTMA overcomes this disadvantage and has almost the same guarantee ratio as DNA with lower arrival rate. When the task load is getting higher, DNA rejects fewer tasks than that of FTMA. This implies that the density heuristic function selects more appropriate tasks to be scheduled when more and more tasks arrived at the system in an interval.

4.2.2 The Effect of Laxity

The effect of task laxity (R) is depicted in Fig. 4.5. The size of feasibility check window, K, ranges from 6 to 10 for FTMA. For HDMA, K ranges from 3 to 8 and the *distance* ranges from 5 to 8. As the laxity increases, the guarantee ration also increases for all algorithms. We



Fig 4.4. Effect of task load. (R = 3, P = 8, FaultP = 0.2)



Fig. 4.5. Effect of laxity. ($\lambda = 0.7, P = 8, FaultP = 0.2$)

can find that the difference of the performance between DNA and the other algorithms is the largest with the smallest laxity, and is getting closed when laxity is bigger. This is because the density heuristic function considers the relationship between computation time and schedulable interval rather than the deadline. It will give the highest priority to the least flexible task whatever the laxity is. Instead, in the other algorithms, the deadline is synthesized as part of the integrated heuristic function directly. In the situation of low laxity,



Fig 4.6. Effect of the number of processor. ($\lambda = 0.7, R = 3, FaultP = 0.2$)

the integrated heuristic function may not select the appropriate task to be scheduled first.

4.2.3 The Effect of the Number of Processor

The effect of varying the number of processors (P) is given in Fig.4.6. The size of feasibility check window, K, ranges from 1 to 12 for FTMA. For HDMA, K ranges from 3 to P and the *distance* ranges from P/2 to P. To increase the number of processor will increases the guarantee ratio for all algorithms. When more processors are available, the difference in guarantee ratio between DNA and the other algorithms is getting large. This is because there are more opportunities for a backup to be overlapped. The DNA could benefit from this situation since the MNO strategy has more opportunities to find less non-overlapped time.

4.2.4 The Effect of Fault Probability

In Fig. 4.7-4.9, the probability that a primary copy encounters a failure (*FaultP*) is varied with three parameters, *R*, and *P*. We compare DNA only with FTMA for simplicity since



Fig. 4.9. Effect of fault probability with various number of processor. (= 0.7, R = 3)

the HDMA does not outperform the other algorithms in the previous simulation results. As *FaultP* increases, the guarantee ratio decreases in any situation. When FautP = 0, there is no

fault in the system, which means that every backup will be deallocated and all the time reserved for backups will be reutilized. When the fault probability increases, more backup copies are active to be executed so that it cannot be overlapped with any backups of new tasks. Thus, there is the most time which can be reutilized as fault probability equals 0, and less and less as fault probability increases. In additional, the results of different degree of parameters,

, *R*, *P*, are the same as the simulation results shown in the above subsections. These figures also show that DNA has higher guarantee ratio than FTMA in any degree of parameters as the fault probability increases.

By the simulation, we have verified the performance of the DNA algorithm. We find that the density heuristic function selects more appropriate tasks to be scheduled, even when the task load is heavy or the deadlines are tight. We also find that the MNO strategy saves more processor time than EFT or overlap as much as possible for new arriving tasks. Furthermore, without any input parameter, DNA still has better guarantee ratio than that of HDMA or FTMA.

Chapter 5. Conclusion and Future Work

In this thesis, we have proposed an algorithm, named DNA, for dynamically scheduling arriving real-time tasks with PB-based fault-tolerant requirement in a heterogeneous multiprocessor system. Through the dynamic simulation, we have evaluated the performance of the proposed algorithm compared with distance myopic algorithm and FTMA. Finally, in this chapter, we make conclusions and describe some future work about our research.

5.1 Conclusion

The integrated heuristic function proposed in [1] is used by most algorithms which dynamically schedule arriving real-time tasks. The integrated heuristic function emphasizes whether a task could be executed earlier. Nevertheless, real-time tasks are not concerned about when to start computation but rather about meeting deadlines. We propose a new heuristic function, named *density*, which indicates the tightness of a task. The density function takes account of the schedulable time and the computation time. A task with the highest density means that it is the least flexible to be scheduled so that it will be selected first for scheduling. The simulation results show that the density function selects more appropriate tasks even with a heavy task load.

The *MNO* strategy for backup scheduling will minimize the processor time reserved for backups. This will also increase the schedulable time for new tasks. Obviously, MNO saves more time than overlapping as much as possible on heterogeneous multiprocessor. Moreover, though simulation, we can find that MNO save more and more time than the EFT strategy when the processor number increases.

Finally, DNA does not need to be adjudged by any input parameters, unlike the distance myopic and FTMA. Though the simulation, DNA gets better results than those of distance myopic and FTMA which are the best among any combination of needed parameters. This means DNA is more general and suitable for any environment.

5.2 Future Work

In additional to the research results we have proposed, there are some issues in the future work.

First, the assumption of our scheduler model is a dedicated processor for scheduling, and the scheduling overhead is ignored. However, the scheduler may have a lot of idle time if the task load is low. This is not economic for a cost-sensitive system. The scheduler may be used for computation while it is idle as well as scheduling tasks. In this way, the scheduling overhead needs to be taken into account for those tasks scheduled on the scheduler. How to define and quantify the scheduling overhead is not trivial and becomes the next extension of this thesis.

Second, most algorithms assume deadlines of tasks are fixed after they are released, i.e. deadlines do not vary with time. For some real-time applications whose high-level requirements may change with time, the model of variable deadlines is required. [26] has proposed a new workload model, called the *state-dependent deadline model*, for this kind of applications. How to modify the density function in our algorithm for the variable deadline model is another future extension of our research.

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