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Effects of ultrasonic waves on linear longitudinal conductivity in non-degenerate semiconductors†

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Abstract. Effects of ultrasonic waves propagating at an angle θ relative to the direction of a DC magnetic field on the linear longitudinal electrical conductivity in non-degenerate semiconductors such as n-type InSb have been studied by using a quantum treatment which is valid at high frequencies and in strong magnetic fields. Numerical results show that the linear longitudinal conductivity depends strongly on the DC magnetic field, the sound frequency, the temperature, and the direction of the propagation of ultrasonic waves relative to that of the field.

1. Introduction

Ultrasonic waves propagating in a semiconducting crystal interact with conduction electrons by way of deformation-potential and piezoelectric coupling mechanisms. When the ultrasonic wave is propagating parallel to a DC magnetic field, the effect of non-parabolicity and the quantum effect of oscillations will appear much more important in the strong magnetic field region (Wu and Spector 1971, Wu and Tsai 1972, Sharma and Phadke 1972, Wu *et al* 1973, Sutherland and Spector 1978). These magnetoacoustic phenomena in semiconductors are dominated by the linear longitudinal electrical conductivity because the longitudinal field is important when the interaction of ultrasonic waves with conduction electrons in semiconductors is by way of either the piezoelectric or deformation-potential coupling mechanisms (Spector 1966). Lifshitz *et al* (1966) obtained directly values for the energy relaxation time by measuring the real and imaginary parts of the complex conductivity at 250 and 400 kHz. They concluded that piezoelectric scattering is predominantly responsible for the energy relaxation and that deformation potential scattering contributes relatively little to the energy relaxation. In our previous works for the wave propagating parallel to the DC magnetic field in non-degenerate semiconductors (Wu and Spector 1971), it has been shown that the linear longitudinal conductivity using the parabolic model is independent of the DC magnetic field while that using the non-parabolic model depends strongly on the DC magnetic

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field. However, in the case of degenerate semiconductors the linear longitudinal conductivity for both parabolic and non-parabolic models depends upon the DC magnetic field and the real part of the linear longitudinal electrical conductivity oscillates more rapidly and saliently than its imaginary part (Wu *et al* 1973). Sutherland and Spector (1978) derived the effective longitudinal conductivity tensor including the phenomena of conduction and diffusion to study magnetoacoustic effects in n-type InSb. They took into account the non-parabolicity of the energy bands and the energy dependence of the electron relaxation time for the longitudinal and transverse configurations. In our present work, we shall study the effect of varying the angle between the direction of ultrasonic propagation and that of the magnetic field on the linear longitudinal electrical conductivity for the non-parabolic band structure in non-degenerate semiconductors such as n-type InSb. The linear longitudinal conductivity is calculated using the quantum treatment which is valid at high frequencies and in strong magnetic fields. Sutherland and Spector (1978) have taken into account the possible energy dependence of the electron relaxation time for the case where acoustic phonon scattering is important. However, experimental results (Lifshitz *et al* 1966, Whalen and Westgate 1972) indicated that piezoelectric scattering is predominantly responsible for energy relaxation and that deformation-potential scattering appears to play no significant role in electron energy relaxation. It has been shown that at frequencies above the microwave region deformation-potential coupling becomes dominant in semimetals and semiconductors because of its stronger frequency dependence (Wu and Tsai 1972, Wu *et al* 1973). Since we are interested in the high-frequency region such that $ql \gg 1$, where $\boldsymbol{q}$ is the ultrasonic wavevector and l is the electron mean free path, the effect of collisions can be neglected and we do not take into account the effect of the electron relaxation time in our present case.

In §2 of this paper, we perform the calculation of the linear longitudinal conductivity for non-parabolic bands in the presence of a DC magnetic field $\boldsymbol{B}$ with which the ultrasonic wavevector $\boldsymbol{q}$ propagates along an angle θ. In §3 some numerical results in n-type InSb are presented and discussed.

2. Theory

In the non-parabolic model, the energy eigenvalue equation for electrons in a uniform DC magnetic field $\boldsymbol{B}$ directed along the z axis is

$$H_0(1 + H_0/E_g)\Psi_{k,n} = (1/2m^*)\,[p_x^2 + (p_y - eBx/c)^2 + p_z^2]\Psi_{k,n} = E_{k,n}(1 + E_{k,n}/E_g)\Psi_{k,n} \tag{1}$$

where m^* is the effective mass of electrons at the minimum of the conduction band, E_g is the energy gap between the conduction and valence bands, and $E_{k,n}$ is the true energy of the system defined by $H_0\Psi_{k,n} = E_{k,n}\Psi_{k,n}$. The eigenfunctions and eigenvalues for equation (1) can be expressed as

$$\Psi_{k,n} = \exp(ik_y y + ik_z z)\Phi_n[x - (\hbar c/eB)k_y] \tag{2}$$

and

$$E_{k,n} = -\tfrac{1}{2}E_g\{1 - [1 + (4/E_g)((n + \tfrac{1}{2})\hbar\omega_c + \hbar^2k_z^2/2m^*)]^{1/2}\}, \tag{3}$$

where $\omega_c = |e|B/m^*c$ is the cyclotron frequency of electrons. The quantum number n can take on any non-negative integral value. Since $(\hbar k_{z\,\max})^2/2m^* \simeq k_BT \ll E_g$ at the low temperatures in which we are interested ($T < 100$ K), equation (3) can be expanded as

$$E_{k,n} \simeq \tfrac{1}{2}E_g + \tfrac{1}{2}E_g a_n + \hbar^2 k_z^2/2m^* a_n \tag{4}$$

with

$$a_n = [1 + (4\hbar\omega_c/E_g)(n + \tfrac{1}{2})]^{1/2}. \tag{5}$$

In our present case we consider those semiconductors in which the principal electron–phonon interaction is by way of either the piezoelectric or deformation-potential coupling mechanisms. Since the transverse electric fields induced by the ultrasonic wave in semiconductors are down by a factor of $(v_s/c)^2$ from the longitudinal electric fields (Spector 1966, Wu and Spector 1972), where v_s is the velocity of sound, the interaction of the charge carriers and the piezoelectric or deformation-potential fields is known to be strongest for longitudinal electric fields. Consequently, the only component of the linear conductivity tensors of interest is σ_{zz}. Following the same method of quantum treatment as our previous paper (Wu and Spector 1971) the linear longitudinal conductivity can be obtained as

$$\sigma_{zz}(\boldsymbol{q}, \omega) = \frac{\omega_p^{*2}}{4\pi i\omega n_0}\left[\sum_{k,n} f_{k,n}\theta_{k,n} - \frac{\hbar^2}{4m^*}\sum_{k,n,n'} \frac{(f_{k,n} - f_{k+q,n'})\theta_{k,n}\theta_{k+q,n'}}{E_{k+q,n'} - E_{k,n} - \hbar\omega} \times (2k_z + q\cos\theta)^2 |M_{n',n}(q\sin\theta)|^2\right] \tag{6}$$

where n_0 is the electron density, $\omega_p^* = (4\pi n_0 e^2/m^*)^{1/2}$ is the plasma frequency of the electron with the effective mass m^*, ω is the frequency of ultrasounds, $f_{k,n}$ is the Boltzmann distribution for non-degenerate semiconductors, θ is the angle between the ultrasonic wavevector $\boldsymbol{q}$ and the magnetic field $\boldsymbol{B}$, and $\theta_{k,n}$ is given by

$$\theta_{k,n} = (1 + 2E_{k,n}/E_g)^{-1}. \tag{7}$$

The function $|M_{n',n}(q\sin\theta)|^2$ is defined by

$$|M_{n',n}(q\sin\theta)|^2 = \left(\frac{n!}{n'!}\right)(\tfrac{1}{2}L^2q^2\sin^2\theta)^{n'-n}\exp(-\tfrac{1}{2}L^2q^2\sin^2\theta)[L_n^{n'-n}(\tfrac{1}{2}L^2q^2\sin^2\theta)]^2 \quad \text{for } n' \geqslant n \tag{8a}$$

and

$$|M_{n',n}(q\sin\theta)|^2 = \left(\frac{n'!}{n!}\right)(\tfrac{1}{2}L^2q^2\sin^2\theta)^{n-n'}\exp(-\tfrac{1}{2}L^2q^2\sin^2\theta)\ [L_{n'}^{n-n'}(\tfrac{1}{2}L^2q^2\sin^2\theta)]^2 \quad \text{for } n' < n \tag{8b}$$

where $L = (\hbar/m^*\omega_c)^{1/2}$ is the classical radius of the lowest Landau level and $L_l^n(x)$ is the associated Laguerre polynomial.

3. Numerical results and discussion

As a numerical example, we shall consider the linear longitudinal conductivity in n-type InSb at high frequencies and low temperatures. The relevant values of physical parameters for n-type InSb are $n_0 = 1{\cdot}75 \times 10^{14}$ cm^{-3}, $m^* = 0{\cdot}013m_0$ (m_0 is the free electron mass), $E_g = 0{\cdot}2$ eV, and $v_s = 4 \times 10^5$ cm s^{-1}. The real and imaginary parts of the linear conductivity σ_{zz} have been plotted as a function of DC magnetic field B for different angles and temperatures as shown in figures 1(*a*) and (*b*). In figure 1(*a*) for $\theta = 0°$, it can be seen that Re[σ_{zz}] increases monotonically with the DC magnetic field but Im[$-\sigma_{zz}$] appears stationary with the field. It can also be seen that Re[σ_{zz}] increases with decreasing temperature and Im[$-\sigma_{zz}$] increases with the temperature. However, when the direction of the propagation of ultrasonic waves is not the same as that of the DC magnetic field, both real and imaginary parts of the linear longitudinal conductivity will appear

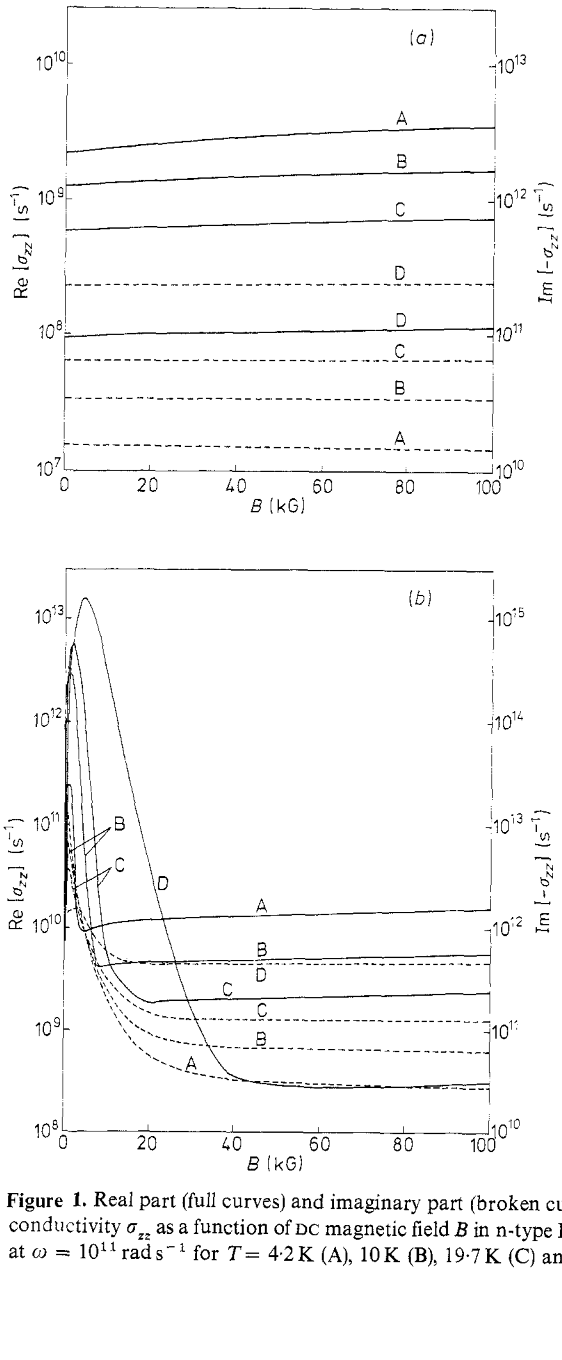

Figure 1. Real part (full curves) and imaginary part (broken curves) of the linear longitudinal conductivity σ_{zz} as a function of DC magnetic field B in n-type InSb with (*a*) $\theta = 0°$ (*b*) $\theta = 45°$ at $\omega = 10^{11}$ rad s^{-1} for $T = 4{\cdot}2$ K (A), 10 K (B), 19·7 K (C) and 77 K (D).

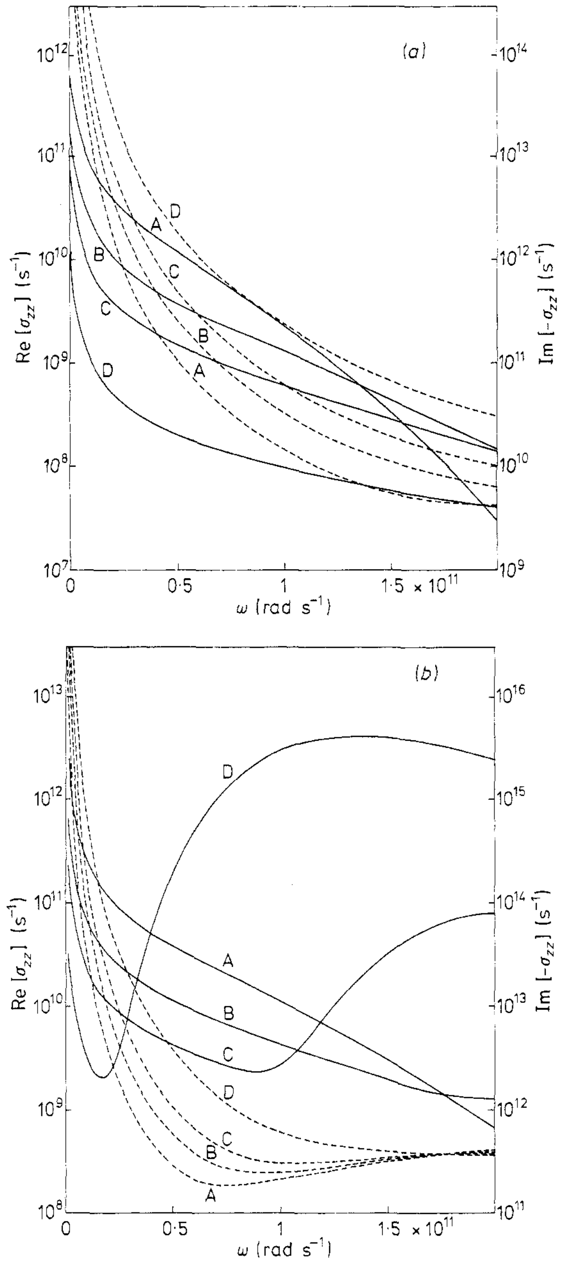

Figure 2. Real part (full curves) and imaginary part (broken curves) of the linear longitudinal conductivity σ_{zz} as a function of sound frequency ω in n-type InSb with (a) $\theta = 0°$ (b) $\theta = 45°$ at $B = 10\,\mathrm{kG}$ for $T = 4{\cdot}2\,\mathrm{K}$ (A), 10 K (B), 19·7 K (C) and 77 K (D).

some different properties in lower magnetic fields. From figure 1(b), we can see that there exist maxima of $\mathrm{Re}[\sigma_{zz}]$ and $\mathrm{Im}[-\sigma_{zz}]$ in lower magnetic fields. After passing the maxima, $\mathrm{Re}[\sigma_{zz}]$ and $\mathrm{Im}[-\sigma_{zz}]$ decrease with increasing DC magnetic field and then increase monotonically with the field. The effect of temperature is the same as that of the case at $\theta = 0°$. In figures 2(a) and (b), we plot $\mathrm{Re}[\sigma_{zz}]$ and $\mathrm{Im}[-\sigma_{zz}]$ as a function of sound frequency ω for different angles θ and temperatures. When the direction of propagation of ultrasonic waves is parallel to the DC magnetic field, both $\mathrm{Re}[\sigma_{zz}]$ and $\mathrm{Im}[-\sigma_{zz}]$ decrease with increasing sound frequency. However, when the angle θ is different from zero, $\mathrm{Re}[\sigma_{zz}]$ could change its sound-frequency dependence with increasing temperature. From figure 2(*b*), it can be seen that $\mathrm{Re}[\sigma_{zz}]$ at $T = 19{\cdot}7$ K decreases with increasing sound frequency and is shifted to increase with the sound frequency in the neighbourhood of $\omega = 10^{11}\ \mathrm{rad\,s^{-1}}$. At $T = 77$ K, $\mathrm{Re}[\sigma_{zz}]$ increases with the sound frequency from $\omega = 2 \times 10^{10}\ \mathrm{rad\,s^{-1}}$. From our numerical results, it can be seen that the direction of ultrasonic propagation with respect to the DC magnetic field will affect the dependence of the linear longitudinal conductivity on the sound frequency and the DC magnetic field. Therefore, we predict that magnetoacoustic effects of the linear longitudinal conductivity for non-parabolic band structure will be quite different from our previous work in non-degenerate semiconductors (Wu and Spector 1971). In order to investigate more clearly the effect of the angle θ between the direction of ultrasonic propagation and that of the DC magnetic field upon the linear longitudinal conductivity, we plot $\mathrm{Re}[\sigma_{zz}]$ and $\mathrm{Im}[-\sigma_{zz}]$ as a function of the angle θ as shown in figures 3(*a*) and (*b*). It is shown that $\mathrm{Re}[\sigma_{zz}]$ has a maximum and a minimum in the region excluding $\theta = 0°$ and $\theta = 90°$. When the temperature decreases, both maximum and minimum will tend to vanish in strong magnetic fields. As the angle θ approaches 90°, the real part of σ_{zz} will be diminished. However, $\mathrm{Im}[-\sigma_{zz}]$ will increase with the angle θ. Since the absorption coefficient due to the interaction between the ultrasonic waves and conduction electrons is proportional to $\mathrm{Re}[\sigma_{zz}]$ and the change in sound velocity due to the interaction between the ultrasonic waves and conduction electrons is proportional to

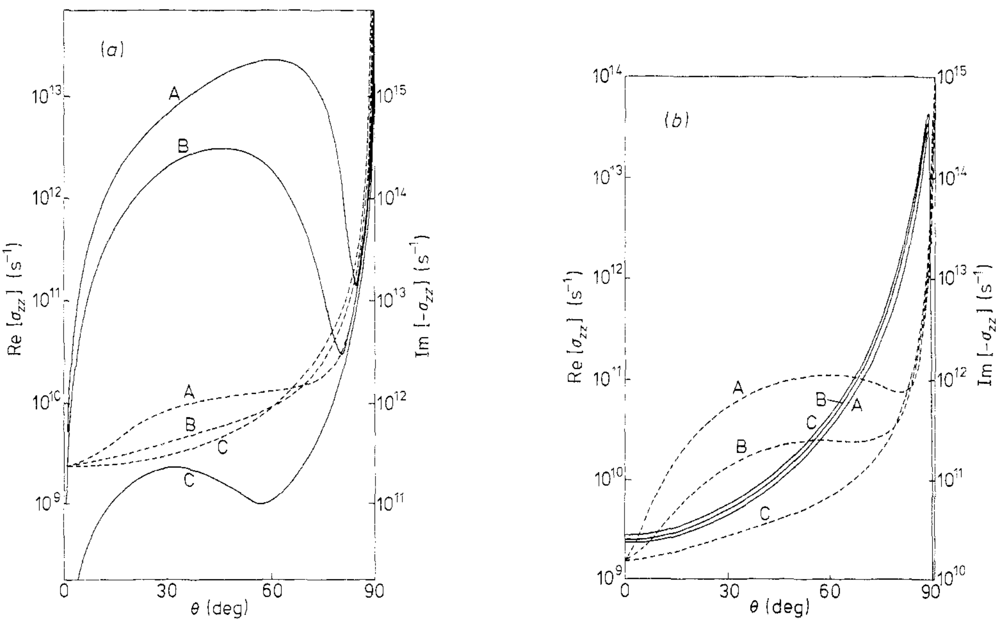

Figure 3. Real part (solid curves) and imaginary part (broken curves) of the linear longitudinal conductivity σ_{zz} as a function of angle θ in n-type InSb at (*a*) $T = 77$ K (*b*) $T = 4{\cdot}2$ K and $\omega = 10^{11}\ \mathrm{rad\,s^{-1}}$ for $B = 5$ kG (A), 10 kG (B) and 30 kG (C).

$(1 - (4\pi \mathrm{Im}[\sigma_{zz}]/\omega\epsilon))$ (Spector 1966), therefore the magnetoacoustic phenomena in semiconductors depend strongly on both real and imaginary parts of the linear longitudinal conductivity. Experimental results (Smith *et al* 1971) showed that the attenuation decreases at $\theta = 90°$ with the DC magnetic field for deformation-potential coupling. Our prediction that the absorption coefficient becomes insignificant at $\theta = 90°$ is in qualitative agreement with experimental results at high frequencies and in strong magnetic fields. However, when the frequency is below the microwave region, the effect of collisions cannot be neglected and the energy dependence of the electron relaxation time should be taken into account for large angles between the propagation of ultrasonic waves and magnetic field directions (Sutherland and Spector 1978).

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