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Transverse magnetoresistance of degenerate semiconductors in strong magnetic fields†

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Abstract. The quantum effect on the transverse magnetoresistance of degenerate semiconductors with isotropic parabolic energy bands has been studied for the case where acoustic phonons are the dominant scattering mechanism. The calculation has been performed taking into account the inelasticity in the electron–phonon scattering due to the finite energy of the phonons involved. Results show that the transverse magnetoresistance oscillates with the DC magnetic field owing to the degeneracy of the electron gas. However, the number of oscillations decreases when the DC magnetic field increases.

1. Introduction

The transverse magnetoresistance for non-degenerate semiconductors with isotropic parabolic energy bands has been investigated for the case where acoustic phonons are the dominant scattering mechanism (Cassiday and Spector 1974). It was shown that the transverse magnetoresistance increases with the DC magnetic field in the quantum limit. Some experimental results for the inelastic scattering mechanism show that the transverse magnetoresistance depends strongly on the DC magnetic field (Aliev *et al* 1975). Arora (1976) found that the transverse magnetoresistance changes dramatically with inelasticity, while the longitudinal magnetoresistance remains essentially unchanged. Consequently, inelasticity may be expected to play an active role and should be included for electronic transport in the transverse configuration.

In this paper we calculate the transverse magnetoresistance of a degenerate semiconductor with isotropic parabolic energy bands throughout the strong-field region in which the splitting of Landau levels is much greater than the average carrier energy. For a degenerate semiconductor, the distribution function of electrons is represented by the Fermi–Dirac statistics. We take into account the inelastic scattering of acoustic phonons from the deformation-potential coupling. The scattering is treated in the Born approximation for strong magnetic fields. Moreover, we assume that inelasticity is the dominant mechanism in resolving the divergence and the cut-off energy due to the inelastic scattering does not change appreciably with the temperature.

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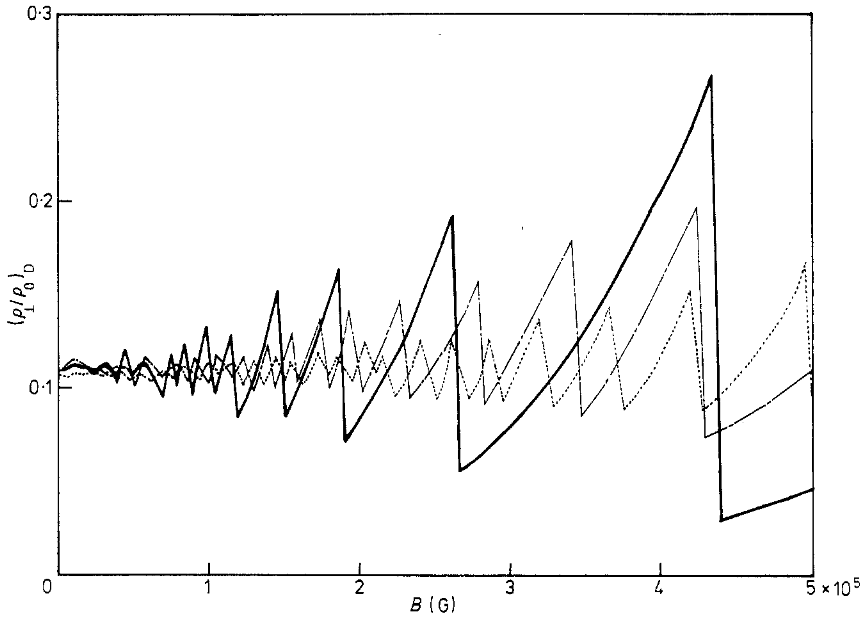

Figure 1. Transverse magnetoresistance $(\rho_\perp/\rho_0)_D$ as a function of DC magnetic field in germanium. Full curve: $n_0 = 3 \times 10^{18}\,cm^{-3}$; chain curve: $n_0 = 10^{19}\,cm^{-3}$; broken curve: $n_0 = 3 \times 10^{9}\,cm^{-3}$.

2. Theoretical development

The Hamiltonian describing an electron of momentum $\boldsymbol{p}$ and charge e in a uniform DC magnetic field $\boldsymbol{B}$ directed along the z axis is (Kittel 1968)

$$H = (1/2m^*)[p_x^2 + (p_y - eBx/c)^2 + p_z^2], \tag{1}$$

where m^* is the effective mass of electrons at the minimum of the conduction band. The eigenfunctions and eigenvalues for equation (1) are given by

$$\psi_{kn} = \phi_n[x - (\hbar c/eB)\,k_y]\exp(\mathrm{i}k_y y + \mathrm{i}k_z z), \tag{2}$$

and

$$E_{kn} = \hbar\omega_c(n + \tfrac{1}{2}) + (1/2m^*)\,k_z^2\hbar^2, \tag{3}$$

where k_y and k_z are the y and z components of the electron wavevector $\boldsymbol{k}$, $\phi_n(x)$ is the harmonic oscillator wavefunction with the quantum number n, and $\omega_c = |e|B/m^*c$ is the cyclotron frequency of electrons.

For the scattering due to acoustic phonons, the dissipative current lying in the direction of the total electric field is given by (Roth and Argyres 1966, Budd 1968, Cassiday and Spector 1974)

$$J_d = \frac{|e|L^2 v_H \hbar}{k_B T} \sum_{kn,\,k'n'} \frac{(k_y - k_y')^2}{2} f_{kn}(1 - f_{k'n'})\,W_{kn\,k'n'}, \tag{4}$$

where $L = (\hbar/m^*\omega_c)^{1/2}$ is the classical radius of the lowest Landau level, $v_H = c(\boldsymbol{E} \times \boldsymbol{B})/B^2$ is the Hall velocity with the applied electric field $\boldsymbol{E}$, f_{kn} is the distribution function of electrons, and $W_{kn,k'n'}$ is the transition probability in the Born aproximation between the Landau states $\boldsymbol{k}n$ and $\boldsymbol{k}'n'$. For the acoustic phonon, the transition probability is given by

$$W_{kn,k'n'} = (2\pi/\hbar) \sum_q |C(q)|^2 \, [|M_{kn,k'n'}(\boldsymbol{q})|^2 \, (N_q + 1) \, \delta(E_{kn} - E_{k'n'} - \hbar\omega_q) + |M_{kn,k'n'}(-\boldsymbol{q})|^2 \, N_q \delta(E_{kn} - E_{k'n'} + \hbar\omega_q)], \tag{5}$$

where $q = (i, \boldsymbol{q})$ denotes collectively the branch and wavevector for the phonon mode with the energy $\hbar\omega_q$, $N_q = [\exp(\hbar\omega_q/k_B T) - 1]^{-1}$ is the Planck distribution function for the phonons in thermal equilibrium, $C(q)$ is the electron–phonon coupling constant, and $\delta(x)$ is the Dirac δ-function. The two terms in equation (5) give the contributions to the scattering rate of the phonon emission and absorption processes, respectively. Now, the matrix element $|M_{kn,k'n'}(\boldsymbol{q})|^2$ in equation (5) can be expressed as (Roth and Argyres 1966, Cassiday and Spector 1974)

$$\begin{aligned} |M_{kn,k'n'}(\boldsymbol{q})|^2 &= |\langle \boldsymbol{k}n| \exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}) |\boldsymbol{k}'n'\rangle|^2 \\ &= (n!/n'!) \exp(-\tfrac{1}{2}L^2 q_\perp^2) \, (\tfrac{1}{2}L^2 q_\perp^2)^{n'-n} [L_n^{n'-n}(\tfrac{1}{2}L^2 q_\perp^2)]^2 \\ &\quad \times \delta(k_y - q_y - k_y') \, \delta(k_z - q_z - k_z'), \qquad \text{for } n' \geqslant n \end{aligned} \tag{6a}$$

and

$$\begin{aligned} |M_{kn,k'n'}(-\boldsymbol{q})|^2 &= |\langle \boldsymbol{k}n| \exp(-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}) |\boldsymbol{k}'n'\rangle|^2 \\ &= (n'!/n!) \exp(-\tfrac{1}{2}L^2 q_\perp^2) \, (\tfrac{1}{2}L^2 q_\perp^2)^{n-n'} \, [L_{n'}^{n-n'}(\tfrac{1}{2}L^2 q_\perp^2)]^2 \\ &\quad \times \delta(k_y + q_y - k_y') \, \delta(k_z + q_z - k_z'), \qquad \text{for } n' < n \end{aligned} \tag{6b}$$

where $L_m^p(x)$ is the associated Laguerre polynomial (Erde'lyi *et al* 1953), and q_z, $q_\perp$, and q_y are the components of the phonon wavevector directed parallel to the DC magnetic field, normal to the DC magnetic field, and in the $\boldsymbol{B} \times \boldsymbol{E}$ direction, respectively. $\delta(E_{kn} - E_{k'n'} - \hbar\omega_q)$ and $\delta(E_{kn} - E_{k'n'} + \hbar\omega_q)$ in equation (5) are given by

$$\delta(E_{kn} - E_{k'n'} - \hbar\omega_q) = (m^*/q_z\hbar^2) \, \delta[k_z - \tfrac{1}{2}q_z - (m^*\omega_c/q_z\hbar)(n' - n) - (m^*\omega_q/q_z\hbar)] \quad \text{for } n' \geqslant n \tag{7a}$$

and

$$\delta(E_{kn} - E_{k'n'} + \hbar\omega_q) = (m^*/q_z\hbar^2) \, \delta[k_z + \tfrac{1}{2}q_z - (m^*\omega_c/q_z\hbar)(n - n') - (m^*\omega_q/q_z\hbar)] \quad \text{for } n' < n. \tag{7b}$$

Making use of the relations

$$f(E_{kn})[1 - f(E_{kn} - \hbar\omega_q)] = [f(E_{kn} - \hbar\omega_q) - f(E_{kn})] \, N_q, \tag{8a}$$

and

$$f(E_{kn})[1 - f(E_{kn} + \hbar\omega_q)] = [f(E_{kn}) - f(E_{kn} + \hbar\omega_q)] \, (N_q + 1), \tag{8b}$$

we obtain the expression for the dissipative current by Fermi–Dirac statistics for the degenerate case

$$J_d = \frac{|e| L^2 v_H m^{*2} \omega_c E_1^2 k_B T}{8\pi^3 \rho v_s^3 \hbar^4} \sum_{p=-P}^{P} \sum_{m=0}^{M} \int_0^\infty q_\perp^2 \, dq_\perp \frac{m!}{(m+p)!} \exp(-\tfrac{1}{2}q_\perp^2 L^2)^p \, [L_m^p(\tfrac{1}{2}q_\perp^2 L^2)]^2 \times \ln\left(\frac{[E_F + \hbar v_s q_\perp - \hbar\omega_c(m + \tfrac{1}{2})]^{1/2} + [E_F - \hbar\omega_c(m + p + \tfrac{1}{2})]^{1/2}}{[E_F - \hbar\omega_c(m + \tfrac{1}{2}]^{1/2} + [E_F - \hbar v_s q_\perp - \hbar\omega_c(m + p + \tfrac{1}{2})]^{1/2}} \right) \tag{9}$$

with the condition

$$\hbar\omega_c(M + P + \tfrac{1}{2}) < E_F. \tag{10}$$

The quantum number m indicates the Landau level n or n' in equation (4), and p is the transition quantum number between n and n'. The correspnding maximum values of m and p are M and P, respectively, which should satisfy the condition in equation (10). In obtaining equation (9), we have used the electron–phonon coupling constant for acoustic-phonon scattering via the deformation-potential coupling mechanism (Roth and Argyres 1966)

$$|C(q)|^2 = E_1^2 q\hbar/2\rho v_s, \tag{11}$$

where E_1 is the deformation potential constant, ρ is the mass density of the crystal, v_s is the sound velocity, and E_F is the Fermi energy. We also employ the high-temperature approximation which is generally satisfied at the temperature where the acoustic-phonon scattering dominates (Kubo *et al* 1965, Roth and Argyres 1966, Cassiday and Spector 1974). Then we have $N_q \simeq k_B T/\hbar\omega_q = k_B T/\hbar v_s|\boldsymbol{q}|$. In strong magnetic fields, the transverse magnetoresistance can be approximated by

$$\rho_\perp = EJ_d/(n_0\, ev_H)^2. \tag{12}$$

The expression for the resistivity in the absence of a DC magnetic field due to the deformation-potential coupling in the degenerate case is (Roth and Argyres1966)

$$\rho_0 = (m^*/n_0 e^2)\,(E_1^2 k_B T/\rho v_s^2)\,(E_F^{1/2}/2\pi\hbar)\,(2m^*/\hbar^2)^{3/2}, \tag{13}$$

where $n_0 = (2m^* E_F)^{3/2}/(3\pi^2\hbar^3)$. From equations (9), (12), and (13), the transverse magnetoresistance due to the deformation-potential coupling can be obtained as

$$\left(\frac{\rho_\perp}{\rho_0}\right)_D = \frac{3\hbar^4\omega_c}{32 v_s (m^* E_F)^2} \sum_{p=-P}^{P} \sum_{m=0}^{M} \int_0^\infty q_\perp^2\, dq_\perp \frac{m!}{(p+m)!} \exp(-\tfrac{1}{2}q_\perp^2 L^2)\,(\tfrac{1}{2}q_\perp^2 L^2)^p\,[L_m^p(\tfrac{1}{2}q_\perp^2 L^2)]^2$$
$$\times \ln\left(\frac{[E_F + \hbar v_s q_\perp - \hbar\omega_c(m+\frac{1}{2})]^{1/2} + [E_F - \hbar\omega_c(m+p+\frac{1}{2})]^{1/2}}{[E_F - \hbar\omega_c(m+\frac{1}{2})]^{1/2} + [E_F - \hbar v_s q_\perp - \hbar\omega_c(m+p+\frac{1}{2})]^{1/2}}\right) \tag{14}$$

with the condition in equation (10).

3. Numerical results and discussion

The expression in equation (14) can be approximated by making use of the conditions for the strong-magnetic-field region (Kubo *et al* 1965, Roth and Argyres 1966, Cassiday and Spector 1974, Arora 1976, Wu and Chen 1978), $\hbar\omega_q \simeq \hbar v_s q_\perp \gg m^* v_s^2$, and $\hbar\omega_c = \hbar^2/m^* L^2 \gg m^* v_s^2$. Using the integral representation (Gradshteyn and Ryzhik 1965)

$$\int_0^\infty \exp(-x)\, x^{n+\beta}\,(L_k^n(x))^2\, dx = \frac{\Gamma(1+n+\beta)\,\Gamma(1+n+k)}{(k!)^2\,\Gamma(1+n)}$$
$$\times \left\{\frac{d^k}{dh^k}\left[(1-h)^\beta F\left(\frac{1+n+\beta}{2}, 1+\frac{n+\beta}{2}; 1+n; \frac{4h}{(1+h)^2}\right)\Big/[(1+h)^{1+n+\beta}]\right]\right\}_{h=0},$$
$$\mathrm{Re}(n+\beta) > -1, \tag{15}$$

where $\Gamma(z)$ is the gamma function and $F(a, b; c; z)$ is a hypergeometric function, we obtain

$$\left(\frac{\rho_\perp}{\rho_0}\right)_\mathrm{D} = \frac{3}{32}(\hbar\omega_\mathrm{c}/E_\mathrm{F})^3\left[\sum_{m=0}^{M}(2m+1)\left(1+\frac{\hbar\omega_\mathrm{c}(2m+1)}{4E_\mathrm{F}}\right)\right.$$
$$\left.+2\sum_{p=0}^{P}\sum_{m=0}^{M}(p+2m+1)\left(1+\frac{\hbar\omega_\mathrm{c}(p+2m+1)}{4E_\mathrm{F}}\right)\right], \quad (16)$$

with the condition in equation (10).

Table 1. Physical parameters for germanium: n_0 is the electron density; m_0 is the mass of free electrons; m^* is the effective mass of electrons at the minimum of the conduction band. (Because germanium does not have isotropic parabolic energy bands, the effective mass m^* used here should be the density-of-states effective mass for electrons: (Kubo *et al* 1965) $m^* = [m_\mathrm{l}^*(m_\mathrm{t}^*)^2]^{1/3}$, where m_l^* and m_t^* are the longitudinal and transverse effective masses respectively)

$n_0(\mathrm{cm}^{-3})$	m^*	$E_\mathrm{F}(\mathrm{eV})$
10^{18}	$0{\cdot}22\,m_0$	0·0166
3×10^{18}	$0{\cdot}31\,m_0$	0·0245
10^{19}	$0{\cdot}42\,m_0$	0·0404

As a numerical example, we consider the transverse magnetoresistance for the deformation-potential coupling in germanium. The numerical values of physical parameters for this material are given in table 1. From our numerical results, it shows that the transverse magnetoresistance oscillates with the DC magnetic field. The electron wavevector, which is defined by $\boldsymbol{k}_n = \hat{z}[(2m^*)^{1/2}/\hbar]\,[E_\mathrm{F} - \hbar\omega_\mathrm{c}(n+\frac{1}{2})]^{1/2}$ because of the degeneracy of the electron gas, plays an important role in the contributions to the scattering role of phonon emission and absorption processes. As the DC magnetic field increases, the quantum numbers M and P in equations (14) or (16) decrease, hence the number of oscillations of $(\rho_\perp/\rho_0)_\mathrm{D}$ with the magnetic field will decrease. However, the amplitudes of oscillations will increase with the DC magnetic field. From equation (10) and table 1, we can also see that the quantum numbers M and P increase with the product of m^* and E_F, and the transition quantum number P will thus increase with the electron density. Consequently, the number of oscillations with the DC magnetic field increases with the electron density. However, the amplitudes of oscillations are enhanced as the electron density decreases. These oscillations can be interpreted as the 'giant quantum oscillations' (Gurevich *et al* 1961, Liu and Toxen 1965) which occur in a degenerate electron gas in the case when the electron level is near the Fermi surface and the sound wavevector $\boldsymbol{q}$ has a component along the DC magnetic field. These oscillations arise because the electrons in semiconductors interact with the acoustic phonons.

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