

Instantaneous control of structures with time-delay consideration

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A modified instantaneous control algorithm with time-delay consideration for the active control of structures is developed in discrete-time formulation. The shift-invariant feedback gain matrix is obtained through an optimization process such that a prescribed instantaneous quadratic performance index is minimized. The effect of time-delay is taken into consideration by introducing a compensation scheme where the feedback gain matrix for the ideal control system is modified by the effective system matrix. It is proved that the real control system with time-delay compensation conserves the eigen-properties of the ideal control system with no time-delay. In the presence of time-delay, the equation of motion of the discrete-time control system remains a set of difference equations which makes the evaluation of the control system simple and straightforward. The feasibility of the proposed control algorithm is verified numerically through eigenvalue, frequency-domain and time-domain analyses by the tendon control of a structure. With the proposed control algorithm, the control system is still effective in spite of the presence of time-delay. © 1997 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The impacts of control technology on different fields of engineering and the lives of mankind have been witnessed worldwide. In the civil engineering community, without exception, the topic of active control for dynamic response reduction originated by Yao¹ has become one of the dominant research fields since the 1970s²⁻⁵. It is very encouraging that the research area has now reached the stage of full-scale implementation⁶⁻⁹. Meanwhile, further research efforts are still needed to thoroughly solve practical problems in the implementation of control systems, such as the discrete-time nature of the control environment, time-delay effects, limited numbers of sensors and controllers, availability of energy source, and reliability issues etc., before widespread applications of this novel technology become possible.

Various control strategies have been proposed. Among

them optimal control law is undoubtedly the most stringent and sound criteria for control design, by which the control signal is determined by minimizing a quadratic performance index over a sufficiently long period of time¹⁰. The effectiveness of this algorithm for seismic response control of structures has been verified by Yang *et al.*¹¹ Yet, the complexity of solving the nonlinear system of Riccati equations has discouraged most civil engineers from appreciating this innovation. In an attempt to simplify the task of control design, Yang¹² later proposed the instantaneous optimal control algorithm that avoids the trouble of solving the Riccati equation while conserving equivalent control efficiency.

Almost all the control systems are now incorporated with a digital computer for both online calculation and data acquisition. This motivates not only the discrete-time formulation of the aforementioned control algorithms but also the development of new digital control strategies¹³. With

the introduction of the digital computer, all information, including input control forces and output measurements, is basically discrete-time in nature. Therefore, it is more logical and more realistic to formulate the control system in the discrete-time fashion.

Prior experimental studies indicated that time-delay results in degradation of the system performance and may lead to instability. Efforts for time-delay compensation have been made mainly through modification of the feedback gains based on the principle of phase-shift^{14,15} or through response prediction of the delayed states by either kinematic or dynamic approaches¹⁶. The former method needs to be performed in the phase space of modal coordinates. Unavoidable numerical errors would be accumulated, as the order of the system grows, during back and forth transformations of the states between physical and modal domains, and which would somehow abate the effect of compensation. The latter method uses either the truncated Taylor's expansion or the online solution of the system's equation for response forecasting. These approaches have been shown to be sensitive to the noise present in the system as reported in Reference 16. Other than the above mentioned strategies, an interesting experimental study by Abdel-Mooty and Roorda¹⁷ proposed a compensation scheme based on response sampling, curve fitting, and extrapolation for vertical vibration control of a simple bridge model. In their study the ideal feedback gains were used and the lagged responses were predicted through a filtering process. Only single mode control was considered in the tests and no spillover effect was observed for the uncontrolled modes. In the presence of time-delay, the governing equation of the active structural control system becomes a differential-difference equation instead of the original differential equation. The analytical method for the evaluation of the stability of the control system is limited to single-degree-of-freedom structures¹⁸.

In this paper, a modified instantaneous control algorithm is developed for the dynamic response control of structures. The proposed algorithm is derived based on the concept of instantaneous control¹² under a discrete-time framework in which first-order discretization of the continuous-time function is considered. The effect of time-delay is taken into consideration and a feedback compensation scheme is presented by simply modifying the gain matrix with the effective system matrix. Unlike the continuous-time system, the effects of time-delay for the discrete-time control system can be easily accommodated in the difference equation and thereby assessed in a more elegant fashion, for a broader category of structures. It is proved through eigenvalue analysis that the real control system with time-delay compensation conserves the properties of the ideal control system with no time-delay. The effectiveness of the proposed control law with time-delay compensation is confirmed through numerical simulations.

2. Control algorithm

When an n -degree-of-freedom (DOF) discrete-parameter structural system is subjected to environmental loads $w(t)$ and counteracted by control forces $u(t)$, its governing equation can be taken as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Bu(t) + Ew(t) \quad (1)$$

where $x(t)$ is the $n \times 1$ displacement vector, M , C , K are

the $n \times n$ mass, damping and stiffness matrix, respectively, B is the $n \times p$ location matrix of control forces, and E is the $n \times q$ location matrix of external loads. Represented in state-space form, the second-order differential equation (1) is changed to a first-order differential equation as

$$\dot{z}(t) = A_c z(t) + B_c u(t) + E_c w(t) \quad (2)$$

where

$z(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$ is the $2n \times 1$ state vector

$A_c = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$ is the $2n \times 2n$ system matrix

$B_c = \begin{bmatrix} \mathbf{0} \\ M^{-1}B \end{bmatrix}$ is the $2n \times p$ control matrix

$E_c = \begin{bmatrix} \mathbf{0} \\ M^{-1}E \end{bmatrix}$ is the $2n \times q$ load matrix.

The control system described by the state equation (2) is linear and time-invariant so that its solution takes the form

$$z(t_2) = e^{A_c(t_2-t_1)}z(t_1) + \int_{t_1}^{t_2} e^{A_c(t_2-\tau)}[B_c u(\tau) + E_c w(\tau)]d\tau \quad (3)$$

where t_1 and t_2 are any two time instants. During real-time control, suppose all information for the online calculation of control forces is sampled with period Δt and the control forces are calculated once every sampling period. Between two consecutive sampling instants, $(k-1)\Delta t$ and $k\Delta t$, the only available information about the control forces is $u((k-1)\Delta t)$ and $u(k\Delta t)$ and the only available information about the external loads is $w((k-1)\Delta t)$ and $w(k\Delta t)$. Therefore, it is reasonable to assume that the control forces and external loads are linear between two consecutive sampling instants as

$$u(\tau) = \frac{k\Delta t - \tau}{\Delta t} u[(k-1)\Delta t] + \frac{\tau - (k-1)\Delta t}{\Delta t} u[k\Delta t], \quad (4a)$$

$$(k-1)\Delta t \leq \tau < k\Delta t$$

$$w(\tau) = \frac{k\Delta t - \tau}{\Delta t} w[(k-1)\Delta t] + \frac{\tau - (k-1)\Delta t}{\Delta t} w[k\Delta t], \quad (4b)$$

$$(k-1)\Delta t \leq \tau < k\Delta t$$

When $t_1 = (k-1)\Delta t$, $t_2 = k\Delta t$, $z[k] = z(k\Delta t)$, $u[k] = u(k\Delta t)$ and $w[k] = w(k\Delta t)$ are assigned, from equation (3), the analytical solution to the state equation (2) is a difference equation as

$$z[k] = Az[k-1] + B_0 u[k-1] + B_1 u[k] + E_0 w[k-1] + E_1 w[k] \quad (5)$$

where

$$A = e^{A_c \Delta t} \text{ is the } 2n \times 2n \text{ discrete-time system matrix}$$

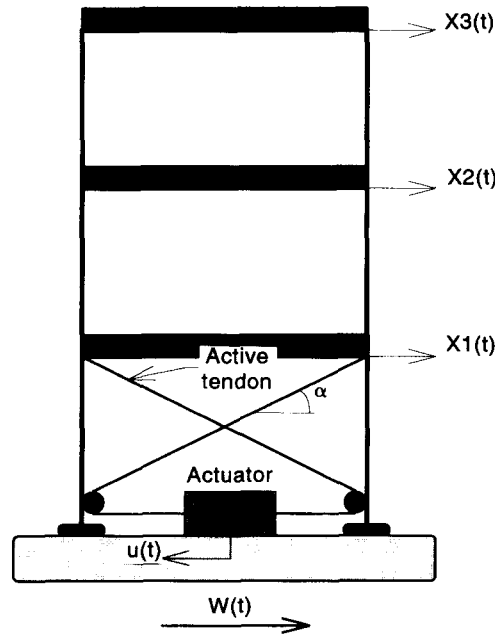


Figure 1 3DOF model structure with control device

$\mathbf{B}_0 = \left[\mathbf{A}_c^{-1} \mathbf{A} + \frac{1}{\Delta t} \mathbf{A}_c^{-2} (\mathbf{I} - \mathbf{A}) \right] \mathbf{B}_c$ is the $2n \times p$ discrete-time control matrix of the previous time step

$\mathbf{B}_1 = \left[-\mathbf{A}_c^{-1} + \frac{1}{\Delta t} \mathbf{A}_c^{-2} (\mathbf{A} - \mathbf{I}) \right] \mathbf{B}_c$ is the $2n \times p$ discrete-time control matrix of the current time step

$\mathbf{E}_0 = \left[\mathbf{A}_c^{-1} \mathbf{A} + \frac{1}{\Delta t} \mathbf{A}_c^{-2} (\mathbf{I} - \mathbf{A}) \right] \mathbf{E}_c$ is the $2n \times q$ discrete-time load matrix of the previous time step

$\mathbf{E}_1 = \left[-\mathbf{A}_c^{-1} + \frac{1}{\Delta t} \mathbf{A}_c^{-2} (\mathbf{A} - \mathbf{I}) \right] \mathbf{E}_c$ is the $2n \times q$ discrete-time load matrix of the current time step

As a whole, it is more logical and more realistic for the structural control system to be modelled in a discrete-time fashion.

The current active control forces are found such that the quadratic objective function

$$J[k] = \mathbf{z}^T[k] \mathbf{Q} \mathbf{z}[k] + \mathbf{u}^T[k] \mathbf{R} \mathbf{u}[k] \quad (6)$$

is minimized. In the above equation, \mathbf{Q} is the $2n \times 2n$ symmetric positive semidefinite weighting matrix for the responses, \mathbf{R} is the $p \times p$ symmetric positive definite weighting matrix for the input control forces, and superscript T denotes transposition of a matrix.

With instantaneous control law, the optimization problem is: searching for the optimal control forces $\mathbf{u}[k]$ that minimize the performance index $J[k]$ in equation (6) subject to the constraint of the discrete-time state equation (5). Incorporated with the constraint equation (5), the Lagrangian J' can be defined as

$$\begin{aligned}
 J' = & \mathbf{z}^T[k] \mathbf{Q} \mathbf{z}[k] + \mathbf{u}^T[k] \mathbf{R} \mathbf{u}[k] + \lambda^T[k] \{ \mathbf{A} \mathbf{z}[k-1] \\
 & + \mathbf{B}_0 \mathbf{u}[k-1] + \mathbf{B}_1 \mathbf{u}[k] + \mathbf{E}_0 \mathbf{w}[k-1] + \mathbf{E}_1 \mathbf{w}[k] \\
 & - \mathbf{z}[k] \} \quad (7)
 \end{aligned}$$

where $\lambda[k]$ is the $2n \times 1$ Lagrangian multiplier vector.

Because the performance index is quadratic, and \mathbf{Q} and \mathbf{R} are positive semidefinite and positive definite matrices, respectively, the necessary and sufficient conditions for the minimization of the Lagrangian J' are

$$\begin{aligned}
 \frac{\partial J'[k]}{\partial \lambda[k]} = & \mathbf{A} \mathbf{z}[k-1] + \mathbf{B}_0 \mathbf{u}[k-1] + \mathbf{B}_1 \mathbf{u}[k] \\
 & + \mathbf{E}_0 \mathbf{w}[k-1] + \mathbf{E}_1 \mathbf{w}[k] - \mathbf{z}[k] = \mathbf{0} \quad (8)
 \end{aligned}$$

$$\frac{\partial J'[k]}{\partial \mathbf{z}[k]} = 2\mathbf{Q} \mathbf{z}[k] - \lambda[k] = \mathbf{0} \quad (9)$$

$$\frac{\partial J'[k]}{\partial \mathbf{u}[k]} = 2\mathbf{R} \mathbf{u}[k] + \mathbf{B}_1^T \lambda[k] = \mathbf{0} \quad (10)$$

From equation (9), the Lagrangian multipliers $\lambda[k]$ are linearly related with the states as

$$\lambda[k] = 2\mathbf{Q} \mathbf{z}[k] \quad (11)$$

From equation (10), the control forces are linearly related with the Lagrangian multipliers $\lambda[k]$ as

$$\mathbf{u}[k] = -\frac{1}{2} \mathbf{R}^{-1} \mathbf{B}_1^T \lambda[k] \quad (12)$$

By substituting equation (11) into (12), the control forces are in turn linearly related to the states as

$$\mathbf{u}[k] = -\mathbf{R}^{-1} \mathbf{B}_1^T \mathbf{Q} \mathbf{z}[k] = \mathbf{G} \mathbf{z}[k] \quad (13)$$

where $\mathbf{G} = -\mathbf{R}^{-1} \mathbf{B}_1^T \mathbf{Q}$ is the $p \times 2n$ feedback gain matrix which is a constant matrix. The control forces $\mathbf{u}[k]$ are simply generated from the structural states $\mathbf{z}[k]$ multiplied by the precalculated constant feedback gain matrix \mathbf{G} .

3. Time-delay compensation

The discrete-time control system described by the state equation (5) has been idealized where the control forces $\mathbf{u}[k]$

Table 1 System parameters of 3DOF control system

System parameter (1)	Parameter value (2)
Mass matrix, \mathbf{M} (kg)	$\begin{bmatrix} 981 & 0 & 0 \\ 0 & 981 & 0 \\ 0 & 0 & 981 \end{bmatrix}$
Stiffness matrix, \mathbf{K} (N/m)	$\begin{bmatrix} 2741700 & -1641600 & 369100 \\ -1641600 & 3022200 & -1624800 \\ 369100 & -1624800 & 1333600 \end{bmatrix}$
Damping matrix, \mathbf{C} (N-s/m)	$\begin{bmatrix} 382.8 & -57.3 & 61.7 \\ -57.3 & 456.9 & -2.6 \\ 61.7 & -2.6 & 437.5 \end{bmatrix}$
Modal frequencies, \mathbf{f} (Hz)	$\begin{bmatrix} 2.24 \\ 6.80 \\ 11.49 \end{bmatrix}$
Modal damping ratios, ζ (%)	$\begin{bmatrix} 1.61 \\ 0.39 \\ 0.36 \end{bmatrix}$
Control force location matrix, \mathbf{B}	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$
External load location matrix, \mathbf{E}	$\begin{bmatrix} -981 \\ -981 \\ -981 \end{bmatrix}$
Tendon stiffness, k_c (N/m)	372 100
Tendon inclination, α ($^\circ$)	36
Response weighting matrix, \mathbf{Q}	$\begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix}$
Control weighting matrix, \mathbf{R}	$\frac{\beta}{16k_c \cos^2 \alpha}$
Control weighting factor, β	0.25

can be applied to the structure simultaneously as the states $z[k]$ are measured. In practice, time has to be consumed during data conditioning, online calculation and control force application. Taking time-delay into consideration, the state equation (5) becomes

$$z[k] = \mathbf{A}z[k-1] + \mathbf{B}_0\mathbf{u}[k-l-1] + \mathbf{B}_1\mathbf{u}[k-l] + \mathbf{E}_0\mathbf{w}[k-1] + \mathbf{E}_1\mathbf{w}[k] \quad (14)$$

where l is the number of delayed time steps. The application of the control forces $\mathbf{u}[k]$ lags the measurement of the state variables $z[k]$ by $l\Delta t$. With time-delay, the instant performance index (equation (6)) becomes

$$J[k] = z^T[k]\mathbf{Q}z[k] + \mathbf{u}^T[k-l]\mathbf{R}\mathbf{u}[k-l] \quad (15)$$

In the presence of time-delay, the optimization problem is converted to the minimization of the performance index (15) subject to the constraint of the state equation (14).

Following the optimization procedures stated in the previous section, the necessary and sufficient conditions for the minimization of the performance index (15) are

$$\frac{\partial J[k]}{\partial \lambda[k]} = \mathbf{A}z[k-1] + \mathbf{B}_0\mathbf{u}[k-l-1] + \mathbf{B}_1\mathbf{u}[k-l] + \mathbf{E}_0\mathbf{w}[k-1] + \mathbf{E}_1\mathbf{w}[k] - z[k] = \mathbf{0} \quad (16)$$

$$\frac{\partial J[k]}{\partial z[k]} = 2\mathbf{Q}z[k] - \lambda[k] = \mathbf{0} \quad (17)$$

$$\frac{\partial J[k]}{\partial \mathbf{u}[k-l]} = 2\mathbf{R}\mathbf{u}[k-l] + \mathbf{B}_1^T\lambda[k] = \mathbf{0} \quad (18)$$

From equations (17) and (18), the time-delay control forces $\mathbf{u}[k-l]$ are linearly related to the current state variables $z[k]$ by

$$\mathbf{u}[k-l] = -\mathbf{R}^{-1}\mathbf{B}_1^T\mathbf{Q}z[k] = \mathbf{G}z[k] \quad (19)$$

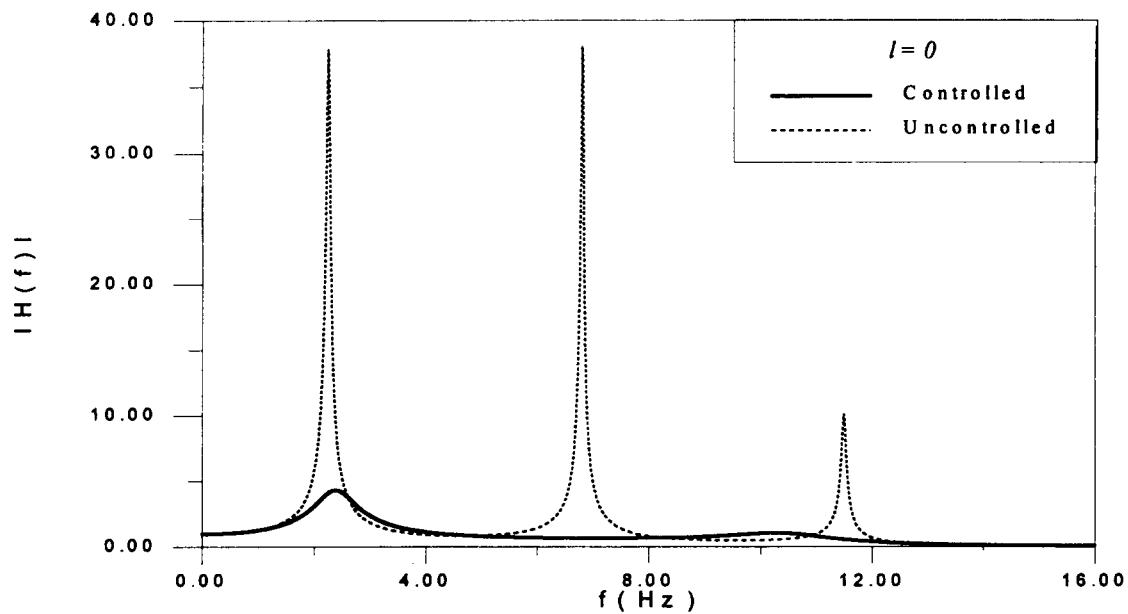


Figure 2 Top-floor absolute acceleration frequency response function at $l=0$

Table 2 Control effect for 3DOF structure

Number of delayed time steps	Modal frequencies, f (Hz)		Damping ratios, ζ (%)	
	No delay consideration	Time delay compensation	No delay consideration	Time delay compensation
$l=0$		$\begin{bmatrix} 2.407 \\ 10.522 \\ 7.929 \end{bmatrix}$		$\begin{bmatrix} 16.2 \\ 8.774 \\ 63.75 \end{bmatrix}$
$l=2$	$\begin{bmatrix} 14.591 \\ 2.465 \\ 9.387 \end{bmatrix}$	$\begin{bmatrix} 2.407 \\ 10.522 \\ 7.929 \end{bmatrix}$	$\begin{bmatrix} -12.85 \\ 13.287 \\ 5.01 \end{bmatrix}$	$\begin{bmatrix} 16.2 \\ 8.774 \\ 63.75 \end{bmatrix}$
$l=4$	$\begin{bmatrix} 9.965 \\ 9.9645 \\ 2.502 \end{bmatrix}$	$\begin{bmatrix} 2.407 \\ 10.522 \\ 7.929 \end{bmatrix}$	$\begin{bmatrix} -17.14 \\ -9.245 \\ 10.523 \end{bmatrix}$	$\begin{bmatrix} 16.2 \\ 8.774 \\ 63.75 \end{bmatrix}$
$l=6$	$\begin{bmatrix} 7.632 \\ 10.625 \\ 2.528 \end{bmatrix}$	$\begin{bmatrix} 2.407 \\ 10.522 \\ 9.161 \end{bmatrix}$	$\begin{bmatrix} -27.048 \\ -0.386 \\ 7.95 \end{bmatrix}$	$\begin{bmatrix} 16.2 \\ 8.774 \\ 23.123 \end{bmatrix}$
$l=8$	$\begin{bmatrix} 6.544 \\ 15.496 \\ 2.547 \end{bmatrix}$	$\begin{bmatrix} 2.407 \\ 10.522 \\ 8.698 \end{bmatrix}$	$\begin{bmatrix} -26.486 \\ -1.596 \\ 5.473 \end{bmatrix}$	$\begin{bmatrix} 16.2 \\ 8.774 \\ 21.168 \end{bmatrix}$

The control forces $u[k-l]$ depend on the state variables $z[k]$ which are not available at the time step $[k-l]$. Consequently, the control law represented by equation (19) cannot be implemented. Instead of the current state variables $z[k]$, the time-delay state variables should be used for the online calculation of the time-delay control forces. In order to evaluate the stability of the control system, the corresponding autonomous system of the time-delay state equation (14) is considered. After substituting equation (19) into the time-delay state equation (14), the current state

variables $z[k]$ and the previous state variables $z[k-1]$ are related by

$$z[k] = (I + B_1 R^{-1} B_1^T Q)^{-1} (A - B_0 R^{-1} B_0^T Q) z[k-1] \quad (20)$$

$$= S z[k-1]$$

where S is the $2n \times 2n$ transition matrix

$$S = (I + B_1 R^{-1} B_1^T Q)^{-1} (A - B_0 R^{-1} B_0^T Q) \quad (21)$$

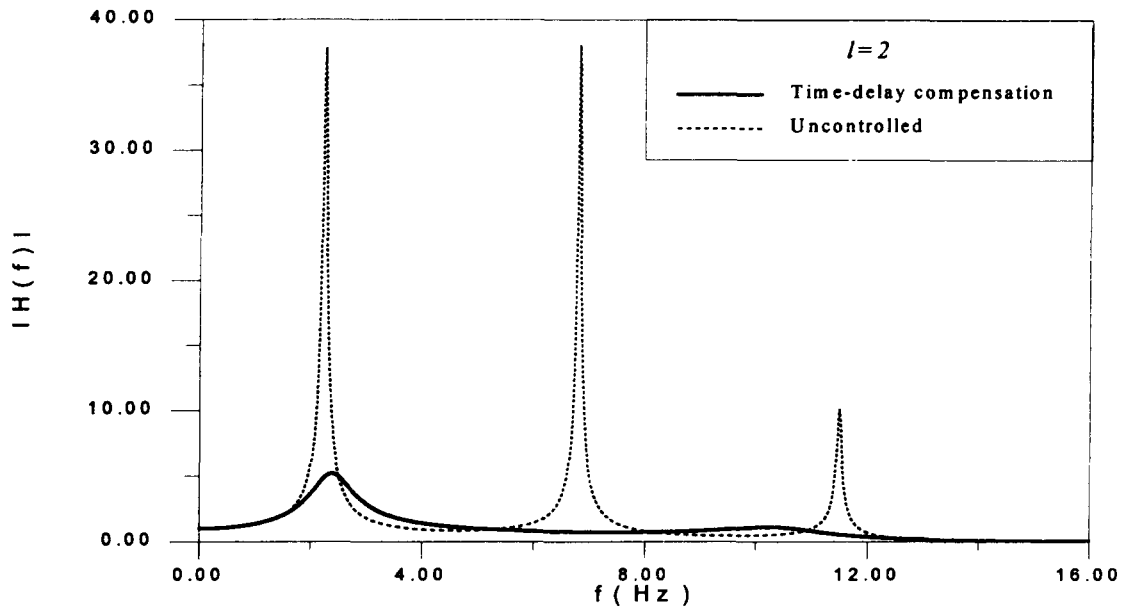


Figure 3 Top-floor absolute acceleration frequency response function at $l=2$

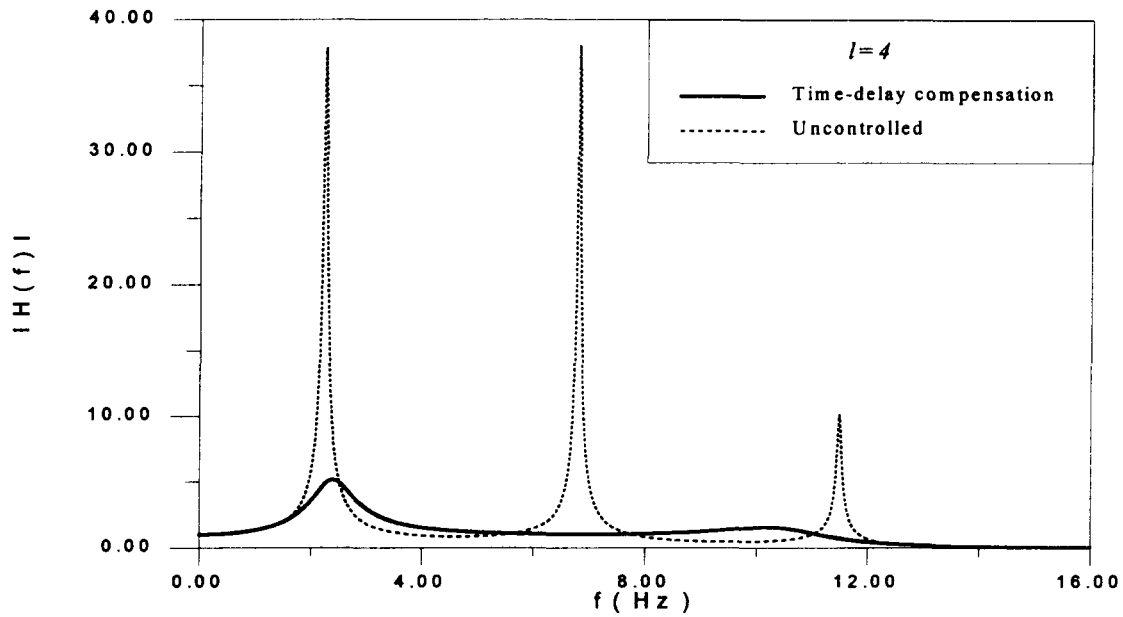


Figure 4 Top-floor absolute acceleration frequency response function at $l=4$

$$= (\mathbf{I} - \mathbf{B}_1 \mathbf{G})^{-1} (\mathbf{A} + \mathbf{B}_0 \mathbf{G})$$

By applying the relation in the above equation repeatedly to equation (19), the time-delay control forces $\mathbf{u}[k-l]$ can be generated from the time-delay state variables $\mathbf{z}[k-l]$ as

$$\mathbf{u}[k-l] = \mathbf{G} \mathbf{S}' \mathbf{z}[k-l] = \mathbf{G}_c \mathbf{z}[k-l] \quad (22)$$

where $\mathbf{G}_c = \mathbf{G} \mathbf{S}'$ is the $p \times 2n$ compensated feedback gain matrix. After time-delay compensation, the control law represented by equation (21) can now be implemented.

4. Evaluation of control system

In eigenvalue analysis, the enhancement of the control system is indicated by the system parameters such as natural frequencies and damping ratios extracted from the eigenval-

ues of the system matrix. The i th eigenvalue γ_i can be expressed as

$$\gamma_i = \rho_i e^{i\theta_i} = e^{-\zeta_i \omega_i \Delta t} e^{j \omega_i \sqrt{1-\zeta_i^2} \Delta t} \quad (23)$$

where $j = \sqrt{-1}$, ω_i is the i th effective natural frequency and ζ_i is the i th damping ratio. Therefore, the i th effective natural frequency and damping ratio can be extracted from the i th eigenvalue of the control system as

$$\omega_i = \frac{1}{\Delta t} \sqrt{(\ln \rho_i)^2 + \theta_i^2} \quad (24)$$

$$\zeta_i = -\frac{\ln \rho_i}{\sqrt{(\ln \rho_i)^2 + \theta_i^2}} \quad (25)$$

When $\rho_i < 1$, the damping ratio ζ_i is positive so that the

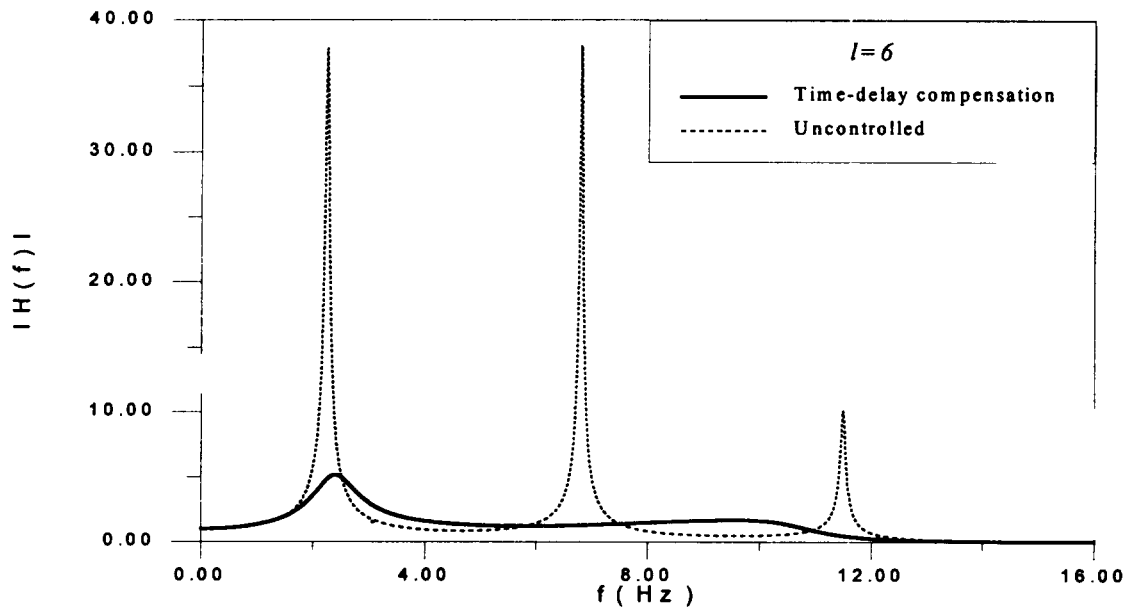


Figure 5 Top-floor absolute acceleration frequency response function at $l=6$

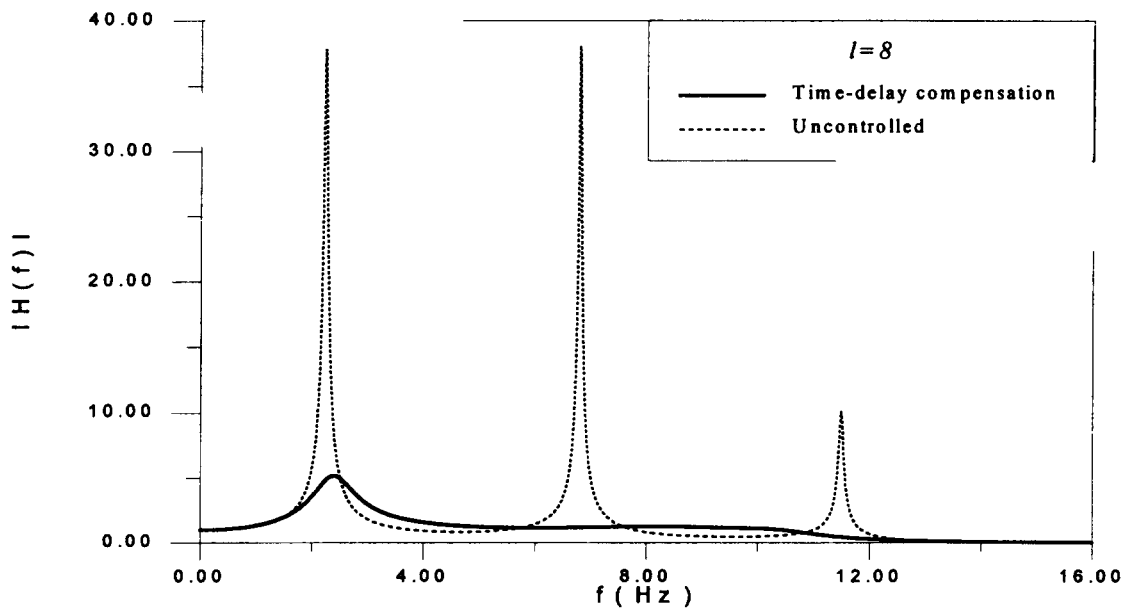


Figure 6 Top-floor absolute acceleration frequency response function at $l=8$

i th mode is stable and convergent. When $\rho_i > 1$, the damping ratio ζ_i is negative so that the i th mode is unstable and divergent. Therefore, if all the eigenvalues are located within the unit circle of the complex plane, the control system is stable and convergent. The stability criterion for a discrete-time system is different from that for a continuous-time system¹⁹.

In the uncontrolled case, the state equation (5) becomes

$$z[k] = Az[k-1] + E_0w[k-1] + E_1w[k] \quad (26)$$

The eigenproperties of the structure can be extracted from the system matrix A . By substituting the control law equation (13) into the state equation (5), the effective state equation for the ideal control system where time-delay does not exist is

$$z[k] = Az[k-1] + B_0Gz[k-1] + B_1Gz[k] \quad (27)$$

$$+ E_0w[k-1] + E_1w[k]$$

From the above equation, the characteristic equation for the ideal control system is

$$|I - A\gamma^{-1} - B_0G\gamma^{-1} - B_1G| = 0 \quad (28)$$

where $|\cdot|$ denotes the determinant of a matrix. The eigenproblem stated in the above characteristic equation is equivalent to solving for the eigenvalues of the effective system matrix

$$A' = (I - B_1G)^{-1}(A + B_0G) = S \quad (29)$$

By substituting the control equation (21) into the state equation (14), the effective state equation for the real system where time-delay does exist is

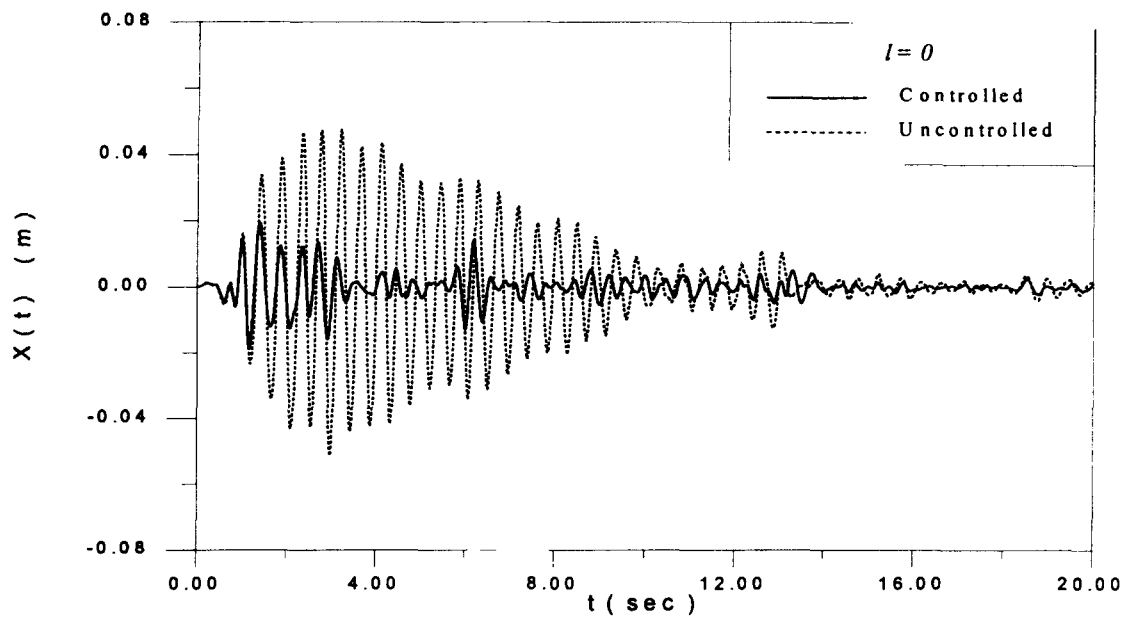


Figure 7 Top-floor relative displacement under El Centro earthquake at $l=0$

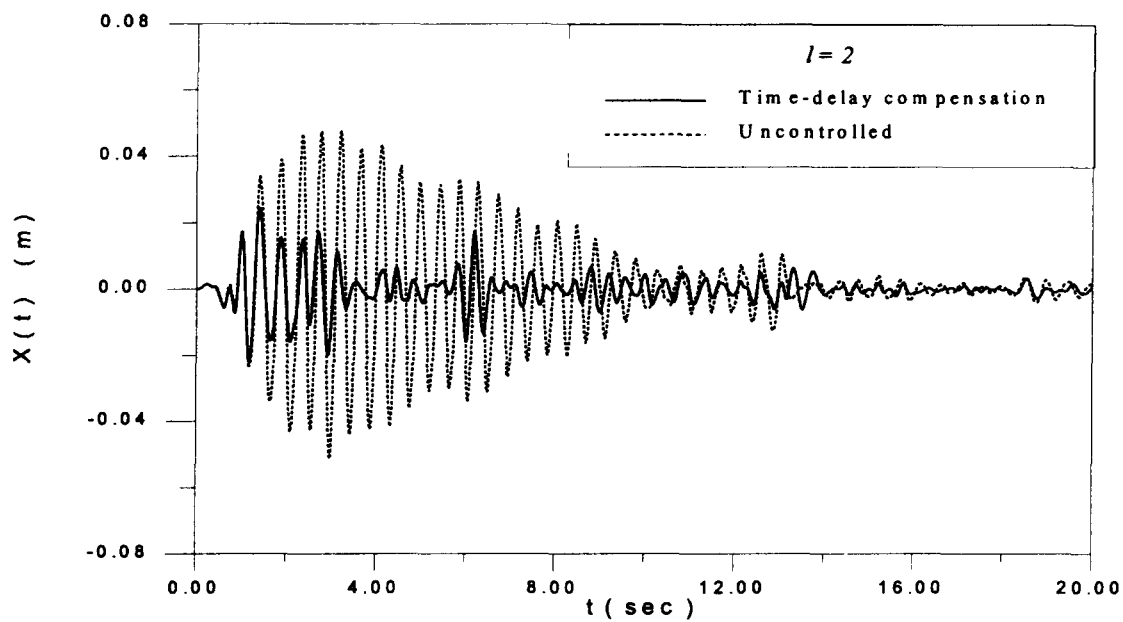


Figure 8 Top-floor relative displacement under El Centro earthquake at $l=2$

$$z[k] = Az[k-1] + B_0 G_c z[k-l-1] + B_1 G_c z[k-l] + E_0 w[k-1] + E_1 w[k] \quad (30)$$

From the above equation, the characteristic equation for the real control system is

$$|I - A\gamma^{-1} - B_0 G(A')^l \gamma^{-l-1} - B_1 G(A')^{l-1} \gamma^{-l}| = |F(\gamma)| = 0 \quad (31)$$

It is found that the effective system matrix A' for the ideal control system satisfies the characteristic equation for the real control system

$$F(A') = 0 \quad (32)$$

Therefore, the real control system with time-delay compen-

sation conserves the eigen-properties of the ideal control system with no time-delay.

5. Numerical verifications

The feasibility of the proposed control algorithm is verified numerically through the tendon control system of a three-degree-of-freedom (3DOF) structure. The dynamic characteristics of the model structure shown in *Figure 1* have been studied thoroughly¹⁵. The three-storey structure is subjected to seismic motion and counteracted by the tendon control device implemented on the first floor. Relevant parameters of the control system are listed in *Table 1*. The control effectiveness is evaluated through eigenvalue, frequency-domain and time-domain analyses. In the eigenvalue analysis, the enhancement of the control system is indicated by the system parameters extracted from the eigenvalues of

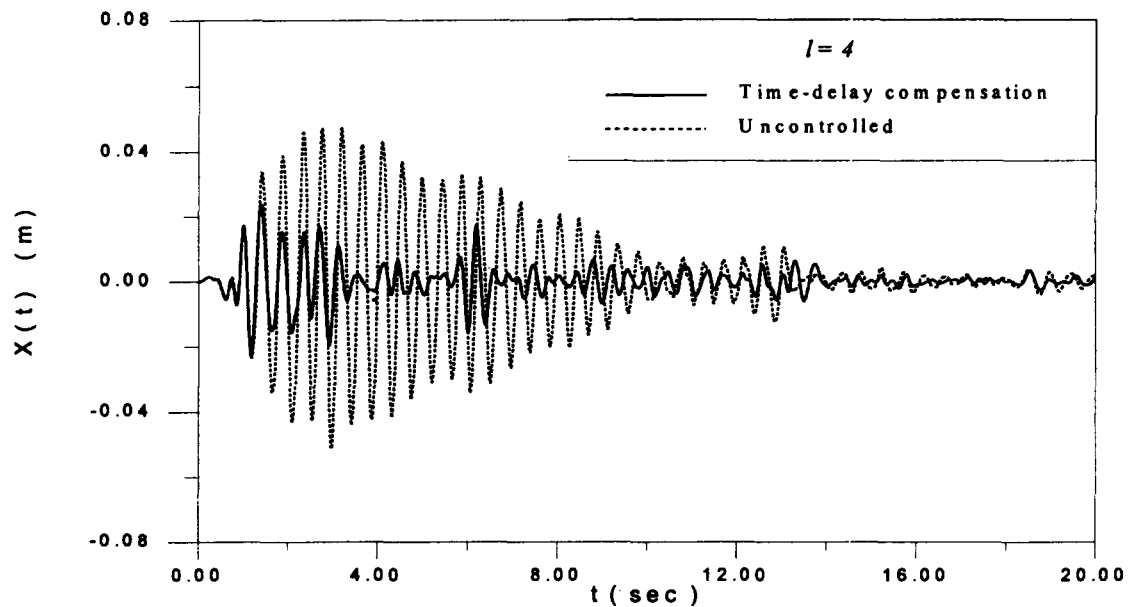


Figure 9 Top-floor relative displacement under El Centro earthquake at $l=4$

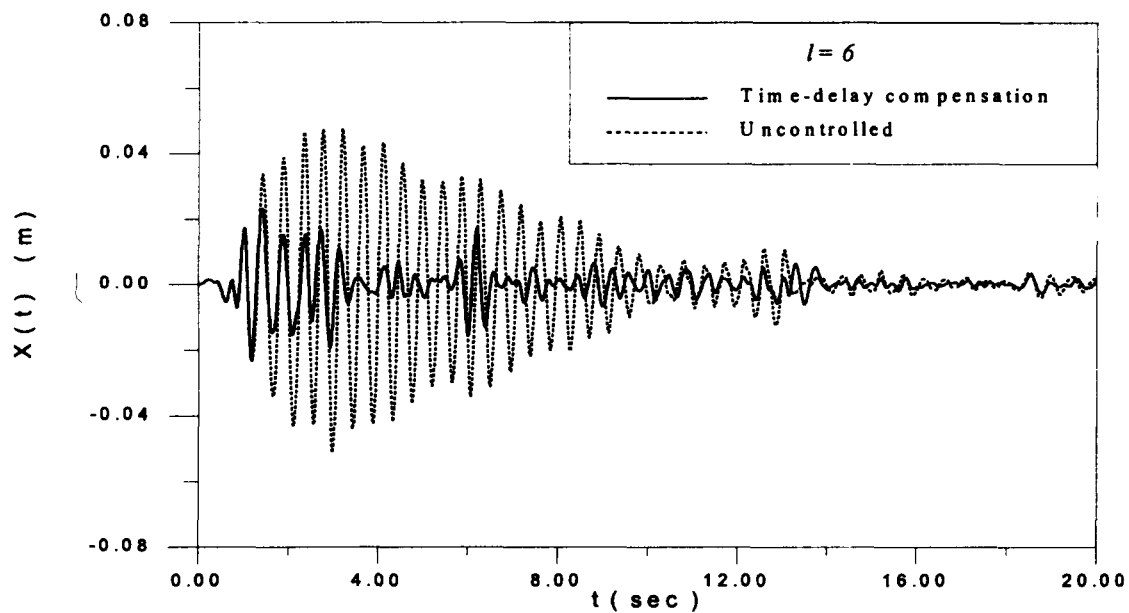


Figure 10 Top-floor relative displacement under El Centro earthquake at $l=6$

the effective system matrix of the control system. In the frequency-domain analysis, the degree of vibration suppression is shown by the peaks of the frequency response function of the control system. Finally, in the time-domain analysis, the reduction of the structural response is demonstrated by subjecting the structure to the 1940 El Centro earthquake (N-S component).

The control effects from the eigenvalue analysis of the control system are listed in Table 2 while the results from the frequency-domain and time-domain analyses are shown in Figures 2–6 and 7–11, respectively. The states of the structure are measured with a sampling period of $\Delta t = 0.01$ s. As the number of delayed time steps l varies, its influences on the different control strategies are discussed below.

(1) When $l=0$, it is the ideal control system where no time-delay exists. Under the action of the control force,

the natural frequencies are changed from 2.24 Hz, 6.80 Hz and 11.49 Hz to 2.41 Hz, 7.93 Hz and 10.52 Hz, respectively, and the damping ratios are increased greatly from 1.61%, 0.39% and 0.16% to 16.20%, 63.75% and 8.77%, respectively, (Table 2). The effectiveness of this control case is obvious from the frequency-domain and time-domain analyses (Figures 2 and 7). In fact, this case possesses the best control result but it is not realistic since time-delay is unavoidable

(2) When $l=2$, the time-delay is 20 ms. If time-delay is neglected, the control system becomes unstable as indicated by the negative damping ratio (Table 2). The frequency response function is not available since the structural response blows up and no steady-state response can be achieved (Figure 8). If time-delay is considered, the control system remains stable. The sys-

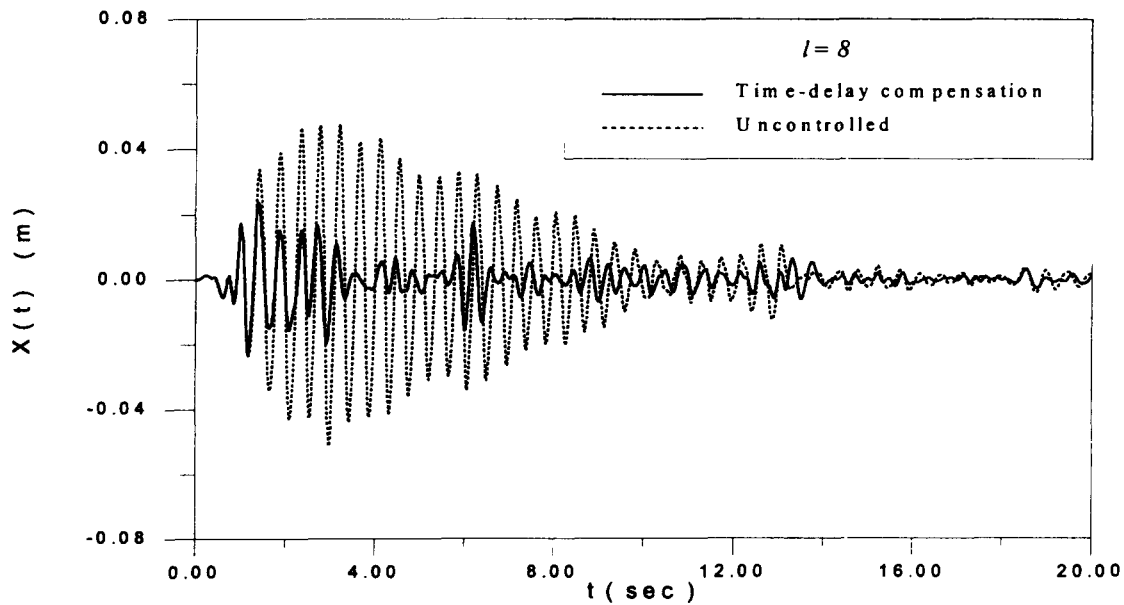


Figure 11 Top-floor relative displacement under El Centro earthquake at $l=8$

tem parameters, modal frequencies and damping ratios, extracted from the three eigenvalues with the largest modulus ρ_i 's are identical to those from the ideal control system (Table 2). In other words, the parameters of the three lowest modes for the real control system with time-delay compensation are identical to those for the ideal control system with no time-delay. The control effectiveness of the real system is close to that of the ideal system as observed from the frequency-domain and time-domain analyses (Figures 3 and 8)

- (3) When $l=4$, the time-delay is increased to 40 ms. If time-delay is neglected, the control system is unstable (Table 2). If time-delay is considered, the control system remains stable and the parameters of the three lowest modes are identical to those of the ideal control system. The control effectiveness of this case is very close to the case of $l=2$ as observed from frequency-domain and time-domain analyses (Figures 4 and 9)
- (4) When $l=6$, the time-delay is increased to 60 ms. If time-delay is neglected, the control system is unstable (Table 2). If time-delay is considered, the control system remains stable. In total there are 21 pairs of complex conjugate eigenvalues and three of the pairs are identical to those of the ideal system but only two out of the three pairs remain in the three lowest modes (Table 2). The control effectiveness of this case is very close to that of $l=4$ as observed from the frequency-domain and time-domain analyses (Figures 5 and 10)
- (5) When $l=8$, the time-delay is further increased to 80 ms. If time-delay is neglected, the control system is unstable (Table 2). If time-delay is considered, the control system remains stable. In total there are 27 pairs of complex conjugate eigenvalues and three of the pairs are identical to those of the ideal system but only two out of the three pairs remain in the three lowest modes (Table 2). The control effectiveness of this case is very close to that of $l=6$ as observed from the frequency-domain and time-domain analyses (Figures 6 and 11)

6. Conclusions

A modified instantaneous control algorithm for the active control of structures is developed in discrete-time formulation which is very efficient when a digital computer is introduced for the real-time calculation of control forces. In real-time control, time is needed for data acquisition, data conditioning, calculation and applications of control forces. If time-delay is neglected, the control system is susceptible to dynamic instability. Therefore, it is better not to put any control action into the structural system before the time-delay is analysed and tackled properly. In this paper, a feedback compensation scheme is developed by simply modifying the feedback gain matrix of the ideal control system with the effective system matrix. It is also proved that the real control system with time-delay compensation conserves the eigen-properties of the ideal control system with no time-delay. In the presence of time-delay, the equation of motion of the discrete-time control system remains a set of difference equations which makes the eigenvalue analysis of the control system simple and straightforward. It is one of the great advantages of discrete-time formulation. The feasibility of the proposed algorithm is verified numerically through eigenvalue, frequency-domain and time-domain analyses. The control forces are simply generated from the time-delay states multiplied by the precalculated and shift-variant modified feedback gain. Such simple online calculation of the control forces makes the proposed control algorithm favourable for real implementation. With the proposed control algorithm, the control system is still effective in spite of the presence of time-delay.

References

- 1 Yao, J. T. P. 'Concept of structural control', *J. Struct. Div., ASCE* 1972, **98**, 1567-1574
- 2 Reinhorn, A. M. and Manolis, G. D. 'Current state of knowledge on structural control', *Shock Vibr. Digest* 1985, **17**, 35-41
- 3 Yang, J. N. and Soong T. T. 'Recent advances in active control of

- engineering structures', *J. Probabilistic Engng Mech.* 1988, **3**, 179–188
- 4 Soong, T. T. 'Active structural control in civil engineering', *Engng. Struct.* 1988, **10**, 74–84
 - 5 Soong, T. T. *Active structural control: theory and practice*, Wiley, New York, 1990
 - 6 Soong, T. T., Reinhorn, A. M., Wang, Y. P. and Lin, R. C. 'Full-scale implementation of active control. I: Design and simulation', *J. Struct. Engng, ASCE* 1991, **117**, 3516–3536
 - 7 Kobori, T., Koshika, N., Yamada, K. and Ikeda, Y. 'Seismic-response-controlled structure with active mass driver system. Part 1: Design', *Earthquake Engng Struct. Dyn.* 1991, **20**, 133–149
 - 8 Kobori, T., Koshika, N., Yamada, K. and Ikeda, Y. 'Seismic-response-controlled structure with active mass driver system. Part 2: Verification', *Earthquake Engng Struct. Dyn.* 1991, **20**, 151–166
 - 9 Reinhorn, A. M., Soong, T. T., Riley, M. A., Lin, R. C., Aizawa, S. and Higashino, M. 'Full-scale implementation of active control. II: Installation and performance', *J. Struct. Engng, ASCE* 1993, **119**, 1935–1960
 - 10 Sage, A. P. and White, C. C. *Optimum systems control*, Prentice-Hall, Englewood Cliffs, NJ, 1977
 - 11 Yang, J. N., Akbarpour, A. and Ghaemmaghmi, P. 'Optimal control algorithms for earthquake excited building structures', *Structural Control, Proc., 2nd Int. Symp. on Struct. Control*, (Ed.) H. H. E. Leipholz, Nijhoff, Amsterdam, The Netherlands, 1985, pp. 748–761
 - 12 Yang, J. N., Akbarpour, A. and Ghaemmaghmi, P. 'New optimal control algorithms for structural control', *J. Engng Mech., ASCE* 1987, **113**, 1369–1386
 - 13 Rodellar, J., Barbat, A. H., and Martin-Sanchez, J. M. 'Predictive control of structures', *J. Engng Mech., ASCE* 1987, **113**, 797–812
 - 14 Chung, L. L., Reinhorn, A. M. and Soong, T. T. 'Experiments on active control of seismic structures', *J. Engng Mech., ASCE* 1988, **114**, 241–256
 - 15 Chung, L. L., Lin, R. C., Soong, T. T. and Reinhorn, A. M. 'Experimental study of active control for MDOF seismic structures', *J. Engng Mech., ASCE* 1989, **115**, 1609–1627
 - 16 McGreevy, S., Soong, T. T. and Reinhorn, A. M. 'An experimental study of time delay compensation in active structural control', *Proc. 6th Intl. Modal Analysis Conf. Society for Experimental Mechanics*, Kissimmee, FL, 1988, pp. 733–739
 - 17 Abdel-Mooty, M. and Roorda, J. 'Time-delay compensation in active damping of structures', *J. Engng Mech., ASCE* 1991, **117**, 2549–2570
 - 18 Zhang, L., Yang, C. Y., Chajes, M. J. and Cheng, H.-D. 'Stability of active-tendon structural control with time delay', *J. Engng Mech., ASCE* 1993, **119**, 1017–1024
 - 19 Kuo, B. C. *Digital control systems*, Holt, Rinehart and Winston, 1980