# Chapter 3 1D PBG SIMULATION



Fig. 3-1 Illustration of 1D photonic crystal

In this Chapter, we will use the Eqs. (2.27), (2.28), (2.29) and (2.30) to calculate the photonic band gap of one-dimensional photonic crystal. The structure we consider is illustrated by Figure 3-1. There are several points important in Figure 3-1:

a.  $x \to \pm \infty$ ,  $y \to \pm \infty$ 

b. Media with  $\varepsilon_1$  and  $\varepsilon_2$  are lossless( $\sigma = 0$ 

c. Normal Incident

## **3-1** The Calculation Flow

The fundamental idea of calculation is described by Figure 3-2. We generate a Dirac-delta function pulse at the start-grid and let it incident into the 1D photonic crystal from the left-hand side. The pulse will propagate in the 1D photonic crystal and go out from the right-hand side of the 1D photonic crystal. We collect the output wave with the receive-grid that is at the rear surface of 1D photonic crystal. We calculate the Fourier transform of both the incident pulse and the output pulse, and we can get two values for every frequency: FAmpi[f] and FAmp[f], where FAmpi[f] is the



Fig. 3-2 The idea of calculation for 1D photonic crystal in this work.

Fourier amplitude of the incident pulse at frequency f and FAmp[f] is the Fourier amplitude of the output pulse at frequency f. And then, we can get the transmission coefficient of every frequency, which we call Trans[f], by calculating the ratio of FAmp[f] and FAmpi[f]:

$$Trans[f] = \frac{FAmp[f]}{FAmpi[f]}$$
(3.1)

Fig. 3-3 is a flow chart of calculation used in this work:

- Input the parameters of the photonic crystal:
   b/a, ε<sub>1</sub>, ε<sub>2</sub>, total layer and time-steps.
- 2. Define the number of total grids (ME) and the structure of photonic crystal (PS and PE).
- 3. Start from T=0, generate a pulse at the start-grid (mc). Note that mc has more 20 grids than at m=0. The reason for this will be explained later. And than, let the pulse incident into the photonic crystal from left side. At the same time, we do Fourier transform of the input pulse in a specific time period (TE) to get the Fourier amplitude of every frequency (*Ampi[f*]).
- Calculate every physic quantity of every grid: D<sub>x</sub>, E<sub>x</sub>, H<sub>y</sub>. Here, we use the Eqs. (2.27), (2.28), (2.29) and (2.30). Apply the Mur's ABCs on the both ends of computation region.
- 5. Do the Fourier transform of transmissible wave at the receive grid (m=PE+50)

and get the Fourier amplitude of every frequency (Amp[f]). We just want to check more output wave data, so we set the receive point is 50 grids after the End of photonic crystal to see more output pulse.

- 6. Use Eq. (3.1) to calculate the transmission coefficient of every frequency, and find the location of photonic band gap..
- 7. Go back to the  $4^{th}$  step until T< time-steps, and repeat step 3 to step 6.





Fig. 3-3 This is the flow chart of the calculation in our program.

## **3-2** Several Important Points

In this simulation, we have to consider several important problems:

#### 1. Source function

We use a Gaussian function to simulate a delta function.

$$E_{x}(mc) = \begin{cases} \exp[-(\frac{4}{t_{0} \cdot (t - t_{0})})^{2}] & \text{when } 0 \le t \le 2t_{0} \\ 0 & else \end{cases}$$
(3.2)

mc is the grid where pulse generated.

For a perfect delta function, the Fourier amplitude of every frequency is 1. When  $t_0 = 1$ , the Fourier Amplitude is 1 at every frequency. So, we can choose the below function to be our source function

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$$E_x(mc) = \begin{cases} \exp\left[-\left(\frac{4}{(t-1)}\right)^2\right] \\ 0 \end{cases}$$
 when  $0 \le t \le 2$   
2. Normalization  $\frac{\varpi a}{2\pi c}$  (3.3)  
When we do Fourier transform:

$$\tilde{E}_x(f) = \int_0^{t_T} E_x(t) \exp(-j\omega t) dt.$$
(3.4)

In Eq. (3.4), we need to normalize  $\omega$ :

$$\omega' = \omega / (\frac{2\pi}{a}c) = \frac{\omega a}{2\pi c}, \qquad (3.5)$$

where  $\omega'$  is the normalized frequency. First we rewrite  $\omega t$  into the difference form

$$\omega t = \omega \cdot n \Delta t \,. \tag{3.6}$$

From Eq. (2.15) we can get

$$\omega t = 2\pi f \cdot n \cdot \frac{\Delta z}{uc} \,. \tag{3.7}$$

Since we let the lattice constant

$$a = a' \cdot \Delta z , \qquad (3.8)$$

the Eq.(3.7) can be rearranged:

$$\omega t = \frac{2\pi n}{ua'} \left(\frac{2\pi f}{2\pi c}a' \cdot \Delta z\right) = \frac{2\pi n}{ua} \left(\frac{\omega a}{2\pi c}\right). \tag{3.9}$$

Therefore, we can rewrite Eq. (3.4) into difference form:

$$\tilde{E}_{x}(f) = \sum_{n=0}^{T} E_{x}(n \cdot \Delta t) \cos(\frac{2\pi}{ua}\omega' n) - \sum_{n=0}^{T} E_{x}(n \cdot \Delta t) \sin(\frac{2\pi}{ua}\omega' n)$$
(3.10)



## **3-3** Simulation Result

In this section we will show several cases in this work including the perfect and defective photonic band gap structure. All the simulation in this section is normal incident. Because the final goal of this work is to design a band pass filter on the *Si* substrate, most of the materials of photonic crystal in this section are *Si* and *Air*.

#### **3-3.1** Perfect Photonic Band Gap Structure

We consider a 1D photonic crystal structure with two materials *Si/AIR*. The relative dielectric constant contrast is  $\varepsilon_1 / \varepsilon_2$ . Fist, we discuss that the ratio of silicon and air layer in a unit cell is 1:1 and there are 5 unit cells of this photonic crystal.





The transmission spectra of a 1D photonic crystal with b/a = 0.5,  $\varepsilon_1 = 11.9$ , and  $\varepsilon_2 = 1$ . The multilayer has 5 layers. The abscissa is the normalized frequency.

Here, we choose the lattice constant a=40, and we will explain why we choose a=40 later. Figure 3-4 is the transmission spectra of this photonic crystal. In Figure 3-4, there are two photonic band gaps. One is between  $0.401(2\pi c/a)$  and  $0.486(2\pi c/a)$  [ $\Delta \omega = 0.058(2\pi c/a)$ ], and another is between  $0.662(2\pi c/a)$  and  $0.712(2\pi c/a)$ 

 $[\Delta \omega = 0.05(2\pi c/a)]$ . From Figure 3-4, we can find that there may be other two photonic band gaps around  $0.1(2\pi c/a)$  and  $0.9(2\pi c/a)$ . So, we consider the same structure  $(b/a = 0.5, \varepsilon_1 = 11.9, \text{ and } \varepsilon_2 = 1)$  with 10 layers. Figure 3-5 shows the transmission spectra of a photonic crystal with 10 layers.





The transmission spectra of a 1D photonic crystal with b/a = 0.5,  $\varepsilon_1 = 11.9$ , and  $\varepsilon_2 = 1$ . The multilayer has 10 layers. The abscissa is the normalized frequency.

In Figure 3-5, the photonic crystal band gaps locate between  $0.164 \sim 0.255$ [  $\Delta \omega = 0.091(2\pi c/a)$  ],  $0.372 \sim 0.517$  [  $\Delta \omega = 0.145(2\pi c/a)$  ],  $0.621 \sim 0.748$ [  $\Delta \omega = 0.127(2\pi c/a)$  ] and  $0.884 \sim 0.926$  [  $\Delta \omega = 0.042(2\pi c/a)$  ] respectively. As we know, the range of the photonic band gap will increase with the increasing of the layers. But the photonic band gap can't increase infinitely, it will converge to a value, which is the photonic band gap with infinite periodic layers. In our simulation, we can't calculate the photonic crystal with infinite layers, but we still use a large and finite number of layers to approach the infinite layers. Table 3-1 illustrates the photonic band gap with several different numbers of layers, and all the data in Table 3-1 is normalized frequency. When we define the range of photonic band gap, we choose the normalized frequency at that the attenuation is more than 40dB. Since the Fourier amplitude of the every normalized frequency of the incident pulse is 1, the Fourier amplitude of the transmissible wave, which's attenuation is more than 40 dB, will small than

$$10^{-\frac{40}{20}} = 0.01$$

We think that it is very small than the Fourier amplitude of the input wave, and can treat it as 0.





Transmission spectra for the crystal with 5 layers, and the spectra of the crystal 10 layers is also shown for comparison. The ordinate is the attenuation in dB.

Figure 3-6 shows the attenuation of the transmitted wave. As can be seen in Figure 3-6, the attenuation increases with increasing the number of the layer, and the range of the band gap also increases. We can check the result by doing more calculation with other number of layers. In Table 3-1, each of the photonic band gaps converge to a specific value as the layers of photonic crystal are larger than 35 layers. So, we can say that we find the photonic band gap of the photonic crystal with infinite multilayer of the *Si/Air* materials.

Layers band gap	5	10	20	30	35	40
1st	Х	0.164~0.255	0.159~0.264	0.158~0.265	0.158~0.265	0.158~0.265
2nd	0.401~0.486	0.372~0.517	0.368~0.522	0.367~0.522	0.367~0.522	0.367~0.522
3rd	0.662~0.712	0.621~0.748	0.615~0.752	0.614~0.752	0.614~0.752	0.614~0.752
4th	Х	Х	0.878~0.927	0.876~0.928	0.875~0.929	0.875~0.929

**Table 3-1** The photonic band gap with several different numbers of the layer. The unit of the data in Table 3-1 is normalized frequency  $(\omega a/2\pi c)$ .

And then, we will take a look at the simulation of other photonic crystal structure

(other ratio of b/a or  $\varepsilon_1/\varepsilon_2$ ):

1.  $\varepsilon_1 / \varepsilon_2 = 11.9/1$  (*Si/Air*) and  $b / a = 0.1 \sim 0.9$ .

ratio	0.1	0.2	0.3	0.4	0.5
	0.071 0.405	0.212 0.459	0 104 0 205	0 169 0 216	0 1 5 9 0 2 6 5
151	0.271~0.495	0.212~0.430	0.104~0.303	0.100~0.310	0.156~0.265
2nd	0.667~0.952	0.646~0.694	0.522~0.640	0.425~0.597	0.367~0.522
3rd		0.883~	0.793~0.916	0.735~0.787	0.614~0.752
4th			396	0.937~	0.875~0.929
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(b)

ratio band gap	0.6	0.7	0.8	0.9	
1st	0.152~0.227	0.148~0.198	0.147~0.175	0.147~0.156	
2nd	0.331~0.453	0.309~0.398	0.296~0.353	0.291~0.317	
3rd	0.535~0.673	0.483~0.595	0.452~0.530	0.437~0.476	
4th	0.752~0.879	0.668~0.788	0.613~0.705	0.584~0.634	
5th	0.972~	0.857~0.977	0.777~0.878	0.731~0.791	
6th			0.942~	0.878~0.947	

**Table 3-2** The list of photonic band gaps of 9 photonic crystals with different b/a. The ratio of b and a is from 0.1 to 0.9, and the range of the ratio is from 0.1 to 0.5 in Table 3-3(a) and from 0.6 to 0.9 in Table 3-3(b). Each ratio is calculated with 40 layers. The unit of all the data in Table 3-1 is normalized frequency  $(\omega a/2\pi c)$ , and the normalized frequency range in this table is between 0 and 1.

2.  $\varepsilon_1 / \varepsilon_2 = 13/1$  (*GaAs/Air*) and  $b/a = 0.1 \sim 0.9$ .

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ratio band gap	0.1	0.2	0.3	0.4	0.5
1st	0.262~0.494	0.203~0.453	0.176~0.375	0.161~0.306	0.151~0.256
2nd	0.657~0.945	0.641~0.67	0.502~0.632	0.408~0.582	0.352~0.504
3rd		0.857~	0.778~0.884	0.705~0.773	0.589~0.731
4th				0.915~	0.838~0.909

(b)

ratio band gap	0.6	0.7	0.8	0.9
1st	0.145~0.219	0.142~0.19	0.14~0.168	0.141~0.149
2nd	0.317~0.436	0.295~0.382	0.283~0.339	0.279~0.303
3rd	0.513~0.648	0.463~0.571	0.432~0.508	0.418~0.456
4th	0.72~0.849	0.639~0.757	0.586~0.675	0.559~0.607
5th	0.932~	0.82~0.939	0.743~0.841	0.699~0.758
6th		J.L.	0.902~	0.841~0.907

**Table 3-3** The list of photonic band gaps of 9 photonic crystals with different b/a. The ratio of b and a is from 0.1 to 0.9, and the range of the ratio is from 0.1 to 0.5 in Table 3-3(a) and from 0.6 to 0.9 in Table 3-3(b). Each ratio is calculated with 40 layers. The unit of all the data in Table 3-1 is normalized frequency  $(\omega a/2\pi c)$ , and the normalized frequency range in this table is between 0 and 1.

## 3-3.2 The Smallest Ratio

We have found the photonic band gap, but we are curious that what is the smallest ratio of  $\varepsilon_1$  and  $\varepsilon_2$ , at that the photonic crystal band gap will exist. In this calculation, air is the fixed material ( $\varepsilon_2 = 1$ ), and we change the value of  $\varepsilon_1$ . The ratio of *b* and *a* is fixed at 0.5, and each of the photonic crystal has 40 layers. Fist, we change the value of  $\varepsilon_1$  from 1 to 11.5 every 0.5 increment, and find that the photonic crystal band gap vanish as the value of  $\varepsilon_1$  is between 1 and 1.5(Table 3-5).

(a)

$\varepsilon_2$ band gap	10.5	9.5	8.5	7.5	6.5
1st	0.168~0.279	0.177~0.291	0.186~0.304	0.198~0.318	0.213~0.335
2nd	0.390~0.548	0.409~0.568	0.431~0.591	0.457~0.616	0.489~0.643
3rd	0.651~0.782	0.683~0.805	0.719~0.829	0.761~0.857	0.813~0.887

(b)

$\varepsilon_2$ band gap	5.5	4.5	3.5	2.5	1.5
1st	0.230~0.355	0.254~0.378	0.286~0.405	0.336~0.437	0.427~0.472
2nd	0.528~0.675	0.578~0.711	0.645~0.754	0.743~0.810	Х
3rd	0.876~0.923	Х	Х	Х	Х

(c)

$\varepsilon_2$ gap range	10.5	9.5	8.5	7.5	6.5	5.5	4.5	3.5	2.5	1.5
1st	0.111	0.114	0.118	0.120	0.122	0.125	0.124	0.119	0.101	0.045
2nd	0.158	0.159	0.160	0.159	0.154	0.147	0.133	0.109	0.058	0.000
3rd	0.131	0.122	0.110	0.096	0.074	0.047	0.000	0.000	0.000	0.000

**Table 3-4** (a) and (b) are the table of photonic band gaps for different  $\varepsilon_2$ .  $\varepsilon_1$  is fixed in this table, and there are 40 layers for every multilayer. (c) is the table of gap range for each  $\varepsilon_2$  in table (a) and (b).

From Table 3-4, we can find that the range of the band gap decreases as the value of  $\varepsilon_2$  decreases, and the band gap vanishes finally. The position of the band gap shifts to higher frequency as the value of  $\varepsilon_2$  decreases. For each band gap, the value of



Fig. 3-7 The range of the band gap decreases as  $\varepsilon_2$  decreases. The abscissa is the value of  $\varepsilon_2$ . The ordinate is the normalized frequency range of the band gap.

 $\varepsilon_2$  at that band gap vanishes is different. In Figure 3-7, the band gap at the higher frequency decay to 0 at the higher  $\varepsilon_2$  (the third band gap decay 0 between  $\varepsilon_2 = 4.5$  and  $\varepsilon_2 = 5.5$ , and the second band gap decay to 0 between  $\varepsilon_2 = 1.5$  and  $\varepsilon_2 = 2.5$ ). Our purpose is to find the smallest  $\varepsilon_2$  that "No" band gaps exist. So, we need to find the value of  $\varepsilon_2$  between 1 and 1.5, at that the first band gap decay to 0. We do the simulation every 0.1 increment from  $\varepsilon_2 = 1$  to  $\varepsilon_2 = 1.5$ , and find that the band gap vanish between 1.3 and 1.4. We continue to do the simulation for every 0.01 increment from 1.31 to 1.39, and find that no photonic band gap exists as  $\varepsilon_2 \leq 1.305$ . We show these results in Table 3-5 and Figure 3-8. (a)

$\mathcal{E}_2$ band gap	1.39	1.38	1.37	1.36	1.35	1.34
1st	0.444~0.474	0.446~0.474	0.448~0.473	0.450~0.473	0.452~0.473	0.454~0.473

(b)

$\mathcal{E}_2$ band gap	1.33	1.32	1.31	1.309	1.308	1.307
1st	0.457~0.472	0.459~0.471	0.463~0.469	0.464~0.469	0.464~0.469	0.465~0.468

(c)

$\mathcal{E}_2$ band gap	1.306	1.305	1.304	1.303	1.302	1.301
1st	0.465~0.468	0	0	0	0	0

**Table 3-5** The range of the band gap of every  $\varepsilon_2$  from 1.390 to 1.301.



**Fig.3-8** The range of the band gap decrease as  $\varepsilon_2$  decreases. The band gap vanishes at  $\varepsilon_2 = 1.306$ . The abscissa is the value of  $\varepsilon_2$ . The ordinate is the normalized frequency range of the band gap.

#### **3-3.3** The Behavior of the Light in the Photonic Crystal

In the previous sections, we discuss the property of the photonic crystal in frequency domain. In this section, we want to discuss the property of the photonic crystal in time domain. We want to know how the light propagates in the photonic crystal. We use the photonic crystal with b/a = 0.3 and  $\varepsilon_1/\varepsilon_2 = 11.9/1$  to be our example.



#### Fig.3-9

The transmission spectrum of the photonic crystal with b/a = 0.3,  $\varepsilon_1/\varepsilon_2 = 11.9/1$ , and 10 layers. The abscissa is the normalized frequency.

Figure 3-9 is the transmission spectrum of the photonic crystal with 5 layers. In Figure 3-9, we can find that there are 3 photonic band gaps for this structure. They are locate between  $0.191\sim0.377$ ,  $0.539\sim0.63$ , and  $0.802\sim0.901$  respectively. In the previous simulation, we use the Gaussian pulse as the incident wave source. Now, we want to switch it to a sinusoidal wave source, and to watch the behavior of the light in the photonic crystal. First, we choose a sinusoidal wave with the normalized frequency 0.3, which is in the photonic crystal band gap. In Figure 3-10, we find that the wave decay to 0.01 about the 6<sup>th</sup> layer and 0.1 about the 3<sup>rd</sup> layer. It's obvious that this wave can't pass the photonic crystal. Next, we choose a sinusoidal wave with the normalized frequency 0.5. Figure 3-11 shows the behavior of the wave propagating in



#### **Fig. 3-10**

This is the behavior of a sinusoidal wave with the normalized frequency 0.3 which propagates in a photonic crystal (b/a=0.3,  $\varepsilon_1/\varepsilon_2 = 11.9/1$ , and 10 layers). The abscissa is the space, and this plot is at T=10000 steps.



the photonic crystal. This wave can pass the photonic crystal, and the waveform of the wave propagating in the photonic crystal is symmetry. And then, we show the plots (Figure 3-13 and Figure 3-14) of several sinusoidal waves with different normalized frequency, which are in the stop band or in the transmission band, and the plots of the other ratio of b and a is 0.5. In Figure 3-12 and 3-13, the position that wave decays to 0 changes with different normalized frequency. This property is important for us to use the defect to design a optical device. We will do more discussion in the next section.



**Fig. 3-12** This are the behavior of the light with normalized frequency 0.6, 0.7, 0.82 and 0.95 in the photonic crystal. The normalized frequency 0.6 and 0.82 are in the stop band, and 0.7 and 0.95 are in the transmission band.





Fig. 3-13 These are the behavior of the light with  $\omega = 0.1, 0.2, 0.3, 0.4, 0.55, 0.7, 0.8$ and 0.85 in the photonic crystal (*b/a*=0.5,  $\varepsilon_1 / \varepsilon_2 = 11.9/1$  and 10 layers). The normalized frequency 0.1, 0.3, 0.55 and 0.8 are in the stop band, and 0.2, 0.4, 0.7 and 0.85 are in the transmission band.

## **3-3.4 Defect Mode**

One of the most important properties of photonic crystals is the emergence of localized defect mode in the gap frequency region when a disorder is introduced to their periodic dielectric structure. In this section, we will present our simulation of the defect mode of the multilayer.

Figure 3-14 shows an example of a defect in the case of 5 layers, where as the defect layer ratio b/a in the 4<sup>th</sup> layer is different from the normal layer. From Figure 3-5, the photonic band gaps locate between 0.158~0.265, 0.367~ 0.522, 0.614~0.752 and 0.875~0.929. If we insert a defect layer, the transmission coefficient of some frequency, which is in the range of photonic band gap, should not be small than 0.01, by which we define the band gap. Therefore, we do the simulation of the structure illustrated in Figure 3-14. First, we insert the defect layer in the 4<sup>th</sup> layer.



Fig. 3-14

This is the illustration of the photonic crystal with a defect layer. The 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> and 5<sup>th</sup> layer are the normal layer with b/a = 0.5, and the 4<sup>th</sup> layer is the defect layer with b/a = 0.3

sert the defect layer in the 3<sup>rd</sup> layer, and insert the defect layer in the 2<sup>nd</sup> layer finally. We should note that there is only one defect layer be inserted into the photonic. We don't insert the defect layer into the photonic, since they are not the defect layer for the photonic crystal. We will show the simulation results to verify our consideration. Figure 3-15 shows the simulation results of the defect mode. Compare Figure 3-5 with Figure 3-15, and we can find that there is almost no defect mode when the defect layer is in the 1<sup>st</sup> and 5<sup>th</sup> layer and the defect mode is very easy to be identified out when the defect mode is in the 2<sup>nd</sup> and 3<sup>rd</sup> layer. And, in Figure 3-15(b), the defect



**Fig. 3-15** The transmittance of the defect mode: (a) The  $2^{nd}$  layer is the defect layer with b/a = 0.3, and others are the normal layer with b/a = 0.5. (b) The  $3^{rd}$  layer is the defect layer with b/a = 0.3, and others are the normal layer with b/a = 0.3, and others are the normal layer with b/a = 0.3, and others are the normal layer with b/a = 0.3, and others are the normal layer with b/a = 0.3, and others are the normal layer with b/a = 0.3, and others are the normal layer with b/a = 0.3, and others are the normal layer with b/a = 0.3, and others are the normal layer with b/a = 0.3, and others are the normal layer by b/a = 0.3.

mode locates around 0.459 is very narrow and sharp, and the transmission coefficient of the center frequency attain to 1. Those characters are very suitable to design a "band pass filter". Before we discuss the possibility of designing a filter with the defect multilayer, we need to discuss the behavior of the wave in the defect multilayer.

In the discussion below, we use the wave with normalized frequency 0.459 for example. From the discussion in the previous stage, we know that the wave, whose frequency is in the photonic band gap, will "inject" into the multilayer for a little layers (we can call them "skin layers"). The number of the skin layers is different for every frequency in the photonic band gap. As we know, the existence of the photonic band gap is resulted from the destructive diffraction caused by the perfect periodic structure. Therefore, the multilayer is not a perfect periodic structure for some frequencies, when we introduce a disorder to the periodic structure. If the defect layer locates among the skin layers, some inner layer disturbs wave propagating more than the layers mear the boundary. The wave could pass the skin layer. Figure 3-16 shows the comparison of the behaviors of the waves in perfect and defect multilayer. From



Fig. 3-16 The frequency of the incident waves in (a) and (b) are both 0.459. (a) This is the behavior of the wave in the perfect multilayer with b/a = 0.5 for each layer.(b) This is the behavior of the wave in the defect multilayer with b/a = 0.3 for the defect layer and b/a = 0.5 for the normal layer.

Figure 3-16(a), the wave decays to zero around the  $3^{rd}$  or  $4^{th}$  layer. We insert the defect layer in the  $3^{rd}$  layer, and the wave, which has the same frequency with the incident wave in Figure 3-16(a), decays as it propagates through the  $1^{st}$  and  $2^{nd}$  layer, but "grows up" in the  $3^{rd}$  layer. So, we can watch the transmitting wave in the back side of the defect multilayer. However, we only talk about the multilayer with 5 layers up to now, and the bottom of the picks is not "clear". If we want to use the defect

multilayer to be a "band pass filter", the peak should be very sharp and we need the bandwidth smaller.

What can and what should we do to modify the result? First, we need to find the simplest structure that has the defect mode, and the defect mode should be suitable for the design of band pass filter. Take a look at Figure 3-16 (a) and (b). In the two figures, there are both two peaks in the band gaps separately. Therefore, we need to compare the difference between these two defect modes and between the two structures. The difference of the structures between Figure 3-16 (a) and (b) is the position of the defect layer. In Figure 3-16 (a), the defect layer is at the 2<sup>nd</sup> one, and, in Figure 3-16 (b), the defect layer is at the 3<sup>rd</sup> one. In order to let the wave, in the defect mode, passes the multilayer, we need to repeat the defect layer for every fixed number of layers.



**Fig. 3-17** (a) The defect layer is at the 8<sup>th</sup> layer. (b) The defect layer is at the 3<sup>rd</sup> layer. There are both 10 layers totally in the (a) and (b). This figure illustrates that we should insert more than one defect layer into the multilayer. Both of the normalized frequency in (a) and (b) is 0.459.

For example, we consider the multilayer with 10 layers totally and defect layer. If we only insert one defect layer, no matter at the head or the tail of the structure, there may be no defect mode. Since the wave will decay to zero in the region of three or four perfect layers, we need to insert a defect layer for every two or three layers at least. Figure 3-17 verify our consideration. In Figure 3-17 (a), we can find that the wave decays to zero as it doesn't arrive the defect layer yet. In Figure 3-17(b), the wave decays to zero around the 7<sup>th</sup> layer even though it grows up again around the defect layer. From the discussion above, we want to define the "sub-lattice" which is the simplest structure with the defect mode, and just need to repeat the sub-lattice to construct the multilayer. Since the wave will decay as it passes through the one or two perfect layer and "grows up" at the perfect layer, we have two kinds of the sub-lattice. One is that the defect layer begin at the 2nd layer, another is that the defect layer begin at the 3<sup>rd</sup> layer. And, for both of these two choices, we define three kinds structure of the sub-lattice. Figure 3-18 illustrates our considerations above. Then, we



**Fig. 3-18** (a), (b) and (c) are the sub-lattices with the defect layer at the 2<sup>nd</sup> one. (d), (e) and (f) are the sub-lattices with the defect layer at the 3<sup>rd</sup> one. We can repeat the sub-lattice to construct the multilayer.

do the simulations of these six sub-lattices. Figure 3-19 and 3-20 illustrate our simulation results. In Figure 3-19, we use five sub-lattices for every kind of sub-lattice, and six sub-lattices for every kind of sub-lattice in Figure 3-20. The reason why we do two kinds of simulation is that, for each kind of sub-lattice, the transmission spectrums are different for a multilayer with odd or even sub-lattices. If we want to find the suitable structure to fabricate a device, all possibilities should be tried.



**Fig. 3-19** (a), (b) and (c) are the transmission spectrums of the defect modes, and the defect layers begin at the 2<sup>nd</sup> layer for these three sub-lattices. (a), (b) and (c) are the transmission spectrums of the defect modes, and the defect layers begin at the 3<sup>rd</sup> layer for these three sub-lattices. For these six calculations, there are five sub-lattices in the multilayer.

In Figure 3-19, the most suitable structure to design a band pass filter, whose center frequency is at  $0.706(2\pi c/a)$ , should be constructed by the sub-lattice illustrated in Figure 3-19(c), and the most suitable sub-lattice for the band pass filter,



**Fig. 3-20** (a), (b) and (c) are the transmission spectrums of the defect modes, and the defect layers begin at the 2<sup>nd</sup> layer for these three sub-lattices. (a), (b) and (c) are the transmission spectrums of the defect modes, and the defect layers begin at the 3<sup>rd</sup> layer for these three sub-lattices. For these six calculations, there are six sub-lattices in the multilayer.

whose center frequency is at  $0.459(2\pi c/a)$ , is illustrated in Figure 3-18(f). In Figure 3-20, there are four kinds of sub-lattice, illustrated in Figure 3-19 (c), (d), (e) and (f), suitable to construct the multilayer for a band pass filter, whose center frequency is at

 $0.459(2\pi c/a)$ , and the sub-lattice, illustrated in Figure 3-18(c), is suitable for the band pass filter with the  $0.706(2\pi c/a)$  center frequency. In the calculation above, we divide the normalized frequency into 1000 parts, and the normalized frequency interval is 0.001. Now, we want to get the more precise data, so, in the next calculation, the normalized frequency region are from 0.450 to 0.470 and 0.690 to 0.710 and we divide these region into 2000 parts, the normalized frequency interval is





These are the transmission spectrums of the defect mode around 0.459. (1), (2), (3) and (4) are the multilayer with 6 layers, and (5) with 5 layers. The abscissa is normalized frequency, and the ordinate is transmission coefficient.

Figure 3-21 and 3-22 show the simulation results. In Figure 3-21, we can find that (4) and (5) seem more suitable for a band pass filter. The center frequency of Figure 3-21(4) and (5) are both  $0.45904(2\pi c/a)$ , and if we want to design a filter with  $1.55 \,\mu m$  wavelength, the bandwidth of Figure 3-21(4) is  $3.29 \times 10^{11} Hz$  and 3-21(5) is  $3.20 \times 10^{11} Hz$ . In Figure 3-22, the center frequency of (1) is 0.705135



Fig. 3-22

These are the transmission spectrums of the defect mode around 0.706. (1) is the multilayer with 5 sub-lattices and (2) with 6 sub-lattices. The abscissa is normalized frequency, and the ordinate is transmission coefficient.

and (2) is 0.705015. Similarly, for a filter with  $1.55 \,\mu m$  wavelength, the bandwidth of Figure 3-22(1) is  $8.20 \times 10^{11}$  Hz, and (2) is  $9.03 \times 10^{11}$  Hz, and the performance of Figure 3-22(1) is better than (2). Compare the results in Figure 3-21 with 3-20. The bandwidths of 3-21 are smaller than 3-22 in substance, and the size of each layer in structure of 3-22 is bigger than 3-21. Table 3-7 shows the parameters that we get from this calculation for a  $1.55 \,\mu m$  filter.

$\searrow$	center wavelength	bandwidth	perfect layer	defect layer
	$(\mu m)$	(Hz)	size (0.5a)	size (0.3a)
Figure 3-20(4)	1.55	3.29×10 <sup>11</sup>	2.235	1.341
Figure 3-20(5)	1.55	$3.20 \times 10^{11}$	2.235	1.341
Figure 3-21(1)	1.55	8.20×10 <sup>11</sup>	3.435	2.061
Figure 3-21(2)	1.55	9.03×10 <sup>11</sup>	3.435	2.061

**Table 3-7** The parameters of a band pass filter with the sub-lattice illustrated in Figure3-19 and 3-20.