Chapter 3 Fundamentals of Mixer

3.1 Fundamentals and design of mixers

There are many different mixer circuit topologies and implementations that are suitable for use in receiver and transmitter systems. We must select the best one depend on our application, system planning and available process technology. For example, the superheterodyne receiver architecture has several frequency downconversion stages to optimize its performance such as image rejection, noise, gain and dynamic range. The mixer in above application must be designed to handle a very wide dynamic range of signal powers at input.

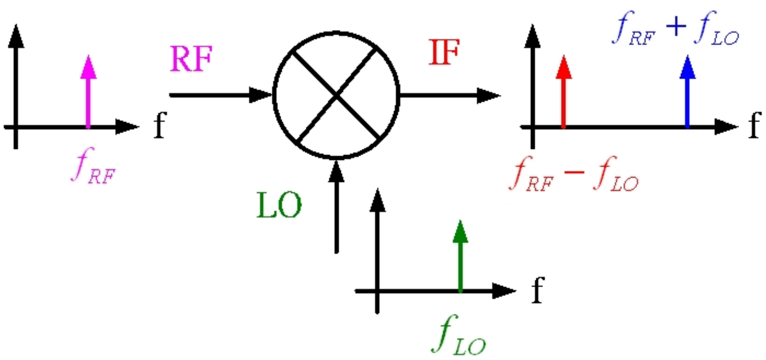

Fig. 3-1 Notation and operation of a mixer.

The mixer is a frequency translation device. The operating function and notation of the mixer can be shown as Fig. 3-1. It performs frequency translations by multiplication of a RF signal with a LO signal.

$$\left(A\cos\omega_1 t\right)\bullet\left(B\cos\omega_2 t\right)=\frac{AB}{2}\left[\cos\left(\omega_1-\omega_2\right)t-\cos\left(\omega_1+\omega_2\right)t\right] \tag{3-1}$$

The relationship (3-1) shows both downconversion and upconversion are obtained simultaneously. So we must remove the undesired term by filtering or other operations.

■ Mixer topologies

Based on the mixer operating mechanism, the mixer can be divided into two topologies [14] [15] [16]:

1. Use a nonlinear device that has a well-known properties and controlled characteristics.
2. Switch the RF signal path on and off at the LO frequency.

The nonlinear mixer is applicable at any frequency where the device presents a known nonlinearity. While the frequencies are low enough and good switches can be built, the switched-type mixer is preferred because it generates fewer spurious.

■ Nonlinear device mixers

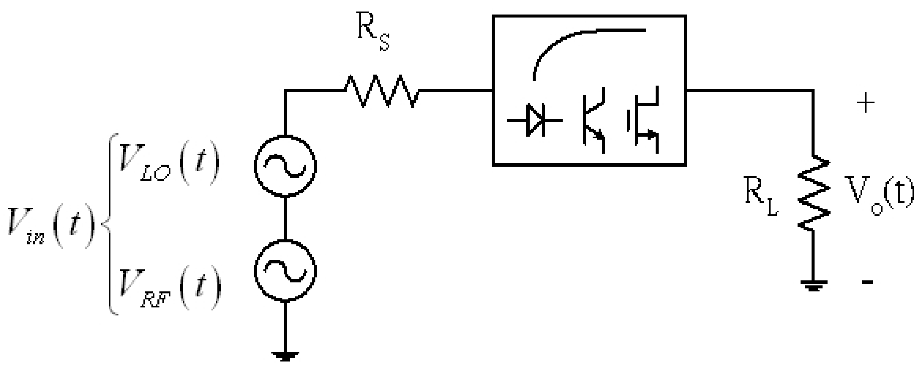

Fig. 3-2 Nonlinear mixer.

$$V_o(t) = a_0 + a_1 V_{in}(t) + a_2 V_{in}^2(t) + a_3 V_{in}^3(t) + \ldots\ldots$$

$$= a_0 + a_1\left[V_{RF}(t) + V_{LO}(t)\right] + a_2\left[V_{RF}(t) + V_{LO}(t)\right]^2 + a_3\left[V_{RF}(t) + V_{LO}(t)\right]^3 + \ldots\ldots \quad (3\text{-}2)$$

We can use a diode or bipolar transistor as the mixer core shown as Fig. 3-2. From the nonlinear transfer function Eq. (3-2), we see the output signal contain a DC term, RF and LO feedthrough and all harmonic terms of the RF and LO frequencies. Only the second-order product term produces the desired output. The spurious output signal strength can be degraded when we use devices that are primarily square-law transfer property such as MOSFET with long channel length. (Eq. 3-3)

$$V_O(t) = g_m R_L = \mu C_{ox}\frac{W}{L}\left(V_{gs} - V_{th}\right)^2 R_L = \mu C_{ox}\frac{W}{L}R_L\left(V_{gs}{}^2 + V_{th}{}^2 + 2V_{gs}V_{th}\right) \quad (3\text{-}3)$$

- Switching type mixers

$$T(t) = square\ wave = V_{LO}(t)$$
$$= \frac{1}{2} + \frac{2}{\pi}\left[\sin(\omega_{LO}t) + \frac{\sin(3\omega_{LO}t)}{3} + \frac{\sin(5\omega_{LO}t)}{5} + \ldots\ldots\right]$$

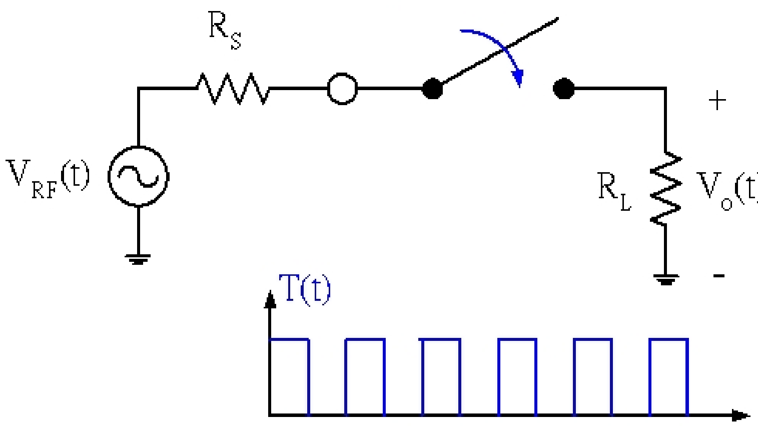

Fig. 3-3 Switching mixer.

$$
\begin{aligned}
V_o(t) &= V_{RF}(t)\bullet V_{LO}(t) \\
&= v_{RF}\cos(\omega_{RF}t)\bullet\left\{\frac{1}{2}+\frac{2}{\pi}\left[\sin(\omega_{LO}t)+\frac{\sin(3\omega_{LO}t)}{3}+\frac{\sin(5\omega_{LO}t)}{5}+\ldots\ldots\right]\right\} \\
&= \frac{v_{RF}}{2}\cos(\omega_{RF}t)+\frac{2v_{RF}}{\pi}\left[\cos(\omega_{RF}t)\bullet\sin(\omega_{LO}t)+\frac{\cos(\omega_{RF}t)\bullet\sin(3\omega_{LO}t)}{3}+\ldots\ldots\right]
\end{aligned}
\tag{3-4}
$$

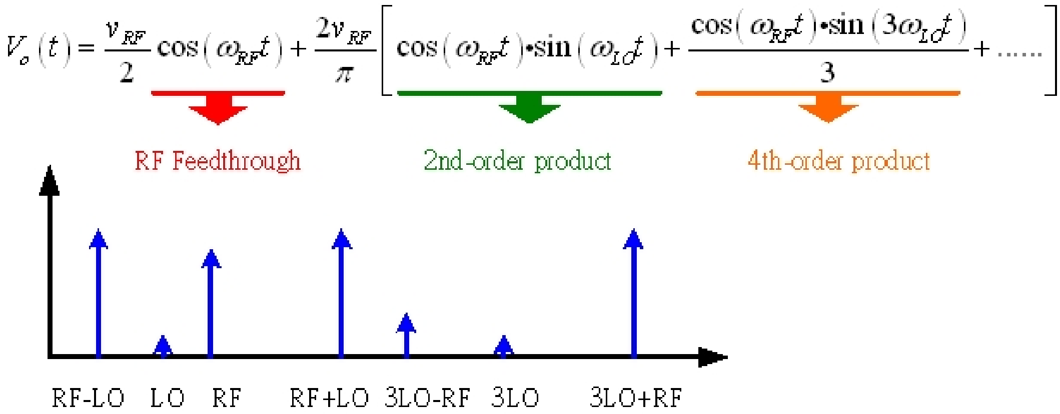

Fig. 3-4 Mixer outputs.

Instead of using nonlinear devices as the frequency translation core, we also can insert a well-controlled switch to realize a mixer which is shown as Fig. 3-3. Assuming a switch with square wave like transfer function with 50% duty cycle are used, then from Eq. (3-4), we see the output signals contain no dc term, but contain RF feedthrough, our desired second-order product term ($\omega_{RF}-\omega_{LO}$ and $\omega_{RF}+\omega_{LO}$) and harmonic terms of the RF and LO frequencies ($n\omega_{LO}-\omega_{RF}$ and $n\omega_{LO}+\omega_{RF}$, where n is odd) which are shown as Fig. 3-4. No even order harmonics is an important property of this kind mixer due to symmetric switching function T(t).

As mentioned before, we prefer the switching type mixer when the RF and LO frequencies are low enough that we can realize proper or good switches.

■ Design considerations of mixer

The downconversion mixer usually has the following design specifications:

(a) Noise figure

(b) IIP3 and OIP3

(c) 1dB gain compression

(d) Image rejection

(e) Conversion loss or gain

(f) Port to port Isolation

(g) Local oscillator power

(h) Power consumption

The properties of noise figure, IIP3, OIP3, 1dB compression point and image rejection are discussed in Chapter 2. But the noise figure in the mixer somewhat is larger than that in the LNA discussed in Chapter 2, we will illustrate why this occur. We illustrate above properties in following pages.

■ Noise figure

We have given the definition of noise figure to be the ratio between total noise output powers over the noise output power due to source only as Eq. (2-3) in chapter 2. In the mixer section, the noise figure were worst than that in LNA since it translate more than two frequencies to IF frequency as shown in Fig. 3-5. The noises come from the desired RF, image and all harmonics will be translated to the IF band. Then they are added to form larger noise floor and degrade the signal to noise ratio at the output port. Hence, a low noise amplifier preceding the mixer is needed since noise performance of an overall receiver is dominated by the first stage, illustrated by Eq. (2-4).

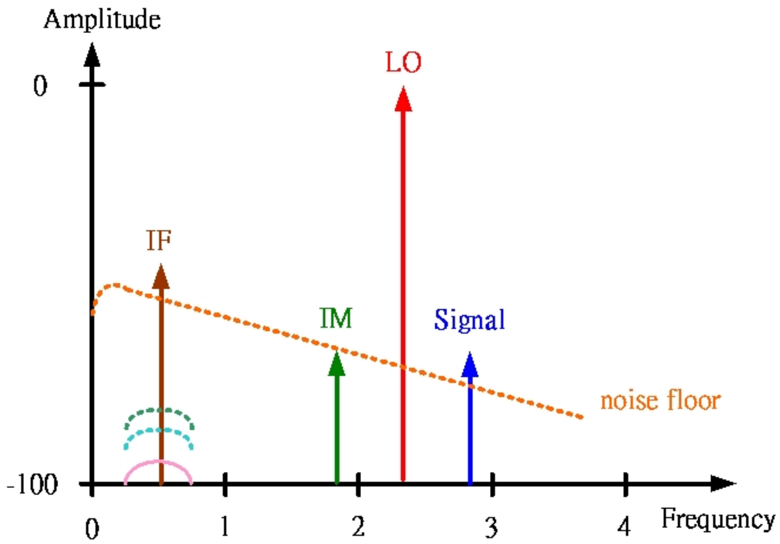

Fig. 3-5 The noises translate to the IF band from RF and IM frequencies. [14]

■ Conversion loss or gain

Conversion loss or gain is usually defined as the ratio of IF output power to the RF input power, or expressed as the ratio of IF output voltage to the RF input voltage that is frequently used in RFIC implementations.

$$\begin{cases} \textit{Conversion Gain (power)} = \dfrac{\textit{Output power at IF}}{\textit{Input power at RF}} = \dfrac{P_{IF}}{P_{IN}} \\ \textit{Conversion Gain (voltage)} = \dfrac{V_{IF}}{V_{IN}} \end{cases} \tag{3-5}$$

For example, we can calculate the conversion gain of simple switching mixer from Fig. 3-4 and Eq. (3-4). Then

$$A_V = \frac{2}{\pi}\cos(\omega_{RF}t)\sin(\omega_{LO}t) = \frac{1}{\pi}\left\{\sin\left[(\omega_{RF}-\omega_{LO})t\right] + \sin\left[(\omega_{RF}+\omega_{LO})t\right]\right\}$$

$$\textit{Conversion Gain }(Power) = \frac{1}{\pi^2} \approx 0.1 \tag{3-6}$$

We can see that the value is small than 1, denoted in loss better than gain.

■ Port to port isolation

Port to port isolation is an important parameter in mixer design. It is desirable to minimize interaction between RF, LO and IF ports. In other words, we must improve isolations among the three posts as possible as we can. Because the LO signal power is generally very large compared with RF signal power, any LO feedthrough to the IF port may cause problems at subsequent stages.

In Low-IF and Zero-IF transceiver architecture the LO to RF reverse isolation is more important than others because RF and LO are located at much higher frequencies than IF, hence this feedthrough may back to the antenna cause interference to other receiver or remix with RF input signal.

Adopting balanced-type mixers such as Gilbert cell mixer can improve the isolation or feedthrough issue, although layout mismatch and package parasitics may degrade isolation.

■ Local oscillator power

The local oscillator power has a crucial effect on the gain and noise figure performance. We must select the proper LO power to preserve good gain and noise performance.

■ Power consumption

In general, larger channel width of MOSFET results in lower noise figure, higher current density and higher power consumption. Low power dissipation with proper low noise performance is preferred since the portable wireless communication systems are usually battery-dependent. We must do trade-off between them.

Following pages, we introduce two popular switching type mixers that using MOSFET to realize the circuits. They are “single-balanced type mixer” and “double-balanced type mixer”, respectively [1] [2] [14] [15] [16].

■ Single balanced mixers

The RF feedthrough problem in simple switching mixer which is shown in Fig. 3-4 can be eliminated by using a differential IF output and a polarity reversing LO switch called as “Single-balanced mixer” shown as Fig. 3-5. It consists of three main stages, the transconductance stage, switching stage and gain stage. The transconductance stage converts the incoming RF voltage into a current with high linearity. The switching stage performs multiplication to translate the RF current into IF current. The gain stage converts the IF current signal into voltage signal linearly by resistive load.

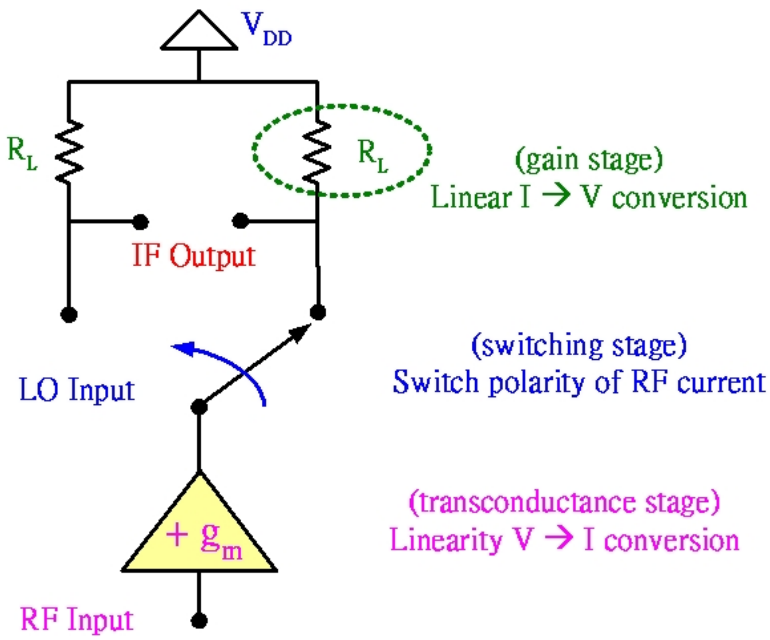

Fig. 3-6 Single balanced mixer.

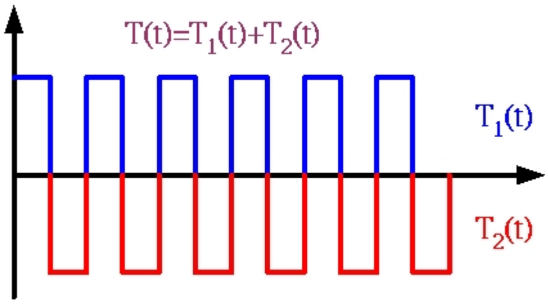

Fig. 3-7 Switching function.

The LO switching function is shown in Fig. 3-7, can be expressed as follow by Fourier series expansion.

$$T_1(t)=\left(+\frac{1}{2}\right)+\frac{2}{\pi}\left[\sin(\omega_{LO}t)+\frac{1}{3}\sin(3\omega_{LO}t)+......\right] \tag{3-7}$$

$$T_2(t)=\left(-\frac{1}{2}\right)+\frac{2}{\pi}\left[\sin(\omega_{LO}t)+\frac{1}{3}\sin(3\omega_{LO}t)+......\right] \tag{3-8}$$

$$T(t)=T_1(t)+T_2(t)=\frac{4}{\pi}\left[\sin(\omega_{LO}t)+\frac{1}{3}\sin(3\omega_{LO}t)+......\right] \tag{3-9}$$

Hence,

$$V_{IF}(t)=g_m R_L v_{RF}\cos(\omega_{RF}t)\frac{4}{\pi}\left[\sin(\omega_{LO}t)+\frac{1}{3}\sin(3\omega_{LO}t)+......\right] \tag{3-10}$$

The second order term:

$$V_{IF}(t)=\frac{2g_m R_L v_{RF}}{\pi}\left[\sin(\omega_{RF}+\omega_{LO})t+\sin(\omega_{RF}-\omega_{LO})t\right] \tag{3-11}$$

Add dc bias condition into Eq. (3-10), then

$$V_{IF}(t)=R_L\left[I_{DC}+g_m v_{RF}\cos(\omega_{RF}t)\right]\bullet\frac{4}{\pi}\left[\sin(\omega_{LO}t)+\frac{1}{3}\sin(3\omega_{LO}t)+......\right]$$

$$=\frac{4R_L}{\pi}\left\{I_{DC}\sin(\omega_{LO}t)+\frac{1}{2}g_m v_{RF}\left[\sin(\omega_{RF}+\omega_{LO})t+\sin(\omega_{RF}-\omega_{LO})t\right]+......\right\}$$

(3-12)

$$Conversion\ gain\ (power)=\left(\frac{2g_m R_L}{\pi}\right)^2=\left(\frac{2}{\pi}\right)^2\quad \text{(assuming } g_m R_L=1) \tag{3-13}$$

Here, comparing with above equation and Eq. (3-6), we see that the conversion gain of a single-balanced mixer is grater than that of a simple switching mixer.

■ Double balanced mixers

See Eq. (3-12), we still get LO feedthrough since the first term can not be eliminated by using single-balanced mixer. To solve this problem, we can combine two single-balanced mixers together to form a double-balance mixer as shown in Fig. 3-8, and the mathematical operation can be seen by Eq. (3-14). Double-balanced mixer removes the LO feedthrough since the DC term cancels.

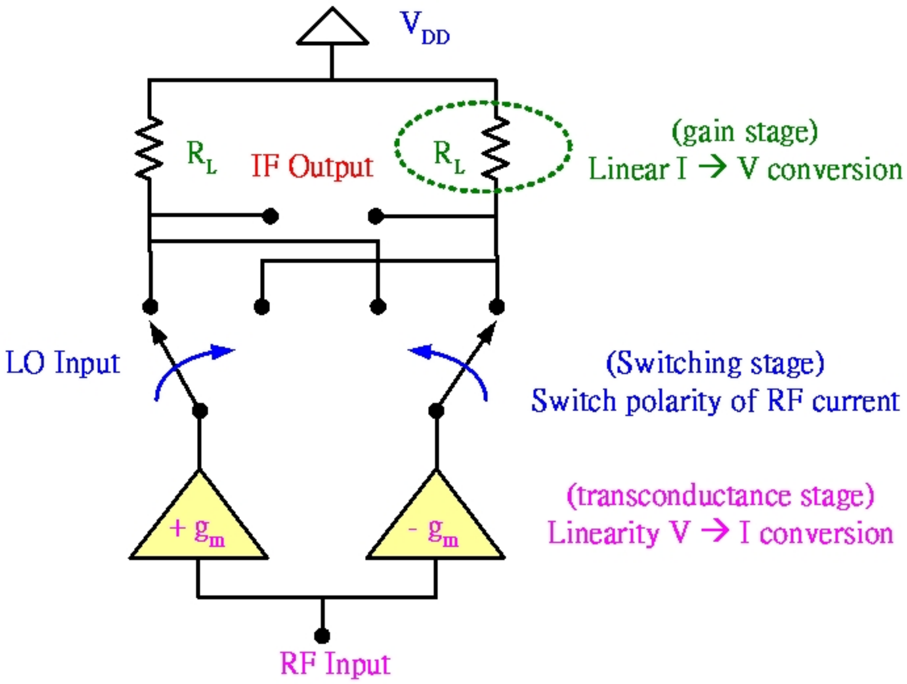

Fig. 3-8 Double balanced mixer.

$$\begin{aligned} V_{IF}(t) &= R_L\left[I_{DC} + g_m v_{RF}\cos(\omega_{RF}t)\right]\bullet\frac{4}{\pi}\left[\sin(\omega_{LO}t)+\frac{1}{3}\sin(3\omega_{LO}t)+......\right] \\ &\quad -R_L\left[I_{DC} - g_m v_{RF}\cos(\omega_{RF}t)\right]\bullet\frac{4}{\pi}\left[\sin(\omega_{LO}t)+\frac{1}{3}\sin(3\omega_{LO}t)+......\right] \\ &= \frac{8}{\pi}R_L g_m v_{RF}\cos(\omega_{RF}t)\left[\sin(\omega_{LO}t)+\frac{1}{3}\sin(3\omega_{LO}t)+......\right] \end{aligned} \tag{3-14}$$

In this thesis, we will implement a double-balance mixer named Gilbert mixer stacked on the top of LNA. Hence, we illustrate the detailed operations, design considerations and design steps of a Gilbert mixer here.

3.2 Fundamentals and design of Gilbert mixer

The Gilbert double-balanced mixer is a compact, efficient approach to combining a differential amplifier with a phase-reversing switch mixer. It is widely used in RFIC applications because of its compact layout and moderately high performance. Many design parameters of a CMOS Gilbert mixer should be concerned such as the linearity of the signal path, the power consumption, the noise, the conversion gain, the 1-dB compression point and therefore the dynamic range of the mixer. We will explore design tradeoffs that include biasing and device sizing, LO power, conversion gain, gain compression, intermodulation distortion and noise.

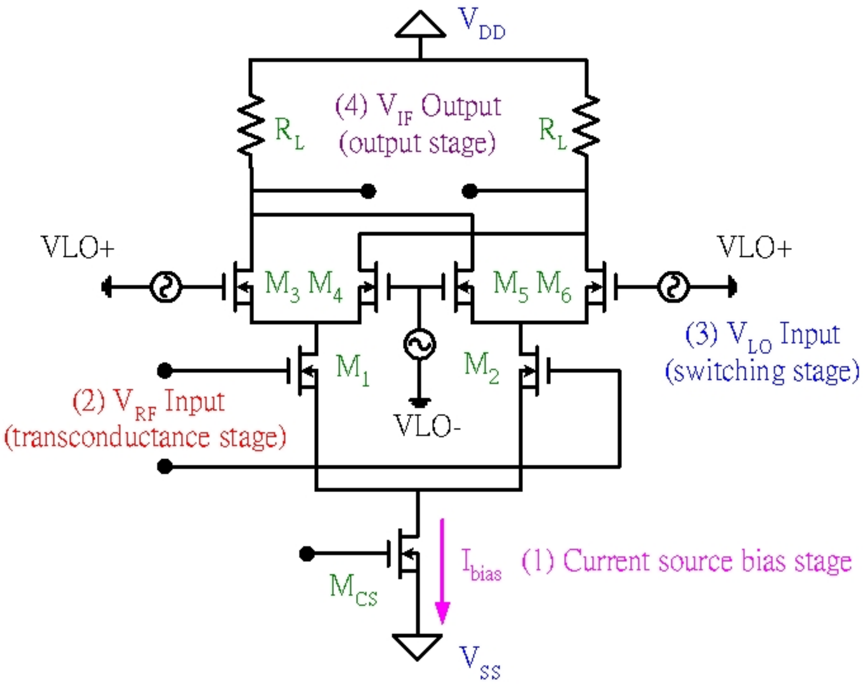

Fig. 3-9 Gilbert mixer implementation by MOSFET.

The circuit implementation of a Gilbert mixer by MOSFET is shown in Fig. 3-9. Here, M_{CS} provides a current source to bias this circuit, M_{1-2} are used as the differential amplifier input and provides a transconductance to transform voltage signal into current signal. M_{3-6} are implemented to be good switchs and translate high frequency signal into IF band. The detail operations is given in the following.

the drain current of M_1 and M_2 are $\begin{cases} i_{D1} = k_1\left(v_{GS1} - v_{th}\right)^2 \\ i_{D2} = k_2\left(v_{GS2} - v_{th}\right)^2 \end{cases}$

then $\sqrt{i_{Dn}} = \sqrt{k_n}\left(v_{GSn} - v_{th}\right)^2$

the total current is $I_{bias} = i_{D1} + i_{D2}$

where $k_n = \frac{1}{2}\mu_n C_{ox}\frac{W_n}{L_n}$ and $v_{GSn} = V_{GSn}\left(DC\right) + v_{gsn}\left(ac\right)$

let $v_{RF} = v_{GS1} - v_{GS2} = v_{gs1} - v_{gs2}$ (because $V_{GS1} = V_{GS2}$), it represents the gate voltage difference between M_1 and M_2. After some manufacture, we can get following expression:

$$\Rightarrow \begin{cases} i_{D1} = \dfrac{I_s}{2} + \sqrt{2k_1 I_s}\left(\dfrac{v_{RF}}{2}\right)\sqrt{1 - \dfrac{v_{RF}^{\;2}}{2\dfrac{I_s}{k}}} \\ i_{D2} = \dfrac{I_s}{2} - \sqrt{2k_1 I_s}\left(\dfrac{v_{RF}}{2}\right)\sqrt{1 - \dfrac{v_{RF}^{\;2}}{2\dfrac{I_s}{k}}} \end{cases} \text{,here } \left(\frac{v_{RF}^{\;2}}{2I_s/k} \le 1\right) \Rightarrow v_{RF} \le \sqrt{2\frac{I_s}{k}}$$

take some approximation $\Rightarrow \begin{cases} i_{D1} \simeq \dfrac{I_s}{2} + \left(\dfrac{I_s}{V_{GS1} - v_{th}}\right)\left(\dfrac{v_{RF}}{2}\right) \\ i_{D2} \simeq \dfrac{I_s}{2} - \left(\dfrac{I_s}{V_{GS2} - v_{th}}\right)\left(\dfrac{v_{RF}}{2}\right) \end{cases}$

set $g_m = \dfrac{I_s}{V_{GSn} - v_{th}} = k_n\left(V_{GSn} - v_{th}\right) \Rightarrow \begin{cases} i_{D1} = \dfrac{I_s}{2} + g_m\left(\dfrac{v_{RF}}{2}\right) \\ i_{D2} = \dfrac{I_s}{2} - g_m\left(\dfrac{v_{RF}}{2}\right) \end{cases}$

Assume the output impedance of IF port is R_L, the input signal $v_{RF} = \sin(\omega_{RF} t)$, and $M_3 - M_6$ are ideal switching operations.

$$\begin{cases} i_{d3} = i_{d5} = (+g_m)\dfrac{v_{RF}}{4}\left(\dfrac{4}{\pi}\right)\left(\sin(\omega_{LO}t) + \dfrac{\sin(3\omega_{LO}t)}{3} + \dfrac{\sin(5\omega_{LO}t)}{5} + \ldots\ldots\right) \\ i_{d4} = i_{d6} = (-g_m)\dfrac{v_{RF}}{4}\left(\dfrac{4}{\pi}\right)\left(\sin(\omega_{LO}t) + \dfrac{\sin(3\omega_{LO}t)}{3} + \dfrac{\sin(5\omega_{LO}t)}{5} + \ldots\ldots\right) \end{cases}$$

the output current of IF port is $i_{IF} = (i_{D3} + i_{D5}) - (i_{D4} + i_{D6})$

$$\Rightarrow v_{IF} = i_{IF} \bullet R_L = g_m R_L v_{RF}\left(\frac{4}{\pi}\right)\left(\sin(\omega_{LO}t) + \frac{\sin(3\omega_{LO}t)}{3} + \frac{\sin(5\omega_{LO}t)}{5} + \ldots\ldots\right)$$

$$= g_m R_L \sin(\omega_{RF}t)\left(\frac{4}{\pi}\right)\left(\sin(\omega_{LO}t) + \frac{\sin(3\omega_{LO}t)}{3} + \frac{\sin(5\omega_{LO}t)}{5} + \ldots\ldots\right)$$

consider the 2nd order term

$$\Rightarrow v_{IF} = \left(\frac{4}{\pi}\right) g_m R_L \sin(\omega_{RF}t)\sin(\omega_{LO}t)$$

$$= \left(\frac{2}{\pi}\right) g_m R_L \left[\cos(\omega_{RF} - \omega_{LO})t - \cos(\omega_{RF} + \omega_{LO})t\right]$$

$$= \left(\frac{2}{\pi}\right) g_m R_L \left[\cos\omega_{IF}t - \cos(\omega_{RF} + \omega_{LO})t\right]$$

The conversion gain of a Gilbert double-balanced mixer is $\left(\dfrac{2}{\pi}\right) g_m R_L$ (3-15)

In power form, the conversion power gain is $\left(\dfrac{2}{\pi}\right)^2 g_m^2 R_L^2$. (3-16)

■ Comparisons of mixers

Up to now, we have illustrated three kinds of mixers in above. We give some comparisons between them in Table 3-1. We must choose the best one depended on our application.

Comparisons of Mixers			
Mixer Type	LO-Balanced	RF-Balanced	Double-Balanced
RF/IF isolation	Good	Poor	Good
LO/IF isolation	Poor	Good	Good
LO/RF isolation	Good	Poor	Good
LO harmonics rejection	Even	All	All
RF harmonics rejection	No	Yes	Yes
Two-tone 2nd order products rejection	No	Yes	Yes

Table 3-1 Comparisons of mixers

■ Design steps of Gilbert cell mixer

A useful design steps are presented here:

(1) Specify the performance needed such as conversion gain, noise figure, linearity, isolation and power consumption.

(2) Construct circuit architecture as Fig. 3-9. Hence, determine the size of M_{CS} and M_{1-6} with noise and power consumption considerations. Here, we determine these sizes based on the method presented in [2] [13]. Then set bias condition to coincide with our assuming power dissipation condition.

(3) For maximum power transfer to the next stage, complex conjugate matching condition at the input stage is needed. So, achieving input matching is the goal here. To improve linearity and matching purpose, we may use source degeneration as shown in Fig. 3-10.

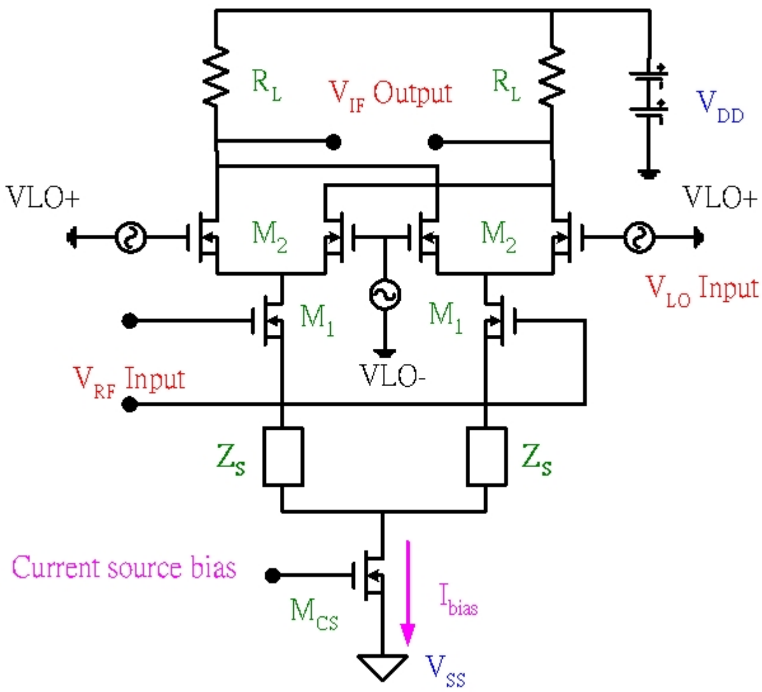

Fig. 3-10 Gilbert cell mixer with resistive source degeneration.

Similar to Chapter 2, the input impedance can be expressed as.

$$z_{in}(j\omega) = \frac{1}{j\omega C_{gs}} + Z_s + \frac{\omega_T}{j\omega} Z_s \tag{3-15}$$

Eq. (3-15) is derived from a simple small signal analysis by neglecting C_{gd}. Different components will produce different impedances, it can be shown below:

Z_s	$\mathrm{Re}[Z_{in}] + \mathrm{Im}[Z_{in}]$
R	$R + \left(\frac{\omega_T R}{j\omega} + \frac{1}{j\omega C_{gs}} \right)$
L	$\omega_T L + \left(\frac{1}{j\omega C_{gs}} + j\omega L \right)$
C	$-\frac{\omega_T}{\omega^2 C} + \left(\frac{1}{j\omega C_{gs}} + \frac{1}{j\omega C} \right)$

Table 3-2 Input impedance of different components. [15]

(4) After choose the device size and achieve input matching, we need to decide the LO power, load and many design factors. Don't forget to check circuit stability, if not stable, do some compensation.

(5) According our specifications such as gain, power consumption, noise figure and isolation, do trade-offs between them to get the optimum condition.

(6) Complete circuit.