Abstract

W. Specht proved that for any matrix A the traces of words in the multiplicative semigroup generated by A and A^* form a complete set of unitary invariants for A. In this paper, we treat some special matrices and try to reduce the number of words that are needed to form a complete set of unitary invariants for them. For an n-by-n matrix A we prove that

$$\{\mathrm{tr}(A), \mathrm{tr}(A^*A)\}, \text{ for } A \text{ with rank } 1$$

and

$$\{\mathrm{tr}(A), \mathrm{tr}(A^2), \mathrm{tr}(A^*A), tr(A^{*2}A), \mathrm{tr}(A^{*2}A^2), \mathrm{tr}(A^*A)^2, \mathrm{tr}(A^{*2}A^2A^*A)\}$$
$$\text{, for } A \text{ with rank } 2$$

form a complete set of unitary invariants respectively. For an n-by-n matrix A with no pair of eigenvectors orthogonal to each other, we prove that

$$\{\mathrm{tr}(A^{*i}A^jA^{*k}A^l) : 0 \le i, j, k, l < n\} \cup \{\mathrm{tr}(A^n)\}$$

forms a complete set of unitary equivalents for such an A.