Electrical conductivity beyond a linear response in layered superconductors under a magnetic field

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The time-dependent Ginzburg-Landau approach is used to investigate nonlinear response of a strongly type-II superconductor. The dissipation takes a form of the flux flow which is quantitatively studied beyond linear response. Thermal fluctuations, represented by the Langevin white noise, are assumed to be strong enough to melt the Abrikosov vortex lattice created by the magnetic field into a moving vortex liquid and marginalize the effects of the vortex pinning by inhomogeneities. The layered structure of the superconductor is accounted for by means of the Lawrence-Doniach model. The nonlinear interaction term in dynamics is treated within self-consistent Gaussian approximation and we go beyond the often used lowest Landau level approximation to treat arbitrary magnetic fields. The *I-V* curve is calculated for arbitrary temperature and the results are compared to experimental data on high-T_c superconductor $YBa_2Cu_3O_{7-\delta}$.

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I. INTRODUCTION

Electric response of a high-temperature superconductor (HTSC) under magnetic field has been a subject of extensive experimental and theoretical investigation for years. Magnetic field in these layered strongly type-II superconductors create magnetic vortices, which, if not pinned by inhomogeneities, move and let the electric field to penetrate the mixed state. The dynamic properties of fluxons appearing in the bulk of a sample are strongly affected by the combined effect of thermal fluctuations, anisotropy (dimensionality) and the flux pinning.[1] Thermal fluctuations in these materials are far from negligible and, in particular, are responsible for existence of the first-order vortex lattice melting transition separating two thermodynamically distinct phases, the vortex solid and the vortex liquid. Magnetic field and reduced dimensionality due to pronounced layered structure (especially in materials such as $Bi_2Sr_2CaCuO_{8+\delta}$) further enhance the effect of thermal fluctuations on the mesoscopic scale. On the other hand the role of pinning in high-T_c materials is reduced significantly compared to the low-temperature one, leading to smaller critical currents. At elevated temperatures the thermal depinning[1] further diminishes effects of disorder.

Linear response to electric field in the mixed state of these superconductors has been thoroughly explored experimentally and theoretically over the last three decades. These experiments were performed at very small voltages in order to avoid effects of nonlinearity. Deviation from linearity, however, are interesting in their own right. These effects have also been studied in low-T_c superconductors experimentally[2,3] and theoretically[4,5] and recently experiments were extended to HTSC compounds.[6,7]

Since thermal fluctuations in the low-T_c materials are negligible compared to the intervortex interactions, the moving vortex matter is expected to preserve a regular lattice structure (for weak enough disorder). On the other hand, as mentioned above, the vortex lattice melts in HTSC over large portions of their phase diagram so the moving vortex matter in the region of vortex liquid can be better described as an irregular flowing vortex liquid. In particular the nonlinear effects will also be strongly influenced by the thermal fluctuations.

A simpler case of a zero or very small magnetic field in the case of strong thermal fluctuations was, in fact, comprehensively studied theoretically[8] albeit in linear response only. In any superconductor there exists a critical region around the critical temperature $|T-T_c| \ll Gi \cdot T_c$, in which the fluctuations are strong (the Ginzburg number characterizing the strength of thermal fluctuations is just $Gi \sim 10^{-10}-10^{-7}$ for low T_c, while $Gi \sim 10^{-5}-10^{-1}$ for HTSC materials). Outside the critical region and for small electric fields, the fluctuation conductivity was calculated by Aslamazov and Larkin[9] by considering (noninteracting) Gaussian fluctuations within Bardeen-Cooper-Schrieffer (BCS) and within a more phenomenological Ginzburg-Landau (GL) approach. In the framework of the GL approach (restricted to the lowest Landau-level approximation), Ullah and Dorsey[10] computed the Ettingshausen coefficient by using the Hartree approximation. This approach was extended to other transport phenomena such as the Hall conductivity[10] and the Nernst effect.[11]

The fluctuation conductivity within linear response can be applied to describe sufficiently weak electric fields, which do not perturb the fluctuations' spectrum.[12] Physically at electric field, which is able to accelerate the paired electrons on a distance on the order of the coherence length ξ, so that they change their energy by a value corresponding to the Cooper pair binding energy, the linear response is already inapplicable.[8] The resulting additional field-dependent depairing leads to deviation of the current-voltage characteristics from the Ohm's law. The non-Ohmic fluctuation conductivity was calculated for a layered superconductor in an arbitrary electric field considering the fluctuations as noninteracting Gaussian ones.[13,14] The fluctuations' suppression effect of high electric fields in HTSC was investigated experimentally for the in-plane paraconductivity in zero magnetic field,[15–17] and a good agreement with the theoretical models[13,14] was found. Theoretically the nonlinear fluctuation conductivity in HTSC has been treated by Puica and Lang.[18] Below we compare their approach and results to ours.

In this paper the nonlinear electric response of the moving vortex liquid in a layered superconductor under magnetic field perpendicular to the layers is studied using the time-dependent GL (TDGL) approach. The layered structure is modeled via the Lawrence-Doniach discretization in the magnetic field direction. In the moving vortex liquid the long-range crystalline order is lost due to thermal fluctuations and the vortex matter becomes homogeneous on a scale above the average intervortex distances. Although sometimes motion tends to suppress the fluctuations, they are still a dominant factor in flux dynamics. The TDGL approach is an ideal tool to study a combined effect of the dissipative (overdamped) flux motion and thermal fluctuations conveniently modeled by the Langevin white noise. The interaction term in dynamics is treated in self-consistent Gaussian approximation which is similar in structure to the Hartree approximation.[8,10,18,19]

First the model of Ref. 18 is physically different from ours. The authors in Ref. 18 believe that the two quantities, layer distance and thickness in the Lawrence-Doniach for HTSC are equal (apparently not the case in HTSC), while we consider them as two independent parameters. Another difference is we use so-called self-consistent Gaussian approximation to treat the model while Ref. 18 used the Hartree approximation.

A main contribution of our paper is an explicit form of the Green's function (GF) incorporating all Landau levels. This allows to obtain explicit formulas without need to cutoff higher Landau levels. In Ref. 18, a nontrivial matrix inversion (of infinite matrices) or cutting off the number of Landau levels is required. Note that the exact analytical expression of Green's function of the linearized TDGL equation in dc field can be even generalized also to ac field. The method is very general, and it allow us to study transport phenomena beyond linear response of type-II superconductor such as the Nernst effect and Hall effect. The renormalization of the models is also different from Ref. 18. One of the main result of our work is that the conductivity formula is independent of ultraviolet (UV) cutoff (unlike in Ref. 18) as it should be as the standard $|\Psi|^4$ theory is renormalizable. Furthermore self-consistent Gaussian approximation used in this paper is consistent to leading order with perturbation theory, see Ref. 20 in which it is shown that this procedure preserved a correct the UV renormalization (is renormalization group invariant). Without electric field the issue was comprehensively discussed in a textbook of Kleinert.[20] One can use Hartree procedure only when UV issues are unimportant. We can also show, if there is no electric field, the result obtained using TDGL model and self-consistent Gaussian approximation will lead the same thermodynamic equation using self-consistent Gaussian approximation.

The paper is organized as follows. The model is defined in Sec. II. The vortex liquid within the self-consistent Gaussian approximation is described in Sec. III. The I-V curve and the comparison with experiment are described in Sec. IV while Sec. V contains conclusions.

II. THERMAL FLUCTUATIONS IN THE TIME-DEPENDENT GL LAWRENCE-DONIACH MODEL

To describe fluctuation of order parameter in layered superconductors, one can start with the Lawrence-Doniach expression of the GL free energy of the two-dimensional (2D) layers with a Josephson coupling between them

$$F_{\mathrm{GL}} = s'\sum_n \int d^2r\left\{\frac{\hbar^2}{2m^*}|\mathbf{D}\Psi_n|^2 + \frac{\hbar^2}{2m_c d'^2}|\Psi_n - \Psi_{n+1}|^2 + a|\Psi_n|^2 + \frac{b'}{2}|\Psi_n|^4\right\}, \tag{1}$$

where s' is the order parameter effective "thickness" and d' distance between layers labeled by n. The Lawrence-Doniach model approximates paired electrons density of states by homogeneous infinitely thin planes separated by distance d'. While discussing thermal fluctuations, we have to introduce a finite thickness, otherwise the fluctuations will not allow the condensate to exist (Mermin-Wagner theorem). The thickness is of course smaller than the distance between the layers (otherwise we would not have layers). The order parameter is assumed to be nonzero within s'. Effective Cooper pair mass in the ab plane is m^* (disregarding for simplicity the anisotropy between the crystallographic a and b axes) while along the c axis it is much larger m_c. For simplicity we assume $a=\alpha T_c^{mf}(t-1)$, $t\equiv T/T_c^{mf}$, although this temperature dependence can be easily modified to better describe the experimental coherence length. The "mean-field" critical temperature T_c^{mf} depends on UV cutoff, τ_c, of the "mesoscopic" or "phenomenological" GL description, specified later. This temperature is higher than measured critical temperature T_c due to strong thermal fluctuations on the mesoscopic scale.

The covariant derivatives are defined by $\mathbf{D}\equiv\nabla+i(2\pi/\Phi_0)\mathbf{A}$, where the vector potential describes constant and homogeneous magnetic field $\mathbf{A}=(-By,0)$ and $\Phi_0=hc/e^*$ is the flux quantum with $e^*=2|e|$. The two scales, the coherence length $\xi^2=\hbar^2/(2m^*\alpha T_c)$, and the penetration depth $\lambda^2=c^2m^*b'/(4\pi e^{*2}\alpha T_c)$ define the GL ratio $\kappa\equiv\lambda/\xi$, which is very large for HTSC. In this case of strongly type-II superconductors the magnetization is by a factor κ^2 smaller than the external field for magnetic field larger than the first critical field $H_{c1}(T)$, so that we take $B\approx H$. The electric current, $\mathbf{J}=\mathbf{J}_n+\mathbf{J}_s$, includes both the Ohmic normal part

$$\mathbf{J}_n = \sigma_n\mathbf{E} \tag{2}$$

and the supercurrent

$$\mathbf{J}_s = \frac{ie^*\hbar}{2m^*}(\Psi_n^*\mathbf{D}\Psi_n - \Psi_n\mathbf{D}\Psi_n^*). \tag{3}$$

Since we are interested in a transport phenomenon, it is necessary to introduce a dynamics of the order parameter. The simplest one is a gauge-invariant version of the "type A" relaxational dynamics.[21] In the presence of thermal fluctuations, which on the mesoscopic scale are represented by a complex white noise,[22] it reads

$$\frac{\hbar^2\gamma'}{2m^*}D_\tau\Psi_n=-\frac{1}{s'}\frac{\delta F_{\rm GL}}{\delta\Psi_n^*}+\zeta_n, \quad (4)$$

where $D_\tau\equiv\partial/\partial\tau-i(e^*/\hbar)\Phi$ is the covariant time derivative with $\Phi=-Ey$ being the scalar electric potential describing the driving force in a purely dissipative dynamics. The electric field is therefore directed along the y axis and consequently the vortices are moving in the x direction. For magnetic fields that are not too low, we assume that the electric field is also homogeneous.[22] The inverse diffusion constant $\gamma'/2$, controlling the time scale of dynamical processes via dissipation, is real, although a small imaginary (Hall) part is also generally present.[23] The variance of the thermal noise, determining the temperature T is taken to be the Gaussian white noise

$$\langle\zeta_n^*(\mathbf{r},\tau)\zeta_m(\mathbf{r}',\tau')\rangle=\frac{\hbar^2\gamma'}{m^*s'}T\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau')\delta_{nm}. \quad (5)$$

Throughout, most of the paper, we use the coherence length ξ as a unit of length and $H_{c2}=\Phi_0/2\pi\xi^2$ as a unit of the magnetic field. The dimensionless Boltzmann factor in these units is

$$\frac{F_{\rm GL}}{T}=\frac{s}{\omega t}\sum_n\int d^2r\left\{\frac{1}{2}|D\psi_n|^2+\frac{1}{2d^2}|\psi_n-\psi_{n+1}|^2-\frac{1-t}{2}|\psi_n|^2+\frac{1}{2}|\psi_n|^4\right\}, \quad (6)$$

where the covariant derivatives in dimensionless units in Landau gauge are $D_x=\frac{\partial}{\partial x}-iby$, $D_y=\frac{\partial}{\partial y}$ with $b=B/H_{c2}$ and the order parameter field was rescaled: $\Psi^2=(2\alpha T_c^{mf}/b')\psi^2$. The dimensionless fluctuations' strength coefficient is

$$\omega=\sqrt{2Gi}\pi, \quad (7)$$

where the Ginzburg number is defined by

$$Gi=\frac{1}{2}(8e^2\kappa^2\xi T_c^{mf}\gamma/c^2\hbar^2)^2. \quad (8)$$

Note that here we use the standard definition of the Ginzburg number different from that in Ref. 24. The relation between parameters of the two models, the Lawrence-Doniach and the three-dimensional anisotropic GL model, is $d'=d\xi_c=d\xi/\gamma$, $s'=s\xi_c=s\xi/\gamma$, where $\gamma^2\equiv m_c/m^*$ is an anisotropy parameter. In analogy to the coherence length and the penetration depth, one can define a characteristic time scale. In the superconducting phase a typical "relaxation" time is $\tau_{\rm GL}=\gamma'\xi^2/2$. It is convenient to use the following unit of the electric field and the dimensionless field: $E_{\rm GL}=H_{c2}\xi/c\tau_{\rm GL}$, $\mathcal{E}=E/E_{\rm GL}$. The TDGL Eq. (4) written in dimensionless units reads

$$\hat{H}\psi_n+\frac{1}{2d^2}(2\psi_n-\psi_{n+1}-\psi_{n-1})-\frac{1-t}{2}\psi_n+|\psi_n|^2\psi_n=\overline{\zeta_n},$$

$$\hat{H}=D_\tau-\frac{1}{2}D^2, \quad (9)$$

while the Gaussian white-noise correlation takes a form

$$\langle\overline{\zeta_n}^*(\mathbf{r},\tau)\overline{\zeta_m}(\mathbf{r}',\tau')\rangle=\frac{2\omega t}{s}\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau')\delta_{nm}. \quad (10)$$

The covariant time derivative in dimensionless units is $D_\tau=\frac{\partial}{\partial\tau}+ivby$ with $v=\mathcal{E}/b$ being the vortex velocity and the thermal noise was rescaled as $\zeta_n=\overline{\zeta_n}(2\alpha T_c^{mf})^{3/2}/b'^{1/2}$. The dimensionless current density is $\mathbf{J}_s=J_{\rm GL}j_s$, where

$$j_s=\frac{i}{2}(\psi_n^*D\psi_n-\psi_nD\psi_n^*). \quad (11)$$

with $J_{\rm GL}=cH_{c2}/(2\pi\xi\kappa^2)$ being the unit of the current density. Consistently the conductivity will be given in units of $\sigma_{\rm GL}=J_{\rm GL}/E_{\rm GL}=c^2\gamma'/(4\pi\kappa^2)$. This unit is close to the normal-state conductivity σ_n in dirty limit superconductors.[25] In general there is a factor k of order one relating the two: $\sigma_n=k\sigma_{\rm GL}$.

III. VORTEX LIQUID WITHIN THE SELF-CONSISTENT GAUSSIAN APPROXIMATION

A. Gap equation

Thermal fluctuations in vortex liquid frustrate the phase of the order parameter, so that $\langle\psi_n(\mathbf{r},\tau)\rangle=0$. Therefore the contributions to the expectation values of physical quantities like the electric current come exclusively from the correlations, the most important being the quadratic one $\langle\psi_n(\mathbf{r},\tau)\psi_n^*(\mathbf{r}',\tau')\rangle$. In particular, $\langle|\psi_n(\mathbf{r},\tau)|^2\rangle$ is the superfluid density. A simple approximation which captures the most interesting fluctuations effects in the self-consistent Gaussian approximation, in which the cubic term in the TDGL Eq. (9), $|\psi_n|^2\psi_n$, is replaced by a linear one $2\langle|\psi_n|^2\rangle\psi_n$

$$\left(\hat{H}-\frac{b}{2}\right)\psi_n+\frac{1}{2d^2}(2\psi_n-\psi_{n+1}-\psi_{n-1})+\varepsilon\psi_n=\overline{\zeta_n}, \quad (12)$$

leading the "renormalized" value of the coefficient of the linear term

$$\varepsilon=-a_h+2\langle|\psi_n|^2\rangle, \quad (13)$$

where the constant is defined as $a_h=(1-t-b)/2$. The average $\langle|\psi_n|^2\rangle$ is expressed via the parameter ε below and will be determined self-consistently together with ε. It differs slightly from a well-known Hartree procedure in which the coefficient of the linearized term is generally different (see Refs. 20 and 22 and Appendix C for details).

Due to the discrete translation invariance in the field direction z, it is convenient to work with the Fourier transform with respect to the layer index

$$\psi_n(\mathbf{r},\tau)=\int_0^{2\pi/d}\frac{dk_z}{2\pi}e^{-ink_zd}\psi_{k_z}(\mathbf{r},\tau),$$

$$\psi_{k_z}(\mathbf{r},\tau)=d\sum_n e^{ink_zd}\psi_n(\mathbf{r},\tau), \quad (14)$$

and similar transformation for $\overline{\zeta}$. In terms of Fourier components the TDGL Eq. (12) becomes

$$\left\{\hat{H}-\frac{b}{2}+\frac{1}{d^2}[1-\cos(k_z d)]+\varepsilon\right\}\psi_{k_z}(\mathbf{r},\tau)=\overline{\zeta_{k_z}}(\mathbf{r},\tau). \tag{15}$$

The noise correlation is

$$\langle\overline{\zeta_{k_z}}^*(\mathbf{r},\tau)\overline{\zeta_{k_z'}}(\mathbf{r}',\tau')\rangle=4\pi\omega t\frac{d}{s}\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau')\delta(k_z-k_z'). \tag{16}$$

The relaxational linearized TDGL equation with a Langevin noise, Eq. (15), is solved using the retarded ($G=0$ for $\tau<\tau'$) GF $G_{k_z}(\mathbf{r},\tau;\mathbf{r}',\tau')$

$$\psi_n(\mathbf{r},\tau)=\int_0^{2\pi/d}\frac{dk_z}{2\pi}e^{-ink_z d}\int d\mathbf{r}'\times\int d\tau' G_{k_z}(\mathbf{r},\tau;\mathbf{r}',\tau')\overline{\zeta_{k_z}}(\mathbf{r}',\tau'). \tag{17}$$

The GF satisfies

$$\left\{\hat{H}-\frac{b}{2}+\frac{1}{d^2}[1-\cos(k_z d)]+\varepsilon\right\}G_{k_z}(\mathbf{r},\mathbf{r}',\tau-\tau')=\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau') \tag{18}$$

and is computed in the Appendix A.

The thermal average of the superfluid density (density of Cooper pairs) is

$$\langle|\psi_n(\mathbf{r},\tau)|^2\rangle=2\omega t\frac{d}{s}\int_0^{2\pi/d}\frac{dk_z}{2\pi}\int d\mathbf{r}'\times\int d\tau'|G_{k_z}(\mathbf{r}-\mathbf{r}',\tau-\tau')|^2=\frac{\omega tb}{2\pi s}\int_{\tau=\tau_c}^{\infty}\frac{f(\varepsilon,\tau)}{\sinh(b\tau)}, \tag{19}$$

where

$$f(\varepsilon,\tau)=\exp\left[\frac{2v^2}{b}\tanh\left(\frac{b\tau}{2}\right)\right]e^{-(2\varepsilon-b+v^2)\tau}e^{-2\tau/d^2}I_0(2\tau/d^2). \tag{20}$$

Here $I_0(x)=(1/2\pi)\int_0^{2\pi}e^{x\cos\theta}d\theta$ is the modified Bessel function. The first pair of multipliers in Eq. (20) is independent of the interplane distance d and exponentially decreases for $\tau>(2\varepsilon-b+v^2)^{-1}$ while the last pair of multipliers depends on the layered structure. The expression in Eq. (19) is divergent at small τ, so an UV cutoff τ_c is necessary for regularization. Substituting the expectation value into the "gap equation," Eq. (13), the later takes a form

$$\varepsilon=-a_h+\frac{\omega tb}{\pi s}\int_{\tau=\tau_c}^{\infty}\frac{f(\varepsilon,\tau)}{\sinh(b\tau)}. \tag{21}$$

B. Renormalization

In order to absorb the divergence into a renormalized value a_h^r of the coefficient a_h, it is convenient to make an integration by parts in the last term for small τ_c

$$b\int_{\tau=\tau_c}^{\infty}\frac{f(\tau)}{\sinh(b\tau)}\simeq-\int_0^{\infty}\ln[\sinh(b\tau)]\frac{d}{d\tau}\left[\frac{f(\varepsilon,\tau)}{\cosh(b\tau)}\right]-\ln(b\tau_c). \tag{22}$$

Physically the renormalization corresponds to reduction in the critical temperature by the thermal fluctuations from T_c^{mf} to T_c. The thermal fluctuations occur on the mesoscopic scale. The critical temperature T_c is defined at $\varepsilon=0$, and $v=0$, and at low magnetic field less than $H_{c1}=\frac{H_{c2}}{2\kappa^2}\ln(\kappa)$ (for a typical high-T_c superconductor, $\kappa\simeq50$, $H_{c1}=7.8\times10^{-4}H_{c2}$), the superconductor is at Meissner phase, $b=0$, leading to

$$T_c=T_c^{mf}\left\{1+\frac{2\omega}{\pi s}[\ln(\tau_c/d^2)+\gamma_E]\right\}, \tag{23}$$

where $\gamma_E=0.577$ is Euler constant, and Eq. (21) can be rewritten as

$$\varepsilon=-a_h^r-\frac{\omega t}{\pi s}\int_0^{\infty}\ln[\sinh(b\tau)]\frac{d}{d\tau}\left[\frac{f(\varepsilon,\tau)}{\cosh(b\tau)}\right]+\frac{\omega t}{\pi s}\{\gamma_E-\ln(bd^2)\}, \tag{24}$$

where $a_h^r=\frac{1-b-T/T_c}{2}$, $t=T/T_c$, and $\omega=\sqrt{2Gi}\pi$, where $Gi=\frac{1}{2}(8e^2\kappa^2\xi T_c\gamma/c^2\hbar^2)^2$ (T_c^{mf} is now replaced by T_c). The formula is cutoff independent. In terms of energy UV cutoff Λ, introduced, for example, in Ref. 11, the cutoff "time" τ_c can be expressed as

$$\tau_c=1/(2e^{\gamma_E}\Lambda). \tag{25}$$

This is obtained by comparing a thermodynamic result for a physical quantity like superfluid density with the dynamic result (see Appendix B). The temporary UV cutoff used is completely equivalent to the standard energy or momentum cutoff Lambda used in thermodynamics (in which the time dependence does not appear). Physically one might think about momentum cutoff as more basic and this would be universal and independent of particular time-dependent realization of thermal fluctuations (TDGL with white noise in our case). Roughly (in physical units) $\Lambda\simeq\varepsilon_F=\hbar^2k_F^2/(2m^*)$. In the next section we will discuss the estimate of T_c^{mf} using this value due to the following reason. For high-T_c materials ordinary BCS is invalid and coherence length is of order of lattice spacing (the cutoff becomes microscopic) and therefore the energy cutoff is of order ε_F. Except the formula to calculate T_c^{mf}, all other formulas in this paper is independent of energy cutoff.

IV. *I-V* CURVE

A. Current density

The supercurrent density, defined by Eq. (11), can be expressed via the Green's functions as

$$j_y^s = i\omega t \frac{d}{s}\int_0^{2\pi/d}\frac{dk_z}{2\pi}\int_{r',\tau'} G_{k_z}^*(\mathbf{r}-\mathbf{r}',\tau-\tau')$$
$$\times \frac{\partial}{\partial y}G_{k_z}(\mathbf{r}-\mathbf{r}',\tau-\tau') + \text{c.c.} \tag{26}$$

Performing the integrals, one obtains

$$j_y^s = \frac{\omega t}{4\pi s}\nu\int_{\tau=0}^{\infty}\frac{f(\varepsilon,\tau)}{\cosh\left(\frac{b\tau}{2}\right)^2}, \tag{27}$$

where the function f was defined in Eq. (20). Consequently the contribution to the conductivity is $\bar{\sigma}^s = j_y^s/\mathcal{E}$. The conductivity expression [Eq. (27)] is not divergent when expressed as a function of renormalized T_c (the real transition temperature), so it is independent of the cutoff. This is considered in detail in Sec. III B and is indeed different from the Ref. 18. In physical units the current density reads

$$J_y = \sigma_n E\left[1 + \frac{\omega t}{4\pi s}\frac{1}{k}\int_{\tau=0}^{\infty}\frac{f(\varepsilon,\tau)}{\cosh\left(\frac{b\tau}{2}\right)^2}\right]. \tag{28}$$

This is the main result of the present paper. We also considered the conductivity expression in 2D in linear response which do match the linear-response conductivity expression derived in our previous work.[11]

$$\bar{\sigma}_{2\text{D}}^s = \frac{\omega t}{4\pi s b}\left\{2 - \left(1 - \frac{2\varepsilon}{b}\right)\left[\psi\left(\frac{\varepsilon}{b}\right) - \psi\left(\frac{1}{2}+\frac{\varepsilon}{b}\right)\right]\right\}, \tag{29}$$

where ψ is the polygamma function.

B. Comparison with experiment

In this section we use physical units while the dimensionless quantities are denoted with bars. The experiment results of Puica *et al.*,[7] obtained from the resistivity and Hall effect measurements on an optimally doped $YBa_2Cu_3O_{7-\delta}$ (YBCO) films of thickness 50 nm and $T_c = 86.8$ K. The distance between the bilayers used the calculation is $d' = 11.68$ Å in Ref. 26. The number of bilayers is 50, large enough to be described by the Lawrence-Doniach model without taking care of boundary conditions. In order to compare the fluctuation conductivity with experimental data in HTSC, one cannot use the expression of relaxation time γ' in BCS which may be suitable for low-T_c superconductor. Instead of this, we use the factor k as fitting parameter. The comparison is presented in Fig. 1. The resistivity

$$\rho = \frac{1}{\sigma_s + \sigma_n}, \tag{30}$$

$$\sigma_s = \frac{\sigma_n}{k}\bar{\sigma}_s \tag{31}$$

curves were fitted to Eq. (30) with the normal-state conductivity measured in Ref. 7 to be $\sigma_n = 1.9 \times 10^4$ $(\Omega\ \text{cm})^{-1}$. The parameters we obtain from the fit are: $H_{c2}(0) = T_c dH_{c2}(T)/dT|_{T_c} = 190$ T (corresponding to $\xi = 13.2$ Å), the Ginzburg-Landau parameter $\kappa = 45.6$, the order parameter effective thickness $s' = 8.5$ Å, and the factor $k = \sigma_n/\sigma_{\text{GL}} = 0.55$, where we take $\gamma = 7.8$ for optimally doped YBCO in Ref. 27. Using those parameters, we obtain $Gi = 1.12 \times 10^{-3}$ (corresponding to $\omega = 0.148$). The order parameter effective thickness s' can be taken to be equal to the layer distance (see in Ref. 28) of the superconducting CuO plane plus the coherence length $2\xi_c = 2\frac{\xi}{\gamma}$ due to the proximity effect: 3.18 Å$+2\frac{13.2}{7.8}$ Å$=6.9$ Å, roughly in agreement in magnitude with the fitting value of s'.

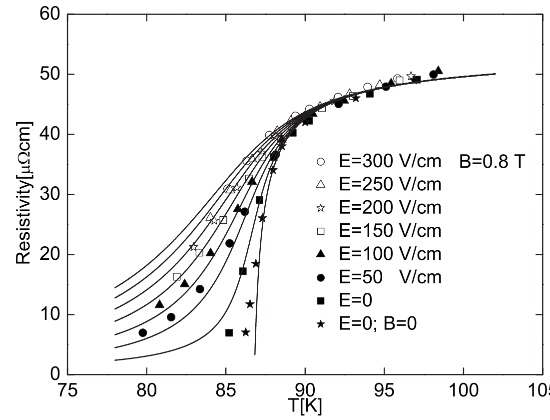

FIG. 1. Points are resistivity for different electric fields of an optimally doped YBCO in Ref. 7. The solid line is the theoretical value of resistivity calculated from Eq. (30) with fitting parameters (see text). The dashed line is the theoretical value of resistivity in linear response with the same parameters.

We will now estimate T_c^{mf} for this sample. For the underdoped YBCO, the radius of the Fermi surface of YBCO was measured in Ref. 29, $k_F = 0.7$ Å^{-1}, while the effective mass is $m^* = 1.9 m_e$. We will assume that the Fermi energy for underdoped YBCO of Ref. 29 is $\varepsilon_F = \hbar^2 k_F^2/(2m^*)$ and is roughly the same for the optimal YBCO studied in this paper. The cutoff time in physical units is then, according to Eq. (25), $\tau_c = 1.39 \times 10^{-17}$ s. Equation (23) gives then $T_c^{mf} = 101.15$ K. Using the parameters specified above we plot several theoretical *I-V* curves. As expected the *I-V* curve shown in Figs. 2 and 3 has two linear portions, the flux flow part for $E \ll E_{\text{GL}}$ and the normal Ohmic part for $E \gg E_{\text{GL}}$. In the crossover region, $E \sim E_{\text{GL}}$, a *I-V* curve becomes nonlinear due to destruction of superconductivity (the normal area inside the vortex cores increases to fill all the space). In Fig. 2 the *I-V* curves are shown for different the magnetic fields, at a fixed temperature $T = 0.75 T_c$. At given electric field, as the magnetic field increases, the supercurrent decreases. When the magnetic field reaches H_{c2}, the *I-V* curve becomes linear. In Fig. 3 the *I-V* curves are shown for different temperatures, at a fixed magnetic field $H = 0.5 H_{c2}$. At given electric field, as the temperature increases, the supercurrent decreases. When the temperature reaches T_c, the *I-V* curve becomes linear. With decreasing temperature the crossover becomes steeper.

V. DISCUSSION AND CONCLUSION

We quantitatively studied the transport in a layered type-II superconductor in magnetic field in the presence of strong

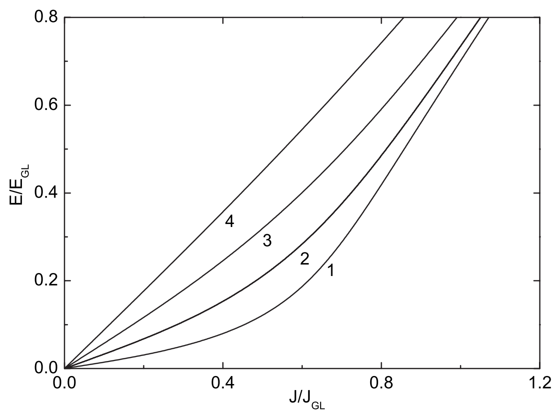

FIG. 2. The current-voltage curves calculated from Eq. (28) by using the parameters (see text) for different magnetic fields $b = B/H_{c2}$: 0.04 (1), 0.1 (2), 0.4 (3), and 1.0 (4) at temperature $t = 0.75$.

thermal fluctuations on the mesoscopic scale beyond the linear response. While in the normal state the dissipation involves unpaired electrons, in the mixed phase it takes a form of the flux flow. Time-dependent Ginzburg-Landau equations with thermal noise describing the thermal fluctuations are used to describe the vortex-liquid regime and arbitrary flux flow velocities. We avoid assuming the lowest Landau-level approximation, so that the approach is valid for arbitrary values the magnetic field not too close to $H_{c1}(T)$.

Our main objective is to study layered high-T_c materials for which the Ginzburg number characterizing the strength of thermal fluctuations is exceptionally high, in the moving vortex matter the crystalline order is lost and it becomes homogeneous on a scale above the average intervortex distances. This ceases to be the case at very low temperature at which two additional factors make the calculation invalid. One is the validity of the GL approach [strictly speaking not far from $T_c(H)$] and another is effect of quenched disorder. The later becomes insignificant at elevated temperature due to a very effective thermal depinning. Although sometimes motion tends to suppress fluctuations, they are still a dominant factor in flux dynamics. The nonlinear term in dynamics is treated using the renormalized self-consistent Gaussian approximation. The renormalization of the critical temperature is calculated and is strong in layered high-T_c materials. The results were compared to the experimental data on HTSC. Our resistivity results are in good qualitative and even quantitative agreement with experimental data on $YBa_2Cu_3O_{7-\delta}$ in strong electric fields.

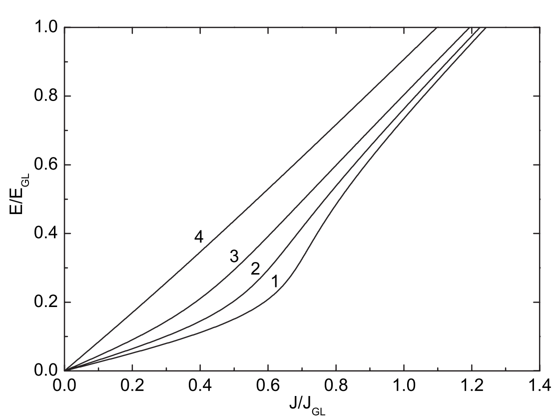

FIG. 3. The current-voltage curves calculated from Eq. (28) by using the parameters (see text) for different temperatures $t = T/T_c$: 0.2 (1), 0.3 (2), 0.4 (3), and 1.0 (4) at magnetic flied $b = 0.5$.

Let us compare the present approach with a widely used Londons' approximation. Since we have not neglected higher Landau levels, as very often is done in similar studies,[1,10] our results should be applicable even for relatively small fields in which the London approximation is valid and used. There is no contradiction since the two approximations have a very large overlap of applicability regions for strongly type-II superconductors. The GL approach for the constant magnetic induction works for $H \gg H_{c1}(T)$ while the Londons' approach works for $H \ll H_{c2}(T)$. Similar methods can be applied to other electric-transport phenomena like the Hall conductivity and thermal-transport phenomena like the Nernst effect. The results, at least in 2D, can be, in principle, compared to numerical simulations of Langevin dynamics. Efforts in this direction are under way.

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APPENDIX A: DERIVATION OF THE GREEN'S FUNCTION OF THE LINEARIZED TDGL EQUATION

In this appendix we outline the method for obtaining the Green's function in strong electric field for the linearized equation of TDGL [Eq. (18)]. The Green's function is a Gaussian

$$G_{k_z}(\mathbf{r},\mathbf{r}',\tau'') = \exp\left[\frac{ib}{2}X(y+y')\right] g_{k_z}(X,Y,\tau''), \quad \text{(A1)}$$

where

$$g_{k_z}(X,Y,\tau'') = C_{k_z}(\tau'')\,\theta(\tau'')\exp\left(-\frac{X^2+Y^2}{2\beta} - vX\right), \quad \text{(A2)}$$

with $X = x - x' - v\tau''$, $Y = y - y'$, and $\tau'' = \tau - \tau'$. $\theta(\tau'')$ is the Heaviside step function, C and β are coefficients.

Substituting the Ansatz Eq. (A1) into Eq. (18), one obtains following conditions condition:

$$\varepsilon - \frac{b}{2} + \frac{v^2}{2} + \frac{1}{d^2}[1-\cos(k_z d)] + \frac{1}{\beta} + \frac{\partial_\tau C}{C} = 0, \quad \text{(A3)}$$

$$\frac{\partial_\tau \beta}{\beta^2} - \frac{1}{\beta^2} + \frac{b^2}{4} = 0. \tag{A4}$$

The Eq. (A4) determines β, subject to an initial condition $\beta(0)=0$,

$$\beta = \frac{2}{b}\tanh(b\tau''/2) \tag{A5}$$

while Eq. (A3) determines C

$$C = \frac{b}{4\pi}\exp\left\{-\left(\varepsilon - \frac{b}{2} + \frac{v^2}{2} + \frac{1}{d^2}[1-\cos(k_z d)]\right)\tau''\right\} \times \sinh^{-1}\left(\frac{b\tau''}{2}\right). \tag{A6}$$

The normalization is dictated by the delta function term in definition of the Green's function Eq. (18).

APPENDIX B: COMPARISON WITH THERMODYNAMICS

From TDGL, we obtain in the case $v=0$

$$\langle|\psi_n(\mathbf{r},\tau)|^2\rangle = \frac{\omega t b}{2\pi s}\int_{\tau=\tau_c}^{\infty} \frac{\exp\left\{-\left(2\varepsilon - b + \frac{2}{d^2}\right)\tau\right\} I_0\left(\frac{2\tau}{d^2}\right)}{\sinh(b\tau)}. \tag{B1}$$

The superfluid density at $b=0$ and $\varepsilon=0$ can be obtained by taking b and ε to zero limit in the above equation

$$\langle|\psi_n(\mathbf{r},\tau)|^2\rangle = \frac{\omega t}{2\pi s}\int_{\tau=\tau_c/d^2}^{\infty} \frac{\exp\{-2\tau\} I_0(2\tau)}{\tau}. \tag{B2}$$

Performing the integration by parts, one obtains

$$\langle|\psi_n(\mathbf{r},\tau)|^2\rangle \simeq -\frac{\omega t}{2\pi s}\{\ln(\tau_c/d^2) + \gamma_E\} + O(\tau_c). \tag{B3}$$

In the case without external electric field (or $v=0$), the equation obtained from TDGL shall approach the thermodynamics result. In thermodynamics method, we shall evaluate the partition function $Z=\int D\psi_n D\psi_n^* e^{-F_{\mathrm{GL}}/T}$, where F_{GL}/T is defined in Eq. (6). The superfluid density in the thermodynamic approach at the phase transition point

$$\langle|\psi_n(\mathbf{r},\tau)|^2\rangle = \frac{\omega t d}{(2\pi)^3 s}\int_0^{k_{\max}} d\mathbf{k}\int_0^{2\pi/d} dk_z \frac{1}{\frac{\mathbf{k}^2}{2} + \frac{1-\cos(k_z d)}{d^2}}$$

$$\simeq \frac{\omega t}{2\pi s}\{\ln\Lambda + \ln(2d^2)\} + O(\Lambda^{-1}), \tag{B4}$$

where $\Lambda = k_{\max}^2/2$.

The relation between the cut-off time τ_c and energy UV cutoff Λ is obtained by comparing Eq. (B3) with Eq. (B4)

$$\tau_c \simeq \frac{1}{2\Lambda e^{\gamma_E}}. \tag{B5}$$

We also remark that in thermodynamic approach, if we use the self-consistent Gaussian approximation, we will get the exact same equation derived in Eq. (24) without electric field derived from TDGL after using Eq. (B5).

APPENDIX C: COMPARISON WITH THE HARTREE APPROACH

Here we explain the difference using an example of thermodynamics. The dynamics is not different since it always can be cast in the Martin-Siggia-Rose form (see Ref. 22).

By using the Hartree approximation, one substitute $|\psi|^4$ by $2\langle|\psi|^2\rangle|\psi|^2$ in the GL free energy Eq. (6) leading the renormalized value of the coefficient of the linear term in the TDGL Eq. (12)

$$\varepsilon = -a_h + \langle|\psi_n|^2\rangle. \tag{C1}$$

In the framework of the variational Gaussian approximation, the GL free-energy Eq. (6) is divided into an optimized quadratic part K, and a "small" part V. Then K is chosen in such a way that the energy of a Gaussian state is minimal.[1] In liquid phase with an arbitrary homogeneous U(1) symmetric state, just one variational parameter ε is sufficient. Thus

$$K = \frac{s}{\omega t}\sum_n \int d^2 r\left[\psi_n^*\left(-\frac{1}{2}D^2 - \frac{b}{2} + \varepsilon\right)\psi_n\right] \tag{C2}$$

and the small perturbation becomes

$$V = \frac{s}{\omega t}\sum_n \int d^2 r\left[(-a_h - \varepsilon)|\psi_n|^2 + \frac{1}{2}|\psi_n|^4\right]. \tag{C3}$$

The eigenvalue of Nth Landau level is

$$-\frac{1}{2}D^2\varphi_n = \left(N + \frac{1}{2}\right)b\varphi_n. \tag{C4}$$

The Gaussian energy which will be minimized therefore is

$$f_{gauss} \equiv -\log\left[\int \mathcal{D}\psi_n \mathcal{D}\bar{\psi}_n \exp(-K)\right] + \langle V\rangle_K, \tag{C5}$$

where

$$\langle V\rangle_K = \sum_n [(-a_h - \varepsilon)\langle|\psi_n|^2\rangle + \langle|\psi_n|^2\rangle\langle|\psi_n|^2\rangle]. \tag{C6}$$

Minimizing the Gaussian energy with respect to ε, we get the gap equation

$$\varepsilon = -a_h + 2\langle|\psi_n|^2\rangle. \tag{C7}$$

While the Hartree method is generally simpler, the Gaussian method applied in its consistent form conserves Ward identities (electric current) and its effective energy is positive definite. In addition it has the correct "large number of components" limit, unlike Hartree method.

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