Simplified Multiaccess Interference Reduction for MC-CDMA With Carrier Frequency Offsets

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***Abstract*—Multicarrier code-division multiple-access (MC-CDMA) system performance can severely be degraded by multiaccess interference (MAI) due to the carrier frequency offset (CFO). We argue that MAI can more easily be reduced by employing complex carrier interferometry (CI) codes. We consider the scenario with spread gain N, multipath length L, and N users, i.e., a fully loaded system. It is proved that, when CI codes are used, each user only needs to combat $2(L-1)$ (rather than $N-1$) interferers, even in the presence of CFO. It is shown that this property of MC-CDMA with CI codes in a CFO channel can be exploited to simplify three multiuser detectors, namely, parallel interference cancellation (PIC), maximum-likelihood, and decorrelating multiuser detectors. The bit-error probability (BEP) for MC-CDMA with binary phase-shift keying (BPSK) modulation and single-stage PIC and an upper bound for the minimum error probability are derived. Finally, simulation results are given to corroborate theoretical results.**

***Index Terms*—Carrier frequency offset, complexity reduction, decorrelating detection, interferometry codes, maximum likelihood detection, multiaccess interference, multicarrier code-division multiple-access (MC-CDMA), multiuser detection, orthogonal codes, parallel interference cancellation.**

I. Introduction

Multicarrier code-division multiple access (MC-CDMA) has emerged as a promising multiaccess technique for high-data-rate communications [2], [10]. MC-CDMA is inherently more robust to intersymbol interference than a conventional CDMA system due to the use of the orthogonal frequency-division multiplexing (OFDM) structure. However, the multipath and/or the carrier frequency offset (CFO) effects tend to destroy orthogonality among users and lead to multiaccess interference (MAI). Thus, the performance of MC-CDMA can greatly degrade.

There has been research on MAI suppression using single-user detection (SUD) techniques. For example, the structural differences of interfering users caused by CFO were exploited at the receiver to suppress MAI in [12]. However, this MAI-suppression technique imposes a computational burden on the receiver since a discrete Fourier transform (DFT) of size larger than N is required due to the oversampling of the received signal in the frequency domain. Another way to reduce MAI is achieved by code design while keeping the structure of MC-CDMA unchanged [19]. In [19], a code-design method based on real Hadamard–Walsh (HW) codes was proposed and shown to achieve zero MAI in a multipath environment in MC-CDMA. However, not all users can enjoy MAI-free communications with this design. In addition, suppression of the MAI due to the CFO effect was not considered in this paper.

Multiuser-detection (MUD) techniques have been developed to mitigate MAI. However, the complexity of MUD techniques is generally high. Much effort has been made to reduce the complexity of the multiuser detectors. Cai *et al.* [4] proposed to assign a set of subcarriers to a group of users while preserving the frequency diversity of MC-CDMA as much as possible. A new maximum-likelihood (ML) MUD scheme called sphere decoding was proposed for MC-CDMA whose complexity is a polynomial function of the user number [3]. However, when the user number is large, the sphere-decoding ML algorithm is cumbersome to perform. Moreover, neither of these techniques are shown to be effective in the presence of CFO.

In this paper, we show how to suppress MAI with simplified techniques for MC-CDMA in CFO environments using carrier-interferometry (CI) codes. CI codewords were introduced to MC-CDMA in [17], which showed that two sets of orthogonal CI codewords can increase user capacity from N to $2N$ in MC-CDMA with negligible performance degradation in a multipath fading channel. CI codes were also used as training sequences for channel estimation to decouple the interantenna interference in a CFO-free MIMO-OFDM system [14].

Here, we first show how to completely eliminate MAI by employing CI codes in MC-CDMA with CFO. That is, for CI codes, when the number of active users is less than or equal to N/G, where G is a power of 2 with $G \geq L$, N is the spread gain, and L is the multipath length, we demonstrate that a proper choice of CI codewords allows MC-CDMA systems to enjoy MAI-free communication.

Second, for a fully loaded MC-CDMA system with CI codes (CI-MC-CDMA), we prove that for $L \leq N/2$, each user receives interference from $2(L-1)$ users only (instead of other $N-1$ active users) in CFO and multipath-fading environments.

Third, we demonstrate that by exploiting the sparsity of the cross-correlation matrix of CI-MC-CDMA, we can lower

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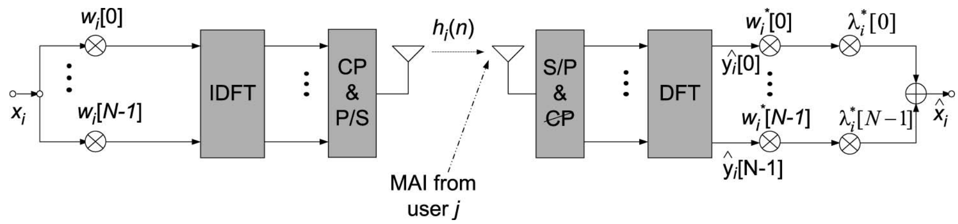

Fig. 1. Block diagram of the uplink transmission of the ith user in an MC-CDMA system.

the complexity of three MUD techniques, namely, parallel interference cancellation (PIC), optimum ML detection, and decorrelating detection, as explained in the succeeding discussion. The complexity for fully loaded PIC (i.e., N active users) is linearly proportional to the channel multipath length L rather than the user number N. Since N is, in general, much larger than L, the complexity is substantially reduced. We also show that it is possible to lower the complexity of ML detectors such that its complexity exponentially grows with L instead of N. Moreover, we explain that, for MC-CDMA with $N-L+1$ users, the cross-correlation matrix can be transformed to a band matrix, which, in turn, can be used to reduce the complexity of the decorrelating detector to $O(2(N-L+1)(L-1)^2)$, which is far less than the $O((N-L+1)^3/3)$ of the decorrelating detector for a general dense matrix of similar dimension.

Finally, we analyze the bit-error-probability (BEP) performance of fully loaded MC-CDMA with single-stage PIC using CI codes. Although CI-MC-CDMA with PIC has previously been considered [21], a detailed BEP analysis of PIC in the presence of CFO has yet to be studied. Moreover, we derive an upper bound on the minimum error probability for an MC-CDMA system in a CFO environment.

The rest of this paper is organized as follows: The MC-CDMA system model is reviewed, and the CFO-induced MAI is derived in Section II. CI codeword schemes are presented, and their MAI-free properties are proved in Section III. The single-stage PIC is described, and its BEP is analyzed in Section IV. We describe the ML detector for MC-CDMA with and without CFO in Section V. In addition, the sparsity of the cross-correlation matrix of CI codes, along with its associated tail-biting trellis (TBT), is studied in this section. In addition, an upper bound on the minimum error probability is derived for MC-CDMA in a CFO environment. In Section VI, we discuss the reduced-complexity decorrelating detector by using a proper subset of CI codes, resulting in a band cross-correlation matrix. Finally, simulation results are shown in Section VII.

II. System Model

Consider K users in an MC-CDMA system. The block diagram of the uplink transmission of the ith user is shown in Fig. 1. As shown in the figure, symbol x_i is spread by an $N \times 1$ codeword in the frequency domain to yield vector

$$y_i[k] = w_i[k]x_i, \qquad 0 \le k \le N-1 \tag{1}$$

where $w_i[k]$ is the kth component of the ith orthogonal code. The kth component of the DFT output $\hat{\mathbf{y}}$ can be expressed by

$$\hat{y}[k] = \sum_{j=0}^{K-1} r_j[k] + n[k] \tag{2}$$

where $n[k]$ is the DFT of additive noise, and $r_j[k]$ is the received signal contributed by the jth user due to the channel fading and CFO effects. We suppose that user j has a normalized CFO ϵ_j, i.e., the actual CFO normalized by $1/N$ of the overall bandwidth, and $-0.5 \le \epsilon_j \le 0.5$. $r_j[k]$ can be written as [15], [19]

$$r_j[k] = \alpha_j \lambda_j[k] y_j[k] + \beta_j \sum_{m=0,\, m\neq k}^{N-1} \{\lambda_j[m] y_j[m] g_j[m-k]\} \tag{3}$$

where $\lambda_j[m]$ is the mth component of the N-point DFT of the channel impulse response of user j, i.e.,

$$\alpha_j = \frac{\sin \pi\epsilon_j}{N \sin \frac{\pi\epsilon_j}{N}} e^{j\pi\epsilon_j \frac{N-1}{N}}$$

$$\beta_j = \sin(\pi\epsilon_j) e^{j\pi\epsilon_j \frac{N-1}{N}}$$

$$g_j[m-k] = \frac{e^{-j\pi\frac{m-k}{N}}}{N \sin \frac{\pi(m-k+\epsilon_j)}{N}}.$$

It is apparent that when there is no CFO, i.e., $\epsilon_j = 0$, $r_j[k] = \lambda_j[k]y_j[k]$. However, if CFO exists, there are two terms as there are in (3). The first term is $\lambda_j[k]y_j[k]$ distorted by α_j, and the second term is the intercarrier interference (ICI) caused by CFO. Since $\beta_j g_j(0) = \alpha_j$, and by (1) and (3), we have

$$\hat{\mathbf{y}} = \mathbf{C}\mathbf{x} + \mathbf{n} \tag{4}$$

where the element in the ith row and the jth column of $\mathbf{C}$ is

$$\mathbf{C}(i,j) = \beta_j \sum_{m=0}^{N-1} g_j(m-i)\lambda_j[m]w_j[m] \tag{5}$$

$$\mathbf{x} = (x_0, x_1, \ldots, x_{K-1})^T$$

$$\hat{\mathbf{y}} = (\hat{y}[0], \hat{y}[1], \ldots, \hat{y}[N-1])^T \tag{6}$$

$$\mathbf{n} = (n[0], n[1], \ldots, n[N-1])^T \tag{7}$$

which is a circularly symmetric complex Gaussian random vector with zero mean and covariance matrix $\sigma^2\mathbf{I}$. Maximum ratio combining (MRC) is usually used to detect the symbol transmitted by the ith user as

$$\begin{aligned}\hat{z}_i &= \sum_{k=0}^{N-1} \hat{y}[k]\lambda_i^*[k]w_i^*[k] \\ &= s_i + \sum_{j=0,\, j\neq i}^{K-1} \mathrm{MAI}_{i\leftarrow j} + \sum_{k=0}^{N-1} n[k]\lambda_i^*[k]w_i^*[k] \end{aligned} \tag{8}$$

where s_i consists of the distorted chip, and ICI caused by CFO for the desired user and can be written as

$$\begin{aligned} s_i &= \sum_{k=0}^{N-1} r_i[k]\lambda_i^*[k]w_i^*[k] \\ &= \alpha_i \sum_{k=0}^{N-1} |\lambda_i[k]|^2 + \beta_i \sum_{m=0,\, m\neq k}^{N-1} \\ &\quad \cdot \{\lambda_i[m]w_i[m]\lambda_i^*[k]w_i^*[k]g_i[m-k]\} \end{aligned} \tag{9}$$

$$\mathrm{MAI}_{i\leftarrow j} = \sum_{k=0}^{N-1} r_j[k]\lambda_i^*[k]w_i^*[k]. \tag{10}$$

$\mathrm{MAI}_{i\leftarrow j}$ is the MAI of user i due to the jth user's CFO and multipath fading channels. With (3) and (10), we can show that the MAI term is given by

$$\mathrm{MAI}_{i\leftarrow j} = \beta_j x_j \sum_{k=0}^{N-1}\sum_{m=0}^{N-1} g_j(m-k)\cdot\{\lambda_j[m]w_j[m]\lambda_i^*[k]w_i^*[k]\} \tag{11}$$

where we used the fact that $\beta_j g_j(0) = \alpha_j$. Equation (10) can be expressed in matrix form as [19]

$$\begin{aligned}\mathrm{MAI}_{i\leftarrow j} &= \beta_j x_j \sum_{p=0}^{N-1} g_j(-p) \\ &\quad \cdot \left\{ \left(\mathbf{h}_i^{(p)}\right)^\dagger \mathbf{F}_0^\dagger \left(\mathbf{W}_i^{(p)}\right)^\dagger \mathbf{W}_j \mathbf{F}_0 \mathbf{h}_j \right\} \end{aligned} \tag{12}$$

where

$$\begin{aligned} \mathbf{W}_i^{(p)} &= \mathrm{diag}\left(w_i[p]\cdots w_i[N-1]w_i[0]\cdots w_i[p-1]\right)^T \\ \mathbf{F}_0 &= \mathbf{F}\begin{pmatrix}\mathbf{I}_L \\ \mathbf{0}\end{pmatrix}_{N\times L} \\ \mathbf{h}_i^{(p)} &= \left(h_i(0)e^{-j\frac{2\pi 0 p}{N}} \cdots h_i(L-1)e^{-j\frac{2\pi(L-1)p}{N}}\right)^T. \end{aligned} \tag{13}$$

$\mathbf{I}_L$ is an $L\times L$ identity matrix, and $\mathbf{F}$ is the $N\times N$ DFT matrix whose element at the kth row and the nth column is $[\mathbf{F}]_{k,n} = (1/\sqrt{N})e^{-j(2\pi/N)kn}$. In addition, $\dagger$ in (12) denotes the matrix Hermitian operation.

III. Orthogonal Codes for Multiaccess Interference-Free Multicarrier Code-Division Multiple Access With Carrier Frequency Offset

A. Requirements on MAI-Free Codes

Theoretical requirements on codes to produce an MAI-free MC-CDMA system in the presence of CFO are implied by (12). That is, to have zero MAI in a frequency-selective channel with CFO, we demand $\mathrm{MAI}_{i\leftarrow j} = 0$. Define

$$\mathbf{R}_{i,j}^{(p)} = \left(\mathbf{W}_i^{(p)}\right)^\dagger \mathbf{W}_j, \quad \mathbf{D}_{ij}^{(p)} = \mathbf{F}^\dagger \mathbf{R}_{i,j}^{(p)}\mathbf{F}.$$

It is well known that $\mathbf{D}_{ij}^{(p)}$ is a circulant matrix [9]. Therefore, its first column, i.e., $(d_{i,j}[0]\cdots d_{i,j}[N-1])^T$, is the N-point inverse DFT (IDFT) of $\mathbf{r}_{i,j}^{(p)}$, where $r_{i,j}^{(p)}[k] = w_i^{(p)}[k]w_j^*[k]$, and $w_i^{(p)}[k] = w_i[((N-p+k))_N]$, for $k = 0, 1, \ldots, N-1$, where $((n))_N$ denotes n modulo N. It was shown in [19] that condition $\mathrm{MAI}_{i\leftarrow j} = 0$ is equivalent to

$$\begin{cases} d_{i,j}[n] = 0, & 0\le n\le L-1 \\ d_{i,j}[N-n] = 0, & 1\le n\le L-1. \end{cases} \tag{14}$$

B. CI Orthogonal Codes

In this section, we study the CI codes of size N, which is of the following form:

$$w_i[k] = e^{j\frac{2\pi}{N}ki}, \qquad k, i = 0, 1, \ldots, N-1. \tag{15}$$

Then, the MAI-free property of this code can be stated as follows.

Theorem 1: Let the channel length be L, and let $G = 2^q \ge L$. There exist N/G CI codewords such that the corresponding MC-CDMA is MAI free in a CFO environment.

Proof: Consider two codewords with indices i and i'. We would like to show that the condition in (14) is met. By taking the IDFT of $\mathbf{r}_{i,j}^{(p)}$, we have

$$r_{i,i'}^{(p)}(n) = \frac{1}{\sqrt{N}}\sum_{m=0}^{N-1} r_{i,i'}^{(p)}[m]e^{j\frac{2\pi}{N}mn}. \tag{16}$$

Note that we use $(.)$ in the time domain and $[.]$ in the frequency domain. Let $m = k + gN/G$, $0\le k\le N/G-1$ and $0\le g\le G-1$. We can rewrite (16) as

$$r_{i,i'}^{(p)}(n) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N/G-1}\sum_{g=0}^{G-1}\cdot\left\{r_{i,i'}^{(p)}[k+gN/G]e^{j\frac{2\pi}{N}(k+gN/G)n}\right\}. \tag{17}$$

Since $w_i^{(p)}[k] = e^{j(2\pi/N)(N-p+k)i}$ and $r_{i,i'}^{(p)}[k+gN/G] = w_i^{(p)}[((k+gN/G))_N]w_{i'}^*[((k+gN/G))_N]$, we have

$$\begin{aligned} r_{i,i'}^{(p)}[k+gN/G] &= e^{j\frac{2\pi}{N}(N-p+k+gN/G)i}e^{-j\frac{2\pi}{N}(k+gN/G)i'} \\ &= e^{j\frac{2\pi}{N}(N-p+k)i}e^{-j\frac{2\pi}{N}ki'}e^{j\frac{2\pi}{G}g(i-i')}. \end{aligned}$$

If $i - i' = mG$, where m can be any nonzero integer, $e^{j(2\pi/G)g(i-i')} = e^{j2\pi mg} = 1$. Then, we have

$$r_{i,i'}^{(p)}[k+gN/G] = e^{j\frac{2\pi}{N}(N-p+k)i}e^{-j\frac{2\pi}{N}ki'} = r_{i,i'}^{(p)}[k]. \tag{18}$$

Using (18), we can rewrite (17) as

$$r_{i,i'}^{(p)}(n) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N/G-1} r_{i,i'}^{(p)}[k]e^{j\frac{2\pi}{N}kn}\sum_{g=0}^{G-1} e^{j\frac{2\pi}{G}gn}. \tag{19}$$

Since

$$\sum_{g=0}^{G-1} e^{j\frac{2\pi}{G}gn} = \begin{cases} G, & n = 0, \pm G, \ldots \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

$r_{i,i'}^{(p)}(n)$ is equal to

$$\begin{cases} \frac{G}{\sqrt{N}} \sum_{k=0}^{N/G-1} r_{i,i'}^{(p)}[k] e^{j\frac{2\pi}{N}kn}, & n = 0, \pm G, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

Furthermore, for $i \neq i'$, we have

$$r_{i,i'}^{(p)}(0) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} r_{i,i'}^{(p)}[k] = \sum_{k=0}^{N-1} e^{j\frac{2\pi(i-i')}{N}(N-p+k)} = 0. \tag{21}$$

Thus, (14) holds, and $\text{MAI}_{i\leftarrow j} = 0$. Since there are N/G codewords such that $i - i' = mG, m = 1, 2, \ldots$, the total number of MAI-free codewords from N CI codes is N/G. ■

The MAI-free property of CI codes in a CFO environment can be expected since a multiuser system with the CI spreading codes in the frequency domain is equivalent to a time-division multiple-access system in the time domain.

Theorem 2: For two CI codewords with indices i and i', i, $i' = 0, 1, \ldots, N-1$, we have $\text{MAI}_{i\leftarrow i'} = 0$ in a CFO environment if $((|i - i'|))_{N-(L-1)} \geq L$, where L is the channel length.

Proof: For $p = 0, 1, \ldots, N-1$, we have

$$\begin{aligned} r_{i,i'}^{(p)}(n) &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}(N-p+k)i} \cdot \left\{ e^{-j\frac{2\pi}{N}i'k} e^{j\frac{2\pi}{N}kn} \right\} \\ &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}(i-i'+n)k} e^{-j\frac{2\pi}{N}ip}. \end{aligned}$$

Therefore, $r_{i,i'}^{(p)}(n)$ can be written as

$$\begin{cases} \sqrt{N} e^{-j\frac{2\pi}{N}ip}, & ((i - i' + n))_N = 0 \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

To meet the condition $((|i - i'|))_{N-(L-1)} \geq L$, $|i - i'|$ should take values from $\{L, L+1, \ldots, N-L\}$. Consider the case where $i - i' \geq 0$. Under the conditions where $i - i'$ is equal to a value in this set, $r_{i,i'}^{(p)}(n) \neq 0$, for $n = N - L, N - L - 1, \ldots, N$ by the preceding equation. On the other hand, if $i - i' \leq 0$, $r_{i,i'}^{(p)}(n) \neq 0$, for $n = N + L, N + L + 1, \ldots, 2N - L$. Since

$$\begin{aligned} & r_{i,i'}^{(p)}(n+N) \\ &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}(N-p+k)i} \cdot \left\{ e^{-j\frac{2\pi}{N}i'k} e^{j\frac{2\pi}{N}k(n+N)} \right\} \\ &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}(N-p+k)i} \cdot \left\{ e^{-j\frac{2\pi}{N}i'k} e^{j\frac{2\pi}{N}k(n)} \right\} \\ &= r_{i,i'}^{(p)}(n) \end{aligned}$$

$r_{i,i'}^{(p)}(n)$ is periodic with period N. We conclude that $r_{i,i'}^{(p)}(n) \neq 0$ for $n = L, L+1, \ldots, N-L$, whereas $r_{i,i'}^{(p)}(0) = r_{i,i'}^{(p)}(1) = \cdots = r_{i,i'}^{(p)}(L-1) = 0$, and $r_{i,i'}^{(p)}(N-L+1) = r_{i,i'}^{(p)}(N-L+2) = \cdots = r_{i,i'}^{(p)}(N-1) = 0$, for $((|i-i'|))_{N-(L-1)} \geq L$. In other words, (14) is satisfied, and $\text{MAI}_{i\leftarrow i'} = 0$. ■

Corollary: For an MC-CDMA system in a CFO environment consisting of N active users with CI codes, if $L \leq N/2$, each user has $2(L-1)$ interfering users only.

Proof: The condition $L \leq N/2$ guarantees that there are i and i' satisfying $((|i - i'|))_{N-(L-1)} \geq L$. For every codeword with index i, there are $L-1$ codewords such as codeword i' for which $i - i' \geq 0$ and $((i - i'))_{N-(L-1)} \geq L$, and $L-1$ codewords such as codeword i'' for which $i - i'' < 0$ and $((i'' - i))_{N-(L-1)} \geq L$. Therefore, the number of interferers is $2(L-1)$. ■

IV. Reduced-Complexity Parallel Interference Cancellation for Carrier Interferometry Multicarrier Code-Division Multiple Access

The receiver for the ith user in an MC-CDMA system with PIC is depicted in Fig. 2. It is assumed that the exact knowledge of channel gains and CFO values of all users is available in the receiver. First, initial bit estimates for all users are derived from the SUD receivers, which is basically the same as that depicted in Fig. 1. We call this stage as stage 0 of the PIC detector and denote detected symbols by $\hat{x}_{0i}, i = 0, \ldots, K-1$. Let $\mathbf{F}_0 \mathbf{h}_j = \boldsymbol{\lambda}_j$ and $\mathbf{F}_0 \mathbf{h}_i^{(p)} = \boldsymbol{\lambda}_i^{(p)}$. From (3)–(9) and (12), we can express the detected symbol for user i at the zeroth stage of PIC as

$$\hat{x}_{0i} = \sum_{k=0}^{N-1} |\lambda_i[k]|^2 x_i + \sum_{j=0,\, j\neq i}^{K-1} x_j \gamma_{i,j} + \hat{n}_i \tag{23}$$

where

$$\gamma_{i,j} = \beta_j \sum_{p=0}^{N-1} g_j[-p] \left(\boldsymbol{\lambda}_i^{(p)}\right)^\dagger \left(\mathbf{W}_i^{(p)}\right)^\dagger \mathbf{W}_j \boldsymbol{\lambda}_j$$

and $\hat{n}_i = \sum_{k=0}^{N-1} n[k] \lambda_i^*[k] w_i^*[k]$. Tentative hard decisions are made on $\hat{x}_{0j}$, $j = 1, 2, \ldots, K$, $j \neq i$, to produce initial bit estimates, namely, $\text{sgn}[\Re\{\hat{x}_{0j}\}]$. Then, the MAI estimate for the desired user i is generated and subsequently subtracted from its received signal $\hat{y}$. The new detected symbol at stage 1 of the PIC detector is given by

$$\begin{aligned} \hat{x}_{1i} = \hat{x}_{0i} - & \sum_{j=0,\, j\neq i}^{K-1} \text{sgn}\left[\text{Re}\{\hat{x}_{0j}\}\right] \beta_j \\ & \cdot \left\{ \sum_{p=0}^{N-1} g_j[-p] \left(\boldsymbol{\lambda}_i^{(p)}\right)^\dagger \left(\mathbf{W}_i^{(p)}\right)^\dagger \mathbf{W}_j \boldsymbol{\lambda}_j + \hat{n}_i \right\}. \end{aligned} \tag{24}$$

We know from Theorem 2 and its corollary that, if MC-CDMA employs CI codes in fading and CFO environments with multipath length L, every user only has $2(L-1)$ interferers. Hence, the PIC complexity with CI codes linearly increases with $2L-1$ instead of N. Since $L \ll N$, in practice, this implies huge savings in the computational cost by employing the CI codes in association with the PIC detector.

If a correct decision is made on a particular interferer's bit, the interference from that user to the ith user can completely be cancelled. On the other hand, if an incorrect decision is made,

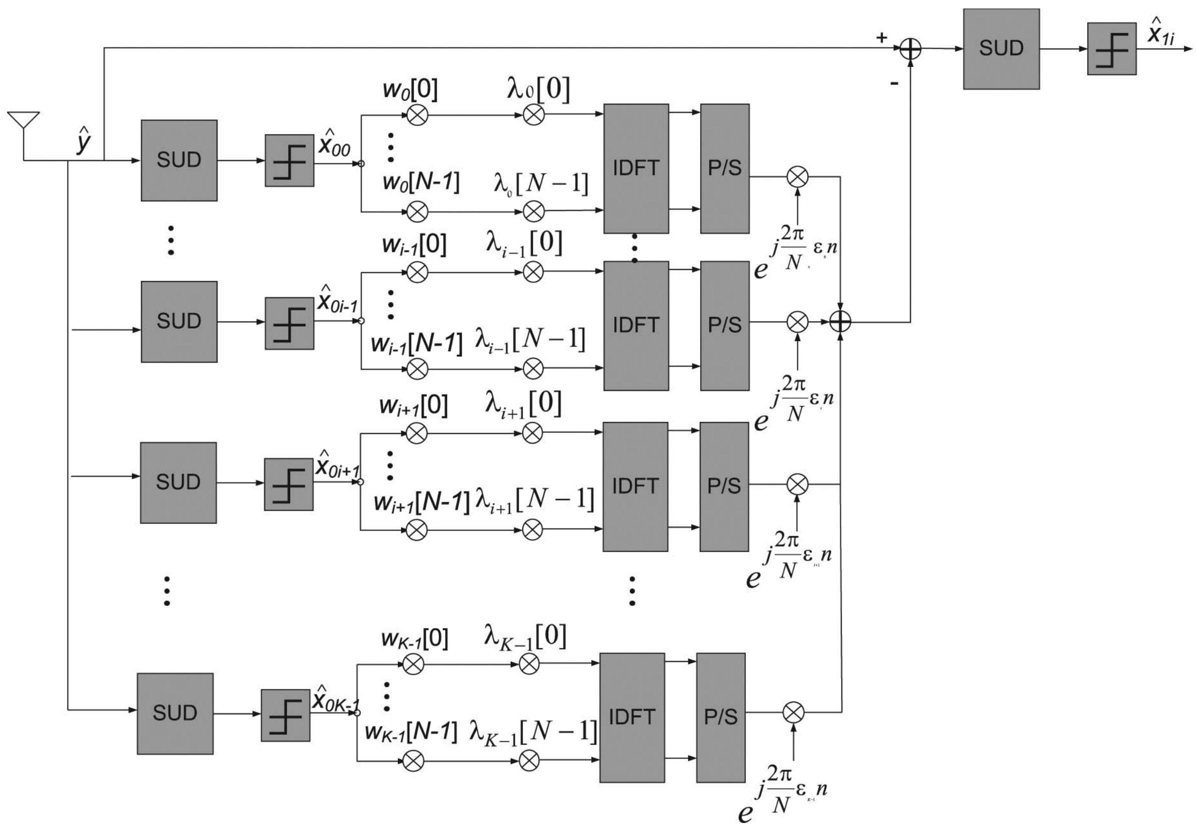

Fig. 2. MC-CDMA receiver with single-stage PIC for the ith user.

the interference from that user will be enhanced rather than cancelled. By substituting $\hat{x}_{0i}$ from (23) in (24), we obtain

$$\hat{x}_{1i}=x_i\sum_{k=0}^{N-1}|\lambda_i[k]|^2+\sum_{j=0,\,j\neq i}^{K-1}\gamma_{i,j}(x_j-\mathrm{sgn}\,[\Re\{\hat{x}_{0j}\}])+\hat{n}_i. \quad (25)$$

If x_j is a binary phase-shift keying (BPSK) symbol, $(x_j - \mathrm{sgn}[\Re\{\hat{x}_{0j}\}])$ is a three-valued random variable (0, 2, −2) whose magnitude represents whether a tentative decision is correctly made on the jth user's bit at the previous stage. It can easily be shown that, given $\boldsymbol{\lambda}_i$, $\hat{n}_i$ is a circularly symmetric zero-mean Gaussian random variable with variance equal to $\sigma^2\sum_{k=0}^{N-1}|\lambda_i[k]|^2$. The random vector $\boldsymbol{\lambda}_i$ consists of N correlated Rayleigh random variables. In other words, $\boldsymbol{\lambda}_i$ is a multivariate Gaussian random vector with zero mean and covariance matrix $\mathbf{R}_i$ whose elements are given by

$$\begin{aligned}\mathbf{R}_i(k,k') &= \mathbb{E}\left\{\lambda_i[k]\lambda_i^*[k']\right\}\\ &=\sum_{l=0}^{L-1}\sum_{l'=0}^{L-1}\mathbb{E}\{h_i(l)h_i^*(l')\}\cdot\left\{e^{-j\frac{2\pi}{N}kl}e^{j\frac{2\pi}{N}k'l'}\right\}. \quad (26)\end{aligned}$$

We assume a uniform model for the multipath intensity (power-delay) profile. In addition, the typical wide-sense stationary uncorrelated scattering channel model [13] is adopted. Then, we have $\mathbb{E}\{h_i(l)h_i^*(l')\}=\sigma_{h_i}^2\delta(l-l')$, and

$$\begin{aligned}\mathbf{R}_i(k,k') &= \sum_{l=0}^{L-1}\sigma_{h_i}^2 e^{-j\frac{2\pi}{N}(k-k')l}\\ &=\begin{cases}\sigma_{h_i}^2 L, & k=k'\\ \sigma_{h_i}^2\frac{1-e^{-j2\pi(k-k')L/N}}{1-e^{-j2\pi(k-k')/N}}, & k\neq k'.\end{cases}\end{aligned}$$

Since $\gamma_{i,j}$ is a linear transform of $\boldsymbol{\lambda}_j$ given $\boldsymbol{\lambda}_i$, $\gamma_{i,j}|\boldsymbol{\lambda}_i$ is a circularly symmetric complex Gaussian random variable with zero mean and variance

$$\begin{aligned}\mathrm{var}[\gamma_{i,j}|\boldsymbol{\lambda}_i] &= |\beta_j|^2\sum_{p,p'=0}^{N-1}\left(\mathbf{W}_i^{(p)}\boldsymbol{\lambda}_i^{(p)}\right)^\dagger\\ &\cdot\left\{\mathbf{W}_j\mathbf{R}_j\mathbf{W}_j^\dagger\left(\mathbf{W}_i^{(p')}\boldsymbol{\lambda}_i^{(p')}\right)g_j(-p)g_j^*(-p')\right\}. \quad (27)\end{aligned}$$

The probability of error for user i at state 0 of the PIC detector using (23) can be written as

$$\begin{aligned}P\left[\mathrm{sgn}\left[\Re\{\hat{x}_{0i}\}\right]\right)\neq x_i] &= P\left[\Re\left\{\hat{n}_i+\sum_{j=0,\,j\neq i}^{K-1}\gamma_{i,j}x_j\right\}\right.\\ &\left.-\sum_{k=0}^{N-1}|\lambda_i[k]|^2>0\right]. \quad (28)\end{aligned}$$

Due to the presence of x_j terms, $\sum_{j=0,\,j\neq i}^{K-1}x_j\gamma_{i,j}$ in (23) is not Gaussian given $\boldsymbol{\lambda}_i$. However, if it is conditioned on all possible x_j, $j\neq i$, we will have a collection of Gaussian random variables that can be approximated by the Gaussian distribution [22]. Hence, we can see that $I_{0i}=\mathrm{Re}\{\hat{n}_i+\sum_{j=0,\,j\neq i}^{K-1}\gamma_{i,j}x_j\}$ in (28) is zero-mean Gaussian conditioned on $\boldsymbol{\lambda}_i$. By using e_{0i} to denote $(x_i-\mathrm{sgn}[\mathrm{Re}\{\hat{x}_{0i}\}])$, the probability of error conditioned on $\boldsymbol{\lambda}_i$ is simply given by

$$P[e_{0i}\neq 0|\boldsymbol{\lambda}_i]=Q\left(\frac{\sum_{k=0}^{N-1}|\lambda_i[k]|^2}{\sqrt{\sum_{j=0,\,j\neq i}^{K-1}\sigma_{\gamma_{i,j}}^2+\sigma_n^2}}\right) \quad (29)$$

where $Q(.)$ is the well-known Q-function, and

$$\sigma^2_{\gamma_{i,j}} = \mathbb{E}\left\{ (\Re\{\gamma_{i,j}\})^2 \left| \boldsymbol{\lambda}_i \right.\right\} = \frac{1}{2}\mathrm{var}[\gamma_{i,j}|\boldsymbol{\lambda}_i]$$
$$\sigma^2_n = \mathbb{E}\left\{ (\Re\{\hat{n}_i\})^2 \right\} = \frac{1}{2}\sigma^2 \sum_{k=0}^{N-1} |\lambda_i[k]|^2 .$$

Then, the BEP can be obtained by averaging $P[e_{0i} \neq 0|\boldsymbol{\lambda}_i]$ over $\boldsymbol{\lambda}_i$ as

$$P[e_{0i} \neq 0] = \int_0^\infty Q\left(\frac{\sum_{k=0}^{N-1} |\lambda_i[k]|^2}{\sqrt{\sum_{j=0,\, j\neq i}^{K-1} \sigma^2_{\gamma_{i,j}} + \sigma^2_n}} \right) P[\boldsymbol{\lambda}_i]\, d\boldsymbol{\lambda}_i. \tag{30}$$

This integral can be calculated by the Monte Carlo method [18]. Now, we are ready to derive the BEP of the detected symbol after stage 1 of PIC. From (25), we have

$$P\left[\Re\{\hat{x}_{1i}\} > 0 | x_i = -1\right] = P\left[\Re\left\{ \hat{n}_i + \sum_{j=0,\, j\neq i}^{K-1} \gamma_{i,j} e_{0j} \right\} > \sum_{k=0}^{N-1} |\lambda_i[k]|^2 \right].$$

To get the closed form for BEP is difficult since e_{0j}'s are dependent with given $\boldsymbol{\lambda}_i$. However, following the arguments in [7] for the BEP derivation of direct-sequence CDMA with PIC, we can assume that e_{0j}'s are actually independent.

A. Derivation of the BEP With a Gaussian Residual Interference Model

Using the aforementioned simplifying assumptions, we can assume that the distribution of the total residual interference (after cancellation) converges to the Gaussian distribution for a sufficiently large number K of users. Under this assumption, we can derive the conditional BEP by noting $(E\{e_{0j}\Re\{\gamma_{i,j}\}|\boldsymbol{\lambda}_i\})^2 = 0$, since

$$\begin{aligned} E&\left\{e_{0j}\Re\{\gamma_{i,j}\}|\boldsymbol{\lambda}_i\right\} \\ &= \sum_{k\in\{0,2,-2\}} \left\{ P[e_{0j} = k] E\left\{ e_{0j}\Re\{\gamma_{i,j}\}|\boldsymbol{\lambda}_i, e_{0j} = k\right\}\right\} \\ &= 2P[e_{0j} = 2]\mathbb{E}\left\{\Re\{\gamma_{i,j}\}|\boldsymbol{\lambda}_i\right\} \\ &\quad - 2P[e_{0j} = -2]\mathbb{E}\left\{\Re\{\gamma_{i,j}\}|\boldsymbol{\lambda}_i\right\} = 0. \end{aligned}$$

In addition, we can use the fact that

$$E\left\{ e^2_{0j}\left(\Re\{\gamma_{i,j}\}\right)^2 \left|\boldsymbol{\lambda}_i\right.\right\} = 4P[e_{0j} \neq 0]\mathbb{E}\left\{ (\Re\{\gamma_{i,j}\})^2 \left|\boldsymbol{\lambda}_i\right.\right\}$$
$$E\left\{ e_{0j}\Re\{\gamma_{i,j}\} e_{0j'}\Re\{\gamma_{i,j'}\}\big|\boldsymbol{\lambda}_i\right\} = 0.$$

Therefore, the conditional BEP is obtained by

$$P[e|\boldsymbol{\lambda}_i] = \left(\frac{\sum_{k=0}^{N-1} |\lambda_i[k]|^2}{\sqrt{\sum_{j=0,\, j\neq i}^{K-1} 4P[e_{0j} \neq 0]\sigma^2_{\gamma_{i,j}} + \sigma^2_n}} \right) \tag{31}$$

and the BEP is given by

$$P[e] = \int_0^\infty P[e|\boldsymbol{\lambda}_i]P[\boldsymbol{\lambda}_i]\, d\boldsymbol{\lambda}_i. \tag{32}$$

B. Derivation of the BEP Using a Non-Gaussian Model for Residual Interference

The assumption of Gaussian residual interference is not true when the number of interfering users is not sufficiently large. For example, as proved before, every user encounters only $2(L-1)$ nonzero interference terms if the CI codewords are used. Thus, for small values of L, the Gaussian assumption for residual interference is not reasonable. The distribution of residual interference must be derived, and therefore, a new BEP formula can be obtained.

To simplify the derivation, we first derive the BEP formula for CI codewords and a channel of length $L = 2$, where the number of interferers is equal to 2, and then extend the result to a generalized case. Suppose users i' and i'' are the two interfering users for user i. The detected received symbol for user i is given by

$$\hat{x}_{1i} = x_i \sum_{k=0}^{N-1} |\lambda_i[k]|^2 + e_{0i'}\gamma_{i,i'} + e_{0i''}\gamma_{i,i''} + \hat{n}_i. \tag{33}$$

Let $e_{0i'}\gamma_{i,i'} = u_1$, $e_{0i''}\gamma_{i,i''} = u_2$, and $\hat{n}_i = \eta$. Then, the probability density function (pdf) of u_1 conditioned on $\boldsymbol{\lambda}_i$, which is denoted by $f_{U_1}(u_1|\boldsymbol{\lambda}_i)$, can be obtained by

$$\begin{aligned} f_{U_1}(u_1|\boldsymbol{\lambda}_1) &= f_{U_1}(u_1|e_{0i'} = 2\boldsymbol{\lambda}_i)P[e_{0i'} = 2|\boldsymbol{\lambda}_i] \\ &\quad + f_{U_1}(u_1|e_{0i'} = -2\boldsymbol{\lambda}_i)P[e_{0i'} = -2|\boldsymbol{\lambda}_i] \\ &\quad + f_{U_1}(u_1|e_{0i'} = 0\boldsymbol{\lambda}_i)P[e_{0i'} = 0|\boldsymbol{\lambda}_i] \\ &= f_{U_1}(2\gamma_{i,i'}|\boldsymbol{\lambda}_i)P[e_{0i'} = 2|\boldsymbol{\lambda}_i] \\ &\quad + f_{U_1}(-2\gamma_{i,i'}|\boldsymbol{\lambda}_1)P[e_{0i'} = -2|\boldsymbol{\lambda}_i] \\ &\quad + \delta(u_1)P[e_{0i'} = 0|\boldsymbol{\lambda}_i]. \end{aligned} \tag{34}$$

We use $N(a,b)$ to denote the Gaussian distribution, with mean a and variance b. Since $f_{U_1}(2\gamma_{i,i'}|\boldsymbol{\lambda}_i) = f_{U_1}(-2\gamma_{i,i'}|\boldsymbol{\lambda}_i) = N(0, 4\sigma^2_{\gamma_{i,i'}})$, we obtain

$$f_{U_1}(u_1|\boldsymbol{\lambda}_i) = P[e_{0i'} \neq 0|\boldsymbol{\lambda}_i]N\left(0, 4\sigma^2_{\gamma_{i,i'}}\right) + \left(1 - P[e_{0i'} \neq 0|\boldsymbol{\lambda}_i]\right)\delta(u_1) \tag{35}$$

where

$$P[e_{0i'} \neq 0|\boldsymbol{\lambda}_i] = Q\left(\frac{\sum_{k=0}^{N-1} |\lambda_{i'}[k]|^2}{\sqrt{\sum_{j=0,\, j\neq i'}^{K-1} \sigma^2_{\gamma_{i',j}} + \sigma^2_n}} \right).$$

Similarly

$$f_{U_2}(u_2|\boldsymbol{\lambda}_i) = P[e_{0i''} \neq 0|\boldsymbol{\lambda}_i]N\left(0, 4\sigma^2_{\gamma_{i,i''}}\right) + \left(1 - P[e_{0i''} \neq 0|\boldsymbol{\lambda}_i]\right)\delta(u_2). \tag{36}$$

Again, $e_{0i'}$ and $e_{0i''}$ are assumed to be independent. Thus, u_1 and u_2 are independent, and given the independence among η and u_1 and u_2, the pdf of their sum can be obtained by convolving their individual pdf's. Thus, the conditional BEP after one stage of PIC detector, with CI codewords and $L = 2$, is given by

$$
\begin{aligned}
P[e|\boldsymbol{\lambda}_i] = & P[e_{0i'} \neq 0|\boldsymbol{\lambda}_i] P[e_{0i''} \neq 0|\boldsymbol{\lambda}_i] \\
& \cdot \left\{ Q\left(\frac{\sum_{k=0}^{N-1} |\boldsymbol{\lambda}_i[k]|^2}{\sqrt{4\left(\sigma^2_{\gamma_{i,i'}} + \sigma^2_{\gamma_{i,i''}}\right) + \sigma_n^2}} \right) \right\} \\
& + P[e_{0i'} \neq 0|\boldsymbol{\lambda}_i] \left(1 - P[e_{0i''} \neq 0|\boldsymbol{\lambda}_i]\right) \\
& \cdot \left\{ Q\left(\frac{\sum_{k=0}^{N-1} |\lambda_i[k]|^2}{\sqrt{4\sigma^2_{\gamma_{i,i'}} + \sigma_n^2}} \right) \right\} \\
& + \left(1 - P[e_{0i'} \neq 0|\boldsymbol{\lambda}_i]\right) P[e_{0i''} \neq 0|\boldsymbol{\lambda}_1] \\
& \cdot \left\{ Q\left(\frac{\sum_{k=0}^{N-1} |\lambda_i[k]|^2}{\sqrt{4\sigma^2_{\gamma_{i,i''}} + \sigma_n^2}} \right) \right\} \\
& + \left(1 - P[e_{0i'} \neq 0|\boldsymbol{\lambda}_i]\right) \left(1 - P[e_{0i''} \neq 0|\boldsymbol{\lambda}_i]\right) \\
& \cdot \left\{ Q\left(\frac{\sum_{k=0}^{N-1} |\lambda_i[k]|^2}{\sigma_n} \right) \right\}. \qquad (37)
\end{aligned}
$$

Now, we extend the aforementioned results to any number I of interferers and any set of codewords. Letting the set of all interfering users' indices for user i be $\zeta_i = \{\zeta_i[1], \zeta_i[2], \ldots, \zeta_i[I]\}$, then we have

$$
\begin{aligned}
\hat{x}_{1i} = x_i \sum_{k=0}^{N-1} |\lambda_i[k]|^2 + e_{0\zeta_i[1]}\gamma_{i,\zeta_i[1]} \\
+ e_{0\zeta_i[2]}\gamma_{i,\zeta_i[2]} + \cdots + e_{0\zeta_i[I]}\gamma_{i,\zeta_i[I]} + \hat{n}_i \qquad (38)
\end{aligned}
$$

and the conditional BEP can be obtained as the equation shown at the bottom of the page, and

$$
P[e] = \int_0^{\infty} P[e|\boldsymbol{\lambda}_i] P[\boldsymbol{\lambda}_i]\, d\boldsymbol{\lambda}_i. \qquad (39)
$$

The analysis of the performance of multiple-stage PIC is even more complicated than the performance analysis given in this section. In Section VII, we shall evaluate the performance of multiple-stage PIC by computer simulation.

V. Complexity Reduction in Maximum Likelihood Multiuser Detection

We consider the ML-detection technique based on the received signal given in (4) in this section. Again, it is assumed that the receiver has the perfect knowledge of channel coefficients and CFO values. We will separately examine the multipath and CFO effects. We will show in this section how the MAI-free property of CI codes can significantly reduce the complexity of the ML-MUD receiver.

A. ML-MUD in a Multipath Fading Channel

For given transmitted signal $\mathbf{x}$, we would like to maximize the likelihood of the received signal. From (4), and since noise $\mathbf{n}$ is Gaussian, the ML estimate can be written as

$$
\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\hat{\mathbf{y}} - \mathbf{C}\mathbf{x}\|^2. \qquad (40)
$$

By expanding the right-hand side of (40) and noting that $\|\hat{\mathbf{y}}\|^2$ is independent of $\mathbf{x}$, we can reformulate the optimization problem as $\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \Omega(\mathbf{x})$, where

$$
\Omega(\mathbf{x}) = \|\mathbf{C}\mathbf{x}\|^2 - 2\Re\left\{\langle \mathbf{C}\mathbf{x}, \hat{\mathbf{y}}\rangle\right\}
$$

$\langle .,. \rangle$ is the inner product of two vectors, and $\mathbf{C}$ is defined in (5) in a CFO environment. If there is no CFO, (5) can be written as

$$
\widetilde{\mathbf{C}}(k,j) = w_j[k]\lambda_j[k]. \qquad (41)
$$

Thus, we have

$$
\langle \widetilde{\mathbf{C}}\mathbf{x}, \hat{\mathbf{y}}\rangle = \mathbf{x}^\dagger \widetilde{\mathbf{C}}^\dagger \hat{\mathbf{y}} = \sum_{j=0}^{K-1} x_j \sum_{k=0}^{N-1} \hat{y}_j[k] w_j^*[k] \lambda_j^*[k].
$$

Note that $\sum_{k=0}^{N-1} \hat{y}_j[k] w_j^*[k] \lambda_j^*[k]$ is actually the estimate of the input signal for user j obtained by MRC (i.e., $\hat{z}_j$). On the other hand, $\|\widetilde{\mathbf{C}}\mathbf{x}\|^2 = \mathbf{x}^\dagger \widetilde{\mathbf{H}} \mathbf{x}$, where $\widetilde{\mathbf{H}} = \widetilde{\mathbf{C}}^\dagger \widetilde{\mathbf{C}}$. Then, the ML optimization problem is equivalent to minimizing

$$
\Omega(\mathbf{x}) = \mathbf{x}^\dagger \widetilde{\mathbf{H}} \mathbf{x} - 2\Re\left\{(\mathbf{x}^\dagger, \hat{\mathbf{z}})\right\}
$$

with respect to $\mathbf{x}$, where $\hat{\mathbf{z}}$ is the output vector of MRC with its ith element given in (8). We denote the MAI from user j to the desired user i without CFO by $\widetilde{\mathrm{MAI}}_{i\leftarrow j}$. Then, we have from (11) that

$$
\widetilde{\mathrm{MAI}}_{i\leftarrow j} = x_j \sum_{k=0}^{N-1} \lambda_j[k] w_j[k] \lambda_i^*[k] w_i^*[k].
$$

It can easily be shown that $\widetilde{\mathbf{H}}(i,j) = \widetilde{\mathrm{MAI}}_{i\leftarrow j}/x_j$. In fact, $\widetilde{\mathbf{H}}$ can be viewed as the cross-correlation channel matrix.

$$
\begin{aligned}
P[e|\boldsymbol{\lambda}_i] = & \sum_{r_1=0}^{1} \sum_{r_2=0}^{1} \cdots \sum_{r_{\zeta_i[I]}=0}^{1} P\left[e_{0\zeta_i[1]} \neq 0|\boldsymbol{\lambda}_i\right]^{r_1} \cdots P\left[e_{0\zeta_i[I]} \neq 0|\boldsymbol{\lambda}_i\right]^{r_{\zeta_i[I]}} \left(1 - P\left[e_{0\zeta_i[1]} \neq 0|\boldsymbol{\lambda}_i\right]\right)^{1-r_1} \cdots \\
& \times \left(1 - P\left[e_{0\zeta_i[I]} \neq 0|\boldsymbol{\lambda}_i\right]\right)^{1-r_{\zeta_i[I]}} Q\left(\frac{\sum_{k=0}^{N-1} |\boldsymbol{\lambda}_i[k]|^2}{\sqrt{4\left(r_1\sigma^2_{\gamma_{i,\zeta_i[1]}} + \cdots + r_{\zeta_i[I]}\sigma^2_{\gamma_{i,\zeta_i[I]}}\right) + \sigma_n^2}} \right)
\end{aligned}
$$

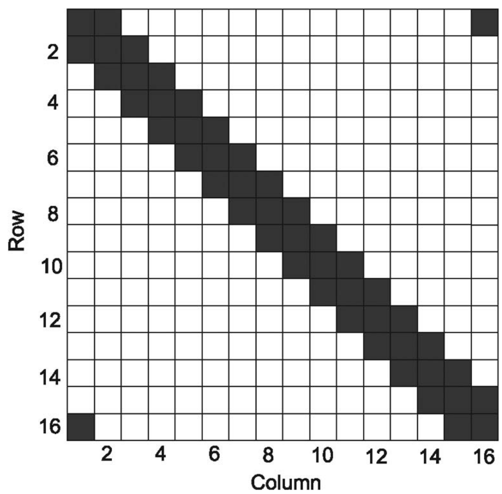

Fig. 3. Cross-correlation matrix of CI codewords with $N = K = 16$ and $L = 2$.

B. ML-MUD in a Multipath Fading Channel With CFO

We rewrite (8) in vector format as

$$\hat{\mathbf{z}} = \mathbf{H}\mathbf{x} + \hat{\mathbf{n}} \tag{42}$$

where the (i, j)th entry of $\mathbf{H}$ is equal to

$$\mathbf{H}(i, j) = \begin{cases} \sum_{k=0}^{N-1} |\lambda_i[k]|^2, & i = j \\ \frac{\mathrm{MAI}_{i \leftarrow j}}{x_j}, & i \neq j. \end{cases}$$

The ML detector has the following form:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \Omega(\mathbf{x}) \tag{43}$$

where

$$\Omega(\mathbf{x}) = \|\hat{\mathbf{z}} - \mathbf{H}\mathbf{x}\|^2 = \sum_{i=0}^{K-1} |\hat{z}_i - \mathbf{h}_i \mathbf{x}|^2 \tag{44}$$

and where $\mathbf{h}_i$ is the ith row of $\mathbf{H}$.

C. VA for the TBT

We know from Theorem 2 that, for $K = N$ active users with CI codewords and $L \leq N/2$, each user has only $2(L-1)$ (instead of $N-1$) interfering users. Both $\mathbf{H}$ and $\widetilde{\mathbf{H}}$ are sparse matrices so that ML-MUD can be performed with a much lower complexity. As shown in Fig. 3, for $N = 16$, $K = 16$, and $L = 2$, the nonzero elements (indicated by black squares) of $\mathbf{H}$ (or $\widetilde{\mathbf{H}}$ for the case without CFO) are concentrated along the three diagonal lines. Elements in the off-diagonal region with $|i - j| \geq L$ are all equal to zero, except for two corners.

The well-known Viterbi algorithm (VA) can be used to solve the ML optimization problem. Generally speaking, its complexity is proportional to the number of states. It turns out that, regardless of how we define the states, the complexity of ML-MUD is $O(2^K)$ for a general nonsparse cross-correlation channel matrix and BPSK modulation. On the other hand, by exploiting the sparsity of $\mathbf{H}$, we can show that the complexity of the VA exponentially grows with $2L - 1$, as explained in the succeeding discussion.

The structure of $\mathbf{H}$ (or $\widetilde{\mathbf{H}}$ for the case without CFO) implies a trellis that is defined on a circulant time axis (or called the TBT [5]). The TBT was defined and discussed for error-correcting codes in [5]. The TBT also arises in the context of ML detection in overloaded array processing [11]. A method for trellis construction for a similar matrix structure was proposed in [11], as explained in the following discussion.

We assume that $K = N$ and denote the state of the trellis at stage i and the state space at the ith stage by $s[i]$ and S_i, respectively. We use $V[i]$ to denote the column indices of nonzero elements on the ith row of $\mathbf{H}$. The ith state is defined as [11]

$$s[i] = \{x_u | u \in V[((i-1))_N] \cap V[i]\}. \tag{45}$$

Using the preceding definition, we obtain

$$\begin{aligned} s[i] \cup s[((i+1))_N] = &\{V[((i-1))_N] \cap V[i]\} \\ &\cup \{V[i] \cap V[((i+1))_N]\} \\ = &V[i]. \end{aligned} \tag{46}$$

In other words, the state sequence $\{s[i]\}$ for the TBT is defined such that, during the ith stage of the VA recursion, $V[i]$ corresponds to symbol indices in both $s[i]$ and $s[i+1]$. From the sequence of states defined by (45), we can construct the trellis by listing state values at stage i and connect the valid state transition from stage i to stage $i+1$. Fig. 4 shows the trellis for the example in Fig. 3. Some approximate ML algorithms with less complexity and satisfactory results for decoding the TBT were discussed in [5] and [11]. In particular, a less-complicated approximate ML algorithm, called the iterative tail-biting VA (ITB-VA), was proposed in [11] that applies the VA iteratively around a TBT multiple times without excluding paths that are not closed. This approach is taken in our work.

We define $N_{\mathrm{stgs}} = \lceil N_{\mathrm{round}} N \rceil$, where $N_{\mathrm{round}} > 1$ is a real number. N_{round} and N_{stgs} are in fact the desired numbers of iterations and stages around the TBT, respectively. After going around the TBT N_{round} times, the optimum path is chosen, and the estimated sequence is translated into a sequence of symbol estimates $\hat{x}_0, \ldots, \hat{x}_{N-1}$. To do this, we recall that the VA has the property that all surviving paths merge after a certain stage Δ [6]. Thus, the optimum surviving path at the ith stage is used to estimate the $(i - \Delta)$th information bits, where Δ is called the *traceback depth* or the *truncation length* [6]. A more detailed description of the ITB-VA can be found in [11].

The number of state transitions per stage determines the complexity of the VA. By assuming that all components of sparse matrix $\mathbf{H}$ (or $\widetilde{\mathbf{H}}$) can be precomputed and their computational complexity is negligible, as compared with the complexity of the ITB-VA, the complexity of the ITB-VA for CI-MC-CDMA and BPSK modulation is $O(2^{2L-1})$ at each stage. Since there are $N_{\mathrm{round}} N$ stages, where N_{round} is usually less than 2 [5], the total complexity for all stages is $O(N_{\mathrm{round}} N 2^{2L-1})$, which is far less than the complexity of a conventional ML-MUD technique [i.e., $O(N 2^N)$].

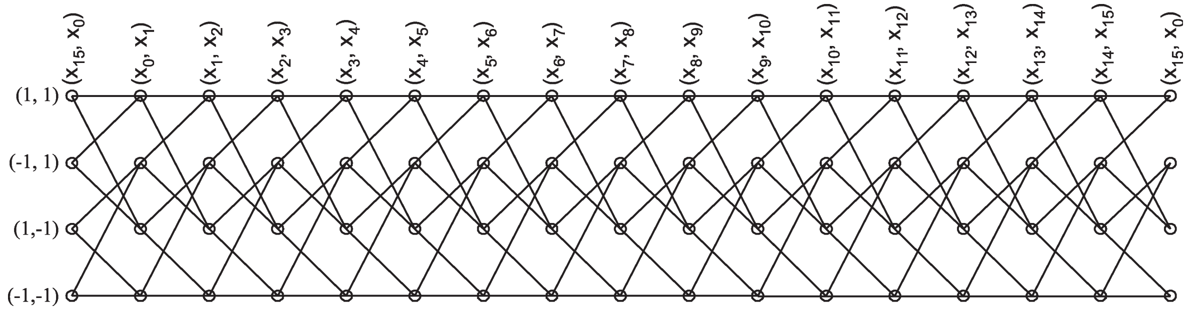

Fig. 4. TBT for the case with $N = K = 16$ and $L = 2$.

D. Upper Bound on the Minimum Error Probability

It is difficult to obtain a closed-form solution of the minimum error probability for MC-CDMA. However, we can derive its upper bound for BPSK transmitted symbols. We take a similar approach to the procedure for synchronous CDMA in the AWGN channel [22] and the fading channel [23] and extend it to synchronous MC-CDMA in a CFO environment. We define E_i to be the set of error vectors that affect the ith user in the form of $E_i = \{\boldsymbol{e} \in \{-1,0,1\}^K, e_i \neq 0\}$, where $e_i = x_i - \hat{x}_i$. The set of *admissible* error vectors that are compatible with the transmitted vector $\mathbf{x} \in \{-1,1\}^K$ is denoted by $A(\mathbf{x}) = \{\boldsymbol{e} \in E, e_i = x_i \text{ or } 0\} = \{\boldsymbol{e} \in E, 2\boldsymbol{e} - \mathbf{x} \in \{-1,1\}^K\}$, where $E = \cup_{i=1}^K E_i$ is the set of nonzero error vectors. We can upper bound the error probability for user i, which is denoted by $P_i(e)$, in the AWGN channel by

$$P_i(e) \leq \sum_{\mathbf{e}\in E_i} P\left\{\Omega(\mathbf{x}-2\boldsymbol{e}) \leq \Omega(\mathbf{x}), \boldsymbol{e} \in A_i(\mathbf{x})\right\} \tag{47}$$

where $A_i(\mathbf{x}) = A(\mathbf{x}) \cap E_i$, and we have used the fact that if $\mathbf{x} - 2\boldsymbol{e}$ is the most likely vector, it is more likely than $\mathbf{x}$. It can easily be shown that when no CFO is present, we have

$$\begin{aligned}\Omega(\mathbf{x}-2\boldsymbol{e})-\Omega(\mathbf{x}) &= 4\Re\{\boldsymbol{e}^T\hat{\mathbf{x}}\} + 4\boldsymbol{e}^T\widetilde{\mathbf{H}}\boldsymbol{e} - 2\mathbf{x}^T\widetilde{\mathbf{H}}\boldsymbol{e} - 2\boldsymbol{e}^T\widetilde{\mathbf{H}}\mathbf{x}\\ &= 4\boldsymbol{e}^T\widetilde{\mathbf{H}}\boldsymbol{e} + 4\Re\{\boldsymbol{e}^T\hat{\mathbf{n}}\}\end{aligned}$$

where the second equality is true since $\hat{\mathbf{z}} = \widetilde{\mathbf{H}}\mathbf{x} + \hat{\mathbf{n}}$. We can see that this event is dependent on noise $\hat{\mathbf{n}}$ only while $\boldsymbol{e} \in \mathbf{A}(\mathbf{x})$ depends on $\mathbf{x}$ only. Thus, we conclude that these two events are independent. Extending (47) to the fading channel, we can express the error probability as

$$P_i(e|\widetilde{\mathbf{H}}) \leq \sum_{\mathbf{e}\in E_i} P\{\Omega(\mathbf{x}-2\boldsymbol{e})-\Omega(\mathbf{x}) \leq 0|\widetilde{\mathbf{H}}\} \cdot \{P\{\boldsymbol{e} \in \mathbf{A}(\mathbf{x})\},\}$$

which follows from the fact that the admissibility of $\boldsymbol{e}$ is independent of $\widetilde{\mathbf{H}}$. For equally likely transmitted bits, we have

$$P\left\{\boldsymbol{e} \in \mathbf{A}(\mathbf{x})\right\} = \prod_{i=0}^{K-1} P\left\{(x_i - e_i)e_i = 0\right\} = 2^{-w(e)}$$

where $w(e) = \sum_{i=1}^K |e_i|$. To compute $P\{\Omega(\mathbf{x}-2\boldsymbol{e}) - \Omega(\mathbf{x}) \leq 0|\widetilde{\mathbf{H}}\}$, we note that, since $\hat{\mathbf{n}}$ is a proper (circularly symmetric) complex Gaussian random vector with zero mean and covariance matrix $\sigma^2\widetilde{\mathbf{H}}$, $\mathbb{E}\{(\Re\{\boldsymbol{e}^T\hat{\mathbf{n}}\})^2\} = (1/2)\mathbb{E}\{\boldsymbol{e}^T\hat{\mathbf{n}}\hat{\mathbf{n}}^\dagger\boldsymbol{e}\} = (1/2)\sigma^2\boldsymbol{e}^T\widetilde{\mathbf{H}}\boldsymbol{e}$. Thus, the error probability for user i is bounded by

$$P_i(e|\widetilde{\mathbf{H}}) \leq \sum_{\mathbf{e}\in E_i} 2^{-w(e)} Q\left(\frac{\sqrt{2\boldsymbol{e}^T\widetilde{\mathbf{H}}\mathbf{e}}}{\sigma}\right). \tag{48}$$

When CFO is present, $\Omega(\mathbf{x}-2\boldsymbol{e}) - \Omega(\mathbf{x}) = 4\boldsymbol{e}^T\mathbf{H}^\dagger\mathbf{H}\boldsymbol{e} + 4\Re\{\boldsymbol{e}^T\mathbf{H}^\dagger\hat{\mathbf{n}}\}$. By taking a similar approach, we can show that the upper bound for the error probability with CFO is

$$P_i(e|\mathbf{H}) \leq \sum_{\mathbf{e}\in E_i} 2^{-w(e)} Q\left(\frac{\sqrt{2}\boldsymbol{e}^T\mathbf{H}^\dagger\mathbf{H}\mathbf{e}}{\sigma\sqrt{\boldsymbol{e}^T\mathbf{H}^\dagger\widetilde{\mathbf{H}}\mathbf{H}\mathbf{e}}}\right). \tag{49}$$

The unconditional BEP for user i can be obtained by

$$P_i(e) = \int_0^\infty P_i(e|\mathbf{H})P\{\mathbf{H}\}\, d\mathbf{H}. \tag{50}$$

This integral can be calculated using the Monte Carlo method [18].

VI. Complexity Reduction in Decorrelating Multiuser Detection

In this section, we consider the complexity reduction for the decorrelating multiuser detector using the CI codes. From Fig. 3, we can see that the channel matrix can be converted to a band matrix if the corner values are reduced to zero. A band matrix is a matrix whose nonzero elements are confined to a diagonal band comprising the main diagonal and several subdiagonals. For band matrix $\mathbf{A}$ with $\mathbf{A}(i,j) = 0$ if $i - j > m_l$ and $j - i > m_u$, integers m_l and m_u are called the lower and upper bandwidths, respectively, and $m = m_l + m_u + 1$ is the total bandwidth.

Channel matrix $\mathbf{H}$ (or $\widetilde{\mathbf{H}}$ for the case without CFO) can be converted to a band matrix by reducing the number of users to $N - (L-1)$ and employing CI codes $w_i[k] = e^{j2\pi ki/N}$, $k = 0, 1, \ldots, N-1$, with the set of indices i, which is either $\{0, 1, \ldots, N-L\}$ or $\{L, L+1, \ldots, N\}$. The bandwidth of the resulting band matrix is $L - 1 + L - 1 + 1 = 2L - 1$. To give an example, for a channel of length $L = 3$, its cross-correlation matrix with $N = 16$ can be transformed into a band matrix of size $N - (L-1) = 14$ and bandwidth $2L - 1 = 5$, as shown in Fig. 5, by omitting the first two CI codes, i.e., $w_0[k] = e^{j2\pi 0k/16}$ and $w_1[k] = e^{j2\pi 1k/16}$, $k = 0, 1, \ldots, 15$.

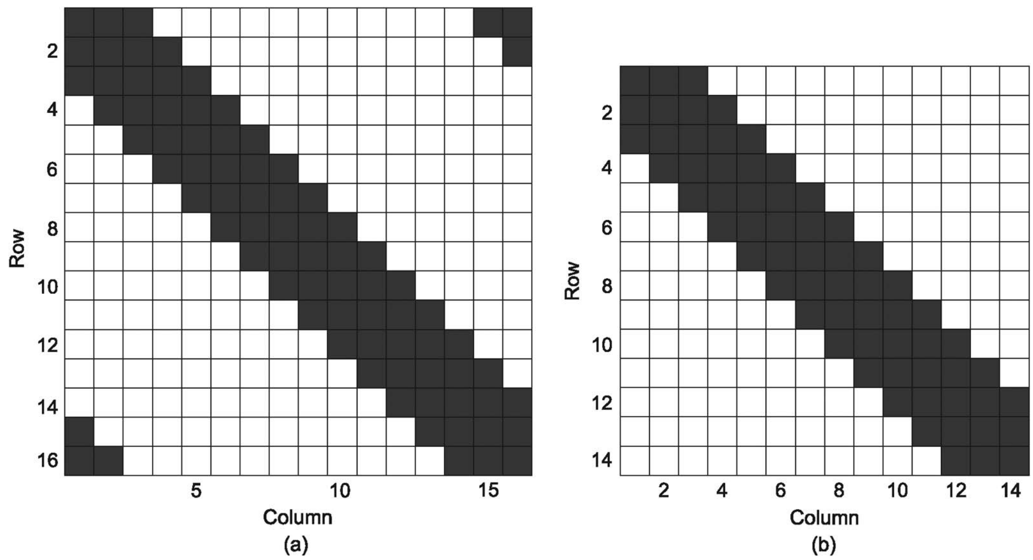

Fig. 5. Conversion of a channel matrix into a band matrix with $N = 16$ and $L = 3$ by reducing the user number to 14.

Band matrices are usually stored by recording only the diagonal entries in the band. Therefore, a band matrix system can be solved by LU decomposition faster and with less storage space than a general dense matrix of the same dimension. Consider an MC-CDMA system with a spreading gain of N and $N_b = N - (L - 1)$ users employing codewords with indices $i = L, L + 1, \ldots, N$ (or $0, 1, \ldots, N - L$) in a CFO environment. By using MRC in the receiver and disregarding the additive noise vector $\hat{\mathbf{n}}$ in (42), the received signal $\hat{\mathbf{z}}$ becomes $\hat{\mathbf{z}} = \mathbf{H}_b\mathbf{x}$, where $\mathbf{H}_b$ is the corresponding cross-correlation matrix. The preceding equation is in fact a linear system equation involving complex band matrix $\mathbf{H}_b$, which can be solved by the Gaussian elimination algorithm with partial pivoting.

The Gaussian elimination algorithm first factors $\mathbf{H}_b$ into the product of an upper triangular matrix $\mathbf{U}$ and a lower triangular matrix $\mathbf{L}$, namely, $\mathbf{H}_b = \mathbf{LU}$. Next, the solution of the system $\mathbf{H}_b\mathbf{x} = \hat{\mathbf{z}}$ can be rewritten by $\mathbf{L}(\mathbf{Ux}) = \hat{\mathbf{z}}$, which demands forward and backward substitutions. The total number of operations required to solve $\mathbf{H}_b\mathbf{x} = \hat{\mathbf{z}}$ depends on the number of pivoting required. Generally speaking, if $N_b \gg m_l + m_u$, the number of operations required by the factorization in $\mathbf{H}_b = \mathbf{LU}$ is $O(N_b m_l(m_l + m_u))$, whereas the number of operations required by solving $\mathbf{x}$ in $\mathbf{L}(\mathbf{Ux}) = \hat{\mathbf{z}}$ by forward/backward substitutions is about $O(N_b(2m_l + m_u))$ [8].

Note that, for the cross-correlation matrix $\mathbf{H}_b$ of our interest, the lower and upper bandwidths are both equal to $L - 1$. Hence, the complexity of the factorization process and the solution process is equal to $O(2(N - L + 1)(L - 1)^2)$ and $O(3(N - L + 1)(L - 1))$, respectively. In contrast, to solve a general linear system of equations with Gaussian elimination with a dense matrix of size $(N - L + 1) \times (N - L + 1)$, it demands $O((N - L + 1)^3/3)$ for the LU factorization and $O((N - L + 1)^2)$ for forward and backward substitutions. The complexity of matrix inversion with fast algorithms is $O((N - L + 1)^2)$. Thus, the complexity of the decorrelating MUD technique for MC-CDMA has considerably been reduced with CI codes in practical channel scenarios, where $N \gg 2(L - 1)^2$.

In the absence of CFO, we denote the cross-correlation matrix by $\widetilde{\mathbf{H}}_b$, where $\widetilde{\mathbf{H}}_b = \widetilde{\mathbf{C}}^\dagger\widetilde{\mathbf{C}}$, and $\widetilde{\mathbf{C}}(k, j)$ is given by (41). $\widetilde{\mathbf{H}}_b$ is a Hermitian positive definite banded matrix with $m = m_l = m_u = L - 1$, for which there is an even faster Gaussian-elimination algorithm for solving the linear system. The total number of operations is approximately equal to $O((N - L + 1)(L - 1)^2/2 - (L - 1)^3/3)$ for the LU factorization and $O(2(N - L + 1)(L - 1) - (L - 1)^2)$ for the forward and backward substitutions [8]. In contrast, the complexity of solving a general dense matrix of the same size is $O((N - L + 1)^3/6)$ for the LU factorization and $O((N - L + 1)^2)$ for the forward and backward substitutions, and the complexity of matrix inversion with fast algorithms is $O((N - L + 1)^2)$.

A. Error Probability for Decorrelating MUD

The detected symbol for the aforementioned MUD technique is given by

$$\hat{\mathbf{x}} = \mathbf{x} + \mathbf{H}_b^{-1}\hat{\mathbf{n}} \tag{51}$$

where $\hat{\mathbf{n}}$ is an $(N - L + 1) \times 1$ Gaussian random vector with zero mean and covariance matrix $\sigma^2\widetilde{\mathbf{H}}$. Hence, $\mathbb{E}\{\mathbf{H}_b^{-1}\hat{\mathbf{n}}\mathbf{H}_b^{-1}\hat{\mathbf{n}}^\dagger\} = \mathbb{E}\{\mathbf{H}_b^{-1}\hat{\mathbf{n}}\hat{\mathbf{n}}^\dagger(\mathbf{H}_b^{-1})^\dagger\} = \sigma^2\mathbf{H}_b^{-1}\widetilde{\mathbf{H}}(\mathbf{H}_b^{-1})^\dagger$. Let $\mathbf{H}_n = \mathbf{H}_b^{-1}\widetilde{\mathbf{H}}(\mathbf{H}_b^{-1})^\dagger$. Since $\hat{\mathbf{n}}$ is a proper complex random vector, $\mathbb{E}\{\Re\{\hat{\mathbf{n}}\hat{\mathbf{n}}^\dagger\}\} = (1/2)\mathbb{E}\{\hat{\mathbf{n}}\hat{\mathbf{n}}^\dagger\} = (1/2)\sigma^2\mathbf{H}_n$. Then, under BPSK modulation, the BEP for the ith user is equal to

$$P_i(e) = \mathbb{E}\left\{Q\left(\sqrt{\frac{2}{\sigma^2[\mathbf{H}_n]_{i,i}}}\right)\right\}.$$

VII. Simulation Results

The Monte Carlo simulation was conducted to corroborate the theoretical results derived in the previous sections. In the simulation, channel taps were generated as independent identically distributed random variables with zero mean and

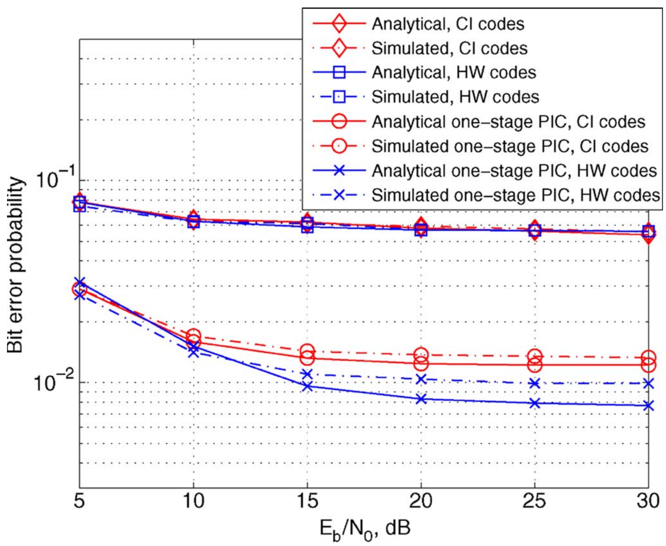

Fig. 6. Analytical and simulated BEP results versus E_b/N_0 with $N = 16$, $L = 2$, and CFO $= \pm 0.1$.

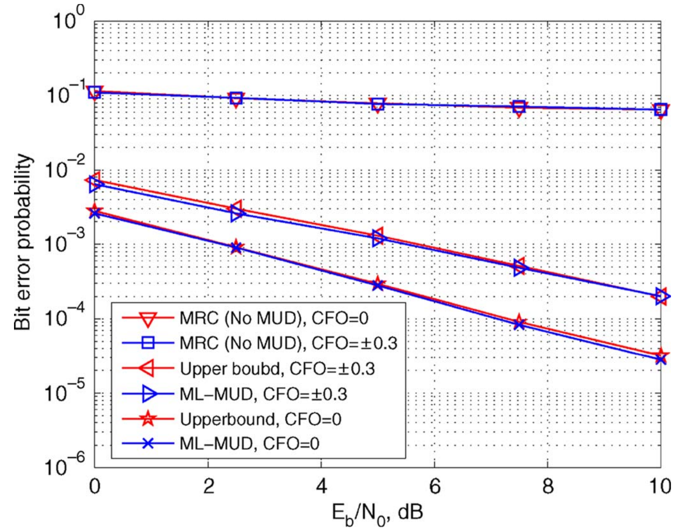

Fig. 7. Upper bound and simulation of the BEP for $N = 8$, $L = 2$, and CFO $= 0, \pm 0.3$.

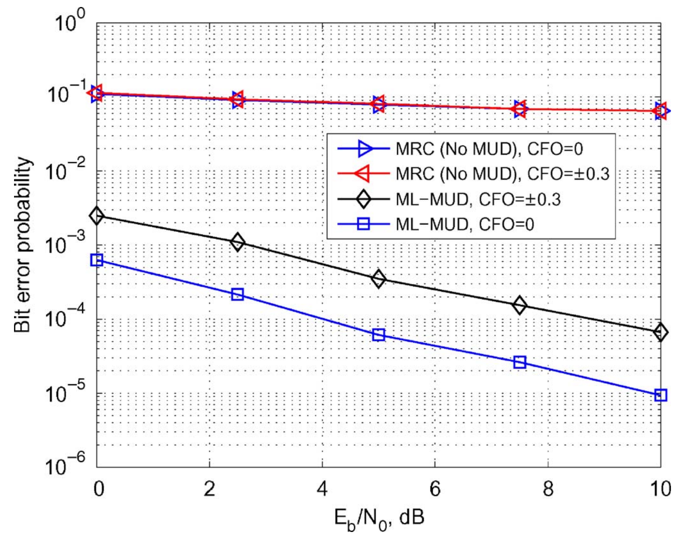

Fig. 8. ML-MUD BEP performance versus E_b/N_0 with $N = 16$, $L = 2$, and CFO $= \pm 0.3$.

unit variance. Every user had his/her own CFO value. To compute the analytical BEP, the Monte Carlo integration method was used [18]. That is, random variables $\lambda_i[k]$, $k = 0, 1, \ldots, N-1$ are generated by taking the DFT of complex Gaussian-distributed channel taps M times. Then, computer-generated samples of $\lambda_i[k]$, $k = 0, 1, \ldots, N-1$ are substituted in the Q-function, and the sum of trials is divided by M.

Example 1: Theoretical Versus Simulated BEP of Fully Loaded MC-CDMA With Single-Stage PIC: In this example, we evaluate the BEP performance of MC-CDMA with PIC in the presence of CFO and examine both analytical and simulated BEP results. We employ two orthogonal codes, namely, orthogonal HW codes and CI codes. Fig. 6 depicts the analytical and simulated BEP results for a fully loaded MC-CDMA system with CI and HW codewords as a function of the SNR value E_b/N_0 under the setting of $N = 16$, $L = 2$, $K = 16$, and CFO $= \pm 0.1$. To shorten the simulation time, only the BEP results for the first users were compared. Since $L = 2$, we can use (37) as the analytical BEP expressions of CI-MC-CDMA. For this case, we observe close agreement between the analytical BEP expression and simulation results. For MC-CDMA with HW codes, the approximate BEP expression, as shown in (31), was used. Due to a higher number of interfering users employing HW codes, the analytical Gaussian model for the total residual user interference can be used. However, the analytical and simulation results for HW codes (the last two curves in the figure) do not have strong agreement, particularly in the high-SNR regime, as shown in Fig. 6.

Example 2: ML-MUD BEP Versus Minimum Probability of Error: Fig. 7 shows the upper bound to the BEP as a function of the SNR value E_b/N_0 for CI-MC-CDMA under the setting of $N = 8$, $L = 2$, and $K = 8$ for both zero CFO and CFO $= \pm 0.3$ cases. The upper bound curves in each case are plotted against their corresponding simulated BEP. To obtain the simulated BEP, $N_{\text{round}} = 1.5$ and $\Delta = 2$ for zero CFO, and $N_{\text{round}} = 1.5$ and $\Delta = 3$ for nonzero CFO values. To shorten the simulation and computation time, only the BEP for the first user was computed. We can see close agreement between the performance of the ML-MUD BEP and the upper bound for the minimum BEP. It is also clear from the figure that ML-MUD performs better in the absence of CFO than in the presence of CFO. This can be explained by noting the fact that the denominator in (49) for the upper bound on the minimum BEP with CFO is $\sqrt{e^T \mathbf{H}^\dagger \widetilde{\mathbf{H}} \mathbf{H} \mathbf{e} \sigma}$, as opposed to just σ in the denominator of (48) for the upper bound on the minimum BEP with no CFO.

Example 3—ML-MUD Performance: Fig. 8 shows a significant performance improvement of the ML detector, where the performance of CI-MC-CDMA with ML-MUD is compared with CI-MC-CDMA with single-user MRC detection. The parameters for the simulated CI-MC-CDMA system were $N = 16$, $L = 2$, and $K = 16$. As compared with Fig. 7 with $N = 8$, we can see that ML-MUD performs better since there were more pairwise MAI-free users. Separate simulations were performed to acquire the BEP performance for CFO $= 0$ and CFO $= \pm 0.3$. We can see that the BEP achieved by ML for both systems is very low when the SNR is close to 10 dB. Again,

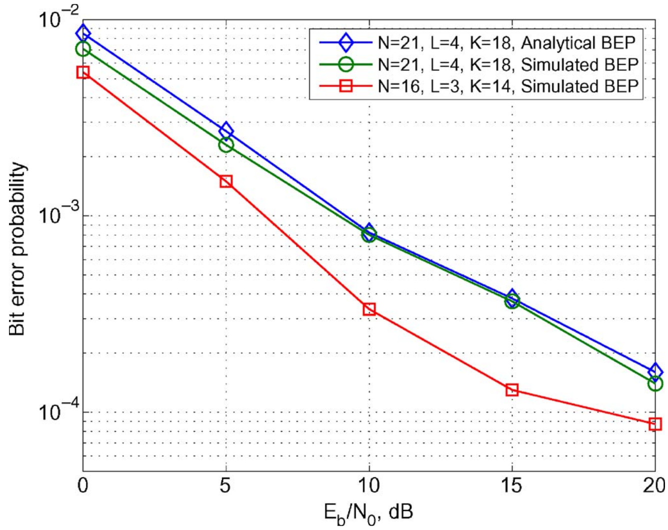

Fig. 9. Analytical and simulated BER performance of the decorrelating detector with the reduced-complexity Gaussian elimination algorithm for MC-CDMA with CFO $= \pm 0.5$.

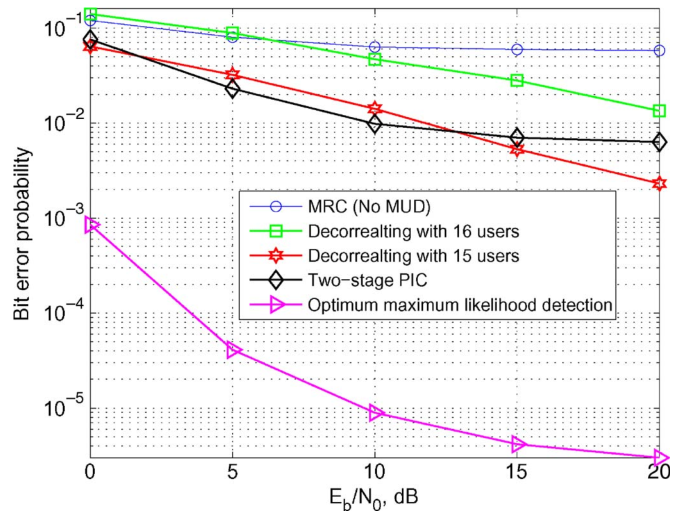

Fig. 10. Comparison of various MUD techniques for $N = 16$, $L = 2$, and CFO $= 0$.

we can see that the performance of ML-MUD is better with no CFO that in the presence of CFO.

Example 4—Decorrelating Detector Performance: Fig. 9 shows the theoretical and simulated BEP results as a function of the SNR value E_b/N_0 in the presence of CFO $= \pm 0.5$ with a decorrelating detector with $N = 21$, $L = 4$, and $K = 18$. To shorten the simulation time, only the BEP for the first user was computed. We can see that simulated and analytical BEP results are in good agreement. The average BEP performance of CI-MC-CDMA and with $N = 16$, $L = 3$, and $K = 14$ is also shown in Fig. 9. We can see that, as the number of users increases from 14 to 18, the BEP performance degrades up to 2.5 dB for $E_b/N_0 < 8$ dB and around 5 dB for $E_b/N_0 > 8$ dB.

Example 5: Performance Comparison of PIC, ML, and Decorrelating Detectors: Fig. 10 compares the BEP performance of CI-MC-CDMA employing second-stage PIC, ML, and decorrelating detectors with the MRC detector as the benchmark for $N = K = 16$, $L = 2$, and CFO $= 0$. As expected, the optimum ML-MUD detector greatly outperforms all other detectors. We also observe that the decorrelating detector with $N - L + 1 = 15$ users outperforms the second-stage PIC detector when $E_b/N_0 > 15$ dB.

It is worthwhile to mention that we concentrated on the performance of CI-MC-CDMA with the simplified multiuser detectors in this work. Although we observed in Fig. 6 that the PIC detectors with CI and HW codes have comparable performance, the simulation results were presented in this section to demonstrate that CI-MC-CDMA with the simplified MUD techniques can successfully suppress MAI, as compared with single-user detection. The performance of MC-CDMA with other codes was compared with CI and HW codes in the previous literature. Particularly, in [17], CI-MC-CDMA was compared with HW, Gold, and quadriphase codes and found to have a better BEP performance for all users. In addition, the BEP performance of MC-CDMA with a special form of HW codes was compared with Shi and Latvaaho codes in [19]. It was shown that the proposed scheme can significantly outperform Shi and Latvaaho codes.

VIII. Conclusion

We have shown in this paper that, for an MC-CDMA system with spread gain N, multipath length L, and N users, when CI codes are used, a proper subset of CI codes leads to a completely MAI-free MC-CDMA system in a CFO environment. We proved that each user only has to combat $2(L - 1)$ (rather than $N - 1$) interferers, even in the presence of CFO. We analyzed the BEP of MC-CDMA in a CFO environment with PIC, ML, and decorrelating multiuser detectors. We also demonstrated that the sparse cross-correlation matrix of the CI codes can be used to considerably reduce the complexity of the aforementioned multiuser detectors. Finally, simulation results were given to corroborate derived theoretical results.

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