Asymptotic Critical Transmission Radius for k-Connectivity in Wireless *Ad Hoc* Networks

Peng-Jun Wan, Chih-Wei Yi, *Member, IEEE*, and Lixin Wang

***Abstract*—A range assignment to the nodes in a wireless *ad hoc* network induces a topology in which there is an edge between two nodes if and only if both of them are within each other's transmission range. The critical transmission radius for k-connectivity is the smallest r such that if all nodes have the transmission radius r, the induced topology is k-connected. In this paper, we study the asymptotic critical transmission radius for k -connectivity in a wireless *ad hoc* network whose nodes are uniformly and independently distributed in a unit-area square or disk. We provide a precise asymptotic distribution of the critical transmission radius for k-connectivity. In addition, the critical neighbor number for k-connectivity is the smallest integer l such that if every node sets its transmission radius equal to the distance between itself and its l-th nearest neighbor, the induced (symmetric) topology is k-connected. Applying the critical transmission radius for k-connectivity, we can obtain an asymptotic almost sure upper bound on the critical neighbor number for k-connectivity.**

***Index Terms*—Asymptotic distribution, critical neighbor number, critical transmission radius, random geometric graph.**

I. Introduction

LET V be the set of radio nodes in a wireless *ad hoc* network. A range assignment to V specifies a transmission radius to each node v in V. The network topology induced by a range assignment is a graph on V with an edge connecting each pair of nodes whose distance is no more than either of their transmission radii. There are two simple range assignment schemes, uniform range assignments and l-nearest-neighbor range assignments, which are both completely determined by a single parameter. In a uniform range assignment with a parameter $r > 0$, every node has the same transmission radius of r. The network topology induced by this range assignment, denoted by $G_r(V)$, is the r-graph on V in which each pair of nodes separated by a distance of at most r is connected by an edge. In a l-nearest-neighbor range assignment with an integer parameter $l > 0$, every node sets its transmission radius equal to the distance between itself and its lth nearest neighbor. The network topology induced by this range assignment, denoted by $H_l(V)$, is the *symmetric* l-nearest-neighbor graph on V in which there is an edge between each pair of nodes which are both one of each other's l nearest neighbors.

In general, a range assignment has to ensure that certain topological properties are met by the induced network topology. Two topological properties of interest are k-connectivity and vertex degree at least k. Let κ and δ denote the connectivity and the smallest vertex degree, respectively, of a graph. Then these two properties can be simply represented by $\kappa \geq k$ and $\delta \geq k$, respectively. Both properties are *monotone-increasing*, which means that all supergraphs of a graph with these properties also have these properties as well. For a monotone-increasing topological property Q, the *critical* (or *hitting*) *transmission radius*, denoted by $\rho(V;Q)$, is the smallest r at which $G_r(V)$ has property Q, and the *critical* (or *hitting*) *neighbor number*, denoted by $\ell(V;Q)$, is the smallest l at which $H_l(V)$ has property Q. Note that $\rho(V;Q)$ is always the distance between some pair of nodes, and $\ell(V;Q)$ is always an integer no more than the size of V. Thus, for those Q which can be tested in polynomial time (such as $\kappa \geq k$ and $\delta \geq k$), both $\rho(V;Q)$ and $\ell(V;Q)$ can be obtained in polynomial time as well.

This paper is concerned with the asymptotic critical transmission radius and critical neighbor number in a random wireless *ad hoc* network. Specifically, the radio devices are represented by a *uniform n-point process* $\mathcal{X}_n$ over a unit-area region Ω, i.e., a set of n independent points each of which is uniformly distributed over Ω. Then both $G_r(\mathcal{X}_n)$ and $H_l(\mathcal{X}_n)$ are random graphs, and both $\rho(\mathcal{X}_n;\kappa \geq k)$ and $\ell(\mathcal{X}_n;\kappa \geq k)$ are random variables. In this paper, the region Ω is assumed to be either a disk or a square. For such Ω, we provide a precise asymptotic distribution of $\rho(\mathcal{X}_n;\kappa \geq k)$ when n goes to infinity. As a corollary, applying the result, we can get an asymptotic almost sure upper bound on $\ell(\mathcal{X}_n;\kappa \geq k)$ for $n \to \infty$.

In what follows, $\|x\|$ is the Euclidean norm of a point $x \in \mathbb{R}^2$, and $|A|$ is shorthand for two-dimensional Lebesgue measure (or area) of a measurable set $A \subset \mathbb{R}^2$. All integrals considered will be Lebesgue integrals. The topological boundary of a set $A \subset \mathbb{R}^2$ is denoted by ∂A. The disk of radius r centered at x is denoted by $D(x,r)$. An event is said to be *asymptotic almost sure* (abbreviated by a.a.s.) if it occurs with a probability converges to one as $n \to \infty$. The symbols $O, o, \sim$ always refer to the limit $n \to \infty$. To avoid trivialities, we tacitly assume n to be sufficiently large if necessary. For simplicity of notation, the dependence of sets and random variables on n will be frequently suppressed.

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P.-J. Wan and L. Wang are with the Department of Computer Science, Illinois Institute of Technology, Chicago, IL 60616 USA (e-mail: wan@cs.iit.edu; wanglix@iit.edu).

C.-W. Yi is with the Department of Computer Science, National Chiao Tung University, Hsinchu City 30010, Taiwan, R.O.C. (e-mail: yi@cs.nctu.edu.tw).

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The remaining of this paper is organized as follows. In Section II, we briefly describe related works. In Section III, we give the precise asymptotic distribution of $\rho(\mathcal{X}_n;\kappa \geq k)$. In Section IV, based on the result of the critical transmission radius, we present an asymptotic almost sure upper bound on $\ell(\mathcal{X}_n;\kappa \geq k)$ as a corollary. Finally, we conclude this paper in Section V.

II. Related Works

Since $\kappa \geq k$ implies that $\delta \geq k, \rho(\mathcal{X}_n;\kappa \geq k)$ is always at least $\rho(\mathcal{X}_n;\delta \geq k)$. A fascinating result proved by Penrose [1], [2] states that they are equal a.a.s. This means when n is big enough, then with high probability, if one starts with isolated points and adds edges connecting the points of $\mathcal{X}_n$ in order of increasing length, then the resulting graph becomes k-connected as soon as the last vertex of degree $k-1$ vanishes. Thus, $\rho(\mathcal{X}_n;\kappa \geq k)$ and $\rho(\mathcal{X}_n;\delta \geq k)$ have the same asymptotic distribution. Although Penrose [1], [2] considered only $\mathcal{X}_n$ over a unit-area square, the same result can be extended to $\mathcal{X}_n$ over a unit-area disk as well with proper modification.

For $k=1$ and $\mathcal{X}_n$ over a unit-area square, the precise asymptotic distribution of $\rho(\mathcal{X}_n;\delta \geq 1)$ has been derived by Dette and Henze [3] much earlier: for any constant c

$$\Pr\left\{\rho(\mathcal{X}_n;\delta \geq 1) \leq \sqrt{\frac{\ln n + c}{\pi n}}\right\} \sim \exp(-e^{-c}).$$

The same asymptotic distribution also holds for $\mathcal{X}_n$ over a unit-area disk. For $k>1$, Penrose [2] presented the following limiting property of $\rho(\mathcal{X}_n;\delta \geq k)$ for Ω being a unit-area square, which also holds for Ω being a unit-area disk.

Theorem 1: [2] Let $k>0$ and $\lambda \in \mathbb{R}$. Then for any sequence $(r_n)_{n\geq 1}$ satisfying

$$\frac{n}{k!}\int_\Omega (n|D(x,r_n)\cap\Omega|)^k e^{-n|D(x,r_n)\cap\Omega|}dx \sim \lambda$$

the probabilities of the two events $\rho(\mathcal{X}_n;\delta \geq k+1) \leq r_n$ and $\rho(\mathcal{X}_n;\kappa \geq k+1) \leq r_n$ both converge to $e^{-\lambda}$ as $n \to \infty$.

A better understanding of Theorem 1 necessitates a brief explanation of the Poissonization technique used by Penrose [2] for the proof. Let $\mathcal{P}_n$ denote a homogeneous Poisson process of intensity n on Ω. Recall that $\mathcal{P}_n$ is characterized by the following property: if $A_1, A_2, \ldots, A_m$ are arbitrarily disjoint regions of Ω, then the numbers of points in $\mathcal{P}_n$ on $A_1, A_2, \ldots, A_m$ are mutually *independent* Poisson random variables with intensity $n|A_1|, n|A_2|, \ldots, n|A_m|$, respectively. The relevance of $\mathcal{P}_n$ to $\mathcal{X}_n$ is that given that there are exactly k points of $\mathcal{P}_n$ in a region A Ω, these k points are independently and uniformly distributed in A. Thus, $\mathcal{X}_n$ can be well approximated by $\mathcal{P}_n$. Due to the extreme independence property, $\mathcal{P}_n$ is much more convenient to be dealt with. Penrose [2] thus first proved a Poissonized version of Theorem 1 in which $\mathcal{P}_n$ is replaced by $\mathcal{X}_n$, and then de-Poissonize this Poissonized version to complete the proof of Theorem 1. The value

$$\frac{n}{k!}\int_\Omega (n|D(x,r_n)\cap\Omega|)^k e^{-n|D(x,r_n)\cap\Omega|}dx$$

in Theorem 1 is exactly the expected number of points of $\mathcal{P}_n$ with degree k in $G_{r_n}(\mathcal{P}_n)$. The value λ is thus the limit of the expected number of points of $\mathcal{P}_n$ with degree k in $G_{r_n}(\mathcal{P}_n)$.

However, Penrose [2] didn't provide the explicit form of r_n, while stating that r_n is not so easy to find because of the dominance of complicated boundary effect. To explain the boundary effect, we define the r-neighborhood of a point x as $D(x,r)\cap\Omega$. The area of such r-neighborhood of a point in $\mathcal{X}_n$ determines the distribution of the number of neighbors in $G_r(\mathcal{X}_n)$. The larger this area, the higher the expected number of neighbors. As a node close to the boundary of Ω has small r-neighborhood, intuitively a node around the boundary have smaller vertex degree. On the other hand, the probability for a node to be around the boundary is small when the node density is large. The overall effect produced by the boundary nodes is thus complicated and even peculiar [4]. In this paper, we will present a partition of Ω to address the boundary effect, based on which we obtain the explicit form of r_n.

Other earlier simulation studies and/or loose analytical results on asymptotic critical transmission radius for connectivity can be found in [5]–[13].

The problem of how many neighbors is desirable for various purposes in a wireless *ad hoc* network whose nodes are specified by a planar Poisson point process has been studied since the 1970s. For the purpose of maximizing the one-hop progress of a packet in the desired direction under the slotted ALOHA protocol, Kleinrock and Silvester [14] proposed that if all nodes have the same transmission power then six was the "magic number," i.e., on average every node should connect itself to its six nearest neighbors. Later, the magic number was revised to eight by Takagi and Kleinrock [15]. The same paper [15] also considered other transmission protocols, which resulted in some other magic numbers five and seven. Hou and Li [16] considered the situation when each node is allowed to adjust its transmission range individually, and obtained the magic numbers six and eight. For the purpose of maximizing the transmission efficiency defined as the ratio between the expected progress and the area covered by the transmission, Hajek [17] suggested that each node should adjust its power to cover about three nearest neighbors on average. Mathar and Mattfeldt [18] analyzed the wireless network generated by a Poisson point process on a line, and also obtained some magic numbers.

However, none of the analyses in [17], [16], [14], [18], and [15] took connectivity into consideration. Based on simulations, Ni and Chandler [10] suggested that six to eight nearest neighbors can make a small size network connected with high probability. But it turns out that as the number of nodes in the network increases, the network becomes disconnected with probability one whether one connects to six or eight nearest neighbors. In fact, Xue and Kumar [19] proved that even if each node connects bidirectionally to $0.074\log n$ nearest neighbors the probability of network disconnectivity is asymptotically equal to one as $n\to\infty$; on the other hand, if each node connects bidirectionally to more than $5.1774\log n$ nearest neighbors, the network is asymptotically connected. Here the bidirectional l nearest neighbor graph means that two nodes have a link if and only if at least one is among the other's l nearest neighbors. In [20], the upper bound was further improved to $(1+\delta)\log_2 n \approx$

$(1+\delta)1.4427\log n$ for any constant $\delta > 0$. Recently, Balister *et al.* [21] proved that the critical number is asymptotically lower bounded by $0.3043\log n$ and upper bounded by $0.5139\log n$. In addition, for a directional version in which node u has a directional link to node v if v is one of u's l nearest neighbors, the two asymptotic bounds are $0.7209\log n$ and $0.9967\log n$, respectively. In this paper, as a corollary of the critical transmission radius, we prove that for any integer $k \geq 1$ and real number $\alpha > 1, \alpha e \log n$ is an upper bound on $\ell(\mathcal{X}_n; \kappa \geq k)$, where $e \approx 2.718$ is the natural base. Note that $\ell(\mathcal{X}_n; \kappa \geq k)$ is defined based on the symmetric l-nearest-neighbor graph, and the symmetric l-nearest-neighbor graph, bidirectional l-nearest-neighbor graph, and directional l-nearest-neighbor graph all are different from each other.

III. Critical Transmission Radius For K-Connectivity

The main results of this section are the following two theorems.

Theorem 2: Assume that Ω is the unit-area square. Let

$$r_n = \sqrt{\frac{\log n + (2k-1)\log\log n + \xi}{\pi n}}$$

where

$$\xi = \begin{cases} -2\log\left(\sqrt{e^{-c}+\frac{\pi}{4}} - \frac{\sqrt{\pi}}{2}\right) & \text{if } k=1 \\ 2\log\frac{\sqrt{\pi}}{2^{k-1}k!} + 2c & \text{if } k>1. \end{cases}$$

Then the probabilities of the two events $\rho(\mathcal{X}_n; \delta \geq k+1) \leq r_n$ and $\rho(\mathcal{X}_n; \kappa \geq k+1) \leq r_n$ both converge to $\exp(-e^{-c})$ as $n \to \infty$.

Theorem 3: Assume that Ω is the unit-area disk. Let

$$r_n = \sqrt{\frac{\log n + (2k-1)\log\log n + \xi}{\pi n}}$$

where

$$\xi = \begin{cases} -2\log\left(\sqrt{e^{-c}+\frac{\pi^2}{16}} - \frac{\pi}{4}\right) & \text{if} k=1 \\ 2\log\frac{\pi}{2^{k}k!} + 2c & \text{if} k>1. \end{cases}$$

Then the probabilities of the two events $\rho(\mathcal{X}_n; \delta \geq k+1) \leq r_n$ and $\rho(\mathcal{X}_n; \kappa \geq k+1) \leq r_n$ both converge to $\exp(-e^{-c})$ as $n \to \infty$.

We notice that in Theorem 2 and Theorem 3, ξ depends on the shape of Ω and parameter k. This can have an intuitive explanation. A node is isolated if and only if there are no other nodes within its transmission range. Based on the Poisson point process assumption, the probability of without neighboring nodes depends on the area of the transmission range. However, if this node is near the boundary of Ω, then its transmission range is not fully contained in Ω and thus with higher probability to be isolated. This is exactly the boundary effect mentioned in the previous section. Actually, comparing the proof for Theorem 2 in the Section III-A with the proof for Theorem 3 in the Section III-B, we can see that the difference in the formulas of ξ is due to the boundary effect. For the case of $k > 1$, the boundary effect is the dominating factor. In other words, nodes with degrees less than k are almost surely near $\partial\Omega$. Moreover, the factor $\frac{\sqrt{\pi}}{2^{k-1}k!}$ in Theorem 2 and $\frac{\pi}{2^k k!}$ in Theorem 3 are proportional to 4 and $2\sqrt{\pi}$ that are the perimeters of a unit-area square and a unit-area disk, respectively. For the case of $k = 1$, the boundary effect is not the only factor. Isolated nodes also can be found in the internal area of Ω with some probability. So, the formula of ξ is decided by the calculation for the internal area and boundary area.

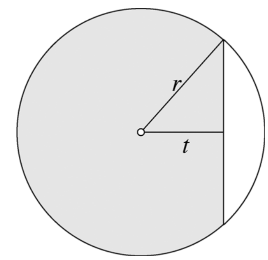

Fig. 1. Area of the shaded region is $a_n(t)$.

Throughout of this section, we use r_n to denote the value given either in Theorem 2 or in Theorem 3 depending on whether Ω is a square or a disk. For any $t \in [0, r_n]$, let

$$a_n(t) = \left|\left\{x=(x_1,x_2) : x_1^2 + x_2^2 \leq r_n^2, x_1 \leq t\right\}\right| \quad (1)$$

the area of the shaded region illustrated in Fig. 1. It is easy to see that $a_n'(t)$ equals to length of the boundary chord, i.e., $2\sqrt{r_n^2 - t^2}$. Remind that we will omit all subscript n for simplicity.

We first present the following technical lemma.

Lemma 4: $n\int_0^{\frac{r}{2}} \frac{(na(t))^k e^{-na(t)}}{k!} dt \sim \frac{\sqrt{\pi}}{2^{k+1}k!} e^{-\frac{\xi}{2}}$.

Proof: It is straightforward to verify that

$$\frac{1}{r}(n\pi r^2)^k e^{-\frac{n\pi r^2}{2}} \sim \sqrt{\pi} e^{-\frac{\xi}{2}}.$$

Let

$$f(t) = na(t). \quad (2)$$

Using integration by parts on the integral yields

$$\begin{aligned} n&\int_0^{\frac{r}{2}} \frac{(f(t))^k e^{-f(t)}}{k!} dt \\ &= n\int_0^{\frac{r}{2}} \frac{(f(t))^k e^{-f(t)}}{k!} \frac{1}{na'(t)} df(t) \\ &= \int_0^{\frac{r}{2}} \frac{1}{a'(t)} d\left(-e^{-f(t)} \sum_{i=0}^{k} \frac{(f(t))^i}{i!}\right) \\ &= -\frac{1}{a'(t)} e^{-f(t)} \sum_{i=0}^{k} \frac{(f(t))^i}{i!}\Bigg|_0^{\frac{r}{2}} \\ &\quad - \int_0^{\frac{r}{2}} \frac{a''(t)}{(a'(t))^2} e^{-f(t)} \sum_{i=0}^{k} \frac{(f(t))^i}{i!} dt \end{aligned}$$

The first term is asymptotically equal to $\frac{1}{2^{k+1}k!}e^{-\frac{\xi}{2}}$ because

$$\begin{aligned}
&\frac{1}{a'\left(\frac{r_n}{2}\right)}e^{-f\left(\frac{r}{2}\right)}\sum_{i=0}^{k}\frac{\left(f\left(\frac{r}{2}\right)\right)^i}{i!}\\
&\quad=\frac{1}{\sqrt{3}r}e^{-\left(\frac{\sqrt{3}}{4\pi}+\frac{2}{3}\right)n\pi r^2}\\
&\qquad\times\sum_{i=0}^{k}\frac{\left(\left(\frac{\sqrt{3}}{4\pi}+\frac{2}{3}\right)n\pi r^2\right)^i}{i!}=o(1),
\end{aligned}$$

and

$$\begin{aligned}
&\frac{1}{a'(0)}e^{-f(0)}\sum_{i=0}^{k}\frac{(f(0))^i}{i!}\\
&\quad=\frac{1}{2r}e^{-\frac{n\pi r_n^2}{2}}\sum_{i=0}^{k}\frac{\left(\frac{n\pi r_n^2}{2}\right)^i}{i!}\\
&\quad\sim\frac{1}{2^{k+1}k!}\frac{1}{r_n}e^{-\frac{n\pi r_n^2}{2}}\left(n\pi r_n^2\right)^k+o(1)\\
&\quad\sim\frac{\sqrt{\pi}}{2^{k+1}k!}e^{-\frac{\xi}{2}}.
\end{aligned}$$

The second term is asymptotically negligible because

$$\begin{aligned}
&\left|\int_0^{\frac{r}{2}}\frac{a''(t)}{(a'(t))^2}e^{-f(t)}\sum_{i=0}^{k}\frac{(f(t))^i}{i!}dt\right|\\
&\quad=\left|\frac{1}{n}\int_0^{\frac{r}{2}}\frac{a''(t)}{(a'(t))^3}e^{-f(t)}\sum_{i=0}^{k}\frac{(f(t))^i}{i!}df(t)\right|\\
&\quad=\frac{1}{4n}\int_0^{\frac{r}{2}}\frac{t}{(r^2-t^2)^2}e^{-f(t)}\sum_{i=0}^{k}\frac{(f(t))^i}{i!}df(t)\\
&\quad\le\frac{1}{8nr^3}\int_0^{\frac{r}{2}}e^{-f(t)}\sum_{i=0}^{k}\frac{(f(t))^i}{i!}df(t)\\
&\quad\le O(1)\frac{1}{nr^3}\int_0^{\frac{r}{2}}e^{-f(t)}\frac{(f(t))^k}{k!}df(t)\\
&\quad=O(1)\frac{1}{nr^3}\int_0^{\frac{r}{2}}d\left(-e^{-f(t)}\sum_{i=0}^{k}\frac{(f(t))^i}{i!}\right)\\
&\quad\le O(1)\frac{1}{nr^3}e^{-f(0)}\sum_{i=0}^{k}\frac{(f(0))^i}{i!}\\
&\quad=O(1)\frac{1}{nr^2}\left(\frac{1}{r}e^{-f(0)}\sum_{i=0}^{k}\frac{(f(0))^i}{i!}\right)\\
&\quad=O(1)\frac{1}{nr^2}=o(1).
\end{aligned}$$

Thus, the lemma follows. ■

In the next two subsections, we give the proofs for Theorem 2 and Theorem 3, respectively.

A. *Proof for Theorem 2*

By Theorem 1, we only need to show that

$$\frac{n}{k!}\int_{\Omega}(n|D(x,r)\cap\Omega|)^k e^{-n|D(x,r)\cap\Omega|}dx\sim e^{-c}.$$

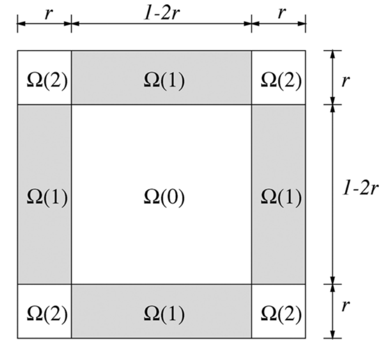

Fig. 2. Partition of the square Ω.

To address the boundary effect of the square region Ω, we partition Ω into three subregions $\Omega(0), \Omega(1)$ and $\Omega(2)$ as illustrated in Fig. 2. For any $0\le i\le 2$, let $\Omega(i)$ denote the set of $x\in\Omega$ satisfying that $D(x,r)$ intersects exactly i sides of Ω. The areas of these three regions are

$$\begin{aligned}
|\Omega(0)|&=(1-2r)^2,\\
|\Omega(1)|&=4r(1-2r)\\
|\Omega(2)|&=4r^2.
\end{aligned}$$

For any $x\in\Omega(i)$

$$|D(x,r)\cap\Omega|\ge 2^{-i}\pi r^2.$$

When $x\in\Omega(1), |D(x,r)\cap\Omega|$ is exactly $a(t)$, where t is the distance between x and the boundary of Ω.

First, we calculate the integration over $\Omega(0)$. If $x\in\Omega(0), |D(x,r)\cap\Omega|=\pi r^2$. Thus

$$\begin{aligned}
&\frac{n}{k!}\int_{\Omega(0)}(n|D(x,r)\cap\Omega|)^k e^{-n|D(x,r)\cap\Omega|}dx\\
&\quad=\frac{n}{k!}(n\pi r^2)^k e^{-n\pi r^2}|\Omega(0)|\\
&\quad\sim\frac{n}{k!}(n\pi r^2)^k e^{-n\pi r^2}\\
&\quad\sim\begin{cases}e^{-\xi} & \text{if } k=1\\ o(1) & \text{if } k>1.\end{cases}
\end{aligned}$$

Now, we calculate the integration over $\Omega(2)$. If $x\in\Omega(2)$

$$|D(x,r)\cap\Omega|\ge\frac{1}{4}\pi r^2.$$

Thus

$$\begin{aligned}
&\frac{n}{k!}\int_{\Omega(2)}(n|D(x,r)\cap\Omega|)^k e^{-n|D(x,r)\cap\Omega|}dx\\
&\quad\le\frac{n}{k!}(n\pi r^2)^k e^{-\frac{n\pi r^2}{4}}|\Omega(2)|\\
&\quad=O(1)(n\pi r^2)^{k+1}e^{-\frac{n\pi r^2}{4}}=o(1).
\end{aligned}$$

Finally, we calculate the integration over $\Omega(1)$. We further partition $\Omega(1)$ into two regions: $\Omega(1,1)$ consists of all points $x\in\Omega(1)$ whose distance from the boundary of Ω is at most $\frac{r}{2}$, and $\Omega(1,2)=\Omega(1)\setminus\Omega(1,1)$. Then for any $x\in\Omega(1,2)$

$$|D(x,r)\cap\Omega|\ge a\left(\frac{r}{2}\right)=\left(\frac{\sqrt{3}}{4\pi}+\frac{2}{3}\right)\pi r^2.$$

Recall that $a(t)$ is defined in (1). Thus

$$\begin{aligned}
&\frac{n}{k!}\int_{\Omega(1,2)}(n|D(x,r)\cap\Omega|)^k e^{-n|D(x,r)\cap\Omega|}dx\\
&\quad\le \frac{n}{k!}(n\pi r^2)^k e^{-\left(\frac{\sqrt{3}}{4\pi}+\frac{2}{3}\right)n\pi r^2}|\Omega(1,2)|\\
&\quad\le O(1)(nr)(n\pi r^2)^k e^{-\frac{2}{3}n\pi r^2}\\
&\quad= O(1)\left(\frac{1}{r}(n\pi r^2)^k e^{-\frac{1}{2}n\pi r^2}\right)\left((n\pi r^2)e^{-\frac{1}{6}n\pi r^2}\right)\\
&\quad= O(1)(n\pi r^2)e^{-\frac{1}{6}n\pi r^2}=o(1).
\end{aligned}$$

The integration over $\Omega(1,1)$ is calculated as follows. A change of integration variable yields

$$\begin{aligned}
&\frac{n}{k!}\int_{\Omega(1,1)}(n|D(x,r)\cap\Omega|)^k e^{-n|D(x,r)\cap\Omega|}dx\\
&\quad=\frac{4n(1-2r)}{k!}\int_0^{\frac{r}{2}}(na(t))^k e^{-na(t)}dt\\
&\quad\sim 4n\int_0^{\frac{r}{2}}\frac{(na(t))^k e^{-na(t)}}{k!}dt\\
&\quad\sim \frac{\sqrt{\pi}}{2^{k-1}k!}e^{-\frac{\xi}{2}}.
\end{aligned}$$

The last asymptotics is given by Lemma 4.

In summary, if $k=1$, the integral is asymptotically equal to

$$e^{-\xi}+\sqrt{\pi}e^{-\frac{\xi}{2}}=e^{-c}.$$

If $k>1$, the integral is asymptotically equal to

$$\frac{\sqrt{\pi}}{2^{k-1}k!}e^{-\frac{\xi}{2}}=e^{-c}.$$

In either case, Theorem 2 holds.

B. *Proof for Theorem 3*

Again by Theorem 1, we only need to show that

$$\frac{n}{k!}\int_{\Omega}(n|D(x,r)\cap\Omega|)^k e^{-n|D(x,r)\cap\Omega|}dx\sim e^{-c}.$$

To address the boundary effect of the disk region Ω, we partition Ω into three subregions $\Omega(0),\Omega(1)$ and $\Omega(2)$ as illustrated in Fig. 3. Without loss of generality, Ω is assumed to be centered at the origin $\mathbf{o}$. $\Omega(0)$ is the disk of radius $\frac{1}{\sqrt{\pi}}-r$ centered at $\mathbf{o}$; $\Omega(1)$ is the annulus of radii $\frac{1}{\sqrt{\pi}}-r$ and $\sqrt{\frac{1}{\pi}-r^2}$ centered at $\mathbf{o}$; and $\Omega(2)$ is the annulus of radii $\sqrt{\frac{1}{\pi}-r^2}$ and $\frac{1}{\sqrt{\pi}}$ centered at $\mathbf{o}$. The areas of these three regions are

$$\begin{aligned}
|\Omega(0)|&=(1-\sqrt{\pi}r)^2\\
|\Omega(1)|&=2\pi r\left(\frac{1}{\sqrt{\pi}}-r\right)\\
|\Omega(2)|&=\pi r^2.
\end{aligned}$$

For any $x\in\Omega(i)$

$$|D(x,r)\cap\Omega|\ge 2^{-i}\pi r^2.$$

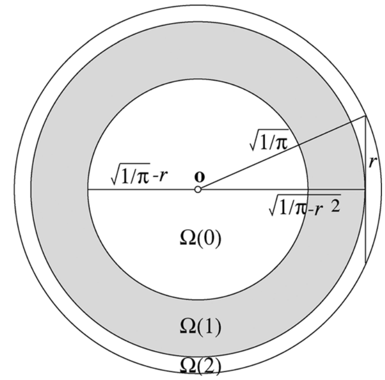

Fig. 3. Partition of a disk region Ω.

Using the same argument as in the proof of Theorem 2, we can show that

$$\frac{n}{k!}\int_{\Omega(0)}(n|D(x,r)\cap\Omega|)^k e^{-n|D(x,r)\cap\Omega|}dx \sim \begin{cases} e^{-\xi} & \text{if } k=1\\ o(1) & \text{if } k>1\end{cases}$$

and

$$\frac{n}{k!}\int_{\Omega(2)}(n|D(x,r)\cap\Omega|)^k e^{-n|D(x,r)\cap\Omega|}dx=o(1).$$

Next, we calculate the integration over $\Omega(1)$.

For any $x\in\Omega(1)$, let $t(x)$ be the distance between x and the chord of the circle $\partial D(x,r)$ through the two intersecting points between $\partial D(x,r)$ and Ω (see Fig. 4). Then

$$\|x\|=\sqrt{\frac{1}{\pi}-r^2+t(x)^2}-t(x).$$

In addition

$$|D(x,r)\cap\Omega|\ge a(t(x))$$
and
$$\begin{aligned}
&|D(x,r)\cap\Omega|\\
&\quad\le a(t(x))+2\sqrt{r^2-t(x)^2}\\
&\qquad\times\left(\frac{1}{\sqrt{\pi}}-\sqrt{\frac{1}{\pi}-r^2+t(x)^2}\right)\\
&\quad=a(t(x))+\frac{2(r^2-t(x)^2)^{\frac{3}{2}}}{\frac{1}{\sqrt{\pi}}+\sqrt{\frac{1}{\pi}-r^2+t(x)^2}}\\
&\quad\le a(t(x))+2\sqrt{\pi}r^3.
\end{aligned}$$

Since $a(t(x))\ge\frac{\pi r^2}{2}$, we further have that

$$\begin{aligned}
|D(x,r)\cap\Omega|&\le a(t(x))\left(1+\frac{2\sqrt{\pi}r^3}{a(t(x))}\right)\\
&\le a(t(x))\left(1+\frac{2\sqrt{\pi}r^3}{\frac{\pi r^2}{2}}\right)\\
&=a(t(x))\left(1+\frac{4}{\sqrt{\pi}}r\right).
\end{aligned}$$

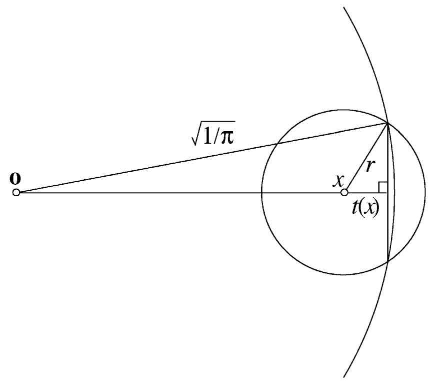

Fig. 4. For $x \in \Omega(1), t(x)$ denotes the distance between x and the chord of the circle $\partial D(x,r)$ through the two intersecting points between $\partial D(x,r)$ and Ω.

Thus, for any $x \in \Omega(1)$

$$\begin{aligned}&(n|D(x,r)\cap\Omega|)^k e^{-n|D(x,r)\cap\Omega|}\\ &\quad\le \left(1+\frac{4}{\sqrt{\pi}}r\right)^k (na(t(x)))^k e^{-na(t(x))}.\end{aligned}$$

and

$$\begin{aligned}&(n|D(x,r)\cap\Omega|)^k e^{-n|D(x,r)\cap\Omega|}\\ &\quad\ge e^{-2\sqrt{\pi}nr^3}(na(t(x)))^k e^{-na(t(x))}.\end{aligned}$$

We partition $\Omega(1)$ into two regions: $\Omega(1,1)$ consists of all points $x \in \Omega(1)$ with $t(x) \le \frac{r}{2}$, and $\Omega(1,2) = \Omega(1) \setminus \Omega(1,1)$. Then for any $x \in \Omega(1,2)$

$$|D(x,r)\cap\Omega| \ge a\left(\frac{r}{2}\right) = \left(\frac{\sqrt{3}}{4\pi}+\frac{2}{3}\right)\pi r^2.$$

Thus, using the same argument as in the proof of Theorem 2, we can show that

$$\frac{n}{k!}\int_{\Omega(1,2)} (n|D(x,r)\cap\Omega|)^k e^{-n|D(x,r)\cap\Omega|}dx = o(1).$$

Finally, we calculate the integration over $\Omega(1,1)$. By the two inequalities just before this theorem

$$\begin{aligned}&\frac{n}{k!}\int_{\Omega(1,1)} (n|D(x,r)\cap\Omega|)^k e^{-n|D(x,r)\cap\Omega|}dx\\ &\qquad\sim \frac{n}{k!}\int_{\Omega(1,1)} (na(t(x)))^k e^{-na(t(x))}dx\end{aligned}$$

Recall that $f(t) = na(t)$ was defined by (2) in the proof of Lemma 4. A change of integration variable yields

$$\begin{aligned}&\frac{n}{k!}\int_{\Omega(1,1)} (na(t(x)))^k e^{-na(t(x))}dx\\ &= 2\pi n\int_0^{\frac{r}{2}} \frac{f(t)^k e^{-f(t)}}{k!}\left(\sqrt{\frac{1}{\pi}-r^2+t^2}-t\right)\\ &\quad\times\left(1-\frac{t}{\sqrt{\frac{1}{\pi}-r^2+t^2}}\right)dt\\ &= 2\pi n\int_0^{\frac{r}{2}} \frac{f(t)^k e^{-f(t)}}{k!}\sqrt{\frac{1}{\pi}-r^2+t^2}\\ &\quad\times\left(1-\frac{t}{\sqrt{\frac{1}{\pi}-r^2+t^2}}\right)^2 dt\\ &\sim 2\sqrt{\pi}n\int_0^{\frac{r}{2}} \frac{f(t)^k e^{-f(t)}}{k!}dt\\ &= \frac{\pi}{2^k k!}e^{-\frac{\xi}{2}}.\end{aligned}$$

The last asymptotics follows Lemma 4.

Therefore, if $k = 1$, the integral is asymptotically equal to

$$e^{-\xi} + \frac{\pi}{2}e^{-\frac{\xi}{2}} = e^{-c}.$$

If $k > 1$, the integral is asymptotically equal to

$$\frac{\pi}{2^k k!}e^{-\frac{\xi}{2}} = e^{-c}.$$

In either case, Theorem 3 holds.

IV. Critical Neighbor Number for k-Connectivity

Based on Theorem 2 and Theorem 3, we can get the following a.a.s. upper bound on the critical neighbor number $\ell(\mathcal{X}_n; \kappa \ge k)$. Remind that the deployment region Ω can be either a disk or a square.

Theorem 5: For any $k \ge 0$ and $\alpha > 1$, the event $\ell(\mathcal{X}_n; \kappa \ge k+1) \le \alpha e \log n$ is a.a.s.

We shall actually prove the following stronger result.

Theorem 6: For any two constants $1 < \beta < \alpha$, the event $G_{\sqrt{\frac{\beta \log n}{\pi n}}}(\mathcal{X}_n) \subseteq H_{\alpha e \log n}(\mathcal{X}_n)$ is a.a.s.

Recall that the critical transmission radius for connectivity was given by Theorem 1 in [1], and the critical transmission radius for $(k+1)$-connectivity is given by Theorem 2 and 3 in this work (for $k \ge 1$). More precisely, the critical transmission radius for $(k+1)$-connectivity is in the form of $r_n = \sqrt{\frac{\log n + \xi}{\pi n}}$ for $k = 0$, and $r_n = \sqrt{\frac{\log n + (2k-1)\log\log n + \xi}{\pi n}}$ for $k \ge 1$. No matter what, for given k and ξ, we have $r_n \le \sqrt{\frac{\beta \log n}{\pi n}}$ if n is sufficiently large. In addition, since the probability of $(k+1)$-connectivity is tend to 1 as $\xi \to \infty$, to show $G_{\sqrt{\frac{\beta \log n}{\pi n}}}(\mathcal{X}_n)$ is $(k+1)$-connected with high probability, we only need to choose ξ large enough. Then, applying Theorem 6, it is a.a.s. that $H_{\alpha e \log n}(\mathcal{X}_n)$ is $(k+1)$-connected. Therefore, we can conclude that Theorem 6 together with the result by Penrose [1], Theorem 2 and Theorem 3 implies Theorem 5.

Throughout this section, we let α and β be fixed constants as in Theorem 6. Pick another constant $\eta \in (\beta, \alpha)$ and let m be the smallest integer which is greater than $1/(1 - \sqrt{\beta/\eta})$. For any integer n, let

$$s_n = \sqrt{\frac{\eta \log n}{\pi n}}.$$

Then

$$\left(1 - \frac{1}{m}\right)s_n > \sqrt{\frac{\beta \log n}{\pi n}}.$$

Let $\mathcal{D}_n$ be the set of all open disks of radius s_n centered at the square grid of side $\frac{s_n}{m}$ with one corner point at the origin which have nonempty intersections with Ω. Let E_n denote the event that all disks in $\mathcal{D}_n$ contains less than $\alpha e \log n$ nodes of $\mathcal{X}_n$.

We first claim that the event E_n implies the event that $G_{\sqrt{\frac{\beta \log n}{\pi n}}}(\mathcal{X}_n) \subseteq H_{\alpha e \log n}(\mathcal{X}_n)$. Assume that then event E_n occurs. For any node $X \in \mathcal{X}_n$, there exist a disk D^* in $\mathcal{D}_n$ such that the distance between X and the center of D^* is less than s_n/m. Thus

$$D\left(X, \sqrt{\frac{\beta \log n}{\pi n}}\right) \subset D\left(X, \left(1-\frac{1}{m}\right) s_n\right) \subset D^*.$$

Since D^* contains less than $\alpha e \log n$ nodes of $\mathcal{X}_n$, so does the disk $D(X, \sqrt{\frac{\beta \log n}{\pi n}})$. This implies that any neighbor of X in $G_{\sqrt{\frac{\beta \log n}{\pi n}}}(\mathcal{X}_n)$ is one of its $\alpha e \log n$ nearest neighbors. Now consider any edge XY in $G_{\sqrt{\frac{\beta \log n}{\pi n}}}(\mathcal{X}_n)$. Then both X and Y are one of each other's $\alpha e \log n$ nearest neighbors. Consequently, XY is also an edge of $H_{\alpha e \log n}(\mathcal{X}_n)$. So our claim is true. Therefore, Theorem 6 would follows if we can prove that E_n is an a.a.s. event. The remaining of this section is devoted to this proof.

We partition $\mathcal{D}_n$ into $(2m)^2$ subsets $\mathcal{D}_n^{ij}$ with $0 \le i, j < 2m$ where $\mathcal{D}_n^{ij}$ consists of all disks in $\mathcal{D}_n$ centered at the square grid of side $2s_n$ with one corner point at $(i\frac{s_n}{m}, j\frac{s_n}{m})$. Correspondingly, for any $0 \le i, j < 2m$ let E_n^{ij} denote the event that all disks in $\mathcal{D}_n^{ij}$ contains less than $\alpha e \log n$ nodes of $\mathcal{X}_n$. Then

$$\mathcal{D}_n = \bigcup_{i=0}^{2m-1} \bigcup_{j=0}^{2m-1} \mathcal{D}_n^{ij}, E_n = \bigcap_{i=0}^{2m-1} \bigcap_{j=0}^{2m-1} E_n^{ij}.$$

Since the intersection of a *constant* number of a.a.s. events is also an a.a.s. event, it is sufficient to show that each E_n^{ij} is an a.a.s. event. We prove this by using the same Poissonization technique as in [19]. Fix two integers $0 \le i, j < 2m$. We denote by $\tilde{E}_n^{ij}$ the event that all disks in $\mathcal{D}_n^{ij}$ contains less than $\eta e \log n$ nodes of $\mathcal{P}_n$. Recall that $\mathcal{P}_n$ denotes a Poisson point process with node density n over the deployment region Ω. Since $\eta < \alpha$, using the similar proof of [19, Lemma 3.2.3] we can show that if $\tilde{E}_n^{ij}$ is an a.a.s. event, so must be E_n^{ij}. Thus, we only to prove that $\tilde{E}_{ij}$ is an a.a.s. event. To prove this, we number the disks in $\mathcal{D}_n^{ij}$ by

$$\mathcal{D}_n^{ij} = \{D_t : t \in I\}$$

where I is the index set. For any $t \in I$, let N_t be the number of points of $\mathcal{P}_n$ which fall in D_t. Then $\tilde{E}_n^{ij}$ can be expressed as $\max_{t \in I} N_t < \eta e \log n$. In the next, we show that

$$\Pr\left\{\max_{t \in I} N_t < \eta e \log n\right\} \sim 1.$$

Let $\lambda = n \cdot \pi s_n^2 = \eta \log n$ and M be a Poisson random variable with rate λ. The following upper bound on the tail distribution of M follows from [19, Lemma 3.2.5] and Stirling's formula:

$$\begin{aligned}
&\Pr\{M \ge e\lambda\} \\
&\quad = \frac{e}{e-1} \frac{\lambda^{e\lambda}}{(e\lambda)!} e^{-\lambda}(1+o(1)) \\
&\quad \le \frac{e}{e-1} \frac{\lambda^{e\lambda}}{\sqrt{2\pi e\lambda}\left(\frac{e\lambda}{e}\right)^{e\lambda}} e^{-\lambda}(1+o(1)) \\
&\quad = \frac{e}{e-1} \frac{e^{-\lambda}}{\sqrt{2\pi e\lambda}}(1+o(1)) \\
&\quad = \Theta\left(\frac{1}{n^{\eta}\sqrt{\log n}}\right).
\end{aligned}$$

This bound implies that $\Pr\{M \ge e\lambda\} = o(1)$. Furthermore, since

$$\mathrm{card}(I) = \Theta\left(\frac{1}{\pi s_n^2}\right) = \Theta\left(\frac{n}{\log n}\right)$$

we have

$$\begin{aligned}
&\mathrm{card}(I)\Pr\{M \ge e\lambda\} \\
&\quad \le \Theta\left(\frac{n}{\log n} \cdot \frac{1}{n^{\eta}\sqrt{\log n}}\right) \\
&\quad = \Theta\left(\frac{1}{n^{\eta-1}(\log n)^{3/2}}\right) \\
&\quad = o(1).
\end{aligned}$$

For any $t \in I$, let $\lambda_t = n|D_t \cap \Omega|$. Then each N_t is a Poisson random variable with rate λ_t. Note that for any integer q, the function $f_q(\mu) = \mu^q e^{-\mu}$ is strictly increasing as long as $0 < \mu < q$, since

$$f_q'(\mu) = q\mu^{q-1}e^{-\mu} - \mu^q e^{-\mu} = \mu^{q-1}e^{-\mu}(q-\mu).$$

As $\lambda_t \le \lambda = \eta \log n$, we have

$$\begin{aligned}
&\Pr\{N_t \ge \eta e \log n\} = \Pr\{N_t \ge e\lambda\} \\
&\quad = \sum_{q=\lceil e\lambda \rceil}^{\infty} \frac{\lambda_t^q}{q!} e^{-\lambda_t} \le \sum_{q=\lceil e\lambda \rceil}^{\infty} \frac{\lambda^q}{q!} e^{-\lambda} \\
&\quad = \Pr\{M \ge e\lambda\}.
\end{aligned}$$

This inequality together with the independence of $\mathcal{P}_n$ implies that

$$\begin{aligned}
\Pr\left\{\max_{t \in I} N_t < \eta e \log n\right\} &= 1 - \Pr\left\{\bigcup_{t \in I} N_t \ge e\lambda\right\} \\
&\ge 1 - \sum_{t \in I} \Pr\{N_t \ge e\lambda\} \\
&\ge 1 - \mathrm{card}(I)\Pr\{M \ge e\lambda\} \\
&\sim 1.
\end{aligned}$$

Therefore, each $\tilde{E}_n^{ij}$ is an a.a.s. event. This completes the proof of Theorem 6.

V. Conclusion

In this paper, we model the wireless *ad hoc* network by a uniform n-point process $\mathcal{X}_n$ over a unit-area disk or square Ω. We derived the precise asymptotic distribution of the critical transmission radius for k-connectivity $\rho(\mathcal{X}_n; \kappa \geq k)$. Based on the result, we also obtained an asymptotic almost sure upper bound on the critical neighbor number for k-connectivity $\ell(\mathcal{X}_n; \kappa \geq k)$.

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Peng-Jun Wan received the B.S. degree from Tsinghua University, the M.S. degree from The Chinese Academy of Science, and the Ph.D. degree from the University of Minnesota, Minneapolis.

He is currently a Full Professor of computer science at the Illinois Institute of Technology, Chicago. His research interests include wireless networks, optical networks, and algorithm design and analysis.

Chih-Wei Yi (M'99) received the B.S. and M.S. degrees from the National Taiwan University and the Ph.D. degree from the Illinois Institute of Technology, Chicago.

He is currently an Associate Professor of computer science with the National Chiao Tung University. He had been a Senior Research Fellow with the Department of Computer Science, City University of Hong Kong. His research focuses on wireless *ad hoc* and sensor networks, vehicular *ad hoc* networks, network coding, and algorithm design and analysis.

Dr. Yi is a member of the ACM. He received the Outstanding Young Engineer Award by the Chinese Institute of Engineers in 2009.

Lixin Wang received the M.S. degree in computer science from the University of Houston at Clear Lake, the M.S. degree in applied math from the University of Houston, Houston, TX, and the M.S. degree in math from the Fudan University, Shanghai, China. He is currently pursuing the Ph.D. degree in computer science at the Illinois Institute of Technology, Chicago.

His research is on wireless networks, and algorithm design, and analysis.