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在比較模式下強診斷性質之研究

Strongly t-diagnosable System under the Comparison Model

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中 華 民 國 九 十 三 年 六 月

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在比較模式下強診斷性質之研究

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摘要

在多處理器系統中，診斷能力是一個重要的性質，以增加系統的可靠度。在比較模式下，n 維度的超立方體家族診斷能力皆是 n，但是我們發現除了當某一點的所有鄰居同時皆是壞點的情形下，其它情況時其實這些超立方體家族診斷能力根本是 n+1 以上。在本篇中，我們提出強診斷性質的觀念，並且證明如下：令 G1 和 G2 擁有相同點數且兩者皆是 t-正則圖形，在 G1 和 G2 之間做一完全配對，形成一配對構成網路 G=G1⊕G2，則 G 在比較模式下不僅是(t+1)-診斷系統並且也是強(t+1)-診斷系統。根據以上結果，我們知道任何一個 n 維度的超立方體家族在比較模式下皆是強 n-診斷系統，當 n≧4。

關鍵字：比較模式，診斷能力，t-診斷能力，強 t-診斷能力，配對構成網路

Strongly t-diagnosable System under the Comparison Model

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Abstract

The diagnosability is an important property on the high-performance signal processing systems. We need to find faulty processors quickly and correctly to make sure the reliability of system. There are many achievements related to diagnosability in recent researchs. Under the comparison model, the diagnosability of n-dimensional cube family is n. But we find that these cubes are almost $(n + 1)$-diagnosable except that all the neighbors of some vertex are faulty simultaneously. In this thesis, we introduce a new concept, called a strongly t-diagnosable system under the comparison model. The goal of this thesis is the following. G_1, G_2 are two t-regular graph with the same number of vertices N, $N \geqq t+1$, for $t \geqq 3$. $order_{G_i}(v) \geqq t$ for every node v in G_i and the connectivity $\kappa(G_i) \geqq t$ for i = 1, 2. We prove that the MCN constructed from G_1 and G_2 is strongly $(t + 1)$-diagnosable system. Applying this result, the Hypercube Q_n, the Crossed cube CQ_n, the Twisted cube TQ_n, and the Mobius cube MQ_n are all strongly n-diagnosable for $n \geqq 4$.

Keywords：Comparison Model, diagnosability, t-diagnosable, strongly t-diagnosable, MCN.

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Chapter 1

Introduction

The *diagnosability* is an important property on the high-performance signal processing systems. It is necessary to find faulty processors quickly and correctly to make sure the reliability of system. A self-diagnosable system is that each processor test and be tested by connected processors. The fault diagnosis topic is widely discussed in many literatures [3, 4, 9, 11, 15, 16, 17, 18, 19, 21, 22]. Different models are presented[3, 16, 17, 19]. Well-known model includes the PMC model, the Comparison model and the BGM model.

A multiprocessor system is made up of a collection of processors and a collection of communication links. A multiprocessor system can be represented by an undirected graph $G = (V, E)$, where each node represents a processor and each undirected edge represents a communication link.

We now introduce the *Matching Composition Networks (MCN)*[14]. The *MCN* is constructed from two graph G_1 and G_2 with the same number of vertices, by adding a perfect matching M between the vertices of G_1 and G_2. The *MCN* family includes many well-known interconnection networks as special cases, such as the Hypercube Q_n, the Crossed cube CQ_n, the Twisted cube TQ_n, and the Möbius cube MQ_n.

A well-known model is so-called PMC model[19] presented by *Preparata*, *Metze*, and *chien* in the self-diagnosable system. Under the PMC model, the status of fault or fault-free of a processor is determined by one processor testing the other processor. The researchers investigated the diagnosability of many well-known interconnection networks under PMC model[2, 9, 10].

The comparison model, which is proposed by *Maeng* and *Malek* [16, 17], is another self-diagnosis model. The faulty or fault-free status of a processor is determined by comparing its response to system tasks with the response to the same tasks produced by other processors in the system. A disagreement between the two responses is an indication of the existence of a fault. There are many studies of diagnosability under the comparison model. For example, Wang[22] performed that the diagnosability of an n-dimensional Hypercube Q_n is n if $n \geq 5$, and the diagnosability of the enhanced Hypercube is $n+1$ if $n \geq 6$. Fan[11] showed that the diagnosability of an n-dimensional crossed cube is n if $n \geq 4$. Araki[1] proved that the k-ary r-dimensional butterfly network $BF(k,r)$ is $2k$-diagnosable for $k \geq 2$ and $r \geq 5$. Suppose that the number of nodes in each component is at least $t+2$, the *order*(which will be defined subsequently) of each node in G_i is t, and the connectivity of G_i is also t ,$i = 1, 2$. Then Lai and Tan [14] et al. proved that the diagnosability of the *MCN* constructed from G_1 and G_2 is $t+1$ under the comparison model for $t \geq 2$.

The diagnosability of n-dimensional cube family is n[14]. We find that these cubes are almost $(n+1)$-*diagnosable* except the case that all the neighbors of some vertex are faulty simultaneously. In this thesis, we introduce a new concept, called *strongly t-diagnosable*, under the comparison model. The goal of this thesis is the following. G_1, G_2 are two t-regular graph with the same number of vertices N, $N \geq 2t+1$, for $t \geq 3$ and $order_{G_i}(v) \geq t$ for every node v in G_i and the connectivity $\kappa(G_i) \geq t$ for $i = 1, 2$. We prove that the *MCN* constructed from G_1 and G_2 is *strongly(*$t+1$*)-diagnosable*.

The organization of this thesis as follows: Chapter 2 includes three sections. The first section gives the basic graph definition and notation, the second section is an introduction of the comparison model, and these preliminaries used in this thesis are presented in section 3. Chapter 3 discusses the concept of a *strongly* t*-diagnosable* system. Some necessary and sufficient conditions for a *strongly* t*-diagnosable* system and our main result are shown in Chapter 3. Finally, some conclusions are discussed in Section 4.

Chapter 2

Terminology and Preliminaries

2.1 Graph definition and notation

In this thesis, We give the basic of graph definition and notation [5]. $G = (V, E)$ is a *graph* if V is a finite set and E is a subset of $\{(u, v)|(u, v)$ is an unordered pair of $V\}$. $V(G)$ or V_G represents *vertex set* and $E(G)$ or E_G represents *edge set*. An element v in V_G is called vertex or node. An element (u, v) in E_G is called edge. $|G|$ represents the number of vertices in the graph G. The *degree* of vertex v in a graph G is the number of edges incident to v. For a vertex v of G, $deg_G(v)$ or $deg(v)$ denotes its *degree* in G. The maximum *degree* in G is denoted by $\Delta(G)$. The minimum *degree* in G is denoted by $\delta(G)$. When $\Delta(G) = \delta(G)$, we call that G is *regular* graph. A graph G is *k-regular* if the degree of any vertex in G is k.

Definition 1 *[23] The components of a graph G are its maximal connected subgraphs. A component is trivial if it has no edges; otherwise it is nontrivial.*

Let $G = (V, E)$. For a set $S \subset V_G$, the notation $G - S$ represents the graph obtained by removing the vertices in S from G and deleting those edges with at least one end vertex in S simultaneously. The *neighbor* of v, written $N_G(v)$ or $N(v)$, is the set of vertices adjacent to v. The *neighborhood set* of V_1 in V_2, denoted by $N(V_2, V_1)$, is defined as $\{x \in V_2|$ there exists a node $y \in V_1$ such that $(x, y) \in E(G)\}$. In graph G, the *connectivity* $\kappa(G)$ is the minimum number of a set S of G such that $G - S$ is disconnected or trivial. A graph G is k-*connected* if its connectivity is not larger than k. Let $G = (V, E)$ be a k-regular graph with connectivity κ. G is *maximum connected* if $\kappa = k$. G is super-connected if it is a complete graph, or it is maximum connected and every minimum vertex cut is $\{(v, x)\}|(v, x) \in E\}$ for some vertex $v \in V_G$. The symmetric difference $F_1 \triangle F_2 = (F_1 - F_2) \bigcup (F_2 - F_1)$.

The Hypercube[20] is a well-known interconnection structure. The Crossed cube[8], the Twisted cube[13], and the Möbius cube[7] are some variations of the Hypercube. We call these *cube family*. For each n-dimensional cube of *cube family* has (i) 2^n vertices, (ii) *n-regular*, (iii) connectivity n ,(iv) be constructed from two copies of $(n-1)$-*dimensional* subcubes by adding a perfect matching between the two subcubes. The difference of these cubes is different perfect matching method between its subcubes. In the following, we briefly define this cubes.

Definition 2 *Let $n > 1$ be an integer. The Hypercube Q_n of dimension n has 2^n nodes. Q_1 is a complete graph with two nodes labeled by 0 and 1, respectively. For $n \geq 2$, an n-dimensional Hypercube Q_n is obtained by taking two copies of $(n-1)$-dimensional subcubes Q_{n-1}, denoted by Q^0_{n-1} and Q^1_{n-1}. For each $v \in V(Q_n)$, insert a 0 to the front* *of $(n-1)$-bit binary string for v in Q^0_{n-1} and a 1 to the front of $(n-1)$-bit binary string for v in Q^1_{n-1}. There are 2^{n-1} edge between Q^0_{n-1} and Q^1_{n-1} as follows:*

Let $V(Q^0_{n-1}) = \{0u_{n-2}u_{n-3}...u_0 : u_i = 0 \text{ or } 1\}$ and $V(Q^1_{n-1}) = \{1v_{n-2}v_{n-3}...v_0 : v_i = 0 \text{ or } 1\}$, where $0 \leq i \leq n-2$. A node $u = 0u_{n-2}u_{n-3}...u_0$ of $V(Q^0_{n-1})$ is joined to a node $v = 1v_{n-2}v_{n-3}...v_0$ of $V(Q^1_{n-1})$ if and only if $u_i = v_i$ for $0 \leq i \leq n-2$.

Definition 3 *[8] The Crossed cube CQ_1 is a complete graph with two nodes labeled by 0 and 1, respectively. For $n \geq 2$, an n-dimensional Crossed cube CQ_n consists of two $(n-1)$-dimensional sub-Crossed cubes, CQ^0_{n-1} and CQ^1_{n-1}, and a perfect matching between the nodes of CQ^0_{n-1} and CQ^1_{n-1} according to the following rule:*

Let $V(CQ^0_{n-1}) = \{0u_{n-2}u_{n-3}...u_0 : u_i = 0 \text{ or } 1\}$ and $V(CQ^1_{n-1}) = \{0v_{n-2}v_{n-3}...v_0 : v_i = 0 \text{ or } 1\}$. The node $u = 0u_{n-2}u_{n-3}...u_0 \in V(CQ^0_{n-1})$ and the node $v = 0v_{n-2}v_{n-3}...v_0 \in V(CQ^1_{n-1})$ are adjacent in CQ_n if and only if

1. *$u_{n-2} = v_{n-2}$ if n is even, and*

2. *$(u_{2i+1}u_{2i}, v_{2i+1}v_{2i}) \in \{(00,00),(10,10),(01,11),(11,01)\}$, for $0 \leq i \leq \lfloor \frac{n-1}{2} \rfloor$*

Definition 4 *[13] The Twisted cube TQ_1 is a complete graph with two nodes, 0 and 1. Let n be an odd integer and $n \geq 3$. The nodes of an n-dimensional Twisted cube TQ_n are decomposed into four sets $S^{0,0}$,$S^{0,1}$, $S^{1,0}$ and $S^{1,1}$. The sets $S^{i,j}$ consists of those nodes $u = u_{n-1}u_{n-2}...u_0$ with $u_{n-1} = i$ and $u_{n-2} = j$, where $(i,j) \in \{(0,0),(0,1),(1,0),(1,1)\}$. The induced subgraph of $S^{i,j}$ in TQ_n is isomorphic to TQ_{n-2}. Edges which connect these four*

$(n-2)$-dimensional subtwisted cubes can be described as follows: Any node $u_{n-1}u_{n-2}..u_0$ with $P_{n-3}(u) = 0$ is connected to $\bar{u}_{n-1}\bar{u}_{n-2}...u_0$ and $\bar{u}_{n-1}u_{n-2}...u_0$; and to $u_{n-1}\bar{u}_{n-2}...u_0$ and $\bar{u}_{n-1}u_{n-2}...u_0$, if $P_{n-3}(u) = 1$.

Definition 5 *[7] $0-MQ_1$ and $1-MQ_1$ are both the complete graph on two nodes whose labels are 0 and 1. For $n \geq 2$, both $0-MQ_n$ and $1-MQ_n$ contain one $0-$type sub-Möbius cube MQ^0_{n-1} and one $1-$type sub-Möbius cube MQ^1_{n-1}. The first bit of every node of MQ^0_{n-1} is 0, and the first bit of every node of MQ^1_{n-1} is 1. For two nodes $u = 0u_{n-2}u_{n-3}...u_0 \in V(MQ^0_{n-1})$ and $v = 1v_{n-2}v_{n-3}...v_0 \in V(MQ^1_{n-1})$,*

1. *u connects to v in $0-MQ_n$ if and only if $u_i = v_i$, for every i, $0 \leq i \leq n-2$*
2. *u connects to v in $1-MQ_n$ if and only if $u_i = \bar{v}_i$, for every i, $0 \leq i \leq n-2$*

Now We formally introduce the *MCN*. The *MCN* is constructed from two graph G_1 and G_2 with the same number of vertices, by adding a perfect matching M between the vertices of G_1 and G_2. We shall call these two graphs G_1 and G_2 as the *M-components* of the *MCN*. We use the notation $G = G_1 \oplus_M G_2$ to denote a *MCN*, which has vertex set $V(G_1 \oplus_M G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \oplus_M G_2) = E(G_1) \cup E(G_2) \cup M$. The *MCN* includes many well-known interconnection networks as special cases, such as the Hypercube Q_n, the Crossed cube CQ_n, the Twisted cube TQ_n, and the Möbius cube MQ_n.

2.2 Comparison Model

The comparison model, the status of fault or fault-free of a processor is determined by sending the same testing task and comparing the response on one processor and the response on another, is proposed by *Maeng* and *Malek* [17, 16]. Because of the names, the comparison model is also called *MM-model*. Under the comparison model, a processor ,which is called *comparator*, sent the same input to two of adjacent processor and compare the responses. Maybe different *comparator* k test the same pair of processors i, j. We define $(i, j)_k$ is that i, j is be compared by *compartor* processor k. A disagreement of the response is defined $r((i, j)_k) = 1$, whereas an agreement of the comparison result is defined $r(i, j)_k) = 0$.

A *comparator* k not always is fault-free. $r((i, j)_k) = 0$ represents that if processor k is fault-free, then i, j are fault-free. In other hand, $r((i, j)_k) = 1$ represents that at least one of i, j, k is faulty. We list all of possible comparison result in Table 2.1.

Other node / comparator	Test Result	
	Faulty free	At least one is faulty
Fault free	0	1
Fault	0 or 1	0 or 1

Table 2.1: The possible result in Comparison

To gain as much information as possible about the faulty status of the system, it was assumed that a comparison is performed by each processor for each pair of distinct neighbors with which it can communicate directly. This special case of *MM-model* is henceforth to as the *MM*-model*. In this thesis, our discussion is under *MM*-model*.

We can use the multigraph $M = (V, C)$ to represent the comparison Model. The set of V in M is the same set of V in G. An edge $(i, j)_k$ in C represents the fact that $\exists i, j, k \in V$,i, j are being compared by a *comparator* k. That is a example in Fig 2.1. It is easy to observe that the same pair of processors i,j can be compared by different *comparator* k. So the comparison Model is multigraph.

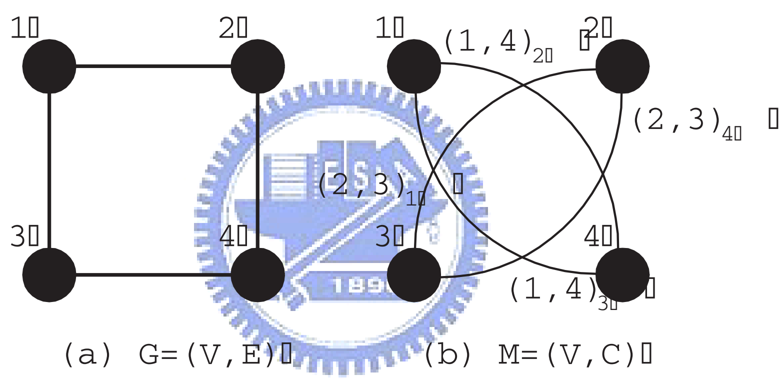

Figure 2.1: (a)A system with four units. (b)all of the testing of (a)

The set of all of the comparison result is called *syndrome*. The *faulty set* in a graph G, written as F, is the set of faulty vertices in a graph G. For example, assume node 1 of Fig 2.1(a) is faulty. node 2,3,4 are faulty-free.(faulty set $F = \{1\}$) We show the possible *syndrome* in Table 2.2.

Hence the same faulty set can make different *syndrome*. A self-diagnosable system

i	j	k	$r((i,j)_k)$
1	4	2	1
1	4	3	1
2	3	1	1
2	3	4	0

or

i	j	k	$r((i,j)_k)$
1	4	2	1
1	4	3	1
2	3	1	0
2	3	4	0

Table 2.2: The possible syndrome of Fig 2.1(faulty set={1})

which is called *t-diagnosable* system is any *syndrome* only mapping one faulty set, when the number of faults does not exceed t.

For example, we list all of possible *syndrome* of Fig 2.1 in table 2.3. We can not tell which node is faulty when seeing *syndrome* 1 because *syndrome* 1 and *syndrome* 7 are the same. So this graph can not diagnosis even if only one node is faulty.

i	j	k	$r((i,j)_k)$							
			fault set={1}		*fault set*={2}		*fault set*={3}		*fault set*={4}	
1	4	2	1	1	0	1	0	0	1	1
1	4	3	1	1	0	0	0	1	1	1
2	3	1	0	1	1	1	1	1	0	0
2	3	4	0	0	1	1	1	1	0	1
syndrome			1	2	3	4	5	6	7	8

Table 2.3: All of possible syndrome of Fig 2.1

Under the comparison Model, there assumptions are made:

1. all faults are permanent;
2. a faulty processor produces incorrect outputs for each of its given tasks;
3. the outcome of a comparison performed by a faulty processor is unreliable;
4. two faulty processors, when given the same inputs and task, do not produce the same output; and,
5. there is an upper bound, t, on the number of faulty processors in the system.

We use $\sigma(F)$ to represent the set of all *syndromes* which F is the *faulty set*. Two distinct sets F_1, F_2 are called to be *indistinguishable* if and only if $\sigma(F_1) \cap \sigma(F_2) \neq \emptyset$. We also say that (F_1, F_2) is an *indistinguishable pair*. Otherwise F_1, F_2 are called to be *distinguishable* or (F_1, F_2) is a *distinguishable pair* if and only if $\sigma(F_1) \cap \sigma(F_2) = \emptyset$.

2.3 Preliminaries

Assume $U \subseteq V(G)$. $G[U]$ denote the subgraph of G induced by the node subset U of G and $\bar{U} = V(G) - U$. A set of vertices in G that covers every edge of G is called a *vertex cover*. A *vertex cover* of minimum cardinality is called *minimum vertex cover*. Given a graph G, let M be the comparison graph of G. For a node $v \in V(G)$, we define X_v to be the set of nodes$\{u|(v,u) \in E(G)\} \cup \{u|(v,u)_w \in E(M)$ for some w$\}$ and Y_v to be the set of edges $\{(u,w)|u,w \in X_v$ and $(v,u)_w \in E(M)\}$.In [21], the *order graph* of node v is defined as $G_v = (X_v, Y_v)$ and the *order* of the node v, denote by $order(v)$, is defined to be the cardinality of a minimum vertex cover of G_v. Let $U \subset V(G)$, we use $T(G,U)$ to denote the set $\{v|(u,v)_w \in E(M)$ and $w,u \in U, v \in \bar{U}\}$. We observe that $T(G,U) = N(\bar{U}, U)$ if $G[U]$ is connected and $|U| > 1$. This observation can be extended to the following lemma.

Lemma 1 *[14]Let U be a subset of $V(G)$ and $G[U_i]$, $1 \le i \le k$, be the connected components of the subgraph $G[U]$ such that $U = \bigcup_{i=1}^{k} U_i$. Then $T(G,U) = \bigcup_{i=1}^{k}\{N(\bar{U}, U_i)|$ $|U_i| > 1\}$.*

We need to use several important way to verify a system whether it is *t-diagnosable* or not. We list several theorems given by *Sengupta* and *Dahbura*[21].

Theorem 1 *[21] For any F_1,F_2 where F_1,$F_2 \subset V$ and $F_1 \neq F_2$, (F_1, F_2) is a distinguishable pair if and only if at least one of the following conditions is satisfied:(See Fig. 2.2)*

1. $\exists i, k \in V - F_1 - F_2$ *and* $\exists j \in (F_1 - F_2) \cup (F_2 - F_1)$ *such that* $(i, j)_k \in C$,

2. $\exists i, j \in F_1 - F_2$ *and* $\exists k \in V - F_1 - F_2$ *such that* $(i, j)_k \in C$, *or*

3. $\exists i, j \in F_2 - F_1$ *and* $\exists k \in V - F_1 - F_2$ *such that* $(i, j)_k \in C$

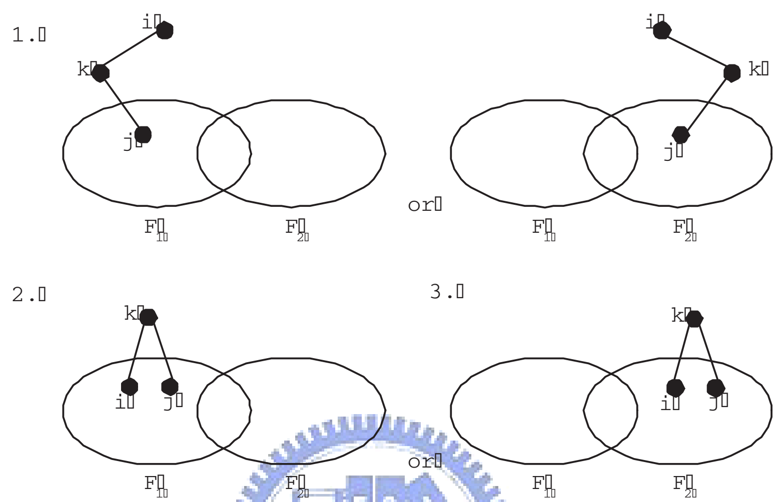

Figure 2.2: Illustrations of a distinguishable pair(F_1, F_2)

Theorem 1 gives a necessary and sufficient condition to ensure distinguishability of a pair of set of vertices (F_1, F_2).The following theorem is necessary and sufficient conditions for ensuring distinguishability.

Theorem 2 *[21] A system is* t*-diagnosable if and only if each node has order at least* t *and for each distinct pair of sets* $F_1, F_2 \subset V$, *such* $|F_1| = |F_2| = t$ *at least one of the conditions of theorem 1 is satisfied.*

The next theorem is a sufficient condition for verifying a system to be t*-diagnosable.*

Theorem 3 *[21] A system with n nodes is t-diagnosable if*

1. $n \geq 2t+1$
2. *each node has order at least t*
3. $|T(G,U)| > p$ *for each* $U \subset V(G)$ *such that* $|U| = N - 2t + p$ *and* $0 \leq p \leq t-1$

Let $G = (V,E)$, there is a component C, $C \subseteq G$. we define that $V_G(C;3) = \{i \in C | deg_G(i) \geq 3\}$.

For any F_1, F_2 where $F_1, F_2 \subset V_G$ and $F_1 \neq F_2$. The following lemma gives a sufficient condition to determine whether (F_1, F_2) is a distinguishable pair. This result is useful for our discussion later.

Lemma 2 *Let $G = (V,E)$ be the graph of a system. For two distinct subsets $F_1, F_2 \subset V(G)$ with $|F_i| \leq t$, $i = 1,2$. Let $S = F_1 \cap F_2$, $|S| = p$, $0 \leq p \leq t-1$. If there exists a component C of $G - S$ such that $V_C \cap (F_1 \triangle F_2) \neq \emptyset$ and $|V_{G-S}(C;3)| \geq 2(t-p)+1$. Then (F_1, F_2) is a distinguishable pair.*

Proof. Let $U = G - F_1 \cup F_2$. Since a component C of $G - S$ such that $V_C \cap (F_1 \triangle F_2) \neq \emptyset$ and $|V_{G-S}(C;3)| \geq 2(t-p)+1$. Hence, there exists a vertex a in $V(C) \cap V(U)$ such that $deg_{G-S}(a) \geq 3$. If $N_{G-S}(a) \cap F_1 \triangle F_2 = \emptyset$. Since component C is connected. Hence, we can find the case such that the condition 1 of Theorem 1 is satisfied. Otherwise, $N_{G-S}(a) \cap F_1 \triangle F_2 \neq \emptyset$. Hence, there exists $(a,b) \in E(G-S)$, $a \in U$ and $b \in F_1 \triangle F_2$,

$deg_{G-S}(a) = 3$. Assume $N_{G-S}(a) \cap U \neq \emptyset$. Hence, the condition 1 of Theorem 1 is satisfied. Otherwise $N_{G-S}(a) \cap U = \emptyset$ and $deg_{G-S}(a) = 3$. It means that the condition 2 or 3 of Theorem 1 is satisfied. This completes the proof of the lemma.

□

By Lemma 2, the following theorem gives a sufficient condition to determine whether a system G is *t-diagnosable*.

Theorem 4 *Let $G = (V, E)$ be the graph of a system. G is t-diagnosable if for each vertex set $S \subset V$ with $|S| = p$, $0 \leq p \leq t-1$, every component C of $G - S$, $|V_{G-S}(C;3)| \geq 2(t-p)+1$.*

Proof.

For any two distinct subsets $F_1, F_2 \subset V(G)$, $|F_i| \leq t$, $i = 1, 2$. We can let $S = F_1 \cap F_2$ with $|S| = p$, $0 \leq p \leq t-1$. Since every component C of $G-S$, $|V_{G-S}(C;3)| \geq 2(t-p)+1$. By Lemma 2, (F_1, F_2) is a distinguishable pair. Hence, G is *t-diagnosable*. This completes the proof of the theorem.

□

The following Theorem is that the diagnosability of the *MCN* constructed from G_1 and G_2 is $t+1$ under the comparison model.

Theorem 5 *[14] For $t \geq 2$, let G_1 and G_2 be two graphs with the same number of nodes N, where $N \geq t+2$. Suppose that $order(v) \geq t$ for every node v in G_i and the connectivity $\kappa(G_i) \geq t$, where $i = 1, 2$. Then the MCN $G = G_1 \oplus_M G_2$ is $(t+1)$-diagnosable.*

Lemma 3 *[6] Assume that t is a positive integer. Let G_1 and G_2 be two k-regular maximum connected graphs with t vertices, and the MCN G is $G = G_1 \oplus_M G_2$. Then, G is $(k+1)$-regular super-connected if and only if (1)$t > k+1$ or (2)$t = k+1$ with $k = 0, 1, 2$.*

Chapter 3

strongly t-diagnosable

In this chapter, we illustrate the concept of *strongly t-diagnosable* and some necessary and sufficient conditions. Finally, We prove that the cube family with n-dimensional are all *strongly n-diagnosable* for $n \geq 4$.

The Hypercube Q_n, the Crossed cube CQ_n are famous n-*diagnosable* but not $(n+1)$-*diagnosable*. For each of these cubes, we observe that for any two distinct sets of vertex F_1 and F_2, $|F_1| \leq n+1$, $|F_2| \leq n+1$, F_1, F_2 are indistinguishable because there exists some vertex v such that $N(v) \subset F_1$ and $N(v) \subset F_2$. In other word, $N(v) \subset F_1 \cap F_2$.

First, we take Q_4 as an example. We know that Q_4 is 4-*diagnosable*[14] but not 5-*diagnosable*. The following Lemma show that Q_4 is almost 5-*diagnosable* except that all the neighbors of some vertex are faulty simultaneously.

A fault-set $F \subset V$ is called a conditional fault-set if $N(v) \nsubseteq F$ for every vertex $v \in V$. Let $F_1, F_2 \subset V$ and $F_1 \neq F_2$. We say (F_1, F_2) is a distinguishable conditional pair(an indistinguishable conditional pair respectively) if F_1 and F_2 are conditional fault sets and are distinguishable(indistinguishable respectively).

Lemma 4 *Let $F_1, F_2 \subset Q_4$, (F_1, F_2) be a conditional pair with $|F_i| \leq 5$, $i = 1, 2$. Then (F_1, F_2) is a distinguishable pair under the comparison model.*

Proof. Let $S = F_1 \cap F_2$ with $|S| = p$, $0 \leq p \leq 4$. Since (F_1, F_2) be a conditional pair. Hence, for each vertex $v \in V(Q_4)$, $N(v) \nsubseteq S$. By Lemma 3, $Q_4 - S$ is connected. That is, the only component of $Q_4 - S$ is itself. Let $C = Q_4 - S$. By theorem 4, we want to prove that the only component C, $|V_{G-S}(C;3)| \geq 2(5-p)+1$, $0 \leq p \leq 4$. We divide this into the following main cases. By Lemma 2, we show that F_1, F_2 in each case is a distinguishable pair.

Case 1: $p = 0$

It is trivial for this case. $deg_{G-S}(v)$ is 4, $v \in G - S$. $|V_{G-S}(C;3)| = 2^4 \geq 2(5-0)+1$. By Lemma 2, F_1, F_2 is a distinguishable pair.

Case 2: $p = 1$

Assume $x \in Q_4$ is faulty. $deg_{G-S}(v)$ is $4 - 1 \geq 3$, $v \in N(x)$. $deg_{G-S}(v)$ is still $4 \geq 3$, $v \in V - N(x) - \{x\}$. $|V_{G-S}(C;3)| = 2^4 - 1 \geq 2(5-1)+1$. By Lemma 2, F_1, F_2 is a distinguishable pair.

Case 3: $p = 2$

The number of nodes which is $deg(v) < 3$ is at most one. $|V_{G-S}(C;3)| \geq 2^4 - 2 - 1 \geq 2(5-2)+1$. By Lemma 2, F_1, F_2 is a distinguishable pair.

Case 4: $p = 3$

Q_4 is composed of Q_3^0 and Q_3^1 by adding a perfect matching. Let $S_0 = S \cap Q_3^0$, $|S_0| = p_0$, $S_1 = S \cap Q_3^1$, $|S_1| = p_1$. We divide the case into two subcases: (4.a) either $p_0 = 0$ and $p_1 = 3$, or, $p_0 = 3$ and $p_1 = 0$. For subcase(4.b) either $p_0 = 1$ and $p_1 = 2$, or, $p_0 = 2$ and $p_1 = 1$.

Subcase 4.a: either $p_0 = 0$ and $p_1 = 3$, or, $p_0 = 3$ and $p_1 = 0$.

Without loss of generality, assume $p_0 = 0$ and $p_1 = 3$. So each vertex in Q_3^0 is faulty free. For each vertex v in Q_3^0, $deg(v) \geq 3$, $|Q_3^0| = 8$. $|V_{G-S}(C;3)| \geq 8 \geq 2(5-3)+1$. By Lemma 2, F_1, F_2 is a distinguishable pair.

Subcase 4.b: either $p_0 = 1$ and $p_1 = 2$, or, $p_0 = 2$ and $p_1 = 1$

Without loss of generality, assume $p_0 = 1$ and $p_1 = 2$. Assume $x_1 \in Q_3^0$ is faulty. For each v in $Q_3^0 - N(x_1) - \{x_1\}$, $deg(v) \geq 3$, $|Q_3^0 - N(x_1) - \{x_1\}| = 4$. Since $p_1 = 2$, assume $x_2, x_3 \in Q_3^1$ are faulty, $|(N(x_2) \cup N(x_3)) \cap Q_3^0| = 2$. Hence, there exists a vertex y in $N(x_1)$ such that z is faulty free, $z \in N(y) \cap Q_3^1$. So $deg_{G-S}(y) = 3$. $|V_{G-S}(C;3)| \geq 4+1 \geq 2(5-3)+1$. By Lemma 2, F_1, F_2 is a distinguishable pair.

Case 5: $p = 4$

Let $U = G - F_1 - F_2$, $|F_1 \triangle F_2| \leq 2(5-p) = 2(5-4) = 2$, $|U| = |V(G)| - |F_1 \cap F_2| \geq 16 - (2 \times 5 - p) = 6 + p = 6 + 4 = 10$. Since $G - S$ is connected, there exists (a, b) in $E(G)$ such that $a \in F_1 \triangle F_2$, $b \in U$. U_i, $1 \leq i \leq k$, be the connected components of subgraph U such that $U = \cup_{i=1}^{k} U_i$. We assume $|U_i| > 1$. We can find the case such that the condition 1 of Theorem 1 is satisfied. Hence, (F_1, F_2) is a distinguishable pair. Otherwise $|U_i| = 1$, for all $1 \leq i \leq k$. Hence, $N_{G-S}(v) \subset F_1 \triangle F_2$, $v \in U$. $\Sigma_{v\in U}|deg_{G-S}(v)| \leq \Sigma_{v\in F_1\triangle F_2}|deg_{G-S}(v)|$. $\Sigma_{v\in U}|deg_{G-S}(v)| \geq (10 \times 4) - 4 \times 4 = 24$. $\Sigma_{v\in F_1\triangle F_2}|deg_{G-S}(v)| \leq 2 \times 4 = 8$. $\Sigma_{v\in U}|deg_{G-S}(v)| > \Sigma_{v\in F_1\triangle F_2}|deg_{G-S}(v)|$. This is a contradiction.

□

Definition 6 *A system G is strongly t-diagnosable if the following two conditions holds:*

1. *G is t-diagnosable, and*
2. *for any two distinct subsets $F_1, F_2 \subset V(G)$ with $|F_i| \leq t+1$, $i = 1, 2$,*

   either (a) (F_1, F_2) is a distinguishable pair;

   or (b) (F_1, F_2) is an indistinguishable pair and there exists a vertex $v \in V$

   such that $N(v) \subseteq F_1$ and $N(v) \subseteq F_2$.

By Theorem 3 and Definition 6, we propose a sufficient condition for checking if a system G is *strongly t-diagnosable* as follows.

Lemma 5 *A system $G = (V, E)$ with $|V| = n$ is strongly t-diagnosable if*

1. $n \geq 2t + 1$

2. each node has order at least t

3. $|T(G, U)| > p$ *for each* $U \subset V(G)$ *such that* $|U| = N - 2t + p$ *and* $0 \leq p \leq t - 1$

4. for any two distinct subsets $F_1, F_2 \subset V(G)$ *with* $|F_i| \leq t + 1$, $i = 1, 2$,

either (a) (F_1, F_2) *is a distinguishable pair;*

or (b) (F_1, F_2) *is an indistinguishable pair and there exists a vertex* $v \in V$

such that $N(v) \subseteq F_1$ *and* $N(v) \subseteq F_2$.

Proof. With conditions 1, 2 and 3, by Theorem 3, G is *t-diagnosable*. Condition 4 is the same as condition 2 of Definition 6. So we complete the proof.

□

Theorem 6 *A system G=(V,E) is strongly t-diagnosable if for each vertex set* $S \subset V$ *with cardinality* $|S| = p$, $0 \leq p \leq t$, *the following two conditions are satisfied*

1. for $0 \leq p \leq t - 1$, *every component* C *of* $G - S$ $|V_{G-S}(C; 3)| \geq 2((t + 1) - p) + 1$

2. for $p = t$, *either every component* C *of* $G - S$ *satisfies* $|V_{G-S}(C; 3)| \geq 3$ *or else* $G - S$ *satisfies at least one trivial component.(Remark:* $2((t + 1) - p) + 1 = 3$ *as* $p = t$*)*

Proof. Assume $S \subset V$, $|S| = p$, $0 \le S \le t-1$, By condition 1, every component C of $G-S$ satisfies $|V_{G-S}(C;3)| \ge 2((t+1)-p)+1 \ge 2(t-p)+1$. By Theorem 4, G is *t-diagnosable*.

In order to prove that G is *strongly t-diagnosable*, we need to show that condition 2 of Definition 6 holds. Assume (F_1, F_2) be an indistinguishable pair, $F_1 \ne F_2$, $|F_1| \le t+1$, $|F_2| \le t+1$. Let $S = F_1 \cap F_2$,$|S| = p$, $0 \le p \le t$. Since F_1 and F_2 are indistinguishable. By Theorem 4, exists component C in $G-S$ is $|V_{G-S}(C;3)| \le 2(t-p)$. By condition 1, p cannot be in the range from 0 to $t-1$. So $p = t$. Because component C $|V_{G-S}(C;3)| \le 2((t+1)-p) = 2((t+1)-t) = 2$. By condition 2, $G-S$ contains at least one trivial component $\{v\}$. So $N(v) \subset S$. It is equal to $v \subseteq F_1$ and $v \subseteq F_2$. Therefore, G is *strongly t-diagnosable*.

□

Theorem 7 *For $t \ge 3$, let $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$ be two t-regular graph with the same number of vertices N, $N \ge 2t+1$. $order_{G_i}(v) \ge t$ for every node v in G_i and the connectivity $\kappa(G_i) \ge t$ for $i = 1, 2$. Then the MCN $G = (V, E) = G_1 \bigoplus_M G_2$ is strongly $(t+1)$-diagnosable.*

Proof. By definition 6, we want to prove the following two conditions: (i)G is $(t+1)$-*diagnosable* (ii) for each indistinguishable pair (F_1, F_2), $F_i \subset V$, $i = 1, 2$, with $|F_i| \le t+2$, it implies that there exists a vertex $v \in V$ such that $N(v) \subseteq F_1$ and $N(v) \subseteq F_2$.

First, by Theorem 5, G is $(t+1)$-*diagnosable*. The condition (i) holds. So we only need to prove condition (ii). Let (F_1, F_2) is an indistinguishable pair, $F_i \subset V$, $i = 1, 2$, with $|F_i| \leq t+2$. Let $S = F_1 \cap F_2$, $|S| = p$, $0 \leq p \leq t+1$. If there exists a vertex $v \in V$, $N(v) \subseteq S$. We finish the proof. Otherwise, $N(v) \nsubseteq S$ for each vertex $v \in V$. We want to show that this is a contradiction. By Lemma 3, $G - S$ is connected. The only component C of $G-S$ is $G-S$ itself. We divide this case into following two main cases: (1)$0 \leq p \leq 3$ and(2)$4 \leq p \leq t+1$.

Case 1: $0 \leq p \leq 3$

We show that (F_1, F_2) in each case is a distinguishable pair.

Subcase 1.1: $p = 0$

It is trivial for this case. $deg_{G-S}(v)$ is $t+1 \geq 3$ for $t \geq 3$, $v \in G-S$. $|V_{G-S}(C;3)| \geq 2(2t+1) \geq 2((t+2)-0)+1$ for $t \geq 3$. By Lemma 2, F_1, F_2 is a distinguishable pair.

Subcase 1.2: $p = 1$

Assume $x \in V$ is faulty. $deg_{G-S}(v)$ is $(t+1)-1 \geq 3$ for $t \geq 3$, $v \in N(x)$. $deg_{G-S}(v)$ is still $(t+1) \geq 3$ for $t \geq 3$, $v \in V - N(x) - \{x\}$. $|V_{G-S}(C;3)| \geq 2(2t+1)-1 \geq 2((t+2)-1)+1$ for $t \geq 3$. By Lemma 2, F_1, F_2 is a distinguishable pair.

Subcase 1.3: $p = 2$

The number of nodes which is $deg(v) < 3$ is at most one. $|V_{G-S}(C;3)| \geq 2^4 - 2 - 1 \geq 2(5-2)+1$. By Lemma 2, F_1, F_2 is a distinguishable pair.

Subcase 1.4: $p = 3$

G is composed of G_1 and G_2 by adding a perfect matching. Let $S_0 = S \cap G_1$, $|S_0| = p_0$, $S_1 = S \cap G_2$, and $|S_1| = p_1$. We divide the case into two subcase: (1.4.1) either $p_0 = 0$ and $p_1 = 3$, or, $p_0 = 3$ and $p_1 = 0$. and (1.4.2) either $p_0 = 1$ and $p_1 = 2$, or, $p_0 = 2$ and $p_1 = 1$.

Subcase 1.4.1: either $p_0 = 0$ and $p_1 = 3$, or, $p_0 = 3$ and $p_1 = 0$.

Without loss of generality, assume $p_0 = 0$ and $p_1 = 3$. So each node in $V(G_1)$ is faulty free. For each vertex v in $V(G_1)$, $deg_{G-S}(v) \geq 3$ for $t \geq 3$. $|V(G_1)| \geq 2t + 1$. $|V_{G-S}(C;3)| \geq 2t+1 \geq 2((t+2)-3)+1$ for $t \geq 3$. By Lemma 2, F_1, F_2 is a distinguishable pair.

Subcase 1.4.2: either $p_0 = 1$ and $p_1 = 2$, or, $p_0 = 2$ and $p_1 = 1$

Without loss of generality, assume $p_0 = 1$ and $p_1 = 2$. Let $x_1 \in V(G_1)$ is faulty. For each v in $V(G_1) - N(x_1) - \{x_1\}$, $deg_{G-S}(v) = t+1 \geq 3$ for $t \geq 3$, $|V(G_1) - N(x_1) - \{x_1\}| \geq 2t + 1 - t - 1 = t$. The number of degree greater than t in $G_1 - N(x_1) - x_1$ is t. For each v in $N(x_1) \cap V(G_1)$. If $N(v) \cap V(G_2)$ is faulty, then $deg_{G-S}(v) = t + 1 - 1 - 1 < t$. There exists at most two vertices $deg_{G-S}(v) = t + 1 - 1 - 1 < t$ because of $p_1 = 2$. The minimum number of degree greater than t in $N(x_1) \cap V(G_1)$ is $t - 2$. G_2 is a t-regular graph with two faulty vertices x_2 and x_3. Then there exists at most $2t$ vertices such that the degree of these vertices is $t - 1$. The minimum number of degree greater than t in G_2 is $2t + 1 - 2t = 1$. $|V_{G-S}(C;3)| \geq t + (t-2) + 1 = 2t - 1 \geq 2((t+2)-3)+1$. By Lemma

2, F_1, F_2 is a distinguishable pair.

Case 2: $4 \leq p \leq t+1$

Let $U = G - F_1 - F_2$, $|F_1 \triangle F_2| \leq 2(t+2-p)$, $|U| = |V(G)| - |F_1 \cap F_2| \geq 2(2t+1) - (2(t+2)-p) = 2t-2+p$. Since $G - S$ is connected, there exists (a,b) in $E(G)$ such that $a \in F_1 \triangle F_2$, $b \in U$. U_i, $1 \leq i \leq k$, be the connected components of subgraph U such that $U = \cup_{i=1}^{k} U_i$. We assume $|U_i| > 1$. We can find the case such that the condition 1 of Theorem 1 is satisfied. Hence, (F_1, F_2) is a distinguishable pair. Otherwise $|U_i| = 1$, for all $1 \leq i \leq k$. Hence, $N_{G-S}(v) \subset F_1 \triangle F_2$, $v \in U$. $\Sigma_{v \in U} |deg_{G-S}(v)| \leq \Sigma_{v \in F_1 \triangle F_2} |deg_{G-S}(v)|$. $\Sigma_{v \in U} |deg_{G-S}(v)| \geq ((2t-2+p) \times t) - p \times t = (2t-2) \times t$. $\Sigma_{v \in F_1 \triangle F_2} |deg_{G-S}(v)| \leq 2(t+2-p) \times t$. $\Sigma_{v \in U} |deg_{G-S}(v)| > \Sigma_{v \in F_1 \triangle F_2} |deg_{G-S}(v)|$, $p \geq 4$. This is a contradiction.

□

Applying Theorem 7, we list the following corollary.

Corollary 1 *The Hypercube Q_n, the Crossed cube CQ_n, the Twisted cube TQ_n, and the Möbius cube MQ_n are all strongly n-diagnosable for $n \geq 4$.*

In the following, we show that Q_3 is not *strongly* 3*-diagnosable*. Let $F_1 = \{010, 100, 111\}$, $F_2 = \{001, 100, 111\}$, $|F_1| = |F_2| = 3$, $S = F_1 \cap F_2$. Since $N(v) \nsubseteq S, v \in V(Q_3)$ and (F_1, F_2) is a distinguishable pair. Hence, Q_3 is not *strongly* 3*-diagnosable*.

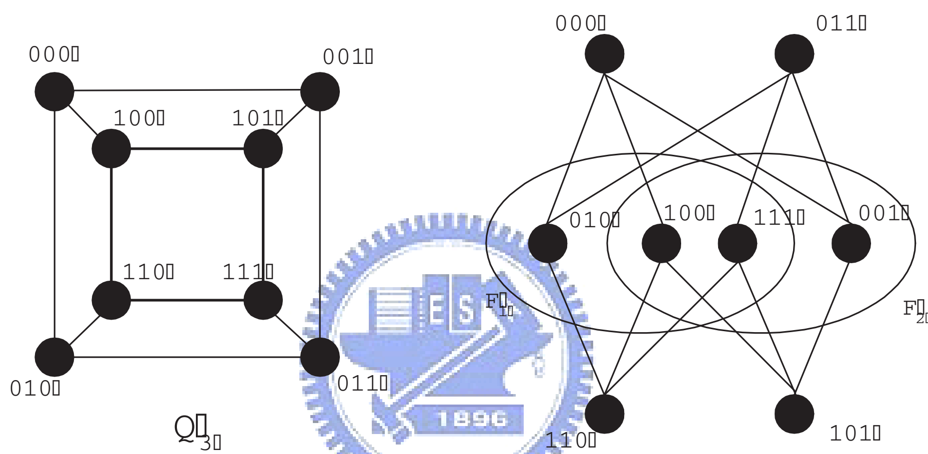

Figure 3.1: An example of *non-strongly* 3-*diagnosable* system

Chapter 4

Conclusions

We observe that cube family are almost $(n+1)$-*diagnosable* except the case that all the neighbors of some vertex are faulty simultaneously. In this thesis, We introduce a new concept, called a *strongly t-diagnosable* system under the comparison model. G_1, G_2 are two t-regular graph with the same number of vertices N, $N \geq 2t+1$, for $t \geq 3$. $order_{G_i}(v) \geq t$ for every node v in G_i and the connectivity $\kappa(G_i) \geq t$ for $i = 1, 2$. We prove that the *MCN* constructed from G_1 and G_2 is *strongly(t+1)-diagnosable*. According to the result, we know that cube family with n-dimensional are all *strongly n-diagnosable* for $n \geq 4$.

In the future work, we can try to solve the problem how large the maximum value of t such that cube family remains t-*diagnosable* under the condition that every fault-set F satisfies $N(v) \nsubseteq F$ for each vertex $v \in V$. For example, $\{v_1, v_2, v_3, v_4\}$ is a subset Q_2 of Q_n. Let $F_1 = \{v_2, v_4\} \cup N(v_1) \cup N(v_2) \cup N(v_3) - \{v_1, v_3\}$, $F_2 = \{v_3, v_4\} \cup N(v_1) \cup N(v_2) \cup N(v_3) - \{v_1, v_2\}$ (See Fig 4.1). $|F_1| = 3(n-2)+2$, $|F_2| = 3(n-2)+2$. Every vertex has at most one good neighbor either F_1 or F_2 is faulty set. Because none of condition of Theorem 1 holds, (F_1, F_2) is an indistinguishable pair. There is an example to show that the conditional diagnosability of the Hypercube Q_n is no greater than $3(n-2)+2$.

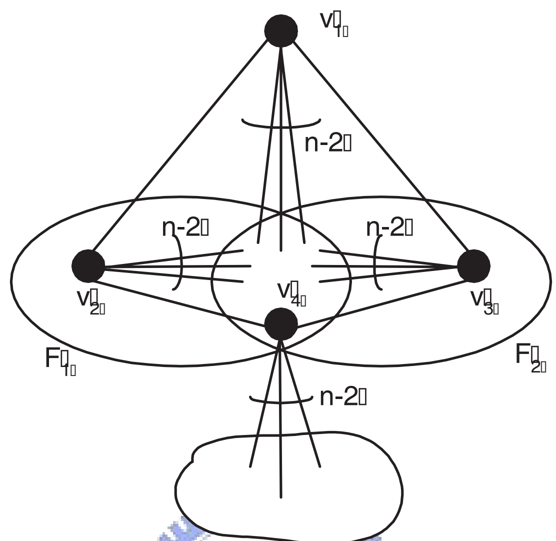

Figure 4.1: (F_1, F_2) is an indistinguishable pair of Q_n under every vertex has one good neighbor condition.

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