Pan-connectivity of augmented cubes

May 25, 2004

Abstract

Augmented cubes, AQ_n, are derivatives of cubes with good geometric nature. A graph G is panconnected if there exists a path of length l joining any two vertices x and y with $d(x, y) \leq l \leq |V(G)| - 1$.In this paper, we make a study of pan-connectivity of a simple graph and prove that augmented cubes are panconnected.

Keywords: pan-connectivity, connectivity, hamiltonian connected, augmented cubes.

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Chapter 1

Introduction

For the good of convenience, we traditionally model a network by a graph, in which nodes stand for processors and edges present the direct communication between two processors. Many *topologies* have been proposed to balance the performance and some cost parameters. *Hypercubes* are widely studied in the interconnection networks[3][4]. Several variations of hypercubes have been proposed to improve the efficiency of hypercube network, like twisted cubes[5][6], twisted N-cubes[7], crossed cubes[8][9], Mobius cubes[10], augmented cubes[2].

1.1 Hamiltonian connected

A graph G is *hamiltonian connected* if there exists a hamiltonian path joining any two vertices of G. all hamiltonian connected graphs except K_1 and K_2 are hamiltonian. A graph G is *k-fault hamiltonian connected* if G-F remains hamiltonian connected for every $F \subset V(G) \bigcup E(G)$ with $|F| \leq k$. The *fault hamiltonian connectivity*, H_f^k(G), is defined to be the maximum integer k such that G is k-fault hamiltonian connected if G is hamiltonian connected and is undefined otherwise.

It can be checked that $H_f^k(G) \leq \delta(G) - 3$ if H_f^k(G) is defined and $|V(G)| \geq 4$. In[11], it is proved that $H_f^k(G) = 2n - 4$ if $n \subsetneq \{1, 3\}$, $H_f^k(AQ_3) = 0$. Again, this result concerning the fault hamiltonian connectivity of the augmented cube AQ_n is optimal since $\delta(AQ_n) = 2n - 1$ and $\delta(AQ_n) - 3 = 2n - 4$. Thus, $H_f^k(AQ_n) = 2n - 4$, if $n \geq 4$. This is described in Theorem 1.

1.2 Pan-connected

A graph G is pan-connected if there exists a path of length $\leq l$ joining any two vertices x and y with $d(x, y) \leq l \leq |V(G)| - 1$. That says, any two processors in the network with pan-connectivity can communicate with each other at all possible distances that varies from the length of the shortest path to that of the hamiltonian path. With that, we can decide the routing ways most possibly, and it is useful in some multi-casting works.

A graph which is hamiltonian connected is not always pan-connected, but a graph is pan-connected is always hamiltonian connected. This is easy to check if we know the definitions of both of them.

The rest of this paper is organized as follows. In chapter 2, we gave definitions of the augmented cubes and discuss some properties of AQ_n. And then we gave a new definition that help discussing pan-connectivity on augmented cubes. In chapter 3, we prove that AQ_2, AQ_3, and AQ_4 are all pan-connected by listing all paths of all lengths of them. In chapter 4, we discussed the pan-connectivity of augmented cubes and then discussed the pan-connectivity of other graphs. Finally, we make some concluding remarks in chapter 5. We also wrote one program - *Graphic* which work for testing pan-connectivity, vertex-pan-cyclic, and fault-vertex-pan-cyclic of a graph. And we briefly state its method on search part and introduce *Graphic* in chapter 6.

Chapter 2

Augmented cubes and pan-connectivity

2.1 Preliminaries

To make a description of simple graphic, we gave some definitions and notations below.

Let$G = (V, E)$ be a graph if V is a finite set and E is a subset of $\{(u, v) \mid (u, v)$ is an unordered pair of $V\}$. V is the *vertex set* and E is the *edge set*. Two vertices u and v are *adjacent* if $(u, v) \in E$. A *path* , denoted by $\langle v_0, v_1, v_2, \ldots, v_k \rangle$, is a sequence of distinct vertices where v_i and v_{i+1} are adjacent for all i, $0 \leq i \leq k-1$. We use $l(P)$ to present the length of a path P and $d(u, v)$ to denote the distance between u and v. A path is a *Hamiltonian path* if its vertices are distinct and span V. A graph G is *pan-connected* if there exists a path of length l joining any two vertices x and y with $d(x, y) \leq l \leq |V(G)| - 1$.

2.2 Augmented cubes

Let $n \geq 1$ be an integer, the n-dimensional augmented cube [2], denoted by AQ_n, has 2^n vertices, each is labelled by an n-bit binary string $V(AQ_n) = \{u_1, u_2 ... u_n \mid u_i \in \{0,1\}\}$. For $n \geq 2$, AQ_n can be recursively constructed by two copies of AQ_{n-1}, denoted by AQ^0_{n-1} and AQ^1_{n-1}, and adding 2^n edges between the two as follows:

Let $V(AQ^0_{n-1}) = \{(0u_2u_3 \dots u_n) \mid u_i = 0 \text{ or } 1 \text{ for } 2 \leq i \leq n\}$ and $V(AQ^1_{n-1}) = \{(1v_2v_3 \dots v_n) \mid v_i = 0 \text{ or } 1 \text{ for } 2 \leq i \leq n\}$. A vertex $\mathbf{u} = (0u_2u_3 \dots u_n)$ of AQ^0_{n-1} is joined to a vertex $\mathbf{v} = (1v_2v_3 \dots v_n)$ of AQ^1_{n-1} if and only if either

(i) $u_i = v_i$, for $2 \leq i \leq n$; in this case, $(\mathbf{u}, \mathbf{v})$ is called a *hypercube edge* and we set $\mathbf{v} = \mathbf{u}^h$, or

(ii) $u_i = \bar{v}_i$, for $2 \leq i \leq n$; in this case, $(\mathbf{u}, \mathbf{v})$ is called a *complement edge* and we set $\mathbf{v} = \mathbf{u}^c$.

In figure 2.1, there are augmented cubes with dimension 1, 2, 3, respectively.

In [11], we have the result theorem 1. Clearly, augmented cubes with dimension n $\geq$ 4 are 1 - fault hamiltonian connected.

To describe theorem 1, we need to explain the followings. In [11], We say a graph G has property $2H$ if it satisfies the following conditions: Let {w,x} and {y,z} be two pairs

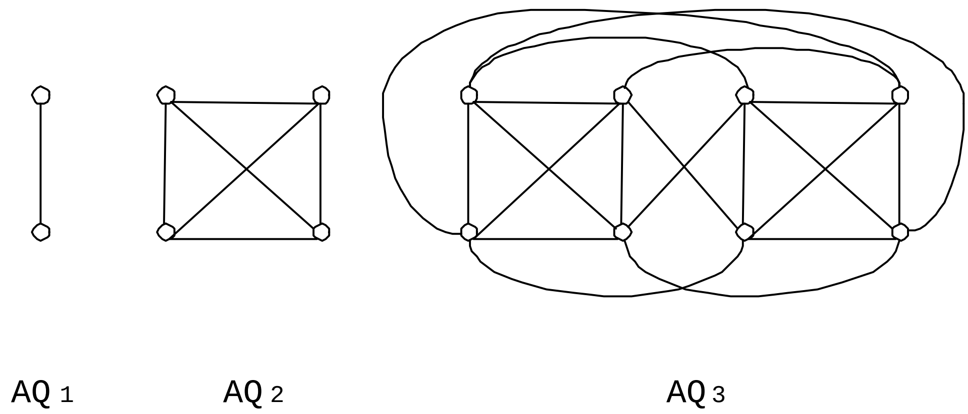

Figure 2.1: The augmented cubes AQ_1, AQ_2, and AQ_3.

of four distinct vertices of G. There exist two disjoint paths P_1 and P_2 of G such that (1) P_1 joins w to x, (2) P_2 joins y to z, and (3) every vertex of G is either on path P_1 or on P_2.

A graph G is *k-fault hamiltonian connected* if G-F remains hamiltonian connected for every $F \subset V(G) \bigcup E(G)$ with $|F| \leq k$. The *fault hamiltonian connectivity*, H_f^k(G), is defined to be the maximum integer k such that G is k-fault hamiltonian connected if G is hamiltonian connected and is undefined otherwise.

Theorem 1 *Let n be a positive integer with $n \geq 4$. Then AQ_n is $(2n-3)$-fault hamiltonian, $(2n-4)$-fault hamiltonian connected, and has property 2H.*

Besides, an augmented cube of dimension n is *vertex transitive* , *(2n-1)-regular*, *(2n-1)-connected*, and has diameter $\lceil n/2 \rceil$.

2.3 T-set

For the convenience of discussing pan-connectivity of AQ_n, we have a new notation : $T - set$, the short writing of $Together - Set$.

Definition 1 *Given a vertex u=$(0u_1, u_2...u_n)$ in AQ^0_{n-1} (or u=$(1u_1, u_2...u_n)$ in AQ^1_{n-1}), we can find a vertex set $\{v, w, x\}$ such that v=$(0u_1, u_2...u_n)$, w=$(1u_1, u_2...u_n)$, x=$(1\overline{u_1}, \overline{u_2}...\overline{u_n})$ (or v=$(1u_1, u_2...u_n)$, w=$(0u_1, u_2...u_n)$, x=$(0\overline{u_1}, \overline{u_2}...\overline{u_n})$). We say that T-set(u)=$\{v, w, x\}$.*

Suppose the set $\{b, c, d\}$ is T-set(a), then $\{a, b, c, d\}$ form a K_4. The identity of node b, c, d is as one is a^c, another is a^h, and the last is $(a^c)^h$ which is equal to $(a^h)^c$.

From the definition, we can easily check the following lemma.

Lemma 1 *For any node v in AQ_n, $n \geq 2$, there must exist one T-set(v).*

For an example in AQ_3, let $a = (000)$ is in AQ^0_2. By definition node a connect both to $b = (100)$ and $c = (111)$. $\{b, c\}$ is in AQ^1_2. The complement node of b, we name it as d, is (011). T-set(a) is $\{b, c, d\}$, T-set(b) is $\{a, c, d\}$, T-set(c) is $\{a, b, d\}$, T-set(d) is $\{a, b, c\}$. The set $\{a, b, c, d\}$ and its connections between each two nodes of them form a K_4.

2.4 Pan-connected graphs

A graph is pan-connected is always with many edges. But it is not only for completed graphs to have the property. Some other graphs have that also. No matter they are regular ones or not.

We have determined that the graphs like HQ, $Star$, and $(n,k)Star$ are all not pan-connected. It is easy to do just with the program - "Graphic", which is described in Appendix.

Additionally, We conclude some necessary conditions for pan-connectivity. They are the properties that a pan-connected graph must not have.

Lemma 2 *if a connected graph G except triangle has any node with degree 2, then G is not pan-connected.*

Proof. For the generality we suppose nodes u and v are the neighbors of the node that has degree 2. Then the path between u and v can not be a hamiltonian path. Therefore, G is not pan-connected.

Lemma 3 *if a connected graph G contains any articulation point, then it is also not pan-connected. In another word, if G can not tolerant of one fault, then G is not a pan-connected graph.*

Proof. We can find two neighbors of the articulation point, u and v. It is impossible to find the hamiltonian path start at u and end at v. So G is not a pan-connected graph.

corollary 1 *For a graph G with order* $n \geq 4$*, if G either has a node that has degree 2 or an articulation point, then G is not a pan-connected graph.*

Chapter 3

Pan-connectivity of AQ_2, AQ_3, and AQ_4

The augmented cubes AQ_2 and AQ_3 and AQ_4 are all pan-connected. For the proof, we list the essential paths of AQ_2, AQ_3 and AQ_4.

To match the program's outcome, we named each nodes in AQ_n from 1 to 2^n. See also figure 3.1.

3.1 Pan-connectivity on AQ_2

Since AQ_n has the property of symmetry and transition. We only to need list the case of starting with some node. Generally, we start at node 1.

from 1 to -	length and path
2	1 : 1,2
	2 : 1,3,2
	3 : 1,3,4,2
3	1 : 1 3
	2 : 1,2,3
	3 : 1 2 4 3
4	1 : 1,5
	2 : 1 4 5
	3 : 1 2 4 5

Table 2.1: paths with different lengths form node 1 to other nodes in AQ_2

3.2 Pan-connectivity on AQ_3

We start at node 1. See also figure 3.1.

from 1 to -	length and path	from 1 to -	length and path
2	1 : 1,2 2 : 1,3,2 3 : 1,3,4,2 4 : 1,3,6,7,2 5 : 1,3,4,5,6,2 6 : 1,3,4,5,6,7,2 7 : 1,3,4,5,6,8,7,2	6	2 : 1 2 6 3 : 1 2 3 6 4 : 1 2 3 7 6 5 : 1,2,3,4,5,6 6 : 1 2 3 4 5 7 6 7 : 1 2 3 4 5 7 8 6
3	1 : 1 3 2 : 1,2,3 3 : 1 2 4 3 4 : 1 2 6 7 3 5 : 1 2 4 5 6 3 6 : 1 2 4 5 6 7 3 7 : 1 2 4 5 6 8 7 3	7	2 : 1 2 7 3 : 1 2 3 7 4 : 1 2 3 6 7 5 : 1,2,3,4,5,7 6 : 1,2,3,4,5,6,7 7 : 1,2,3,4,5,6,8,7
4	1 : 1,4 2 : 1 2 4 3 : 1,2,3,4 4 : 1 2 6 3 4 5 : 1 2 3 6 5 4 6 : 1 2 3 6 5 8 4 7 : 1 2 3 6 5 7 8 4	8	1 : 1,8 2 : 1 4 8 3 : 1 2 4 8 4 : 1 2 3 4 8 5 : 1 2 3 4 5 8 6 : 1,2,3,4,5,6,8 7 : 1,2,3,4,5,6,7,8
5	1 : 1,5 2 : 1 4 5 3 : 1 2 4 5 4 : 1,2,3,4,5 5 : 1 2 3 4 8 5 6 : 1 2 3 4 8 6 5 7 : 1 2 3 4 8 6 7 5		

Table 2.1: paths with different lengths form node 1 to other nodes in AQ_3

With the table above, we complete the discussion of pan-connectivity of AQ_3.

3.3 Pan-connectivity on AQ_4

AQ_4 only need to be discussed in the case in which one end is node 1.

See figure 3.2, 3.3, 3.4, 3.5, 3.6.

The "1" in the final matrix represents the path from 1 to other node u that is the node-descriptor shown on that row exists.

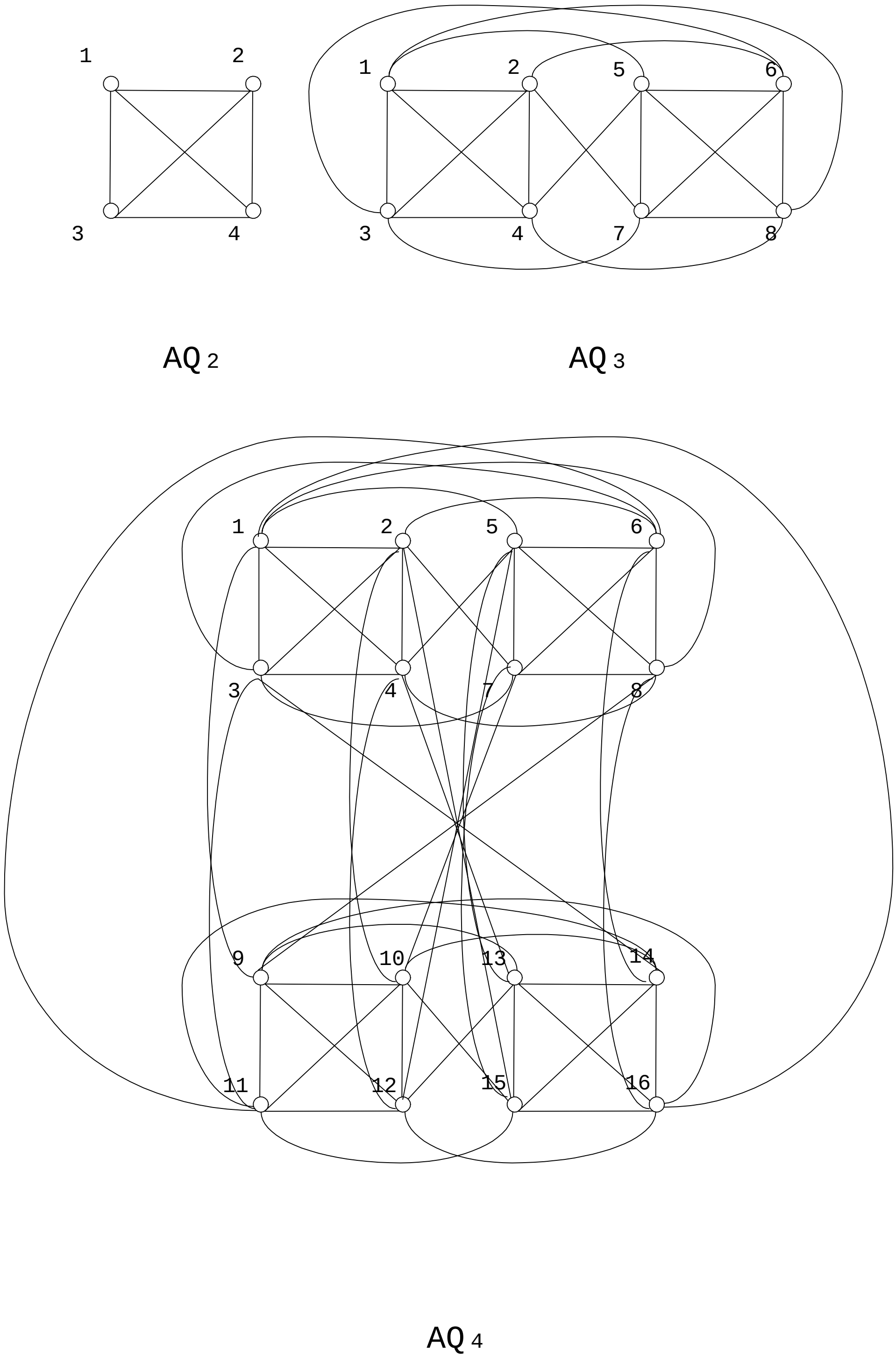

Figure 3.1: AQ_2, AQ_3, AQ_4

```
1 2 ,from 1 to 2 length is 1
1 2 3 ,from 1 to 3 length is 2
1 2 3 4 ,from 1 to 4 length is 3
1 2 3 4 5 ,from 1 to 5 length is 4
1 2 3 4 5 6 ,from 1 to 6 length is 5
1 2 3 4 5 6 7 ,from 1 to 7 length is 6
1 2 3 4 5 6 7 8 ,from 1 to 8 length is 7
1 2 3 4 5 6 7 8 9 ,from 1 to 9 length is 8
1 2 3 4 5 6 7 8 9 10 ,from 1 to 10 length is 9
1 2 3 4 5 6 7 8 9 10 11 ,from 1 to 11 length is 10
1 2 3 4 5 6 7 8 9 10 11 12 ,from 1 to 12 length is 11
1 2 3 4 5 6 7 8 9 10 11 12 13 ,from 1 to 13 length is 12
1 2 3 4 5 6 7 8 9 10 11 12 13 14 ,from 1 to 14 length is 13
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 ,from 1 to 15 length is 14
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 ,from 1 to 16 length is 15
1 2 3 4 5 6 7 8 9 10 11 12 13 14 16 ,from 1 to 16 length is 14
1 2 3 4 5 6 7 8 9 10 11 12 13 14 16 15 ,from 1 to 15 length is 15
1 2 3 4 5 6 7 8 9 10 11 12 13 15 ,from 1 to 15 length is 13
1 2 3 4 5 6 7 8 9 10 11 12 13 15 14 ,from 1 to 14 length is 14
1 2 3 4 5 6 7 8 9 10 11 12 13 15 16 14 ,from 1 to 14 length is 15
1 2 3 4 5 6 7 8 9 10 11 12 13 16 ,from 1 to 16 length is 13
1 2 3 4 5 6 7 8 9 10 11 12 16 ,from 1 to 16 length is 12
1 2 3 4 5 6 7 8 9 10 11 12 16 13 ,from 1 to 13 length is 13
1 2 3 4 5 6 7 8 9 10 11 12 16 14 13 ,from 1 to 13 length is 14
1 2 3 4 5 6 7 8 9 10 11 12 16 14 15 13 ,from 1 to 13 length is 15
1 2 3 4 5 6 7 8 9 10 11 14 ,from 1 to 14 length is 11
1 2 3 4 5 6 7 8 9 10 11 14 13 12 ,from 1 to 12 length is 13
1 2 3 4 5 6 7 8 9 10 11 14 13 15 16 12 ,from 1 to 12 length is 15
1 2 3 4 5 6 7 8 9 10 11 14 13 16 12 ,from 1 to 12 length is 14
1 2 3 4 5 6 7 8 9 10 11 14 15 ,from 1 to 15 length is 12
1 2 3 4 5 6 7 8 9 10 11 15 ,from 1 to 15 length is 11
1 2 3 4 5 6 7 8 9 10 11 15 14 ,from 1 to 14 length is 12
1 2 3 4 5 6 7 8 9 10 12 ,from 1 to 12 length is 10
1 2 3 4 5 6 7 8 9 10 12 11 ,from 1 to 11 length is 11
1 2 3 4 5 6 7 8 9 10 12 13 ,from 1 to 13 length is 11
1 2 3 4 5 6 7 8 9 10 12 13 14 11 ,from 1 to 11 length is 13
1 2 3 4 5 6 7 8 9 10 12 13 14 15 11 ,from 1 to 11 length is 14
1 2 3 4 5 6 7 8 9 10 12 13 14 16 15 11 ,from 1 to 11 length is 15
1 2 3 4 5 6 7 8 9 10 12 16 ,from 1 to 16 length is 11
1 2 3 4 5 6 7 8 9 10 14 ,from 1 to 14 length is 10
1 2 3 4 5 6 7 8 9 10 14 11 12 ,from 1 to 12 length is 12
1 2 3 4 5 6 7 8 9 10 14 15 11 ,from 1 to 11 length is 12
1 2 3 4 5 6 7 8 9 10 15 ,from 1 to 15 length is 10
1 2 3 4 5 6 7 8 9 11 ,from 1 to 11 length is 9
1 2 3 4 5 6 7 8 9 11 10 ,from 1 to 10 length is 10
1 2 3 4 5 6 7 8 9 11 12 10 ,from 1 to 10 length is 11
1 2 3 4 5 6 7 8 9 11 12 13 14 10 ,from 1 to 10 length is 13
1 2 3 4 5 6 7 8 9 11 12 13 14 15 10 ,from 1 to 10 length is 14
1 2 3 4 5 6 7 8 9 11 12 13 14 16 15 10 ,from 1 to 10 length is 15
```

Figure 3.2: pan-connectivity of AQ_4 part 1

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1 2 3 4 5 6 7 8 9 11 14 15 10 ,from 1 to 10 length is 12
1 2 3 4 5 6 7 8 9 12 ,from 1 to 12 length is 9
1 2 3 4 5 6 7 8 9 12 13 ,from 1 to 13 length is 10
1 2 3 4 5 6 7 8 9 12 16 ,from 1 to 16 length is 10
1 2 3 4 5 6 7 8 9 13 ,from 1 to 13 length is 9
1 2 3 4 5 6 7 8 9 16 ,from 1 to 16 length is 9
1 2 3 4 5 6 7 8 16 ,from 1 to 16 length is 8
1 2 3 4 5 6 7 8 16 9 ,from 1 to 9 length is 9
1 2 3 4 5 6 7 8 16 12 9 ,from 1 to 9 length is 10
1 2 3 4 5 6 7 8 16 12 10 9 ,from 1 to 9 length is 11
1 2 3 4 5 6 7 8 16 12 10 11 9 ,from 1 to 9 length is 12
1 2 3 4 5 6 7 8 16 12 10 11 14 13 9 ,from 1 to 9 length is 14
1 2 3 4 5 6 7 8 16 12 10 11 14 15 13 9 ,from 1 to 9 length is 15
1 2 3 4 5 6 7 8 16 12 10 14 11 9 ,from 1 to 9 length is 13
1 2 3 4 5 6 7 8 16 14 ,from 1 to 14 length is 9
1 2 3 4 5 6 7 8 16 15 ,from 1 to 15 length is 9
1 2 3 4 5 6 7 10 ,from 1 to 10 length is 7
1 2 3 4 5 6 7 10 9 8 ,from 1 to 8 length is 9
1 2 3 4 5 6 7 10 9 11 12 13 14 15 16 8 ,from 1 to 8 length is 15
1 2 3 4 5 6 7 10 9 11 12 13 14 16 8 ,from 1 to 8 length is 14
1 2 3 4 5 6 7 10 9 11 12 13 16 8 ,from 1 to 8 length is 13
1 2 3 4 5 6 7 10 9 11 12 16 8 ,from 1 to 8 length is 12
1 2 3 4 5 6 7 10 9 12 16 8 ,from 1 to 8 length is 11
1 2 3 4 5 6 7 10 9 16 8 ,from 1 to 8 length is 10
1 2 3 4 5 6 7 10 11 ,from 1 to 11 length is 8
1 2 3 4 5 6 7 10 12 ,from 1 to 12 length is 8
1 2 3 4 5 6 7 10 14 ,from 1 to 14 length is 8
1 2 3 4 5 6 7 10 15 ,from 1 to 15 length is 8
1 2 3 4 5 6 7 15 ,from 1 to 15 length is 7
1 2 3 4 5 6 7 15 10 ,from 1 to 10 length is 8
1 2 3 4 5 6 7 15 13 ,from 1 to 13 length is 8
1 2 3 4 5 6 8 ,from 1 to 8 length is 6
1 2 3 4 5 6 8 7 ,from 1 to 7 length is 7
1 2 3 4 5 6 8 9 ,from 1 to 9 length is 7
1 2 3 4 5 6 8 9 10 7 ,from 1 to 7 length is 9
1 2 3 4 5 6 8 9 10 11 12 13 14 15 7 ,from 1 to 7 length is 14
1 2 3 4 5 6 8 9 10 11 12 13 14 16 15 7 ,from 1 to 7 length is 15
1 2 3 4 5 6 8 9 10 11 12 13 15 7 ,from 1 to 7 length is 13
1 2 3 4 5 6 8 9 10 11 14 15 7 ,from 1 to 7 length is 12
1 2 3 4 5 6 8 9 10 11 15 7 ,from 1 to 7 length is 11
1 2 3 4 5 6 8 9 10 15 7 ,from 1 to 7 length is 10
1 2 3 4 5 6 8 16 ,from 1 to 16 length is 7
1 2 3 4 5 6 11 ,from 1 to 11 length is 6
1 2 3 4 5 6 11 9 8 ,from 1 to 8 length is 8
1 2 3 4 5 6 11 10 7 ,from 1 to 7 length is 8
1 2 3 4 5 6 11 12 ,from 1 to 12 length is 7
1 2 3 4 5 6 11 14 ,from 1 to 14 length is 7
1 2 3 4 5 6 14 ,from 1 to 14 length is 6
1 2 3 4 5 6 14 11 ,from 1 to 11 length is 7
1 2 3 4 5 6 14 13 ,from 1 to 13 length is 7
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Figure 3.3: pan-connectivity of AQ_4 part 2

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1 2 3 4 5 7 ,from 1 to 7 length is 5
1 2 3 4 5 7 6 ,from 1 to 6 length is 6
1 2 3 4 5 7 8 6 ,from 1 to 6 length is 7
1 2 3 4 5 7 8 9 10 11 6 ,from 1 to 6 length is 10
1 2 3 4 5 7 8 9 10 11 12 13 14 6 ,from 1 to 6 length is 13
1 2 3 4 5 7 8 9 10 11 12 13 15 14 6 ,from 1 to 6 length is 14
1 2 3 4 5 7 8 9 10 11 12 13 15 16 14 6 ,from 1 to 6 length is 15
1 2 3 4 5 7 8 9 10 11 14 6 ,from 1 to 6 length is 11
1 2 3 4 5 7 8 9 10 11 15 14 6 ,from 1 to 6 length is 12
1 2 3 4 5 7 8 9 11 6 ,from 1 to 6 length is 9
1 2 3 4 5 7 10 ,from 1 to 10 length is 6
1 2 3 4 5 7 10 11 6 ,from 1 to 6 length is 8
1 2 3 4 5 7 15 ,from 1 to 15 length is 6
1 2 3 4 5 8 ,from 1 to 8 length is 5
1 2 3 4 5 8 9 ,from 1 to 9 length is 6
1 2 3 4 5 8 16 ,from 1 to 16 length is 6
1 2 3 4 5 12 ,from 1 to 12 length is 5
1 2 3 4 5 12 13 ,from 1 to 13 length is 6
1 2 3 4 5 13 ,from 1 to 13 length is 5
1 2 3 4 5 13 12 ,from 1 to 12 length is 6
1 2 3 4 8 ,from 1 to 8 length is 4
1 2 3 4 8 5 ,from 1 to 5 length is 5
1 2 3 4 8 6 5 ,from 1 to 5 length is 6
1 2 3 4 8 6 7 5 ,from 1 to 5 length is 7
1 2 3 4 8 6 7 10 9 11 12 5 ,from 1 to 5 length is 11
1 2 3 4 8 6 7 10 9 11 12 13 5 ,from 1 to 5 length is 12
1 2 3 4 8 6 7 10 9 11 12 16 13 5 ,from 1 to 5 length is 13
1 2 3 4 8 6 7 10 9 11 12 16 14 13 5 ,from 1 to 5 length is 14
1 2 3 4 8 6 7 10 9 11 12 16 14 15 13 5 ,from 1 to 5 length is 15
1 2 3 4 8 6 7 10 9 12 5 ,from 1 to 5 length is 10
1 2 3 4 8 6 7 10 12 5 ,from 1 to 5 length is 9
1 2 3 4 8 6 11 12 5 ,from 1 to 5 length is 8
1 2 3 4 8 9 ,from 1 to 9 length is 5
1 2 3 4 8 16 ,from 1 to 16 length is 5
1 2 3 4 12 ,from 1 to 12 length is 4
1 2 3 4 12 10 ,from 1 to 10 length is 5
1 2 3 4 12 11 ,from 1 to 11 length is 5
1 2 3 4 13 ,from 1 to 13 length is 4
1 2 3 4 13 14 ,from 1 to 14 length is 5
1 2 3 4 13 15 ,from 1 to 15 length is 5
1 2 3 6 ,from 1 to 6 length is 3
1 2 3 6 5 4 ,from 1 to 4 length is 5
1 2 3 6 5 7 8 4 ,from 1 to 4 length is 7
1 2 3 6 5 7 8 9 10 11 12 4 ,from 1 to 4 length is 11
1 2 3 6 5 7 8 9 10 11 12 13 4 ,from 1 to 4 length is 12
1 2 3 6 5 7 8 9 10 11 12 16 13 4 ,from 1 to 4 length is 13
1 2 3 6 5 7 8 9 10 11 12 16 14 13 4 ,from 1 to 4 length is 14
1 2 3 6 5 7 8 9 10 11 12 16 14 15 13 4 ,from 1 to 4 length is 15
```

Figure 3.4: pan-connectivity of AQ_4 part 3

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1 2 3 6 5 7 8 9 10 12 4 ,from 1 to 4 length is 10
1 2 3 6 5 7 8 9 12 4 ,from 1 to 4 length is 9
1 2 3 6 5 7 10 12 4 ,from 1 to 4 length is 8
1 2 3 6 5 8 4 ,from 1 to 4 length is 6
1 2 3 6 7 ,from 1 to 7 length is 4
1 2 3 6 11 ,from 1 to 11 length is 4
1 2 3 6 14 ,from 1 to 14 length is 4
1 2 3 7 ,from 1 to 7 length is 3
1 2 3 7 6 ,from 1 to 6 length is 4
1 2 3 7 10 ,from 1 to 10 length is 4
1 2 3 7 15 ,from 1 to 15 length is 4
1 2 3 11 ,from 1 to 11 length is 3
1 2 3 11 9 ,from 1 to 9 length is 4
1 2 3 14 ,from 1 to 14 length is 3
1 2 3 14 16 ,from 1 to 16 length is 4
1 2 4 ,from 1 to 4 length is 2
1 2 4 3 ,from 1 to 3 length is 3
1 2 4 5 ,from 1 to 5 length is 3
1 2 4 5 6 3 ,from 1 to 3 length is 5
1 2 4 5 6 7 3 ,from 1 to 3 length is 6
1 2 4 5 6 7 8 9 10 11 3 ,from 1 to 3 length is 10
1 2 4 5 6 7 8 9 10 11 12 13 14 3 ,from 1 to 3 length is 13
1 2 4 5 6 7 8 9 10 11 12 13 15 14 3 ,from 1 to 3 length is 14
1 2 4 5 6 7 8 9 10 11 12 13 15 16 14 3 ,from 1 to 3 length is 15
1 2 4 5 6 7 8 9 10 11 14 3 ,from 1 to 3 length is 11
1 2 4 5 6 7 8 9 10 11 15 14 3 ,from 1 to 3 length is 12
1 2 4 5 6 7 8 9 11 3 ,from 1 to 3 length is 9
1 2 4 5 6 7 10 11 3 ,from 1 to 3 length is 8
1 2 4 5 6 8 7 3 ,from 1 to 3 length is 7
1 2 4 8 ,from 1 to 8 length is 3
1 2 4 12 ,from 1 to 12 length is 3
1 2 4 13 ,from 1 to 13 length is 3
1 2 6 ,from 1 to 6 length is 2
1 2 6 3 4 ,from 1 to 4 length is 4
1 2 6 7 3 ,from 1 to 3 length is 4
1 2 7 ,from 1 to 7 length is 2
1 2 7 10 ,from 1 to 10 length is 3
1 2 7 15 ,from 1 to 15 length is 3
1 2 10 ,from 1 to 10 length is 2
1 2 10 9 ,from 1 to 9 length is 3
1 2 15 ,from 1 to 15 length is 2
1 2 15 16 ,from 1 to 16 length is 3
1 3 ,from 1 to 3 length is 1
1 3 2 ,from 1 to 2 length is 2
1 3 4 2 ,from 1 to 2 length is 3
1 3 4 5 6 2 ,from 1 to 2 length is 5
1 3 4 5 6 7 2 ,from 1 to 2 length is 6
1 3 4 5 6 7 8 9 10 2 ,from 1 to 2 length is 9
1 3 4 5 6 7 8 9 10 11 12 13 14 15 2 ,from 1 to 2 length is 14
1 3 4 5 6 7 8 9 10 11 12 13 14 16 15 2 ,from 1 to 2 length is 15
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Figure 3.5: pan-connectivity of AQ_4 part 4

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 1 3 4 5 6 7 8 9 10 11 12 13 15 2 ,from 1 to 2 length is 13
 1 3 4 5 6 7 8 9 10 11 14 15 2 ,from 1 to 2 length is 12
 1 3 4 5 6 7 8 9 10 11 15 2 ,from 1 to 2 length is 11
 1 3 4 5 6 7 8 9 10 15 2 ,from 1 to 2 length is 10
 1 3 4 5 6 7 10 2 ,from 1 to 2 length is 7
 1 3 4 5 6 7 10 15 2 ,from 1 to 2 length is 8
 1 3 6 7 2 ,from 1 to 2 length is 4
 1 3 11 ,from 1 to 11 length is 2
 1 3 14 ,from 1 to 14 length is 2
 1 4 ,from 1 to 4 length is 1
 1 4 5 ,from 1 to 5 length is 2
 1 4 8 ,from 1 to 8 length is 2
 1 4 12 ,from 1 to 12 length is 2
 1 4 13 ,from 1 to 13 length is 2
 1 5 ,from 1 to 5 length is 1
 1 8 ,from 1 to 8 length is 1
 1 8 9 ,from 1 to 9 length is 2
 1 8 16 ,from 1 to 16 length is 2
 1 9 ,from 1 to 9 length is 1
 1 16 ,from 1 to 16 length is 1

The pan_cont_matrix (1)
 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1


It is Pan-connected!
```

Figure 3.6: pan-connectivity of AQ_4 part 5

Chapter 4

Pan-connectivity of augmented cubes

4.1 Pan-connectivity of AQ_n

In this chapter, we will prove augmented cubes are pan-connected by induction.

Theorem 2 *The augmented cubes with dimension $n \geq 1$, AQ_n, are pan-connected.*

Base on the results of chapter 3, AQ_2, AQ_3, and AQ_4 are pan-connected, we suppose that AQ_n is pan-connected when $n \geq 4$. Then we prove that AQ_{n+1} is also pan-connected.

Let a, b be any two vertices in AQ_{n+1}, we want to find the path P of length $\leq l$ join a and b with $d(a, b) \leq l \leq 2^{n+1} - 1$.

case 1: a, b are in AQ_n^0 or a and b are in AQ_n^1.

Without loss of generality, let a and b are in AQ_n^0. By induction, there exist the paths of length from $d(a, b)$ to $2^n - 1$ joining a to b.

case 1.1: a and b are not adjacent

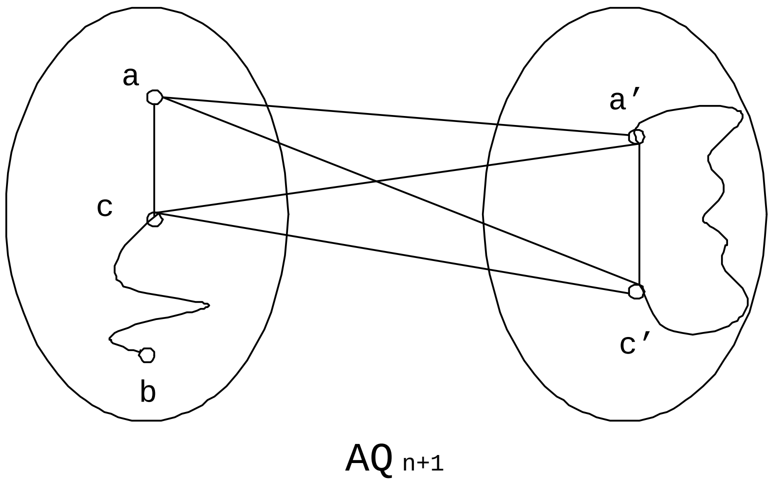


Figure 4.1: a and b are not adjacent in the augmented cubeAQ_{n+1}.

Let T-set(a)=$\{c, a', c'\}$. Without loss of generality, let (a, a'), (c, c') are hypercube edges and a, c are in AQ_n^0.

By theorem 1, there exist a hamiltonian path P_1 of $AQ_n^0 - \{a\}$ joining c to b. Hence, $l(P_1) = 2^n - 2$. And by induction, there exist paths Q_i of lengths from 1 to $2^n - 1$ in AQ_n^1 joining a' to c'. $l(Q_i) = 1 \sim 2^n - 1$.

We trace P in the following sequences to get different lengths:

First, to get the path of length 2^n:

Let $P = \langle a, a', c, P_1, b \rangle$, and let $l(\langle c,..., b \rangle) = 2^n - 2$. Then $l(P) = 2^n$.

Second, to get the path of lengths from $2^n + 1$ to $2^{n+1} - 1$:

Let $P_i = \langle a, a', Q_i, c', c, P_1, b \rangle, i = 1 \sim 2^n - 1$. $l(P_i) = 2^n + 1 \sim 2^{n+1} - 1$. So this case is completed.

case 1-2: a and b are adjacent

case 1-2-1: b is in a's T-set

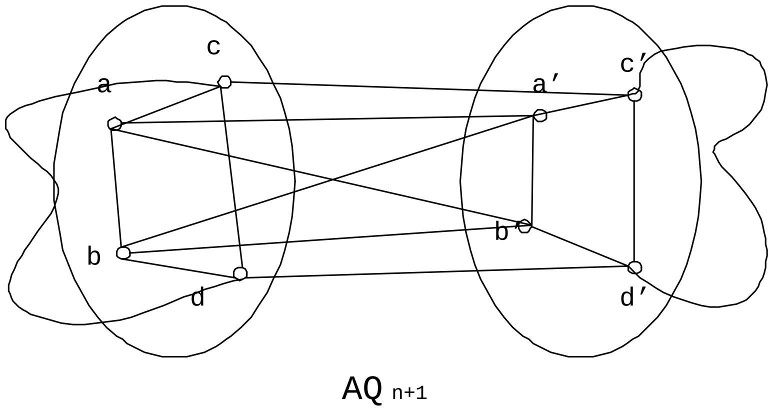


Figure 4.2: b is in a's T-set

Lemma 4 *For any node a in AQ_n, $n \geq 4$, suppose $\{a', b, b'\}$ is T-set(a) and both a and b are in AQ^0_{n-1} or AQ^1_{n-1}. There exist two pairs of distinct nodes, c, d in AQ^0_{n-1} and c', d' in AQ^1_{n-1}, that have the following relations: (1). c and d are adjacent and c is adjacent to a and d is adjacent to b. (2). c' and d' are adjacent and c' is adjacent to a' and d' is adjacent to b'. (3). c is adjacent to c' and d is adjacent to d'.*

Proof. Without loss of generality, let a and b are in AQ_{n-1}^0 and $a = (0, v_1, v_2,...v_i...v_{n-2}), b = (0, \overline{v_1}, \overline{v_2},...\overline{v_i}..., \overline{v_{n-2}})$. Let $c = (0, v_1, v_2,...\overline{v_i}...v_{n-2})$, $d = (0, \overline{v_1}, \overline{v_2},...v_i..., \overline{v_{n-2}})$, $c' = (1, v_1, v_2,...\overline{v_i}...v_{n-2})$, and $d\prime = (1, \overline{v_1}, \overline{v_2},...v_i..., \overline{v_{n-2}})$ for some i, $1 \le i \le 2^n - 2$. Such c, d, c', d' conform to the conditions (1), (2), (3) of lemma 4.

This case can be constructed as Figure 4.2. Let T-set(a)=$\{b, a', b'\}$. Without loss of generality, let (a, a'), (b, b') are hypercube edges and a, b are in AQ_n^0.

First, to get the path of length 2^n:

By induction, there exist the path P_1 of length $l = 2^n - 2$ in AQ_n^1 joining a' to b'. Let $P = \langle a, a', P_1, b', b\rangle$, then $l(P) = 2^n$.

Second, to get the path of length $2^n + 1 \sim 2^{n+1} - 1$:

By theorem 1, there exist the hamiltonian path P_a in AQ_n^0 joining a to c and not pass through b. Thus, $l(P_a) = 2^n - 2$. By induction, there exist paths Q_i of lengths from 1 to $2^n - 1$ in AQ_n^1 joining c' to a'.

Let $P_i = \langle a, P_a, c, c', Q_i, a', b\rangle$, $1 \le i \le 2^n - 1$. $l(P_i) = 2^n + 1 \sim 2^{n+1} - 1$. Which completes this case.

case 1-2-2: b is not in $a's$ T-set

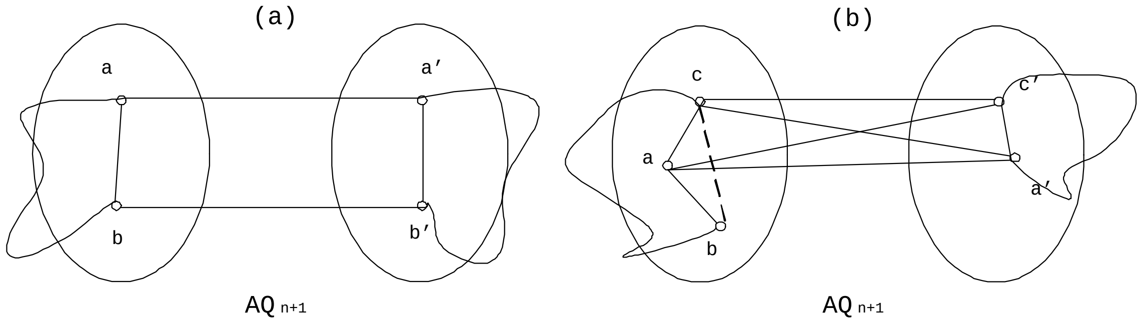


Figure 4.3: b is not in $a's$ T-set

In AQ_n, since AQ_{n-1}^0 and AQ_{n-1}^1 are isomorphic, we have the following lemma.

Lemma 5 *If a and b are adjacent in AQ_{n-1}^0, let $a' = (a^h)$ and $b' = (b^h)$, then a' and b' are also adjacent in AQ_{n-1}^1.*

By lemma 5, we can trade a and b as in figure 4.3.a in this case. But it is not enough to explain all subcases. Thus, we need a more detail descriptions about a and b. We see that b is not in a's T-set, so there exists another node $c \neq b$ and c is in a's T-set. Watch that c and b are not always adjacent, so we dotted it in figure 4.3.b. Of course, there exist another two nodes c' and a' which are both in T-set(a). Without loss of generality, let a and c are both in AQ_n^0.

We begin our subcases that to find the paths of lengths from 2^n to $2^{n+1} - 1$.

First, to get the path of length 2^n:

See figure 4.3.a. By induction, there exist a path P_1 of length $2^n - 2$ in AQ_n^1 joining c' to a'. Let $P = \langle a, a', P_1, b', b \rangle$. As a result, $l(P) = 2^n$.

Second, to get the path of lengths $2^n + 1 \sim 2^{n+1} - 1$:

See figure 4.3.b. By induction, there exist paths Q_i of lengths from 1 to $2^n - 1$ in AQ_n^1 joining a' to c'. And by theorem 1, there exist the hamiltonian path P_1 that does not go through a joining c to b. $l(P_1) = 2^n - 2$. Let $P_i = \langle a, a', Q_i, c', c, P_1, b \rangle$, $i = 1 \sim 2^n - 1$. Thus, $l(P_i) = 2^n + 1 \sim 2^{n+1} - 1$. And it completes this case.

case 2: a is in AQ_n^0 and b is in AQ_n^1, or a is in AQ_n^1 and b is in AQ_n^0

Without loss of generality, let a is in AQ_n^0 and b is in AQ_n^1.

case 2-1: a, b are adjacent

In this case, it is obvious to see that b is in T-set(a). And $d(a, b) = 1$. Let T-set(a)=$\{a', b, b'\}$. It is shown in figure 4.4.

By induction, there exist paths Q_i of lengths from 1 to $2^n - 1$ in AQ_n^0 joining a to a', $1 \leq i \leq 2^n - 1$. $l(Q_i) = 1 \sim 2^n - 1$.

First, to get the path of lengths $2 \sim 2^n$:

Let $P_i = \langle a, Q_i, a', b \rangle$, $1 \leq i \leq 2^n - 1$. $l(P_i) = 2 \sim 2^n$.

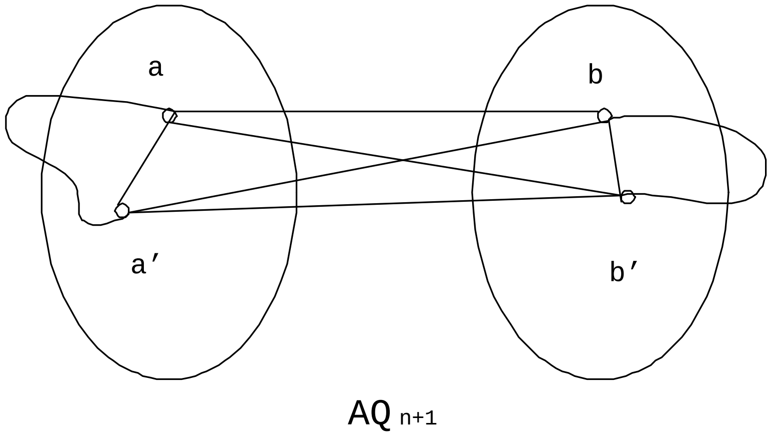


Figure 4.4: a, b are adjacent

Second, to get the path of lengths $2^n + 1 \sim 2^{n+1} - 1$:

By induction, there exist the hamiltonian path P_1 in AQ_n^0 joining a to a'. Also, there exist paths Q_i of lengths from 1 to $2^n - 1$ in AQ_n^1 joining b to b'.

Let $P_i = \langle a, P_1, a', b, Q_i, b' \rangle$, $1 \leq i \leq 2^n - 1$. $l(P_i) = 2^n + 1 \sim 2^{n+1} - 1$.

case 2-2: a, b are not adjacent

Without loss of generality, let a be in AQ_n^0, and let $d(a, b) = k \geq 2$. Since a is not adjacent to b, there exist another node $c \neq a$ in AQ_n^0 such that c is in T-set(b), and b' in AQ_n^1 is also in T-set(b). So $d(a, c) = k - 1$.

Without loss of generality, let (c, b) be the hypercube edge. When tracing a path joining a to b, if we take each step when going from node u to v as a bit-operation,

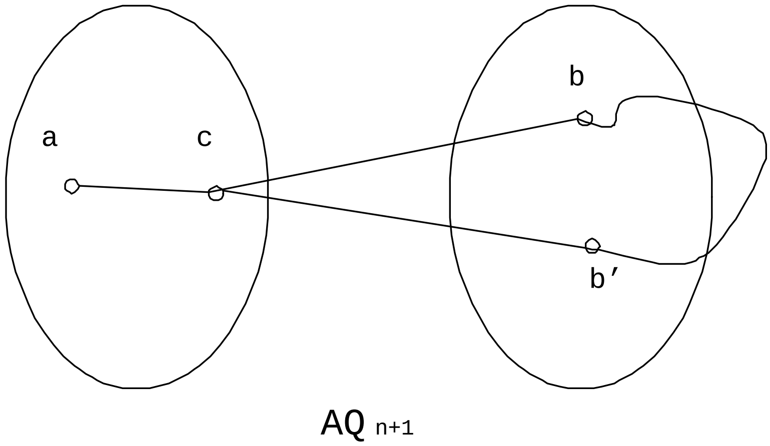


Figure 4.5: a, b are not adjacent

where u and v are adjacent on the path. Then tracing a path from a to b is as many bit-operations. And any combination of those bit-operations have the same result. For example, if $a = (000)$ and $b = (110)$, then we can go from a to b by the following steps : (1). Change the most right bit to 1, which acts as go from a=(000) to (001).(2). Inverse all bits, which acts as go from (001) to (110)=b. Or we can do the (2)-step first, and then do the (1)-step. It does not matter to have any combinations. See figure 4.5. As a result, we can say that if the path joining a to b exist, then the path going from a to c and from c to b' and from $b\prime$ to b exist, and the passing nodes in the segment from a to c are all in AQ_n^0, and the passing nodes in the segment from b' to b are all in AQ_n^1.

By induction, there exist paths Q_i of lengths from $k-1$ to $2^n - 1$ in AQ_n^0 joining a to c. Also, there exist paths R_j of lengths from 1 to $2^n - 1$ in $2^n - 1$ in AQ_n^1 joining b' to b.

To get the paths of lengths $k + 1 \sim 2^{n+1} - 1$:

Let $P_{ij} = \langle a, Q_i, c, b', R_j, b\rangle$, $k-1 \leq i \leq 2^n-1, 1 \leq j \leq 2^n-1$. $l(P_{ij}) = k+1 \sim 2^{n+1}-1$.

With all above, it is completed of proof of theorem 2.

4.2 Other pan-connected graphs

As we see, not only completed graphs, but also some other graphs still are pan-connected. However, the cost be paid are large degrees of nodes.

Besides augmented cubes, there are still some other graphs found are also pan-connected. We just introduce them in this section.

We find that $C(n, 1, 2, n-2, n-1)$, n is from 5 to 10, are also pan-connected. All nodes in these graphs all have a regular degree 4. Among them, $C(5, 1, 2, 3, 4)$ is equal to K_5. Figure 4.6 and figure 4.7 below are displayed for them.

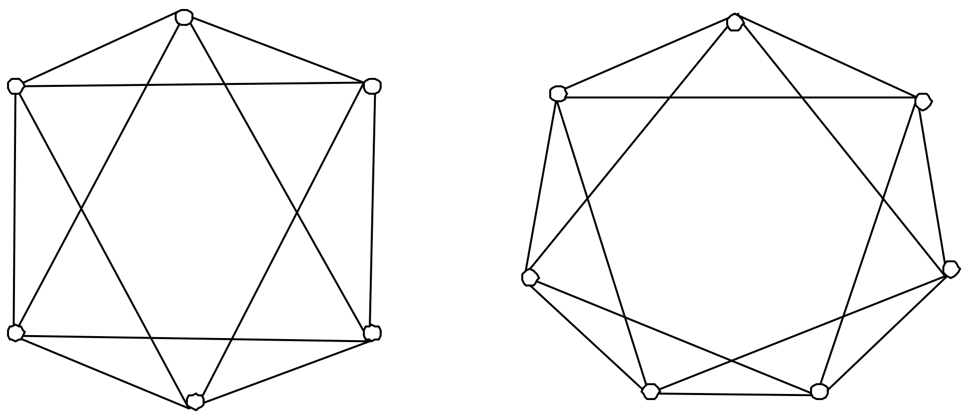


Figure 4.6: $C(6, 1, 2, 4, 5)$ and $C(7, 1, 2, 5, 6)$

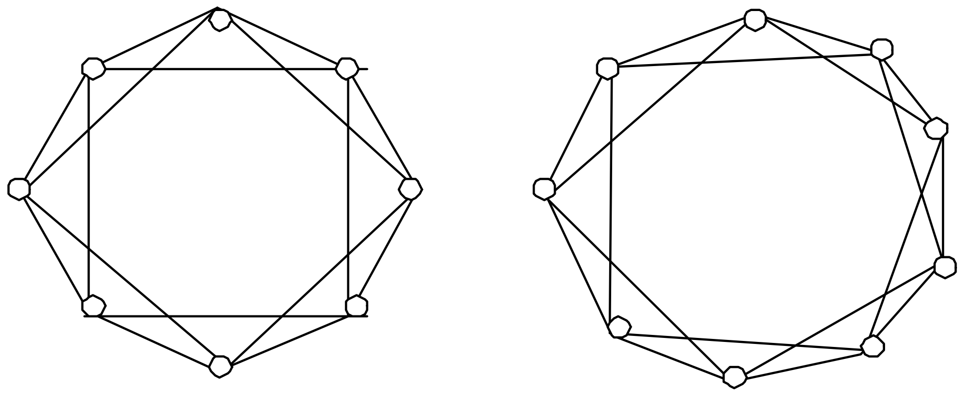
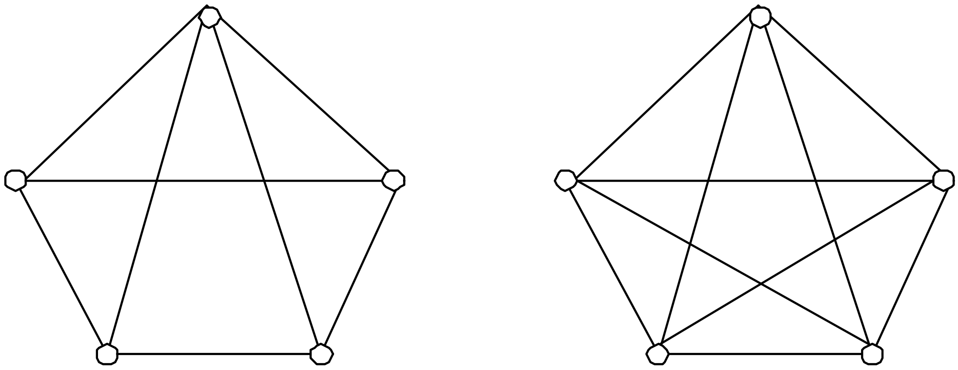


Figure 4.7: $C(8, 1, 2, 6, 7)$ and C(9,1,2,7,8)

But $C(5, 1, 2, 3, 4)$ is not the pan-connected graph with fewest edges when the order is 5. The graph shown in the left of figure 4.8 is another pan-connected graph with fewer edges than $C(5, 1, 2, 3, 4)$.

It is also not true that a pan-connected graph $G_1 \times$ (cross-product) G_2, another pan-connected graph, is also pan-connected.

We disprove it by a contrary example of $K_2 \times K_3$, although $K_3 \times K_3$ is pan-connected.

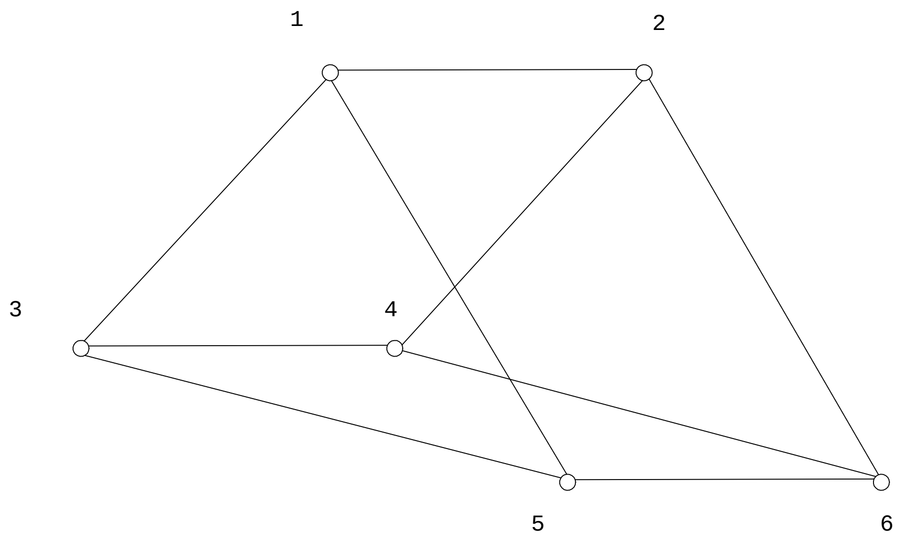


(both are) pan-cont-matrix (1)

	1	2	3	4
2	1	1	1	1
3	1	1	1	1
4	1	1	1	1
5	1	1	1	1

TRUE

Figure 4.8: $C(5, 1, 2, 3, 4)$ and another pan-connected graph



	1	2	3	4	5
2	1	0	1	1	1
3	1	1	1	1	1
4	0	1	1	1	1
5	1	1	1	1	1
6	0	1	1	1	1

FALSE

Figure 4.9: $K_2 \times K_3$ is not pan-connected

Chapter 5

Conclusion

In this paper, we have proved that augmented cubes are pan-connected. It can be implemented on the network like the priority queue of routing paths from one processor to another.

Augmented cubes have other good properties that have been demonstrated like vertex symmetry, maximum connectivity ($2n-1$, equals to degree), best possible wide diameter ($\lceil n/2 \rceil + 1$), routing and broadcasting procedures with linear time complexity.

It remains space to discover more pan-connected graphs. Any attempt may cause a new result. But we still do not know what is the minimum number of edges should a pan-connected graph of order n have. Although the graphs mentioned in the figures 4.6, 4.7 have fewer degrees than augmented cubes, we don't discuss them in this paper.

Chapter 6

Appendix

6.1 The tool - "Graphic"

The tool - *Graphic* is written in VC++. With this GUI tool, we can build a graph and edit it. An edge is presented as two nodes. A user just needs to draw the nodes and connect the edges to build a graph. The actions "*move*" and "*delete*" for nodes and edges are ready if one user clicks both "$node - button$" and "$move - button$" for moving a node or "*edge*" and "*delete*" for deleting an edge, etc. It can be applied to find the isomorphic one of the original graph. The main point is, users can test pan-connectivity of any graph in it. Anybody needs only draw the graph and clicks the button "$Pan - connected$", then he will get the result of "true" or "false". Besides, "Graphic" can determine if a graph is vertex-pan-cyclic, and fault-vertex-pan-cyclic or not.

The algorithm of testing the pan-connectivity of a graph is to trace every path from each node, and check if all lengths from the shortest one to hamiltonian path exist. The way to trace a path is by D.F.S (Depth First Search).

Initially, it is must be done to establish the topology. In "Graphic", we record the topology in iterators which save the graph as linked lists. The second, we deal with the signals when the state on the drawing board is changed. "Depth First Search" is adapted to trace the graph. We show the mainly function of search part in figure 6.1.

6.2 Pan-cont-matrix

We use pan-cont-matrix(v) in our program, where v is a description-number of any node v in graph. We explain it as follows.

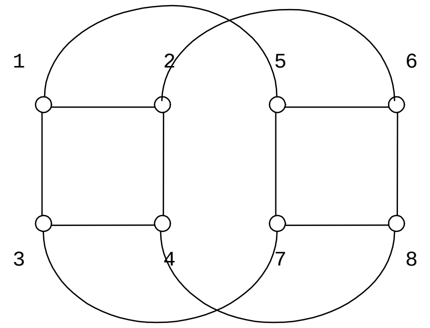
The pan-cont-matrix(v) records each lengths of paths connect to v and presents as a matrix. The row of it presents the other node's description-number, and the column of it stands for the distance between v to other node. In a graph G, if two nodes u and v have the path with length l, then we record the content on the matrix on row u and column l to be 1. Obviously, a matrix of a completed graph is filled with 1. Any hypercube's pan-cont-matrices looks as '0' and '1' occurs alternatively. We demonstrate the pan-cont-matrix of HQ_3 and AQ_3 in figure 6.2.

According to the definition of pan-connectivity, it's known that for a connected graph G with order n, it's node set V is $\{v_0, v_1, ...v_n\}$, if pan-cont-matrix($v_i, i = 1$ to n) has a series of 1 from an positive integer $k, k \leq n-1$ to n-1, then We call such pan-cont-matrix(v_i) is $true$ or $false$ otherwise. Obviously, if pan-cont-matrix($v_i, i = 1$ to n) is *true*, G is pan-connected. On the other way, if G is pan-connected, then all its pan-cont-matrix($v_i, i = 1$ to n) is *true*.

```
void CGraphicDoc::PathAll(int n)
{
      GraphTopo();          // topology initialization

      POSITION P;           // use iterator to record path
      int v=n,len=-1,k=0;
      Cnode Pnode;          // also use iterator to record a node
      P=eGraph[v].GetHeadPosition();
      Pnode.ID=v;           // ID is an alias of a node
      Path.AddTail(Pnode);
      visit[k]=v;
      if(eGraph[v].IsEmpty())
            return;
      old_visit=0;

      while(P!=NULL){       // when P is NULL,
                              // the search from some node is over
            if(old_visit!=0){     // use old_visit
                                      // to record privious walking
                  for(;eGraph[v].GetAt(P).ID!=old_visit;\
eGraph[v].GetNext(P));
                eGraph[v].GetNext(P);

            }
            for(;P!=NULL;eGraph[v].GetNext(P)){
                if(!Visited(eGraph[v].GetAt(P).ID)){
                  break;
                }
            }
            if(P==NULL){        // back search
                if(v==n)
                  return;
                Visit_Record();
                Path.RemoveTail();
                for(k=0;visit[k]!=-1;k++);
                old_visit=visit[k-1];
                visit[k-1]=-1;
                v=Path.GetTail().ID;
                P=eGraph[v].GetHeadPosition();
                continue;
            }
            old_visit=0;
            Pnode.ID=eGraph[v].GetAt(P).ID;     // search new
            Path.AddTail(Pnode);
            for(k=0;visit[k]!=-1;k++);
            visit[k]=Pnode.ID;
            len=Path.GetCount()-1;
            pan_cont[Pnode.ID][len]=TRUE;
            v=Pnode.ID;
            P=eGraph[v].GetHeadPosition();
      }
}
```

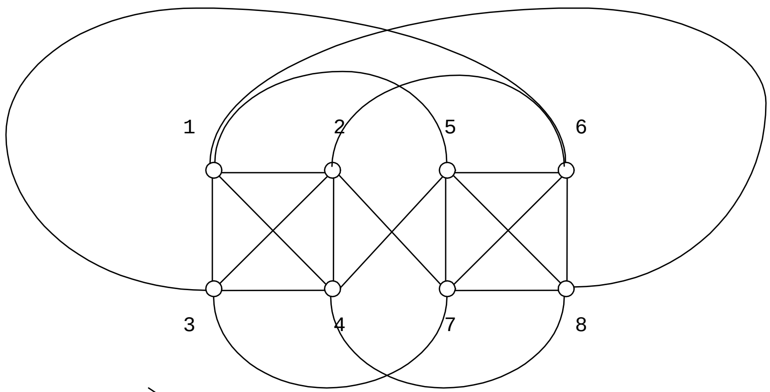
Figure 6.1: the search-path function

HQ$_3$ and the pan-cont-matrix(1):



node \ length	1	2	3	4	5	6	7
(node_2)	1	0	1	0	1	0	1
(node_3)	1	0	1	0	1	0	1
(node_4)	0	1	0	1	0	1	0
(node_5)	1	0	1	0	1	0	1
(node_6)	0	1	0	1	0	1	0
(node_7)	0	1	0	1	0	1	0
(node_8)	0	0	1	0	1	0	1

AQ$_3$ and the pan-cont-matrix(1):



node \ length	1	2	3	4	5	6	7
(node_2)	1	1	1	1	1	1	1
(node_3)	1	1	1	1	1	1	1
(node_4)	1	1	1	1	1	1	1
(node_5)	1	1	1	1	1	1	1
(node_6)	0	1	1	1	1	1	1
(node_7)	0	1	1	1	1	1	1
(node_8)	1	1	1	1	1	1	1

Figure 6.2: two pan-cont-matrices

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