Nonlinear pricing of taxi services

Hai Yang [a,*], C.S. Fung [a], K.I. Wong [b], S.C. Wong [c]

[a] Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, PR China
[b] Department of Transportation Technology & Management, National Chiao Tung University, Taiwan
[c] Department of Civil Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong, PR China

ARTICLE INFO

ABSTRACT

This paper examines the effects of nonlinear fare structures in taxi markets using an extended taxi model with an explicit consideration of perceived profitability. The expected profit, defined as the profit per unit time (inclusive of both occupied and vacant taxi times), that a taxi driver expects to receive from picking up a customer in a particular zone or location, has great impact on the taxi driver's choice of location in the search for customers. The fare structure directly governs the profitability of taxi rides of different distances originating from different locations. With these explicit considerations, the extended model is intended to look into the market effects of adopting a nonlinear fare structure with declining incremental charges. The proposed nonlinear fare structure could help restore a level-playing field for taxi operators whose businesses have been affected by some taxi drivers who resort to practices such as offering fare discounts or accepting requests for discounted fares from passengers for long-haul trips. Analysis of sensitivity of social welfare and profit gain as well as taxi/customer wait/search times is conducted with respect to the parameters in the nonlinear fare structure for the Hong Kong taxi market, and Pareto-improving nonlinear fare amendments are identified that neither disadvantage any customer nor reduce the taxi operators' profits.

1. Taxi service and pricing issues in Hong Kong

The taxi service has been an indispensable part of the transportation system in Hong Kong. While railways, buses and mini-buses provide mass transit services at a low cost, taxis are classified as a high-end transportation mode in which point-to-point service is provided with a 24-h-a-day availability. At present, there are 15,250 urban (red) taxis, 2838 New Territories (green) taxis and 50 Lantau (blue) taxis. These taxis combine to serve a little more than one million passengers each day, which is approximately 10% of the total passenger transportation volume in Hong Kong (HKSAR Government, 2007). The efficiency of taxis has improved throughout the past 10 years. The maximum passenger waiting time at 80% of taxi stands has reduced from 15 min in 1997 to 7 min in 2006. Nevertheless, the recent urban development in Hong Kong has had a great impact on taxi service. The rapid increase in the number of new towns and infrastructure located in remote areas has resulted in an increase in the demand for long-haul transportation service. As the other transportation modes offer discounts and benefits for long-haul travelers, taxis are finding it harder to attract travelers, particularly under the current "linear" fare structure based on a front-loaded flag-fall charge and a linear component proportional to travel distance and stopping time.

To improve their competitiveness and in view of considerable revenue from long-distance trips such as the airport-based trips, 'taxi discount gangs', as they are commonly known, have emerged. Some drivers offer special discounts to passengers

---

of long-distance service illegally, including discounts on metered fare, preset trip fare upon bargaining and car-pooling. They would hint their willingness to offer fare concessions by distributing name cards or through taxi radio stations. An increasing number of taxi drivers, succumbing to competition pressure, have joined their ranks, and more and more passengers bargain with the taxi drivers on boarding the taxis, rendering the taximeters almost useless (HKSAR, 2007). This phenomenon has resulted in a number of undesirable consequences as outlined below (HKSAR, 2008).

Firstly, it wastes resources. The existing "linear" fare structure makes long-distance trips more profitable. A considerable number of taxis are drawn to the airport waiting for customers for hours on end. Over-supply of taxis at the airport results in substantial waste of valuable taxi service hours. Diverting excess taxi supply from the airport to other areas will certainly enhance overall taxi service quality and thus make better use of taxi service resources. Secondly, it results in unfair competition. Due to the lower fare charged for airport-based trips by discount gangs, a considerable number of passengers are captured by this group, which has resulted in a 60% drop in service provided by legal taxis, according to the representative of taxi organization. At the same time, legal taxis need to wait for about 4 h for their next customers at taxi stands at the airport; while discount gangs can provide service after receiving dial without having to queue. Thirdly, it results in dangerous driving. Since discount gangs provide service after receiving dials, they are used to talking on phones while driving. Their attention is distracted, inducing danger to other road-users as well.

In tackling problems raised by discount gangs and addressing the concern from the taxi associations, the Transport Advisory Committee (TAC) launched a review to examine the operations of taxi services, particularly the existing 'linear' fare structure in Hong Kong and changes in the operating environment (HKSAR, 2007). Through broad public consultation, the review sought to identify feasible and appropriate improvement measures to broaden the taxi trade's business opportunities, and at the same time benefit the public through the provision of competitive taxi services to meet their needs. More than 100 specific fare restructuring proposals were received from individuals or organizations including taxi associations during the consultation. Among these proposals, there were more views in support of raising short-haul fares and lowering long-haul fares. With regard to the current taxi operating condition and the views of the public and the trade, TAC recommended that the policy on the taxi fare structure be changed from "front-loaded and subsequent incremental charges being calculated at the same rate" to "front-loaded and thereafter on a varying descending scale for incremental changes". This recommended change was supposed to align the taxi fare structure with the fare structures of other public transport modes such as railways, franchised buses and green mini-buses to increase the taxis' competitiveness. The proposed change could also help restore a level-playing field for taxi operations whose businesses have been affected by the discount gangs (HKSAR, 2008).

2. Review of relevant studies

A substantial number of analytical studies are available in the literature concerning the models and economics of taxi services under various types of regulation such as entry restriction and price control. A few notable contributions include the early studies by Douglas (1972) and De vany (1975), and recent studies by Cairns and Liston-Heyes (1996) and Arnott (1996), and more recent studies by Yang et al. (2005a, 2005b), Flores-Guri (2005), Fernandez et al. (2006) and Moore and Balaker (2006). Most studies have adopted a highly aggregate model originally proposed by Douglas (1972) without consideration of the spatial structure of the market. In this case, trips are in fact considered to be of constant duration and distance and hence of a constant fare. Yang et al. (2005a) indeed considered variable taxi fare as a linearly increasing function of travel delay due to congestion but average taxi ride length is assumed to be constant. Arnott (1996) provided a specific "spatially structural" model of dispatching taxi service. Taxi rides are randomly and uniformly distributed over space in a spatially homogeneous two-dimensional city. In this case, variable trip distance is thus considered and measured by Euclidean distance without congestion, but, as in Douglas, the fare per unit time for occupied taxis or per unit travel distance is constant.

It is commonly realized that a principal characteristic that distinguishes the taxi market from the idealized market of conventional economic analyses is the role of bilateral customer and taxi waiting/search time (De vany, 1975). In the taxi market, the equilibrium quantity of service supplied (total taxi-hours) will be greater than the equilibrium quantity demanded (occupied taxi-hours) by a certain amount of slack (vacant taxi-hours). It is this amount of slack that governs the average customer waiting time and average taxi utilization. From the consumers' perspective, the expected customer waiting time is generally considered as an important value or quality of the services offered and thus as part of the full price of a taxi ride. This variable affects customers' decisions as to whether or not to take a taxi. From the suppliers' perspective, taxi drivers will operate in response to market profitability, which depends on the taxi utilization rate as well as trip revenues and costs.

It is thus evident that bilateral customer and taxi waiting/search times play a crucial role in the determination of the resulting demand–supply equilibrium of the market. With high expected revenue per taxi ride and customer-taxi point meetings, the above-mentioned long taxi waiting time of up to a couple of hours at the airport is the outcome of the market equilibrium in terms of the spatially equal profitability under a linear fare structure. Namely, through the long waiting time, the airport taxi profitability will eventually be driven to become identical with that of urban taxi services. This market inefficiency or failure will inevitably occur in a spatially heterogeneous market with a linear fare structure. The emergence of "taxi discount gangs" is also largely attributed to the market spatial heterogeneity and the linear fare structure. Through their long-distance fare discounts, a win–win situation can emerge: on one hand, taxi drivers can achieve a higher profitability per unit time by avoiding long waiting times at the airport, and on the other hand, customers benefit from the discounted taxi fare. Evidently, modeling and talking about these issues require explicit consideration of the spatial structure of the

market. In this respect, a network equilibrium model of customer demand and taxi supply have been developed and refined over the years by Yang and Wong and their research group (Yang and Wong, 1998; Yang et al., 2002, in press; Wong et al., 2008).

3. Proposed nonlinear fare structure

To partially rectify the market deficiency and crack down on the black market of discount gangs, here we consider the effectiveness of a nonlinear fare structure in a spatially heterogeneous taxi market. In economics, nonlinear pricing refers to any case in which the tariff is not strictly proportional to the quantity purchased. It has been studied and used in other transportation fields (railway tariffs and airline fares) and many other industries (electricity and telephone tariffs). Block-declining tariffs are generally adopted in which the marginal prices of successive units decline in steps. The simplest example of a nonlinear tariff is a two-part tariff, in which the customer pays an initial fixed fee for the first unit, plus a smaller constant price for each unit after the first. Multipart tariffs that are piecewise linear in different intervals or volume bands are also prevalent in practice (Wilson, 1992).

Instead of a piecewise linear multipart tariff, we consider a nonlinear fare structure in which price varies continuously with travel distance. The following nonlinear parabolic equation of fare structure is proposed[1]:

$$F(x) = ax^2 + bx + c \tag{1}$$

where x is the distance of a taxi ride and $F(x)$ is the fare charge for serving a taxi ride of distance x; c is the initial flag-fall charge; a and b are the parameters to be determined.

In order to ensure a desirable fare structure, parameter a should be negative so that the parabolic curve is concave in order to mitigate the fare bargaining problem mentioned earlier. For a sensitivity analysis, a positive value of parameter a is considered as well. The marginal fare revenue for one unit of additional distance is given by the derivative of taxi fare with respect to distance.

$$\frac{\mathrm{d}F}{\mathrm{d}x} = 2ax + b \tag{2}$$

For the equation to be realistically sensible, taxi fare should be an increasing function of distance within a realistic interval of trip distance: $0 \leqslant x \leqslant x_{max}$, where x_{max} is the maximum taxi ride length in a given study area. This means that parameters a and b should satisfy the following relation:

$$2ax_{max} + b \geqslant 0 \tag{3}$$

For a comparative analysis, we also consider the following "linear" fare structure:

$$F^*(x) = b^*x + c^* \tag{4}$$

where c^* is the initial flag-fall charge and b^* is a constant charging rate per unit distance.[2]

Note that, for a concave parabolic curve in Eq. (1) with $a < 0$, the marginal revenue is decreasing with distance as $\mathrm{d}^2F/\mathrm{d}x^2 = 2a < 0$ and the decreasing rate is constant (constant rate of change of slope). As depicted in Fig. 1, by selecting appropriate values of parameters a, b and c, one may increase taxi fares for short-distance trips and decrease those for long-distance trips and thus achieve the same or even higher market profitability in comparison with the case with a linear fare structure. Therefore, the taxi drivers' incentives in favor of serving long-distance trips will be reduced and thus the long taxi waiting time at the problem sites such as the airport will be mitigated.

4. Profitability-based taxi service model

The taxi model used in this study is adapted from Yang and Wong (1998) and Wong et al. (2003). The basic network model of taxi movement was set up in Yang and Wong (1998). Wong et al. (2003) considered expected profit for a taxi ride from a given zone or location in describing taxi driver's choice of destination in the search of next customer. A minor extension is made here by considering the time required for realizing the expected profit for a taxi ride from a given location/zones. Namely, the market profitability is measured by the expected profit per unit time and adopted to ascertain taxi drivers' destination choice behavior. For completeness, we present a brief description of the model.

Consider a road network with a set of customer origin zones I and a set of customer destination zones J. In any given hour, the demand Q_{ij} (trip/h) of taxi rides from zone $i \in I$ to zone $j \in J$ is assumed to depend on the customer waiting time, W_i, at zone i, monetary cost or fare, F_{ij}, travel time, h_{ij}, and the potential customer demand, $\bar{Q}_{ij}$, from i to j, and is denoted by:

---

[1] This simple quadratic fare structure with few parameters is sufficient to capture the profit and consumer surplus forgone by using any higher degree polynomial. Also, the nonlinear fare equation, once established, can easily be used to derive a practical multipart tariff that realizes most of the advantages of nonlinear pricing with a few distance segments.

[2] Strictly speaking from an economics point of view, the fare structure (4) is a two-part tariff, which can be regarded as an approximation of the fare structure adopted in Hong Kong (as of 30 November 2008). The Hong Kong taxi fare structure (as of 30 November 2008) was in fact a three-part tariff with a front-loaded flag-fall charge for the first 2 km and a constant incremental charge for every subsequent 0.2 km/every minute of waiting time.
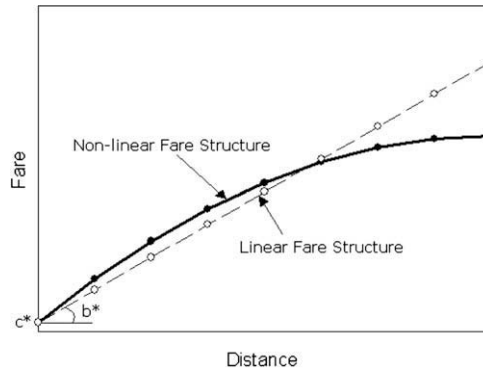
Fig. 1. Linear versus nonlinear taxi fare structure.

$$Q_{ij} = Q_{ij}(W_i, F_{ij}, h_{ij}, \bar{Q}_{ij}), \quad i \in I, \ j \in J \tag{5}$$

The following trip end constraints are satisfied:

$$O_i = \sum_{j \in J} Q_{ij}, \quad i \in I \tag{6}$$

$$D_j = \sum_{i \in I} Q_{ij}, \quad j \in J \tag{7}$$

where O_i and D_j are the total customer demands generated from origin zone $i \in I$ and the total demand attracted to destination zone $j \in J$, respectively. Since the focus of this study is on the effect of nonlinear fare structure, traffic congestion is not considered for simplicity, this implies that travel time, h_{ij}, $i \in I$, $j \in J$, is a constant independent of link flow.

Suppose that there are N cruising taxis operating in the network, in which taxi operations in the network and customer demand are considered to be in a stationary state in one unit period (1 h). Let T_{ij}^o be the occupied taxi movements (veh/h) from zone $i \in I$ to zone $j \in J$ along the shortest route. Clearly, we have $T_{ij}^o = Q_{ij}$, $i \in I$, $j \in J$, giving the total occupied time of all taxis equal to $\sum_{i \in I} \sum_{j \in J} T_{ij}^o h_{ij}$. After dropping customers at destination zone $j \in J$, a taxi becomes vacant and then either stays within the same zone or moves to another zone to find the next customer, thus, the total unoccupied taxi time is given by $\sum_{j \in J} \sum_{i \in I} T_{ji}^v (h_{ji} + w_i)$, where T_{ji}^v is the vacant taxi movements (veh/h) from zone $j \in J$ to zone $i \in I$ along the shortest route, and w_i, $i \in I$ is the taxi waiting/search time for a customer at zone $i \in I$. The sum of the total occupied taxi-hours and total vacant taxi-hours is equal to the total taxi service time (Yang and Wong, 1998):

$$\sum_{i \in I} \sum_{j \in J} T_{ij}^o h_{ij} + \sum_{j \in J} \sum_{i \in I} T_{ji}^v (h_{ji} + w_i) = N \tag{8}$$

In deciding whether to cruise in the same zone or going to other zones to seek the next customer after completing a taxi ride, each taxi driver is assumed to attempt to maximize the expected profit per unit time to be gained from serving customers. This consideration is essential in order to investigate the effects of alternative fare structures. Consider a taxi driver who goes to zone $i \in I$ to meet the next customer after dropping off a customer at zone $j \in J$. The expected revenue for serving a customer from zone i is denoted by $\hat{F}_i$ and can be estimated as:

$$\hat{F}_i = \frac{\sum_{j^* \in J} T_{ij^*}^o F_{ij^*}}{\sum_{j^* \in J} T_{ij^*}^o}, \quad i \in I \tag{9}$$

where $T_{ij^*}^o / \sum_{j^* \in J} T_{ij^*}^o$ can be regarded as the probability of a customer from zone i destined for zone j^* and F_{ij^*} is the total taxi fare for the ride from zone $i \in I$ to zone $j^* \in J$. Similarly, the expected ride time of a customer from zone i is denoted by $\hat{h}_i$ and can be estimated as:

$$\hat{h}_i = \frac{\sum_{j^* \in J} T_{ij^*}^o h_{ij^*}}{\sum_{j^* \in J} T_{ij^*}^o}, \quad i \in I \tag{10}$$

where h_{ij^*} is the (shortest) travel time from zone $i \in I$ to zone $j^* \in J$. The total expected time spent from dropping the last customer at zone $j \in J$ to finding and serving the next customer from zone $i \in I$ is thus equal to $h_{ji} + w_i + \hat{h}_i$. The total operating cost of the taxi driver during this period for finding and serving a customer from zone $i \in I$ is given as $\phi_1 h_{ji} + \phi_2 w_i + \phi_3 \hat{h}_i$, where ϕ_1, ϕ_2 and ϕ_3 are parameters denoting the unit time cost of vacant cruising, waiting and occupied running taxis, respectively. It is reasonably assumed that $\phi_1 \approx \phi_3$, $\phi_2 < \phi_1$ and $\phi_2 < \phi_3$. The expected profit or net revenue received by the taxi driver is thus equal to $\hat{F}_i - (\phi_1 h_{ji} + \phi_2 w_i + \phi_3 \hat{h}_i)$. Therefore, the expected profit per unit time for a taxi driver originating from zone $j \in J$ and meeting and serving a customer at zone $i \in I$ can be defined as:

$$\hat{\pi}_{i/j} = \frac{\hat{F}_i - (\phi_1 h_{ji} + \phi_2 w_i + \phi_3 \hat{h}_i)}{h_{ji} + w_i + \hat{h}_i} \tag{11}$$

In Eq. (11), the expected profit is a random variable due to the perception errors of taxi drivers and the random arrivals of customers. Assuming this random variable is identically distributed with a Gumbel density function, the probability that a vacant taxi originating from zone $j \in J$ to seek the next customer in zone $i \in I$ is specified by the following logit model[3]:

$$P_{i/j} = \frac{\exp\left\{\theta \frac{\hat{F}_i - (\phi_1 h_{ji} + \phi_2 w_i + \phi_3 \hat{h}_i)}{h_{ji} + w_i + \hat{h}_i}\right\}}{\sum_{m \in I} \exp\left\{\theta \frac{\hat{F}_m - (\phi_1 h_{jm} + \phi_2 w_m + \phi_3 \hat{h}_m)}{h_{jm} + w_m + \hat{h}_m}\right\}}, \quad i \in I, \ j \in J \tag{12}$$

where θ is a nonnegative parameter that can be calibrated from observational data (Yang et al., 2001). The value of θ (h/HK\$) reflects the degree of uncertainty in customer demand and taxi services in the whole market from the perspective of individual taxi drivers. When θ approaches zero, vacant taxis will tend to be distributed uniformly in the network; when θ approaches infinity, all taxis ending services in zone j will be attracted to one single zone that has the highest expected profit.

Note that our focus here is on the strategic location choices of vacant taxi drivers at the zonal level to make the model operational in practice. Such a macroscopic choice behavior is of first-order significance and is evidenced by a recent stated preference survey, in which 400 taxi drivers were interviewed (Sirisoma et al., 2010). In reality, taxi drivers may pass and search through some intermediate locations before meeting a customer. Such a micro-searching process may be modeled by virtue of a Markov chain approach (Lagos, 2000; Wong et al., 2005) at the expense of model complexity and operability.

In a stationary equilibrium state, movements of vacant taxis within the network should meet the customer demands at all origin zones (supply equal to demand), meaning that every customer will eventually receive taxi service after an acceptable waiting and searching process, and the customers who give up taxis due to excessive waiting time will be assumed to be diverted to the other travel modes. As there are D_j taxis to complete services at destination zone $j \in J$ per hour, the movement of vacant taxis must satisfy the following trip balance constraints:

$$\sum_{i \in I} T_{ji}^v = D_j, \quad j \in J \tag{13}$$

$$\sum_{j \in J} T_{ji}^v = \sum_{j \in J} D_j \cdot P_{i/j} = O_i, \quad i \in I \tag{14}$$

Customer waiting time, which is an endogenous variable of the model, varies across locations/zones. Within each zone, the expected waiting time of customers, denoted by W_i, depends on the density of vacant taxis in the area, $W_i = W_i(n_i, w_i)$, where n_i is the number of vacant taxis per hour that meet customers at zone $i \in I$, in which $n_i = O_i$ at equilibrium, and w_i is the expected average cruising/waiting time that a vacant taxi spends in zone $i \in I$ in finding a customer, in which w_i is determined from the above-mentioned network equilibrium of taxi movements (Wong and Yang, 1998). This specification of customer waiting time function depends on the distribution of taxi stands over individual zones. In case the taxi stand distribution is continuous, in which taxis can pick up customers anywhere on the streets, we can assume that vacant taxis move randomly through the street, and the expected average customer waiting time is proportional to the area of the reference zone[4] and inversely proportional to the cruising vacant taxi-hours.

$$W_i = \eta \frac{A_i}{n_i w_i} = \eta \frac{A_i}{O_i w_i}, \quad i \in I \tag{15}$$

where A_i is the area of zone $i \in I$, $O_i w_i (= n_i w_i)$ is the cruising vacant taxi-hours at zone $i \in I$, and η is a scale parameter of searching and meeting frictions that is assumed to be common for all zones and can be calibrated from taxi survey data. This approximate distribution of customer waiting time can be derived theoretically in the ideal situation mentioned above (Douglas, 1972; Yang et al., 2002).[5]

Note that Eq. (15) can well characterize taxi-customer point meetings at such locations as the airport or a railway station. By letting ηA_i approach zero and in view of O_i being of a limited value, we thus arrive at $W_i w_i = 0$. It is thus readily observed that: if $W_i > 0$, then $w_i = 0$, or alternatively if $w_i > 0$, then $W_i = 0$, which means that either customers are waiting for taxis or taxis are waiting for customers at location $i \in I$ (Yang et al., in press).[6]

---

[3] We mention that the meetings between unserved customers and vacant taxis in the various zones are characterized by a logit-based probability; similar but complicated probabilistic search models can be derived based on an intervening opportunity type model (Akwawua and Pooler, 2001) or from an entropy-maximization distribution model of vacant taxi movements.

[4] A good measure would be the total number of street kilometers that can be assumed to be proportional to the area of the reference zone.

[5] The customer waiting time function (15) is, in fact, an instance of the Cobb–Douglas type production function of the bilateral taxi-customer searching and meeting relationships with increasing returns to scale (Yang et al., in press).

[6] The result, $W_i w_i = 0$, is valid at some spot markets such as the airport or a railway station only if the simultaneous embarking capacity is unlimited. Otherwise, ηA_i is of a limited value or both $w_i > 0$ and $W_i > 0$. In fact, in the case of a spot market with a limited embarking capacity, the taxi and customer waiting times can be derived from a double-ended queuing model (Wong et al., 2005).

5. A simplified case study

5.1. Problem settings

Since our focus is on a strategic policy analysis of taxi fare structures within the whole territory, we divide the whole city into four major zones only shown in Fig. 2 for simplicity, including Hong Kong Island (HK Island), Kowloon (KL), New Territories (NT) and the Airport (Airport). Here, HK Island and KL are noted as urban areas; NT is noted as a rural area; the airport is located in an outlying island. The data in Travel Characteristics Survey (TCS) 2002 (HKSAR, 2002a,b) is used for partial calibration of the model. The survey captures the considerable changes in Hong Kong's economic, infrastructure, land use, technological developments and the relocation of the airport, which provides more complete information as compared with earlier TCSs. It covers a sample of 1.4% of all households in Hong Kong, together with providing supplementary information of tourist behavior.

The customer demand given in Eq. (5) for all origin–destination pairs is assumed to be proportional to the potential demand, $\bar{Q}_{ij}$ (trips/h), but decreasing with customer waiting time, W_i (h), monetary cost or fare, F_{ij} (HK\$), and travel time, h_{ij} (h), and specified as follows:

$$Q_{ij} = Q_{ij}(W_i, F_{ij}, h_{ij}) = \bar{Q}_{ij} \exp\{-\alpha(F_{ij} + v_1 W_i + v_2 h_{ij})\}, \quad i \in I, \ j \in J \tag{16}$$

where v_1 (HK\$/h) and v_2 (HK\$/h) are the customers' values of in-vehicle time and waiting time, respectively; α ($\alpha > 0$) (1/HK\$) is a parameter of sensitivity of customer demand to full trip price. From TCS (HKSAR, 2002a,b), v_1 and v_2 are found to be 100 HK\$/h and 50 HK\$/h (1US\$ $\approx$ 7.8HK\$), respectively, parameter α is derived to be 0.03 (1/HK\$).

The average inter-zonal and intra-zonal travel times, h_{ij}, $i \in I$, $j \in J$, in Eq. (16) are given in Table 1. Note that the two urban zones of Hong Kong Island and Kowloon have a shorter average intra-zonal travel distance or travel time, while New Territories, a remote zone, has a longer average intra-zonal travel distance or travel time. Inter-zonal trips going through the cross-harbor tunnels have to pay a toll charge of 25 HK\$, which is the average fee of the three cross-harbor tunnels weighed by traffic volume. These fees are also included in the customers' travel cost.

Daily intra-zonal and inter-zonal customer demands by taxis are approximately derived with reference to the Annual Traffic Census 2006 (HKSAR, 2006). With an average of 1.2 persons per taxi ride and assuming that daily demand is equivalent to 18 times the peak hour demand, an hourly taxi demand matrix is approximated and given in Table 2. Note that, as we have used highly aggregated zones of the whole study area, intra-zone trips that represent short-distance trips within the same zone accounts for a large portion of the total taxi rides as shown in Table 2. In Hong Kong, over 50% of trips made by
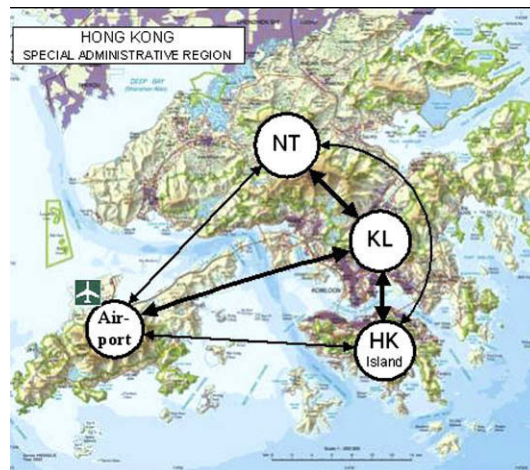

**Fig. 2.** A highly aggregated and simplified network of Hong Kong taxi services.

Table 1
Inter-zonal and intra-zonal travel time matrix (h).

	HK Island	KL	NT	Airport
HK Island	0.24	0.36	0.79	1.00
KL	0.36	0.24	0.45	0.80
NT	0.79	0.45	0.32	0.86
Airport	1.00	0.80	0.86	N/A

Table 2
Inter-zonal and intra-zonal base customer trip matrix (persons/h).

	HK Island	KL	NT	Airport	Total
HK Island	17,006	945	50	302	18,303
KL	945	25,510	100	478	27,032
NT	50	700	4252	100	5102
Airport	251	503	126	N/A	880
Total	18,253	27,657	4527	880	51,317
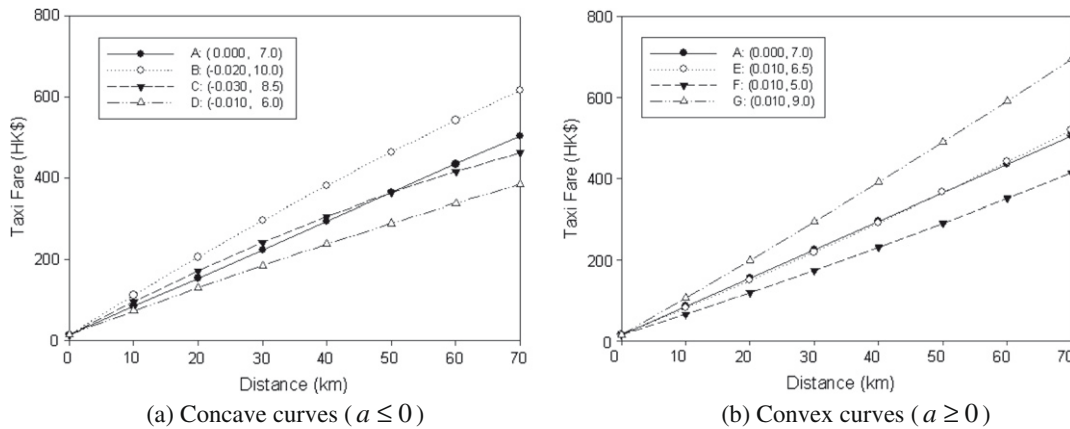
Fig. 3. Curves of alternative parabolic taxi fare equations.

urban taxis have a travel distance of less than 4 km (HKSAR, 2008). The demand matrix in Table 2 is regarded as the base matrix and used to derive the potential customer demand $\bar{Q}_{ij}, i \in I, j \in J$ in the demand function. In particular, the potential demands are estimated such that the resulting realized demands from the taxi service model under the existing fare structure closely match with the base trip matrix in Table 2.

With reference to the calibration and validation results of the network equilibrium taxi model for Hong Kong (Yang et al., 2002), the following parameter values are adopted. In the logit-based probability model (12) of taxi driver search behaviors, parameter θ is assumed to be 0.1 (h/HK\$). For the customer waiting time Eq. (15), $\eta A_i, i = 1, 2$, are estimated to be 5.0 veh/h for the urban areas of Hong Kong Island and Kowloon (as most taxis are cruising), $\eta A_i, i \in 3$ is assumed to have a larger value of 10.0 veh/h as the taxis and customers are sparsely dispersed in the rural area, $\eta A_i, i \in 4$ is assumed to have a much smaller value of 0.01 veh/h due to the nature of point meeting at the airport. Supplemented with the statistics from the Hong Kong Annual Digest of Statistics in 2002, the operating and prorated capital cost of a taxi is computed as 84.0 HK\$/h of service time on average, including occupied and vacant taxi service time. Thus, ϕ_1 and ϕ_3 are both 84.0 HK\$/h while ϕ_2 is assumed to be 42.0 HK\$/h. Finally, throughout the numerical study, the current fleet size of 15,250 urban taxis is adopted.

5.2. Results and discussions

Our aim is to look into the effect of the nonlinear fare Eq. (1): $F(x) = ax^2 + bx + c$, in comparison with the linear one (4): $F^*(x) = b^*x + c^*$, where F is in HK\$, x in km, a in HK\$/km^2, b and b^* in HK\$/km and c and c^* in HK\$. For a meaningful comparison, we set the initial charge $c = c^* = 15$ (HK\$) for both linear and nonlinear fare structures.[7] As mentioned in Eq. (3), taxi fare should always increase with distance. Assuming a maximum travel distance of 70 km, the feasible domain of (a, b) are given by: $2a(70) + b > 0$ or $140a + b > 0$. For a comparative analysis, we specifically examine the following seven selected specific sets of fare structures within the feasible domain of (a, b)

A $(0.000, 7.0)$, B $(-0.020, 10.0)$, C $(-0.030, 8.5)$, D $(-0.010, 6.0)$ E $(0.010, 6.5)$, F $(0.010, 5.0)$, G $(0.010, 9.0)$

The fare curves are plotted in Fig. 3, where fare equation A is a straight line, approximately representing the fare structure in Hong Kong (as of 28 February 2008). In Fig. 3a, fare equations B, C and D show concave parabolic curves, B and D always give higher and lower fares respectively in comparison with the linear one for all distances considered. In contrast, C gives

---

[7] As of 28 February 2008 when this study was being carried out in Hong Kong, the initial charge of urban taxis for the first 2 km was 15.0 HK\$ and then 1.4 HK\$ incremental charge for every 0.2 km thereafter and every period of 1 min waiting time.

higher fares for short-distance trips but lower fares for long-distance trips, a desirable fare structure in Hong Kong. Similarly, three convex curves E, F and G are given and plotted in Fig. 3b for sensitivity analysis.

For various feasible combinations of parameters a and b, we can obtain the corresponding producer surplus and customer surplus as well as taxi/customer waiting/search times with the taxi model. Figs. 4–7 portray these measures graphically in
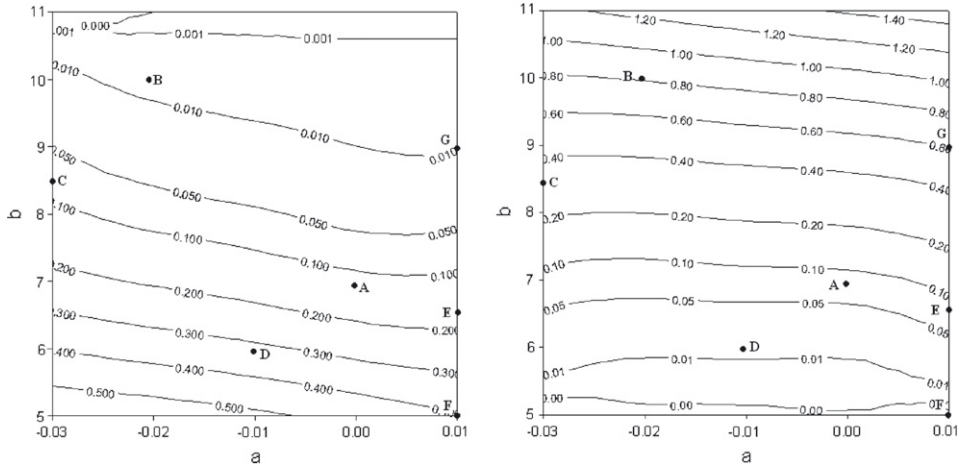
(a-1) Customer waiting time in the urban area   (b-1) Taxi waiting time in the urban area
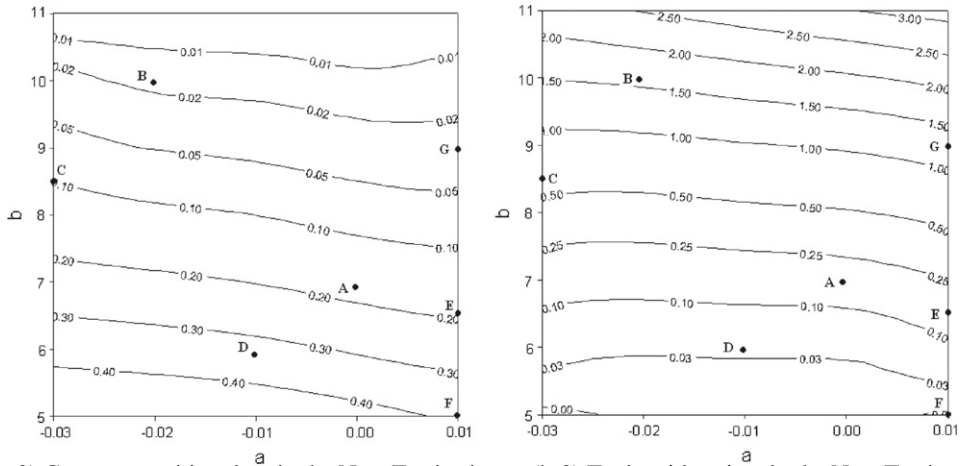
(a-2) Customer waiting time in the New Territories   (b-2) Taxi waiting time in the New Territories
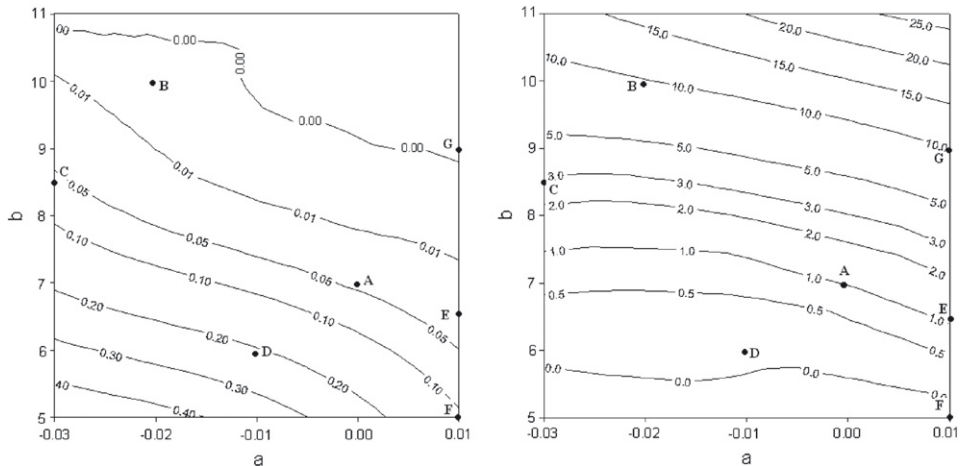
(a-3) Customer waiting time at airport   (b-3) Taxi waiting time at airport

Fig. 4. Customer and taxi wait/search times (h).

(a) Consumer surplus for non-airport-based customers

(b) Consumer surplus for airport-based customers
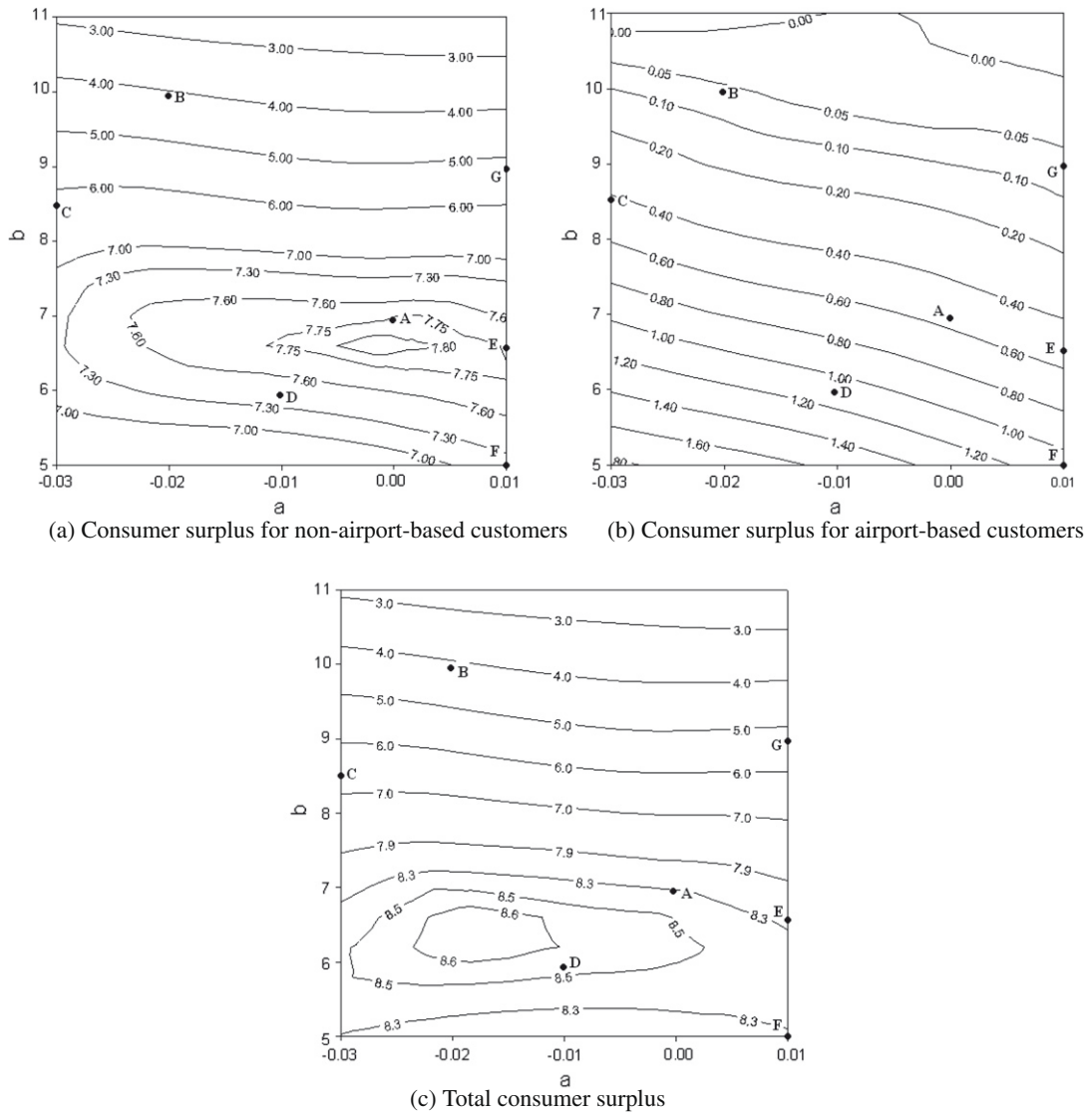
(c) Total consumer surplus

Fig. 5. Contour lines of consumer surplus ($\times 10^5$ HK$) for airport- and non-airport-based customers.

contour lines within the feasible domain of the (a, b) two-dimensional space, together with the results at seven selected specific sets of fare structures. In these figures, "$5.0 \leqslant b \leqslant 11.0$" is considered in view of the current charging rate per kilometer in Hong Kong; "$-0.03 \leqslant a \leqslant 0.01$" is chosen to embrace both concave and convex nonlinear fare structures as represented in Fig. 3, with a focus on the former. With these numerical results, we are ready to look into the principal operational characteristic of the taxi market.

5.2.1. Customer waiting time

Customer waiting times are shown in Fig. 4a-1–a-3. In both urban areas (Hong Kong Island and Kowloon) and the New Territories, customer waiting times decrease with parameter b, which is, as expected, a result of decreasing customer demand due to increasing taxi fare. Within the feasible domain considered, customer waiting time generally decreases with the value of parameter a but the change is quite marginal. As parameter a increases, taxi fare will increase and hence customer demand decreases, resulting in lower customer waiting times. Nevertheless, parameter a in a parabolic nonlinear taxi fare structure will have different relative impact on the short-distance and long-distance fares and hence on the vacant taxi supply and consequently on the customer waiting time in the urban area, New Territories and the airport. Consider Point C and Point A in Fig. 4a-1 for the urban customer waiting time. As shown in Fig. 3, the concave parabolic taxi fare at Point C makes short-distance fare higher and long-distance fare lower in comparison with the linear taxi fare at Point A, but customer waiting time at Point C is lower than that at Point A, this is because, under such a nonlinear fare structure, more taxis
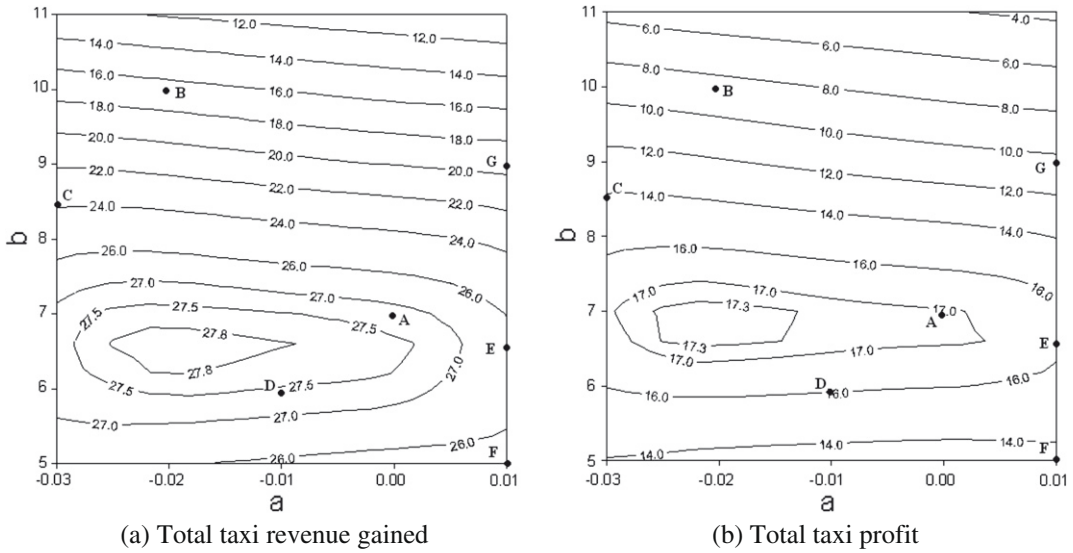
(a) Total taxi revenue gained   (b) Total taxi profit

Fig. 6. Contour lines of total taxi revenue and profit gained ($\times 10^5$ HK$).

(a) The win-win domain   (b) The win-win-win domain
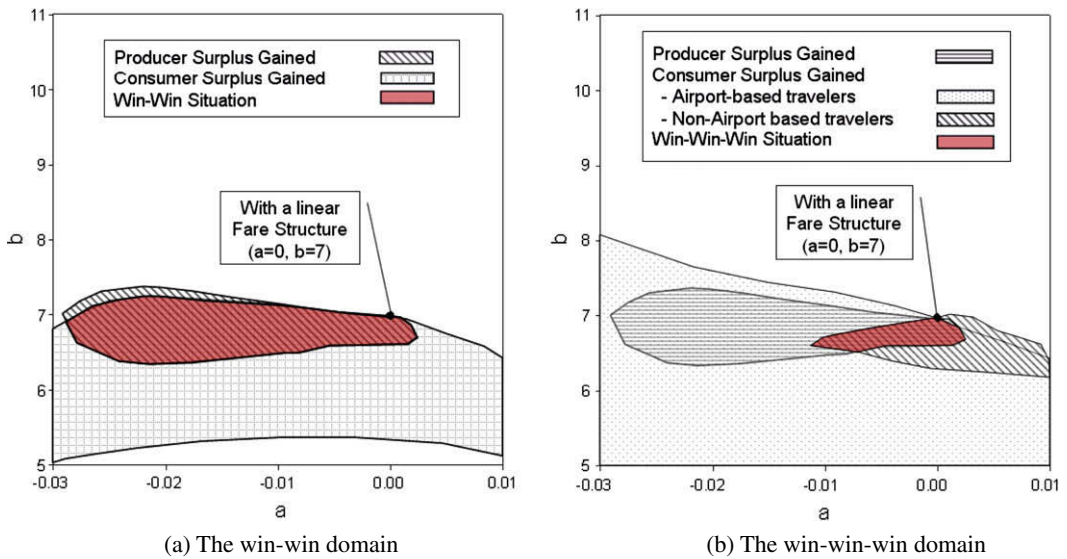
Fig. 7. The Pareto-improving win–win and win–win–win domains in comparison with the linear fare equation ($a = 0$, $b = 7.0$).

will be attracted to cruise in the urban area instead of waiting at the airport, resulting in improved service quality in the urban area. This also explains why the change of customer waiting time with parameter a is insignificant largely due to the complicated, opposite effects of nonlinear pricing on customer waiting times at different zones or locations. Finally, it is noted that, customer waiting time at the airport changes very little within the limited feasible domain of parameters a and b, this is due to the nature of point meetings at the airport and the long-distance profitable taxi service will always attract sufficient taxi supply.

5.2.2. Taxi waiting time

Taxi waiting times are shown in Fig. 4b-1–b-3. As expected and opposite to the customer waiting times, taxi waiting times increase with parameter b in both urban areas (Hong Kong Island and Kowloon) and the New Territories due to increased taxi fare and thus decreased customer demand. But parameter a has almost no impact on taxi waiting time in the urban areas and New Territories. This is also due to the complicated, opposite effects of nonlinear pricing on taxi waiting times at different zones or locations. At the airport, taxi waiting time generally increases in parameter a (with parameter b as well), because the nonlinear taxi fare with a larger value of a will make long-distance service more profitable and thus attract more taxi supply at the airport.

5.2.3. Consumer surplus

With the negative exponential demand function in Eq. (16), consumer surplus of customers between a given origin–destination pair is directly proportional to its realized demand. As portrayed in Fig. 5, introducing nonlinear pricing of taxi services will give quite different impacts on the consumer surplus (and demand as well) of long-distance, airport-based and short-distance, non-airport-based customers. For the non-airport-based customers, taxi fare influences customer demand in two opposite manners. On one hand, increasing taxi fare will result in direct decrease in demand. On the other hand, increase in taxi fare will attract more taxi supply and thus reduce customer waiting time, which in turn induces more customer demand. The latter effect is generally secondary, but it is not always true under a nonlinear fare structure. Consider Point D (concave fare curve) and Points E and F (both convex curve) in Fig. 5a. Short-distance taxi fares under all three sets of parameters are lower than that under the linear taxi fare at Point A, but the consumer surpluses at Points D, E and F are lower than that at Point A. Thus, the effect of decrease in taxi fare on customer demand can be either over-offset or under-offset by that of increase in customer waiting time in the urban areas. It is unclear which force will dominate the other when a nonlinear fare structure has different impact on long- and short-distance fares and allocation of taxi supply at different zones and locations. This explains why the non-airport-based customer demand (consumer surplus) does not change monotonically with parameter b. Nevertheless, there indeed exists an optimal combination of parameters (a, b) that gives the highest non-airport-based consumer surplus within the domain examined. In contrast, consumer surplus for the airport-based customers, as shown in Fig. 5b, decreases with both parameters a and b. This is simply due to the fact that airport customer demand is mainly governed by taxi fare and the effect of customer waiting time with airport point meeting is trivial. Finally, the total consumer surplus, as shown in Fig. 5c, achieves a maximum value at an interior point of the feasible domain of parameters (a, b). The change pattern of total consumer surplus depends on the relative magnitudes of the airport- and non-airport-based customer demands, which vary with taxi fare structure quite differently. In the current case, the majority of taxi rides are non-airport-based (in particular, the intra-zonal trips carry a large portion of the overall demand).

5.2.4. Taxi revenue and profit

Total taxi revenue and profit are displayed in Fig. 6. Note that the profit per taxi per unit time is indifferent to specific zones or locations and individual taxis, as a result of spatially equal profitability at equilibrium. It is thus sufficient to look at the aggregate (rather than location-specific) taxi revenue and profit under linear and nonlinear taxi fare structures. Total taxi revenue, which depends on both taxi fare structure and customer demand, exhibits a clear concave pattern within the two-dimensional space of parameters (a, b). Also, the greatest overall profit is obtained under a concave nonlinear fare structure near $(a, b) = (-0.02, 6.80)$. Note that the revenue and profit contour lines exhibit slightly different pattern, which is due to the different unit time operating cost for running and waiting taxis.

5.2.5. Win–win and win–win–win situations

We now look at the gain in total consumer surplus and total producer surplus (taxi profit) under a nonlinear fare structure in comparison with those under the linear one. Fig. 7a plots the domain of the win–win situation with positive consumer surplus and producer surplus gains. In view of the different impacts of a nonlinear fare structure on short and long-distance trips, consumer surplus gains for airport-based and non-airport-based customers are examined separately; Fig. 7b plots the domain of the win–win–win situation, which is, of course, included in the win–win domain in Fig. 7a. The win–win–win domain represents Pareto-improving nonlinear fare structures over the current linear one. From Fig. 7, it is interesting to note that the Pareto-improving nonlinear taxi fare structures exist for values of parameter b slightly less than $b = 7.0$ (adopted in the old linear fare as of 28 February 2008), and mostly for *negative* value of parameter a. This means that the old linear fare structure in Hong Kong was sub-optimal; there is room for taxi fare reduction (overall fare reduction with less distance-proportional charge for all trips and relatively more percentage fare reduction with a concave cure of $a < 0$ for long-distance trips). Pareto-improvements can be made mostly with a concave fare structure as proposed by the taxi drivers and the government.[8] As seen again from Fig. 4b-3, within the Pareto-improving domain, wasteful taxi waiting times at the airport can be reduced substantially, which means appropriate concave nonlinear fare structures can induce more efficient equilibrium allocation of taxi services in different service zones or locations.

6. Conclusions

This article proposed a nonlinear fare structure with a continuously declining charge rate per unit distance in a taxi market, which is intended to address the problems of taxi fare discount gangs and long taxi waiting queues at problem spots such as the airport. The nonlinear fare structure is built into the network taxi service model which explicitly takes into account the perceived profitability of taxi drivers in search of customers. As demonstrated with a simplified Hong Kong case study, introducing a nonlinear fare structure into the taxi market will likely create a Pareto-improving win–win situation in

---

[8] On 30 November 2008 (after submission of the initial version of this paper), the Hong Kong government commenced implementation of the following new nonlinear fare structure for urban taxi services which raises short-haul fares and lowers long-haul fares: the flag-fall fare for the first 2 km was increased by 3.0 HK$ from 15.0 HK$ to 18.0 HK$, the incremental charge for every subsequent 0.2 km or every period of 1 min waiting time was increased by 0.1 HK$ to 1.5 HK$ before meter fare of 70.5 HK$. From meter fare of 70.5 HK$ onwards, the incremental charge was reduced to 1.0 HK$.

the sense that both the customers and taxi drivers benefit. The total social welfare is thus improved through more efficient allocation of taxi service resources in different zones or locations of the whole territory.

Acknowledgements

References

Akwawua, S., Pooler, J.A., 2001. The development of an intervening opportunities model with spatial dominance. Journal of Geographical Systems 3 (1), 69–86.

Arnott, R., 1996. Taxi travel should be subsidized. Journal of Urban Economics 40 (3), 316–333.

Cairns, R.D., Liston-Heyes, C., 1996. Competition and regulation in the taxi industry. Journal of Public Economics 59 (1), 1–15.

De vany, A.S., 1975. Capacity utilization under alternative regulatory constraints: an analysis of taxi markets. Journal of Political Economy 83 (1), 83–94.

Douglas, G.W., 1972. Price regulation and optimal service standards: the taxicab industry. Journal of Transport Economics and Policy 6 (2), 116–127.

Fernandez, J.E., De Cea, J., Briones, J., 2006. A diagrammatic analysis of the market for cruising taxis. Transportation Research Part E 42 (6), 498–526.

Flores-Guri, D., 2005. Local exclusive cruising regulation and efficiency in taxicab markets. Journal of Transport Economics and Policy 39 (2), 155–166.

HKSAR Government, 2002a. Travel Characteristics Survey 2002 Final Report. Transport Department, HKSAR Government.

HKSAR Government, 2002b. Hong Kong Annual Digest of Statistics. Hong Kong Government Printer.

HKSAR Government, 2006. Annual Traffic Census 2006, TTSD Publication No. 07CAB1. Transport Department, HKSAR Government.

HKSAR Government, 2008. Report on Review of Taxi Operation. Transportation Advisory Committee, HKSAR Government.

HKSAR Government, 2007. Inviting Views from the Trade and the Public for Quality Taxi Services. Transportation Advisory Committee, HKSAR Government.

Lagos, R., 2000. An alternative approach to search frictions. Journal of Political Economy 108 (51), 851–873.

Moore, A.T., Balaker, T., 2006. Do economists reach a conclusion on taxi deregulation? Economical Journal Watch 3 (1), 109–132.

Sirisoma, R.M.N.T., Wong, S.C., Lam, W.H.K., Wang, D.G., Yang, H., Zhang, P., 2010. Empirical evidence for taxi customer-search model in Hong Kong. Proceedings of the Institution of Civil Engineers, Transport.

Wilson, R.B., 1992. Nonlinear Pricing. Oxford University Press, New York.

Wong, S.C., Yang, H., 1998. A network equilibrium model of urban taxi services: an improved algorithm. Transportation Research Record 1623, 27–30.

Wong, K.I., Wong, S.C., Yang, H., Tong, C.O., 2003. The effect of perceived profitability on the level of taxi service in remote areas. Journal of the Eastern Asia Society for Transportation Studies 5, 79–94.

Wong, K.I., Wong, S.C., Bell, M.G.H., Yang, H., 2005. Modeling the bilateral micro-searching behavior for urban taxi services using the absorbing Markov chain approach. Journal of Advanced Transportation 39 (1), 81–104.

Wong, K.I., Wong, S.C., Yang, H., Wu, J.H., 2008. Modeling urban taxi services with multiple user classes and vehicle types. Transportation Research Part B 42 (10), 985–1007.

Yang, H., Wong, S.C., 1998. A network equilibrium model of urban taxi services. Transportation Research Part B 32 (4), 235–246.

Yang, H., Wong, K.I., Wong, S.C., 2001. Modeling urban taxi services in road networks: progress, problem and prospect. Journal of Advanced Transportation 35 (3), 237–258.

Yang, H., Wong, S.C., Wong, K.I., 2002. Demand–supply equilibrium of taxi services in a network under competition and regulation. Transportation Research Part B 36 (9), 799–819.

Yang, H., Ye, M., Tang, W.H., Wong, S.C., 2005a. Regulating taxi services in the presence of congestion externalities. Transportation Research Part A 39 (1), 17–40.

Yang, H., Ye, M., Tang, W.H., Wong, S.C., 2005b. A multi-period dynamic model of taxi services with endogenous service intensity. Operations Research 53 (3), 501–515.

Yang, H., Leung, W.Y., Wong, S.C., Bell, M.G.H., in press. Equilibria of bilateral taxi-customer searching and meeting on networks. Transportation Research Part B. doi:10.1016/j.trb.2009.12.010.