國 立 交 通 大 學

資訊科學系

碩 士 論 文

增 強 立 方 體 之 容 錯 泛 圈 性 質 研 究

Fault-Tolerant Pancyclicity of Augmented Cubes

研 究 生：史偉華

指導教授：徐力行　教授

中 華 民 國 九 十 三 年 六 月

增 強 立 方 體 之 容 錯 泛 圈 性 質 研 究
Fault-Tolerant Pancyclicity of Augmented Cubes

研　究　生：史偉華　　　　Student：Wei-Hua Shih

指導教授：徐力行　　　　Advisor：Lih-Hsing Hsu

國 立 交 通 大 學
資 訊 科 學 研 究 所

碩 士 論 文

A Thesis

Submitted to Institute of Computer and Information Science

College of Electrical Engineering and Computer Science

National Chiao Tung University

in partial Fulfillment of the Requirements

for the Degree of

Master

in

Computer and Information Science

June 2004

Hsinchu, Taiwan, Republic of China

中華民國九十三年六月

增強立方體之容錯泛圈性質研究

研究生：史偉華　　　　　　　　指導教授：徐力行　博士

國 立 交 通 大 學 資 訊 科 學 研 究 所

摘要

增強立方體，AQ_n 是利用超立方體 Q_n 加上額外的連線而得到原本超立方體所沒有的性質，在本篇中我們將研究增強立方體在維度大於等於 4 時的容錯泛圈性質，假設當 $n \geq 4$ 時 $F \subseteq V(AQ_n)\,Y\,E(AQ_n)$，若$|F| \leq 2n-3$，我們可以證明 $AQ_n - F$ 是泛圈圖

關鍵字：容錯，泛圈圖，增強立方體

Fault-Tolerant Pancyclicity of Augmented Cubes

Student : Wei-Hua Shih Advisor : Lih-Hsing Hsu

Institute of Computer and Information Science

National Chiao Tung University

Abstract

Augmented cubes, AQ_n is a graph which adding some edges to hypercube Q_n to improve some properties according to some rule. In this thesis, we consider the fault-tolerant pancyclicity of the augmented cubes AQ_n for $n \geq 4$. Assume that $F \subseteq V(AQ_n) \cup E(AQ_n)$ for $n \geq 4$. We prove that AQ_n - F is pancyclic if $|F| \leq 2n-3$.

Keywords：fault-tolerant, pancyclicity, augmented cubes.

感謝

衷心地感謝我的指導老師徐力行教授以及譚建民教授在這兩年來的教誨，以及對這篇論文幫助最大的陳玉專學長，沒有他們我無法完成這篇論文，感謝在平時給我許多建議與幫忙的學長姐們，李增奎學長、賴寶蓮學姊、楊明堅學長、許弘駿學長、鄭斐文學長，江良志學長、張晉學長、陳永穆學長；一起研究課業甘苦與共的同學們，力中、岳倫、哲維、國晃、元翔；以及有話直說毫不保留的學弟倫閔。

此外，我更要感謝我的父母，因爲他們的支持我才能完成我的學業

史偉華 06/09/2004

Contents

List of Figures

Chapter 1

Introduction

Network topology is a crucial factor for the interconnection networks since it determines the performance of the networks. Many interconnection network topologies have been proposed in the literature for the purpose of connecting thousands of processing elements. When we design the topology of a interconnection network, there are a lot of requirements which affect the characteristics of network. Network topology is usually represented by a graph where vertices represent processors and edges represent links between processors.

There are a lot of literature to propose the interconnection network topologies. The hamiltonian properties of a network are important aspects of designing a network. A lot of related works have appeared in the following literature. D.Barth et al. proved that there are two disjoint hamiltonian cycles in the butterfly graph [2]. J.C. et al. showed that Caley graphs of degree 4 can be decomposed to disjoint hamiltonian cycles [3]. The hypercube is one of the most popular network since it has a simple structure and is easy to implement. The n-dimensional hypercube [19, 25], denoted by Q_n, is a popular one. Several variations of the hypercubes have been investigated to improve the efficiency of the hypercubes, such as twisted N-cubes [10], twisted cubes [1, 12], crossed cubes [8, 9], Möbius cubes [7]. The augmented cubes AQ_n, recently proposed by Choudum and Sunitha [6], is one of such variations. For any positive integer n, AQ_n is a vertex transitive, $(2n-1)$-regular, and $(2n-1)$-connected graph with 2^n vertices. The pancycle problem asks if a cycle of length l is a subgraph of a given graph with a given positive integer l. Hwang [16] and Fan [11] et al. studied this problem on butterfly graphs and Möbius cubes, respectively. But they did not consider the possibilities of failures of vertices and/or edges. Fault tolerant ability is also desirable on network which have relatively high probability of failure. There are many researches related to fault-tolerant hamiltonian properties of networks [15, 13, 17, 18, 20, 21, 22, 23, 24, 26, 27].

In this thesis, we consider the important property fault-tolerant pancyclicity of the augmented cubes.

A graph G is *pancyclic* if and only if there are cycles of length from 3 to N in G where N is the vertex number of G. A pancyclic graph G is *k-fault pancyclic* if $G - F$ remains pancyclic for every $F \subset V(G) \cup E(G)$ with $|F| \leq k$. The *fault pancyclicity*, $\mathcal{P}_f(G)$, is defined to be the maximum interger k such that G is k-fault pancyclic. In this thesis, we prove that the fault pancyclicity of the augmented cube AQ_n is exactly $2n-3$ for $n \geq 4$.

The rest of this thesis is organized as follows. In Chapter 2, we give some definitions and notations. In Chapter 3, we give the definition of the augmented cubes and discuss some properties of them. In Charpter 4, we prove the fault pancyclicity of the augmented cubes. Finally, we make some concluding remarks in Chapter 5.

Chapter 2

Some definitions and notations

In this chapter, we introduce several definitions and notations.

For the graph definitions and notations we follow [4]. $G=(V,E)$ is a *graph* if V is a finite set and E is a subset of $\{(u,v) \mid (u,v)$ is an unordered pair of $V\}$. We say that V is the *vertex set* and E is the *edge set*. For any vertex x of V, $deg_G(x)$ denotes its degree in G. We use $\delta(G)$ to denote $min\{deg_G(x) \mid x \in V(G)\}$. Two vertices u and v are *adjacent* if $(u,v) \in E$. A *path*, denoted by $\langle v_0, v_1, v_2, ..., v_k \rangle$, is a sequence of distinct vertices where v_i and v_{i+1} are adjacent for all $0 \leq i \leq k-1$. The *length* of a path P, $l(P)$, is the number of edges in P. We also write the path $\langle v_0, v_1, v_2, ..., v_k \rangle$ as $\langle v_0, P_1, v_i, v_{i+1}, ..., v_j, P_2, v_t, ..., v_k \rangle$, where P_1 is the subpath $\langle v_0, v_1, ..., v_i \rangle$ and P_2 is the subpath $\langle v_j, v_{j+1}, ..., v_t \rangle$. In this thesis, it is possible to write a path as $\langle v_0, v_1, P, v_1, v_2, ..., v_k \rangle$ if $l(P)$ is 0. Sometime, a path can also be represented by $\langle v_0, v_1, ..., v_i, e, v_{i+1}, ..., v_n \rangle$ to emphasize that e is the edge (v_i, v_{i+1}). We use $d(u,v)$ to denote the distance between u and v. That is the length of the shortest path joining u and v. Let a path be a *hamiltonian path* if its vertices are distinct and included the whole V. Let a path be named a *cycle* with at least three vertices if the first vertex is the same as the last vertex. Let the cycle be denoted C_n with length n for $n \geq 3$.

A graph G is *pancyclic* if and only if there are cycles of length from 3 to N in G where N is the vertex number of G. A pancyclic graph G is *k-fault pancyclic* if $G - F$ remains pancyclic for every $F \subset V(G) \cup E(G)$ with $|F| \leq k$. The *fault-tolerant pancyclicity*, $\mathcal{P}_f(G)$, is defined to be the maximum interger k such that G is k-fault pancyclic. In this thesis, we abbreviate k-fault-tolerant pancyclicity as $k - faultpancyclicity$.

A *hamiltonian cycle* of G is a cycle that traverses every vertex of G exactly once. A graph is *hamiltonian* if it has a hamiltonian cycle. A hamiltonian graph G is *k-fault hamiltonian* if $G - F$ remain hamiltonian for every $F \subset V(G) \cup E(G)$ with $|F| \leq k$. The *fault hamiltonianicity*, $\mathcal{H}(G)$, is define to be the maximum integer k such that G is k-fault hamiltonian if G is hamiltonian.

For a set $S \subset V(G)$, the notation $G - S$ represents the graph obtained by removing the vertices in S from G and deleting those edges with at least one end vertex in S simultaneously. In graph G, the *connectivity* κ is the minimum number of a set S of G such that $G - S$ is disconnected or trivial.

A graph G is *hamiltonian connected* if there exists a hamiltonian path joining any two vertices of G. All hamiltonian connected graphs except K_1 and K_2 are hamiltonian. A graph G is *k-fault hamiltonian connected* if $G - F$ remains hamiltonian connected for every $F \subset V(G) \cup E(G)$ with $|F| \leq k$. The *fault hamiltonian connectivity*, $\mathcal{H}_f^k(G)$, is defined to be the maximum integer k such that G is k-fault hamiltonian connected if G is hamiltonian connected.

Chapter 3

Augmented cubes

3.1 Definition and notation

First, we shows the define of hypercube and several examples of hypercube which are illustrated in Fig.3.1.

Definition 1 *Let $n > 1$ be an integer. The Hypercube Q_n of dimension n has 2^n nodes. Q_1 is a complete graph with two nodes labeled by 0 and 1, respectively. For $n \geq 2$, an n-dimensional Hypercube Q_n is obtained by taking two copies of $(n-1)$-dimensional subcubes Q_{n-1}, denoted by Q^0_{n-1} and Q^1_{n-1}. For each $v \in V(Q_n)$, insert a 0 to the front of $(n-1)$-bit binary string for v in Q^0_{n-1} and a 1 to the front of $(n-1)$-bit binary string for v in Q^1_{n-1}. There are 2^{n-1} edge between Q^0_{n-1} and Q^1_{n-1} as follows:*

Let $V(Q^0_{n-1}) = \{0u_{n-2}u_{n-3}...u_0 : u_i = 0 \text{ or } 1\}$ and $V(Q^1_{n-1}) = \{1v_{n-2}v_{n-3}...v_0 : v_i = 0 \text{ or } 1\}$, where $0 \leq i \leq n-2$. A node $u = 0u_{n-2}u_{n-3}...u_0$ of $V(Q^0_{n-1}$ is joined to a node $v = 1v_{n-2}v_{n-3}...v_0$ of $V(Q^1_{n-1})$ if and only if $u_i = v_i$ for $0 \leq i \leq n-2$.

Let $n \geq 1$ be an integer. The *n-dimensional augmented cube* [6], denoted by AQ_n, has 2^n vertices, each labeled by an n-bit binary string $V(AQ_n) = \{u_1 u_2 ... u_n |\ u_i \in \{0,1\}\}$. AQ_1 is the complete graph K_2 with vertex set $\{0,1\}$. For $n \geq 2$, we write this recursive construction of AQ_n symbolically as $AQ_n = AQ^0_{n-1} \Diamond AQ^1_{n-1}$ and by adding 2^n edges between AQ^0_{n-1} and AQ^1_{n-1} as follows:

Let $V(AQ^0_{n-1}) = \{(0u_2u_3...u_n) \mid u_i = 0 \text{ or } 1 \text{ for } 2 \leq i \leq n\}$ and $V(AQ^1_{n-1}) = \{(1v_2v_3...v_n) \mid v_i = 0 \text{ or } 1 \text{ for } 2 \leq i \leq n\}$. A vertex $\mathbf{u} = (0u_2u_3...u_n)$ of AQ^0_{n-1} is joined to a vertex $\mathbf{v} = (1v_2v_3...v_n)$ of AQ^1_{n-1} if and only if either

(i) $u_i = v_i$ for $2 \leq i \leq n$; in this case, $(\mathbf{u}, \mathbf{v})$ is called a *hypercube edge* and we denote $\mathbf{v} = \mathbf{u}^h$,or

(ii) $u_i = \bar{v}_i$ for $2 \leq i \leq n$; in this case, $(\mathbf{u}, \mathbf{v})$ is called a *complement edge* and we denote $\mathbf{v} = \mathbf{u}^c$.

3.2 Some basic properties of augmented cubes

There are several augmented cubes (ex:AQ_1, AQ_2, and AQ_3) that are illustrated in Fig.3.2 By definition, AQ_n is a vertex transitive, $(2n-1)$-regular, and $(2n-1)$-connected graph with 2^n vertices for any positive integer n. Let $E^h_n = \{(\mathbf{u}, \mathbf{u}^h) |\ \mathbf{u} \in \mathbf{V}(AQ^0_{n-1})\}$ and $E^c_n = \{(\mathbf{u}, \mathbf{u}^c) |\ \mathbf{u} \in \mathbf{V}(AQ^0_{n-1})\}$. Obviously, E^h_n and E^c_n are two perfect matchings between the vertices of AQ^0_{n-1} and AQ^1_{n-1}. Then, $|E^h_n|$ and $|E^c_n|$ are both equal to 2^{n-1}. Let $C^*_n =$

$\{(u,v) \mid u = 0u_1...u_n$ and $v = 0\bar{u_1}...\bar{u_n}\}$. In other words, C_n^* is the set of all complement edges in AQ_{n-1}^0. Let $F \subseteq V(AQ_n) \cup E(AQ_n)$ be the set of faults. We divide F into five parts:

(1)F_v^0=$F \cap V(AQ_{n-1}^0)$, (2)F_e^0=$F \cap E(AQ_{n-1}^0)$,

(3)F_v^1=$F \cap V(AQ_{n-1}^1)$, (4)F_e^1=$F \cap E(AQ_{n-1}^1)$,

(5)F_e^x=$F - F(AQ_{n-1}^0) \cup F(AQ_{n-1}^1)$

By the way, we let $F_v = F_v^0 \cup F_v^1$ and $F_e = F_e^0 \cup F_e^1 \cup F_e^x$. Let f=$|F|$, f_v=$|F_v|$, f_v^0=$|F_v^0|$, f_e^0=$|F_e^0|$, f_v^1=$|F_v^1|$, $f_e^1 = |F_e^1|$ and f_e^x=$|F_e^x|$.

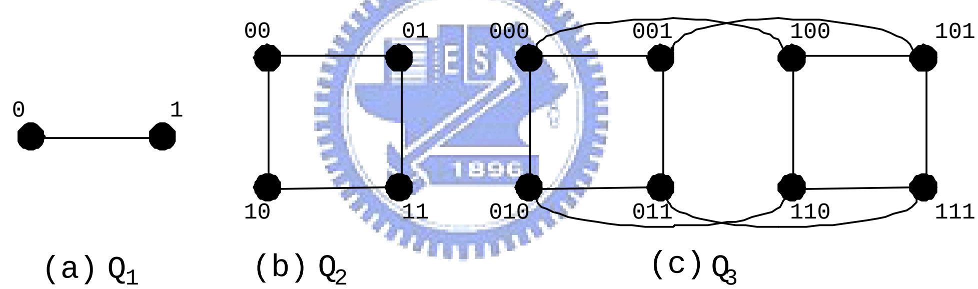

Figure 3.1: The hypercubes Q_1, Q_2, and Q_3.

The folowing lemmas are derived directly from the definition.

Lemma 1 *Given a graph AQ_n for $n \geq 4$, let* $\mathbf{u}$ *and* $\mathbf{v}$ *be two distinct vertices in which are both in AQ_{n-1}^0 or both in AQ_{n-1}^1. Then $\mathbf{u}^h, \mathbf{u}^c, \mathbf{v}^h$, and $\mathbf{v}^c$ are distinct if and only if* $(\mathbf{u}, \mathbf{v}) \notin \mathbf{C_n^*}$.

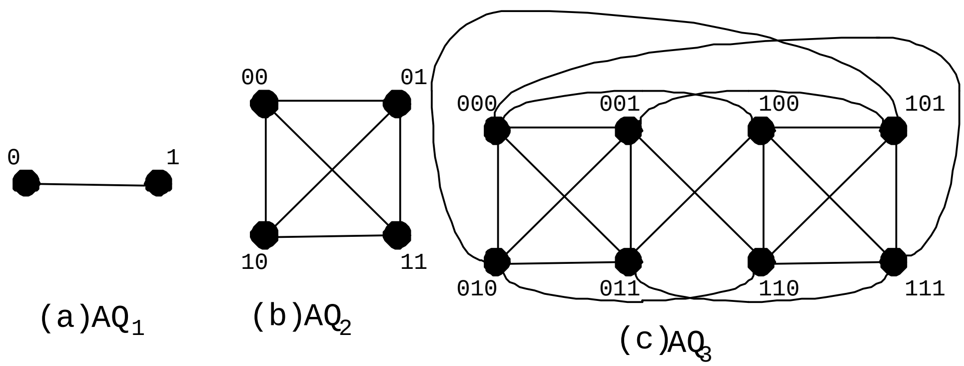

Figure 3.2: The augmented cubes AQ_1, AQ_2, and AQ_3.

Lemma 2 *Assume that* $(\mathbf{u}, \mathbf{v}) \in AQ^0_{n-1}$. *Then* $(\mathbf{u}^h, \mathbf{v}^h) \in AQ^1_{n-1}$ *and* $(\mathbf{u}^c, \mathbf{v}^c) \in AQ^1_{n-1}$.

Lemma 3 *Given a vertex* $\mathbf{u} \in AQ^0_{n-1}$. *Let* $A = \{\mathbf{v} | (\mathbf{u}, \mathbf{v}) \in E(AQ^0_{n-1})\}$ *and* $B = \{\mathbf{v}^h, \mathbf{v}^c | \mathbf{v} \in A\}$. *Then* $|A| = 2n - 3$ *and* $|B| = 4n - 8$.

For example, given a vertex 00000 in AQ^0_5. Let $A = \{00001, 00010, 00100, 01000, 00011, 00111, 01111\}$ and $B = \{10001, 11110, 10010, 11101, 10100, 11011, 11000, 10111, 10011, 11100, 11111, 10000\}$. Hence, only two vertices in A which are in C^*_n.

Chapter 4

Fault-tolerant pancyclicity of augmented cubes

To discuss the fault-tolerant pancyclicity of augmented cubes, we need to introduce the following term for graphs. A graph G has *property 2H* if it satisfies the following conditions: *Let* $\{w, x\}$ *and* $\{y, z\}$ *be two pairs of four distinct vertices of* G. *There exist two disjoint paths* P_1 *and* P_2 *of* G *such that (1)* P_1 *joins* w *to* x, *(2)* P_2 *joins* y *to* z, *and (3) every vertex of* G *is either on path* P_1 *or on* P_2.

Lemma 4 *[5] Let* $n \geq 4$. AQ_n *is* $(2n-3)$*-fault hamiltonian,* $(2n-4)$*-fault hamiltonian connected and has property 2H.*

Property 1 *Given a path* $\mathbf{P}$. $\mathbf{P}$ *has* $i+1$ *subpaths of length* $l(\mathbf{P})-i$ *for* $0 \leq i \leq l(\mathbf{P})-\mathbf{1}$.

Now, we are going to discuss the fault-tolerant pancyclicity of augmented cube. We can find that AQ_3 is not fault-tolerant pancyclicity with our computer programs. The base case is AQ_n for $n = 4$. We translated the proof to computer programs and we put these programs and their outputs on the web site "http://140.113.167.44/aq4.html". Let $F \subset V(AQ_n) \cup E(AQ_n)$ be any faulty set of AQ_n. An edge $(\mathbf{u}, \mathbf{v})$ is called *F-fault free* if $(\mathbf{u}, \mathbf{v}) \notin F$, $\mathbf{u} \notin F$, and $\mathbf{v} \notin F$; otherwise it is called F-fault. Let $H = (V', E')$ be a subgraph of AQ_n. We use $F(H)$ to denote the set $(V' \cup E') \cap F$

Theorem 1 *Let $n \geq 5$. Suppose AQ_{n-1} is $(2n-5)$-fault pancyclic. Then, AQ_n is $(2n-3)$-fault pancyclic*

Proof. Let F be any subset of $V(AQ_n) \cup E(AQ_n)$ with $|F| \leq 2n-3$. Without loss of generality, we assume that $|F(AQ_{n-1}^0)| \geq |F(AQ_{n-1}^1)|$. We discuss this problem with $|F| = 2n-3$. If the fault of augmented cubes is less then $2n-3$, we can add some edges into fault set which are fault free . To show that AQ_n is $(2n-3)$-fault pancyclic, we shall find cycles of lengths from 3 to $|V(AQ_n)| - |F_v(AQ_n)|$. We divide the proof into three cases. Because we suppose augmented cubes is $(2n-3)$-fault pancyclic, AQ_{n-1} is $(2n-5)$-fault pancyclic.

Case 1: *Cycles of lengths from 3 to $|V(AQ_{n-1}^1)| - f_v^1$.*

Since AQ_{n-1} is $(2n-5)$-fault pancyclic, AQ_{n-1}^1 contains of lengths from 3 to $|V(AQ_{n-1}^1)| - f_v^1$ for $n \geq 5$. Clearly, $AQ_n - F$ also contains cycles of these lengths.

Case 2: *A cycle of length $|V(AQ_{n-1}^1)| - f_v^1 + 1$.*

We shall claim that there exists a vertex u in AQ^0_{n-1} such that both $(\mathbf{u}, \mathbf{u}^h)$ and $(\mathbf{u}, \mathbf{u}^c)$ are F-fault free. Suppose it is not exist, $|F| = 2n-3 \geq 2^{n-1}/2 = 2^{n-2}$, which is a contradiction for $n \geq 5$. In addition, AQ^1_{n-1} is $(2n-6)$-fault hamiltonian connected, $AQ^1_{n-1} - (F^1_v \cup F^1_e)$ has a hamiltonian path $\langle \mathbf{u}^h, \mathbf{P}, \mathbf{u}^c \rangle$. Thus, $\langle \mathbf{u}, \mathbf{u}^h, \mathbf{P}, \mathbf{u}^c, \mathbf{u} \rangle$ is a cycle of length $|V(AQ^1_{n-1})| - f^1_v + 1$.

We follow [5] to construct cycles of lengths from $|V(AQ^1_{n-1})| - f^1_v + 2$ to $|V(AQ_n)| - f_v$.

Case 3: *Cycles of lengths from* $|V(AQ^1_{n-1})| - f^1_v + 2$ *to* $|V(AQ_n)| - f_v$

Subcase 3.1: $|F(AQ^0_{n-1})| = 2n-3$. Thus, $|F - F(AQ^0_{n-1})| = 0$. Let f_1 and f_2 be any two elements in $F(AQ^0_{n-1})$. Since AQ_{n-1} is $(2n-5)$-fault hamiltonian, there exists a hamiltonian cycle $C = \langle \mathbf{u}, f'_1, \mathbf{v}, \mathbf{P_1}, \mathbf{w}, f'_2, \mathbf{x}, \mathbf{P_2}, \mathbf{u} \rangle$ in $AQ^0_{n-1} - (F(AQ^0_{n-1}) - \{f_1, f_2\})$ where $f'_1 = f_1$ if f_1 is on C or f'_1 is an arbitrary edge of C otherwise and $f'_2 = f_2$ if f_2 is on C or f'_2 is an arbitrary edge of C otherwise. Without loss of generality, we assume that $l(\mathbf{P_2}) \leq l(\mathbf{P_1})$.(See Fig.4.1)

Subcase 3.1.1: $l(\mathbf{P_2}) \geq 1$. First, we take $\mathbf{P_1}$ to construct the cycles of lengths from $|V(AQ^1_{n-1})| - f^1_v + 2$ to $|V(AQ^1_{n-1})| - f^1_v + 1 + l(\mathbf{P_1})$. Let $\mathbf{P'_1} = \langle \mathbf{m}, \mathbf{P}"_\mathbf{1}, \mathbf{n} \rangle$ be the subpaths of the path $\mathbf{P_1}$ have lengths from 1 to $l(\mathbf{P_1})$ by Property 1. Since vertices $\mathbf{m}$ and $\mathbf{n}$ are distinct in AQ^0_{n-1}, there are two vertices $\mathbf{m}^h$ and $\mathbf{n}^h$ which are distinct in AQ^1_{n-1} by Lemma 1. By Lemma 4, there exists one hamiltonian path $H = \langle \mathbf{m}^h, \mathbf{P_3}, \mathbf{n}^h \rangle$ in AQ^1_{n-1}. Thus, $\langle \mathbf{m}, \mathbf{P}_1", \mathbf{n}, \mathbf{n}^h, \mathbf{P}_3, \mathbf{m}, \mathbf{m}^h \rangle$ forms cycles of lengths from $|V(AQ^1_{n-1})| - f^1_v + 2$ to

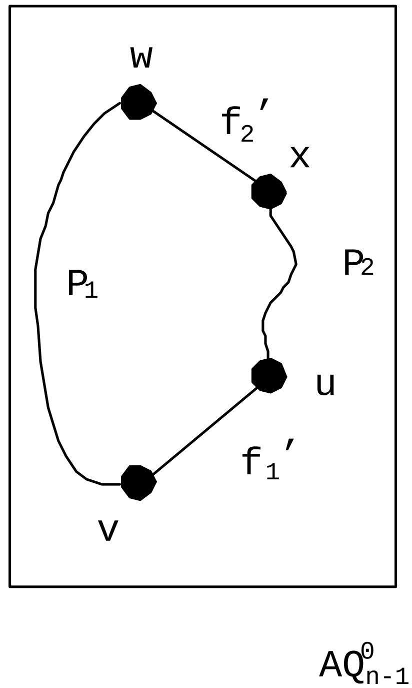

Figure 4.1: The example of Subcase 3.1.

$|V(AQ^1_{n-1})| - f^1_v + 1 + l(\mathbf{P_1})$.(See Fig.4.2)

Now, let $\mathbf{P'_1} = \langle \mathbf{m}, \mathbf{P^{"}_1}, \mathbf{n} \rangle$ of the path $\mathbf{P_1}$ which have length $l(\mathbf{P_1}) - \mathbf{2}$ and the set of subpaths $\mathbf{P'_2} = \langle \mathbf{k}, \mathbf{P^{"}_2}, \mathbf{q} \rangle$ of $\mathbf{P_2}$ which have lengths from 1 to $l(\mathbf{P_2})$ by Property 1. Since AQ_{n-1} has property 2$\mathbf{H}$, there exist two disjoint spanning paths $\langle \mathbf{k}^h, \mathbf{P_3}, \mathbf{m}^h \rangle$ and $\langle \mathbf{n}^h, \mathbf{P_4}, \mathbf{q}^h \rangle$ in AQ^1_{n-1}. Thus, the set of cycles $= \langle \mathbf{n}, \mathbf{P^{"}_1}, \mathbf{m}, \mathbf{m}^h, \mathbf{P_3}, \mathbf{k}^h, \mathbf{k}, \mathbf{P^{"}_2}, \mathbf{q}, \mathbf{q}^h, \mathbf{P_4}, \mathbf{n}^h, \mathbf{n} \rangle$ which have lengths from $|V(AQ^1_{n-1})| - f^1_v + 2 + l(\mathbf{P_1})$ to $|V(AQ_n)| - f_v - 2$.(See Fig.4.3) Finally, we take the two kind of subpaths P'_1 which have lengths $l(\mathbf{P_1}) - \mathbf{1}$ to $l(\mathbf{P_1})$. Thus, the set of cycles $= \langle \mathbf{n}, \mathbf{P^{"}_1}, \mathbf{m}, \mathbf{m}^h, \mathbf{P_3}, \mathbf{k}^h, \mathbf{k}, \mathbf{P^{"}_2}, \mathbf{q}, \mathbf{q}^h, \mathbf{P_4}, \mathbf{n}^h, \mathbf{n} \rangle$ are results of Subcase 3.1.1.

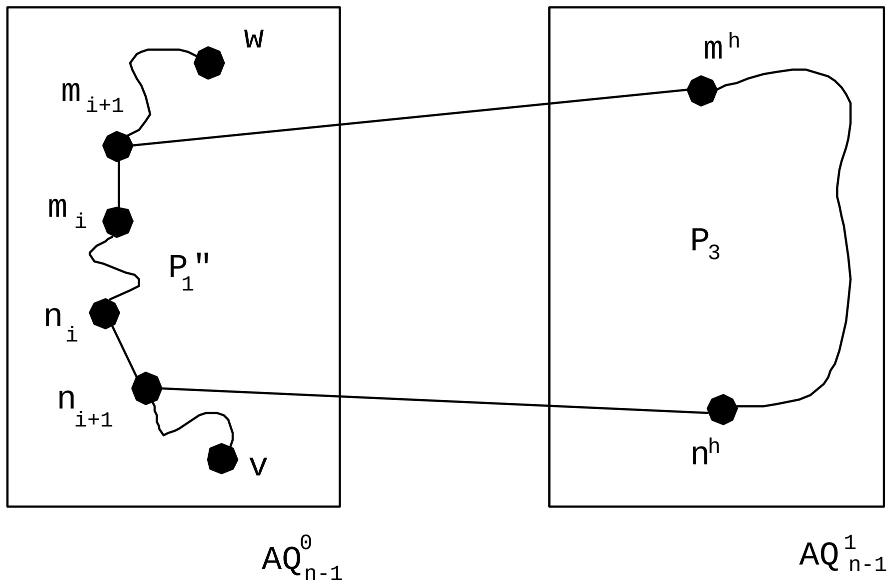

Figure 4.2: The cycles of lengths from $|V(AQ^1_{n-1})| - f^1_v + 2$ to $|V(AQ^1_{n-1})| - f^1_v + 1 + l(\mathbf{P_1})$.

Subcase 3.1.2: $l(\mathbf{P_2}) = \mathbf{0}$. Then $\mathbf{u} = \mathbf{x}$. By Property 1, we use the set of subpaths $\mathbf{P'_1} = \langle \mathbf{a}, \mathbf{P^{"}_1}, \mathbf{b} \rangle$ of path $\mathbf{P_1}$ have lengths from 1 to $l(\mathbf{P_1})$ by Property 1 with the hamiltonian path $\mathbf{P_2} = \langle \mathbf{a}^h, \mathbf{P'_2}, \mathbf{b}^h \rangle$ of AQ^1_{n-1} to construct the cycles= $\langle \mathbf{a}, \mathbf{P^{"}_1}, \mathbf{b}, \mathbf{b}^h, \mathbf{P'_2}, \mathbf{a}^h, \mathbf{a} \rangle$ of lengths from $|V(AQ^1_{n-1})| - f^1_v + 2$ to $|V(AQ_n)| - f_v - 1$.(See Fig.4.4) Since the hamiltonian cycle is constructed in Lemma 4, this completes the proof of Subcase 3.1.

Subcase 3.2: $|F(AQ^0_{n-1})| = 2n-4$. Thus, $|F - F(AQ^0_{n-1})| = 1$. Since AQ_{n-1} is $(2n-5)$-fault hamiltonian, there exists a hamiltonian cycle C_i of $AQ^0_{n-1} - (F(AQ^0_{n-1}) - \{f_i\})$ for every $f_i \in F(AQ^0_{n-1})$. We can write C_i as $\langle \mathbf{u}_i, \mathbf{P}_i, \mathbf{v}_i, f_i, \mathbf{u}_i \rangle$. Since $|F(AQ^0_{n-1})| \geq 6$ and $|F - F(AQ^0_{n-1})| \leq 1$, there exists an index i such that $(\mathbf{u}_i, \mathbf{u}^h_i)$ and $(\mathbf{v}_i, \mathbf{v}^h_i)$ are F-fault free. Since AQ_{n-1} is $(2n-6)$-fault hamiltonian connected, there is a hamiltonian

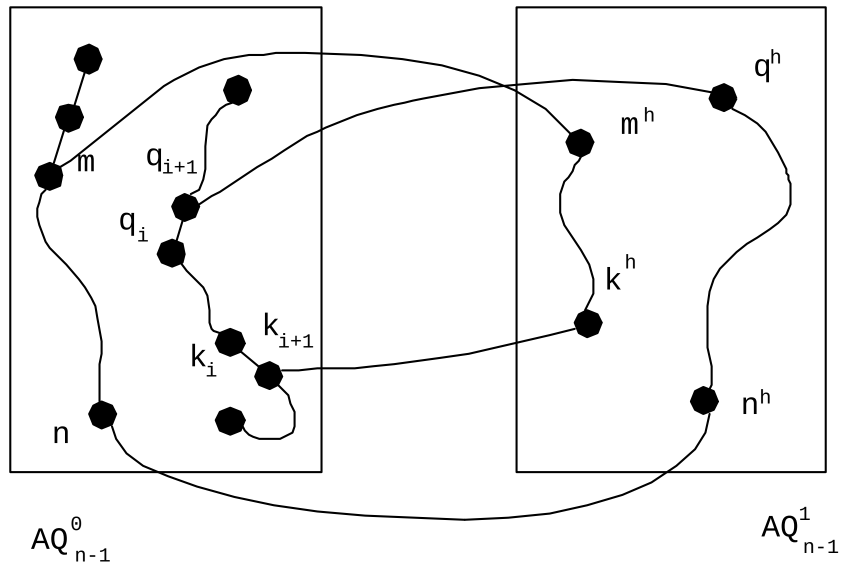

Figure 4.3: The cycles of lengths from $|V(AQ^1_{n-1})| - f^1_v + 2 + l(\mathbf{P_1})$ to $|V(AQ_n)| - f_v - 2$.

path Q_1 in $AQ^1_{n-1} - F(AQ^1_{n-1})$ joining $\mathbf{u}^h_i$ and $\mathbf{v}^h_i$. Thus, $\langle \mathbf{u}_i, \mathbf{u}^h_i, Q_1, \mathbf{v}^h_i, \mathbf{v}_i, \mathbf{P}_i, \mathbf{u}_i \rangle$ forms a hamiltonian cycle in $AQ_n - F$. By Property 1, we can find out the necessary set of subpaths $\mathbf{P'_i} = \langle \mathbf{a}, \mathbf{P^{"}_i}, \mathbf{b} \rangle$ of $\mathbf{P_i}$. We claim that edges $(\mathbf{a}, \mathbf{a}^h)$ and $(\mathbf{b}, \mathbf{b}^h)$ are F-fault free, too. Since AQ_{n-1} is $(2n-6)$-fault hamiltonian connected for $n \geq 5$, there is a hamiltonian paths $\langle \mathbf{a}^h, \mathbf{P_2}, \mathbf{b}^h \rangle$ of $AQ^1_{n-1} - F(AQ^1_{n-1})$. Then the cycles $\langle \mathbf{a}, \mathbf{a}^h, \mathbf{P_2}, \mathbf{b}^h, \mathbf{b}, \mathbf{P^{"}_1}, \mathbf{a} \rangle$ are cycles of length from $|V(AQ^1_{n-1})| - f^1_v + 2$ to $|V(AQ_n)| - f_v - 1$ of $AQ_n - F$. This completes the proof of Subcase 3.2.

Subcase 3.3: $|F(AQ^0_{n-1})| \leq 2n-5$. Thus, $|F(AQ_n) - F(AQ^0_{n-1})|$ is $2n-3-|F(AQ^0_{n-1})|$. Since AQ_{n-1} is $(2n-5)$-fault hamiltonian, there is a hamiltonian cycle C in $AQ^0_{n-1} - F(AQ^0_{n-1})$. We want to select a path P_1 which has lengths from 1 to $|V(AQ^0_{n-1})| - f^0_v - 1$

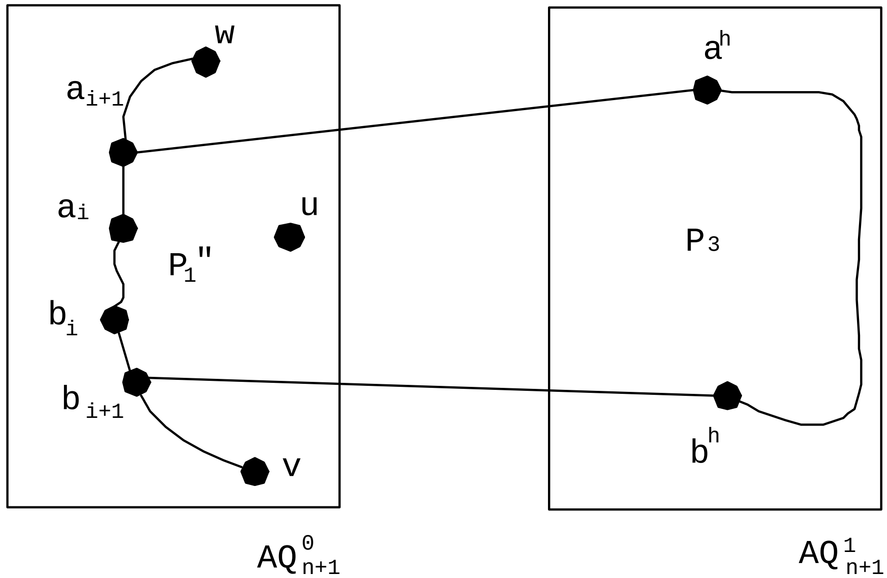

Figure 4.4: The cycles of lengths from $|V(AQ^1_{n-1})| - f^1_v + 2$ to $|V(AQ_n)| - f_v - 1$.

in AQ^0_{n-1}. We wish that there exists the set of subpath $\mathbf{P_1} = \langle \mathbf{u}, \mathbf{P'_1}, \mathbf{v} \rangle$ in C such that both $(\mathbf{u}, \mathbf{u}^h)$ and $(\mathbf{v}, \mathbf{v}^h)$ are fault free. We can find out $\lfloor (2^{n-1} - f^0_v)/2 \rfloor$ pairs of vertices to construct the subpaths of lengths from 1 to $|V(AQ^0_{n-1})| - f^0_v - 1$ in this hamiltonian cycle. Because $|F(AQ_n) - F(AQ^0_{n-1})|$ is at most $2n-3-|F(AQ^0_{n-1})|$, we can find out that $\lfloor (2^{n-1} - f^0_v)/2 \rfloor > 2n-3-|F(AQ^0_{n-1})|$ for $n \geq 5$. We decide that there exist two vertices $\{\mathbf{u}, \mathbf{v}\}$ which construct necessary subpaths of lengths from 1 to $|V(AQ^0_{n-1})| - f^0_v - 1$ in C such that both $(\mathbf{u}, \mathbf{u}^h)$ and $(\mathbf{v}, \mathbf{v}^h)$ are fault free. Since AQ_{n-1} is $(2n-6)$-fault hamiltonian connected, there is a hamiltonian path $\langle \mathbf{u}^h, \mathbf{P_2}, \mathbf{v}^h \rangle$ of $AQ^1_{n-1} - F(AQ^1_{n-1})$. Then $\langle \mathbf{u}, \mathbf{u}^h, \mathbf{P_2}, \mathbf{v}^h, \mathbf{v}, \mathbf{P'_1}, \mathbf{u} \rangle$ can construct subpaths of lengths from $|V(AQ^1_{n-1})| - f^1_v + 2$ to $|V(AQ_n)| - f_v$. This completes the proof of Subcase 3.3(See Fig.4.5).

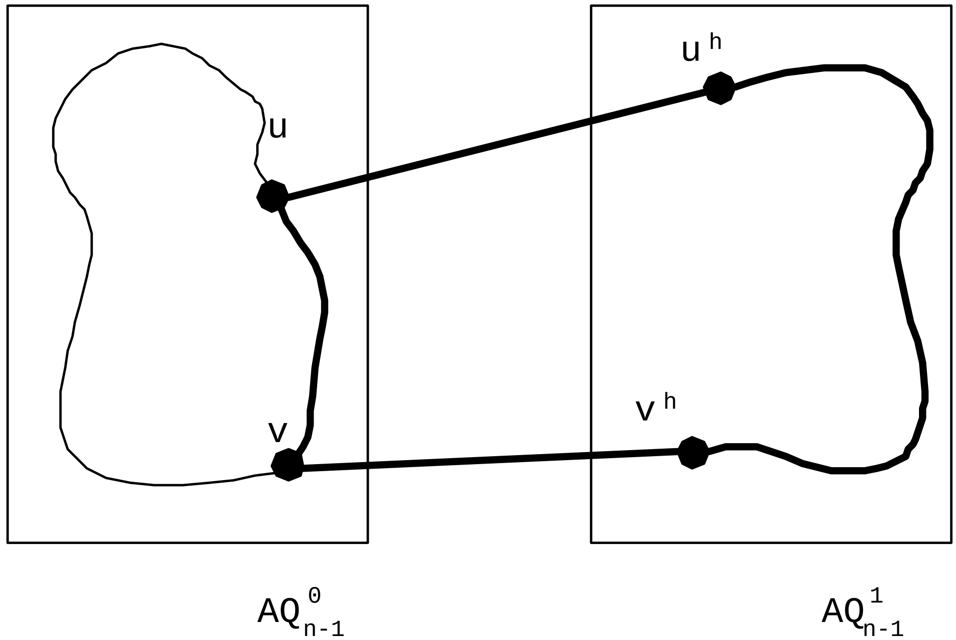

Figure 4.5: The example of Subcase 3.3.

This completes the proof of the theorem.

□

Corollary 1 *By Theorem 1 and our computer programs, AQ_n is $(2n-3)$-fault pancyclic for $n \geq 4$.*

Chapter 5

Conclusion

There are a lot of good properties in hamiltonian cubes. Various hamiltonian properties of hypercube-based graphs have been proposed in literature. In addition, fault tolerance is also considered. The augmented cubes is introduced as an alternative to hypercubes. In this thesis, we discuss the fault-tolerant pancyclicity of the augmented cubes. We proved that AQ_4 is 5-fault pancyclic for induction base by computer programs, then by induction, we proved that AQ_n is $(2n-3)$-fault pancyclic for $n \geq 4$.

Bibliography

[1] S. Abraham and K. Padmanabhan, "The twisted cube topology for multiprocessors: A study in network asymmetry", *Journal of Parallel and Distributed Computing,* vol. 13, pp. 104-110, 1991.

[2] D. Barth, and A. Raspaud, "Two disjoint Hamiltionian cycles in the butterfly graph", *Information Processing Letters,* vol. 51, pp.175-179, 1994.

[3] J.C. Bermond, O. Favaron, and M. Maheo, "Hamiltonian decomposition of caley graphs of degree 4", *Journal of Combinatorial Theory,* vol. B 46, pp. 142-153, 1989.

[4] J.A. Bondy and U.S.R. Murty, *Graph theory with applications*, North Holland, New York, 1980.

[5] L.C. Chiang, "Fault hamiltonicity of augmented cubes" A Thesis submitted to N.C.T.U in Taiwan for the Degree of Master, 2003.

[6] S.A. Choudum and V.Sunitha, "Augmented cubes", *Networks*, vol. 40(2), pp. 71-84, 2002.

[7] P. Cull and S.M. Larson, "The Möbius cubes", *IEEE Transactions on Computers,* vol. 44, no. 5, pp. 647-659, 1995.

[8] K. Efe, "A variation on the hypercube with lower diameter", *IEEE Transactions on Computers,* vol. 40, no. 11, pp. 312-316, 1991.

[9] K. Efe, "The crossed cube architecture for parallel computing", *IEEE Transations on Parallel and Distributed System,* vol. 3, no. 5, pp. 513-524, 1992.

[10] A. Estafahanian, L. Ni. and B. Sagan, "The twisted n-cube with application to multiprocessing", *IEEE Transactions on Computers,* vol. 40, no. 1, pp.88-93, 1991.

[11] J. Fan, "Hamiltonian-connectivity and cycle-embedding of the Möbius cubes", *Information Processing Letters* vol. 82, pp.113-117, 2002.

[12] P.A, J. Hilbers, M.R.J. Koopman, and J.L.A. van de Snepscheut, "The twisted cube, in parallel architectures and languages europe", *Lecture Notes in Computer Science,* pp. 591-604, 1987.

[13] S.Y. Hsieh, G.H. Chen, and C.W. Ho, "Fault-tolerant ring embedding in faulty arrangement graph", *IEEE Tranactions on Parallel Distributed Systems,* vol. 10, pp. 223-237, 1999.

[14] W.T. Huang, J.M. Tan, C.N. Hung, and L.H. Hsu, "Fault-tolerant hamiltonicity of twisted cubes", *Jounal of Parallel and Distributed Computing,* vol. 62, pp. 591-604, 2002.

[15] W.T. Huang, Y.C. Chuang, Jimmy J.M. Tan, and L.H. Hsu, "Fault-free hamiltonian cycle in faulty Möbius cubes", *Journal of Computing and Systems,* vol. 4, pp. 106-114, 2000.

[16] S.C. Hwang and Y.C. G.H. Chen, "Cycles in butterfly graphs", *Network,* vol.35(2) pp.161-171, 2000.

[17] H.C. Keh and J.C. Lin, "On fault-tolerant embedding of Hamiltonian cycles, linear arrays and rings in a Flexible Hypercube", *Parallel Computing,* vol. 26, pp. 769-781, 2000.

[18] S. Latifi, S.Q. Zheng, and N. Bagherzadeh, "Optimal ring embedding in hypercubes with faulty link", *Proceedings of the IEEE Symposium on Fault-Tolerant Computing,* vol. 42, pp. 178-184, 1992.

[19] F.T. Leighton, "Introduction to parallel algorithms and architectures: arrays, trees, hypercubes", San Mateo, Chalif., Morgan Kaufmann, 1992.

[20] Y.R. Leu and S.Y. Kuo, "Distributed fault-tolerant ring embedding reconfiguration in hypercubes", *IEEE Transactions on Computers*, vol. 48, no. 1, pp. 81-88, 1999.

[21] R.S. Lo and G.H. Chen, "Embedding hamiltonian paths in faulty arrangement graphs with the backtracking method", *IEEE Tranactions on Parallel and Distributed Systems,* vol. 12, No. 2, pp. 209-221, 2001.

[22] R.S. Lo and G.H. Chen, "Embedding longest fault-free paths in arrangement graph with faulty vertices", *Networks,* vol. 37(2), pp. 84-93, 2001.

[23] G.M. Megson, X. Liu, and X. Yang, "Fault-tolerant ring embedding in a honeycom torus with node failures", *Parallel Processing Letters,* vol. 9, No. 4, pp. 551-561, 1997.

[24] R.A. Rowley and B. Bose, "Fault-tolerant ring embedding in de-Bruijn networks", *Proceedings of the IEEE Symposium on Fault-Tolerant Computing,* vol 42, pp. 178-184, 1992.

[25] Y. Saad and M.H. Schultz, "Topological properties of hypercubes", *IEEE Transactions on Computers*, vol. 34, pp. 867-872, 1988.

[26] C.H. Tsai, T. Liang, and L.H. Hsu, "Fault-free ring embedding in faulty wrapped butterfly graphs", *Proceedings of the ICS2000 Workshop on Algorithms and Theory of Computation,* pp. 70-73, 2000.

[27] C.H. Tsai, Jimmy J.M. Tan, Y.C. Chuang, and L.H. Hsu, "Fault-free cycles and links in faulty recursive circulant graphs", *Proceedings of the ICS2000 Workshop on Algorithms and Theory of Computation*, pp. 74-77, 2000.

[28] M.C. Yang "Fault-tolerant cycle-embedding of crossed cubes" *ELSEVIER*, Information Processing Letters 88 pp. 149-154, 2003.