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碩 士 論 文

在 PMC 模式下對強診斷系統之研究

Strongly t-Diagnosable System Under the PMC Model

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摘要

科技技術的迅速發展，使得一個系統中的處理機數目越來越多。為了維持系統的可靠度，當系統中有壞掉的處理機時，我們希望能將這些處理機找出來，所以診斷能力扮演著一個相當重要的角色。令 G_1 和 G_2 為兩個 t-診斷系統且有相同的點數。在 G_1 和 G_2 之間做一完全配對，形成一配對構成網路 $G = G_1 \oplus_M G_2$。在本篇論文中，我們證明了 G 在 PMC 模式下不僅是(t+1)-診斷系統並且也是強(t+1)-診斷系統。所以我們可以知道任何一個 n 維度的超方體系列在 PMC 模式都為強 n-診斷系統， $n \geq 4$。

關鍵字：t-診斷能力，PMC 模式，超方體，強 t-診斷能力

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Abstract

The rapid development in digital technology has resulted in developing systems including a very large number of processors. In order to maintain the reliability of a multiprocessors system, the faulty processors in the system have to be replaced by fault-free processors, hence the diagnosability has played an important role. Let G_1 and G_2 be two t-diagnosable systems with the same number of vertices. A family of interconnection network, called the Matching Composition Network (MCN), which can be constructed from G_1 and G_2, by adding a perfect matching M between the vertices of G_1 and G_2. We use the notation $G = G_1 \oplus_M G_2$ to denote a MCN, which has vertex set $V(G) = V(G_1) \cup V(G_2)$ and edge set $E(G) = E(G_1) \cup E(G_2) \cup M$. In this thesis, we prove that the MCN G is not only (t+1)-diagnosable but also strongly (t+1)-diagnosable under the PMC model. According to the result, we can know that the cube family with n-dimensional are all strongly n-diagnosable for $n \geq 4$.

Keywords：t-diagnosable，PMC Model，hypercube，strongly t-diagnosable

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Chapter 1

Introduction

The rapid development in digital technology has resulted in developing systems including a very large number of processors. The processors work on a problem simultaneously at very high speeds. Thus, it is inevitable that the processors in the system become faulty. In order to maintain the reliability of a multiprocessors system, the faulty processors in the system have to be replaced by fault-free processors. Before being replaced, the faulty processor in the multiprocessors system must be diagnosed. The process of identifying these faulty processors is called the *fault diagnosis*. The maximum number of faulty processors that the system can guarantee to identify is called the *diagnosability*.

For convenience, the architecture of a multiprocessor system is usually represented as a graph. The vertices and edges in a graph correspond to the processors and communication links in a multiprocessor system, respectively. For the graph definition we follow [2]. Let $G = (V, E)$ represents a graph, where V represents the *vertex set* of G and E the *edge set* of G. The degree of vertex v in a graph G, written as $d_G(v)$ or $deg(v)$, is the number of edges incident to v. The maximum degree is denoted by $\Delta(G)$, the minimum degree is $\delta(G)$, and G is regular if $\Delta(G) = \delta(G)$. It is k-regular if the common degree is k. The neighborhood of v, written $N_G(v)$ or $N(v)$, is the set of vertices adjacent to v. The connectivity $\kappa(G)$ of a graph $G(V, E)$ is the minimum number of vertices whose removal results in a disconnected or a trivial graph. A graph G is k-connected if its connectivity is at least k.

In recent years, researchers have considered a large number of strategies for self-diagnosis in multiprocessor systems [11], [10], [12], [9], [4]. Much of the work is based on the PMC model proposed by Prepaarata et al. [21]. In this thesis, we use the widely-adopted PMC model as fault diagnosis model, and present a new concept that is called the *strongly t-diagnosable*.

Firstly, we introduce the hypercube [22]. The hypercube is a famous interconnection network. The n-dimensional hypercube is denoted by Q_n, is an undirected graph consisting of 2^n vertices and $n2^{n-1}$ edges. we usually use n-bit binary strings to represent the vertices of the hypercube. Using notation $\{0,1\}^n$ to denote the set $\{u_{n-1}u_{n-2}\ldots u_0 \mid u_i \in \{0,1\}\ for\ 0 \leq i \leq n-1\}$ and $h(u,v)$ to denote the number of different bits between two given vertices u and v in $\{0,1\}^n$. $h(u,v)$ is called the *Hamming distance* of u and v. The following definition 1 is more formally for hypercube.

Definition 1 *An n-dimensional hypercube* $Q_n = (V, E)$, *where*

1. $\mid V \mid = 2^n$

2. $E = \{(u, v) \mid u, v \in V \text{ and } h(u, v) = 1\}$

Let $e = (u, v)$ is an edge in Q_n. The edge e is called dimension d if u and v differ in bit position d. Thus, each vertex connects to n neighbors. For example, vertex 0000 in Q_4 connects to 0001, 0010, 0100 and 1000. Figure 1.1 shows the Q_0, Q_1, Q_2 and Q_3.

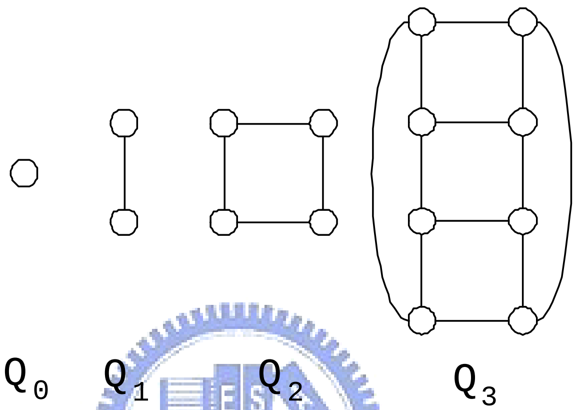

Figure 1.1: The structure of Q_0, Q_1, Q_2 and Q_3.

Let G_1 and G_2 be two t-diagnosable systems with the same number of vertices. A family of interconnection network, called the *Matching Composition Network* (MCN)[15], which can be constructed from G_1 and G_2, by adding a perfect matching M between the vertices of G_1 and G_2. We use the notation $G = G_1 \bigoplus_M G_2$ to denote a MCN, which has vertex set $V(G) = V(G_1) \bigcup V(G_2)$ and edge set $E(G) = E(G_1) \bigcup E(G_2) \bigcup M$. Figure 1.2 shows the MCN $G = G_1 \bigoplus_M G_2$. In this thesis, we prove that the MCN G is not only $(t+1)$-diagnosable but also strongly $(t+1)$-diagnosable under the PMC model. According to the result, we can know that the cube family with n-dimensional are all strongly n-diagnosable for $n \geq 4$. The MCN includes many famous interconnection network, such as the Hypercube Q_n [22], the Crossed cube CQ_n [6], the Twisted cube TQ_n [13] and the Möbius cube MQ_n [3].

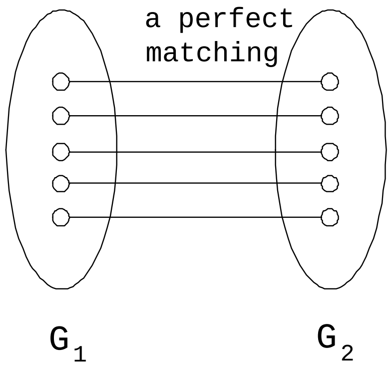

Figure 1.2: Graph $G = G_1 \bigoplus_M G_2$.

The rest of this thesis is organized as follow: In chapter 2, we describe backgrounds and definitions for diagnosable system and some preliminaries. In chapter 3, The strongly t-diagnosable system is formally defined. Besides, we will prove that the cube family with n-dimensional are all strongly n-diagnosable for $n \geq 4$. Finally, we discuss some problems in chapter4.

Chapter 2

The PMC Model and Some Preliminaries

Definition 2 *The components of a graph G are its maximal connected subgraph. A component is trivial if it has no edges; otherwise it is nontrivial.*

Let $G = (V, E)$. For a set $F \subset V$, the notation $G - F$ represents the graph obtained by removing the vertices in F from G and deleting those edges with at least one end vertex in F simultaneously. If $G - F$ is disconnected, then F is called a *vertex cut* or a *separating set*. Let G_1, G_2 be two subgraph of G, if there are ambiguities, we shall write the vertex set of G_1 as V_{G_1} or $V(G_1)$. The neighborhood set of the vertex set V_{G_1} is defined as $N(V_{G_1}) = \{y \in V(G) \mid \text{there exists a vertex } x \in V_{G_1} \text{ such that } (x, y) \in E(G)\} - V_{G_1}$. The restricted neighborhood set of V_{G_1} in G_2, is defined as $N(V_{G_1}, G_2) = \{y \in V(G_2) \mid \text{there exists a vertex } x \in V_{G_1} \text{ such that } (x, y) \in E(G)\} - V_{G_1}$. For $v \in V$, let $\Gamma(v) = \{v_i \mid (v, v_i) \in E\}$ and $\Gamma(X) = \{\bigcup_{v \in X} \Gamma(v) - X\}, X \subset V$. The number of edge directed toward vertex v in G is denoted by $d_{in}(v)$. We use $\mid X \mid$ to denote the cardinality of set X.

The PMC model is presented by Preparata, Metze and Chien. In this model, a system is decomposed into n units $u_1, u_2, \ldots, u_n$. Each unit u is test a subset of system that is connection with u.

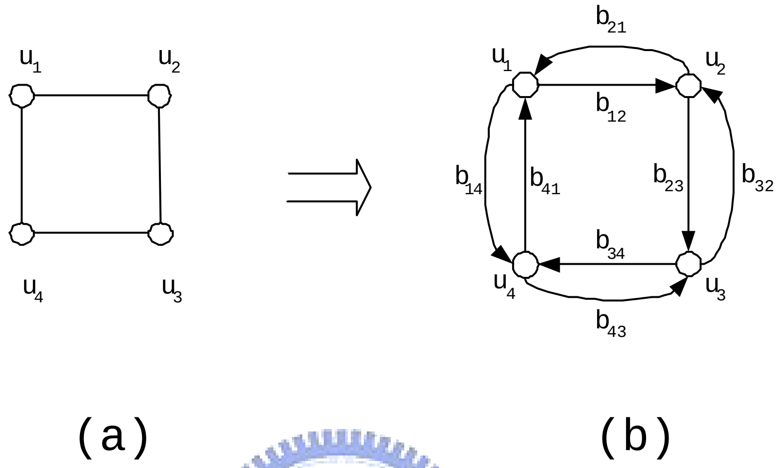

Figure 2.1: (a) A system with four units. (b) The testing graph of (a).

In Figure 2.1(a), each unit u_i of the system will be a vertex of the graph. The Figure 2.1(b) is the testing graph of Figure 2.1(a). A testing link b_{ij} is presented that vertex u_i evaluates vertex u_j. In this situation, u_i is called the tester and u_j is called the tested vertex. The weight associated with b_{ij} will be 0, 1 or x. We noted the weight of b_{ij} is $\omega(b_{ij})$. $\omega(b_{ij})$ is zero if under the hypothesis that u_i is fault-free, u_j is also fault-free; $\omega(b_{ij})$ is one if under the same hypothesis that u_i is fault-free, u_j is faulty; $\omega(b_{ij})$ is x if that u_i is faulty. i.e. x can be 0 or 1. The PMC model assumes that a fault-free should always give correct test-result, whereas the test-result given by a faulty node is unreliable.

Definition 3 *A syndrome σ of the system is represented by the set of test outcomes*

$\omega(b_{ij})$.

Example 1 *Let us consider a system with four units u_1, u_2, u_3 and u_4. The testing link is b_{12}, b_{14}, b_{21}, b_{23}, b_{32}, b_{34}, b_{43} and b_{41} as shown in Figure 2.2.*

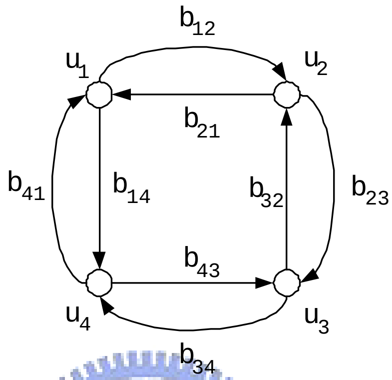

Figure 2.2: A testing graph with four nodes

The syndromes of the system will be represented as the 8-bits vector.

$$\langle \omega(b_{12}), \omega(b_{14}), \omega(b_{21}), \omega(b_{23}), \omega(b_{32}), \omega(b_{34}), \omega(b_{43}), \omega(b_{41}) \rangle$$

Assume exactly two of the units, say u_1 and u_4 are faulty. Then

$$\omega(b_{23}) = \omega(b_{32}) = 0$$

$$\omega(b_{21}) = \omega(b_{34}) = 1$$

i.e. u_2 and u_3 correctly identifies u_1 and u_4 as the faulty, respectively.

$$\omega(b_{12}) = \omega(b_{14}) = \omega(b_{43}) = \omega(b_{41}) = x \text{ i.e. } 0 \text{ or } 1$$

Since u_1 and u_4 are faulty, may or may not diagnose u_2 and u_3 properly. Thus the syndrome for exactly one of the four units being faulty can only be of the form

$$\langle x, x, 1, 0, 0, 1, x, x \rangle$$

In other words, there are sixteen syndromes can be produced by the testing graph of Figure 2.2 under the PMC model.

Definition 4 *Let $G = (V, E)$ is a testing graph, and $S \subset V$. We use the symbol σ_s to represent the set of all syndromes which could be produced if S is the set of faulty vertices.*

Definition 5 *Given a multiprocessor system and one syndrome σ. If we can indicate an only vertex set S such that $\sigma \in \sigma_s$. Then the system is called diagnosable. In other words, a system $G = (V, E)$ is not diagnosable if and only if exist two distinct sets of vertex S_1 and S_2 such that $\sigma_{S_1} \cap \sigma_{S_2} \neq \emptyset$.*

Definition 6 *Let G=(V,E) is a testing graph. Two distinct sets of vertex $S_1, S_2 \subset V$ are said to be indistinguishable if and only if $\sigma_{S_1} \cap \sigma_{S_2} \neq \phi$; otherwise, S_1, S_2 are said distinguishable. Besides, we say (S_1, S_2) is an indistinguishable-pair if $\sigma_{S_1} \cap \sigma_{S_2} \neq \phi$, else (S_1, S_2) is a distinguishable-pair.*

We know that for any two distinct sets of vertex $S_1, S_2 \subset V$ are distinguishable iff they have no same syndrome(s). By the method of diagnosing a system, for any two distinct sets of vertex $S_1, S_2 \subset V$, $\sigma_{S_1} \cap \sigma_{S_2} = \emptyset$ if and only if there exists at least one edge connecting the two disjoint vertex sets, $V - (S_1 \bigcup S_2)$ and $(S_1 - S_2) \bigcup (S_2 - S_1)$. Let $X = V - (S_1 \bigcup S_2)$ and the symmetric difference $S_1 \Delta S_2 = (S_1 - S_2) \bigcup (S_2 - S_1)$. We state the method as follows:

Lemma 1 *Let G=(V,E) is a testing graph. For any two distinct sets of vertex $S_1, S_2 \subset V$, (S_1, S_2) is a distinguishable-pair if and only if $\exists a \in X$ and $\exists b \in S_1 \Delta S_2$ such that $(a, b) \in E$ (see Figure 2.3)*

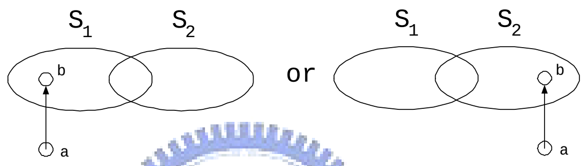

Figure 2.3: Illustrations of a distinguishable pair (S_1, S_2)

Inversely, the two kinds of situation are not exist if and only if (S_1, S_2) is indistinguishable-pair. The definition of t-diagnosable system and related concepts are listed as follows:

Definition 7 *Given a system G=(V,E). If any two distinct sets of vertex $S_1, S_2 \subset V$ are distinguishable, then the system is diagnosable.*

Now, we have a problem. How many faulty vertices can causing that the indistinguishable situation in always. The maximum number is noted by t.

Definition 8 *[21] A system of n units is t-diagnosable if all faulty units can be identified without replacement provided that the number of faults present does not exceed t.*

By the above definition, we obtain the following lemma.

Lemma 2 *A system is t-diagnosable if and only if for each distinct pair of sets* $S_1, S_2 \subset V$ *such that* $| S_1 | \leq t$ *and* $| S_2 | \leq t$*, then* S_1 *and* S_2 *are distinguishable.*

An equivalent way of stating the above lemma is the following:

Lemma 3 *A system is t-diagnosable if and only if for each indistinguishable pair* $S_1, S_2 \subset V$*,* $| S_1 | > t$ *or* $| S_2 | > t$*.*

The following two lemmas are presented by Hakimi et al. [10], and Preparata et al. [21], respectively.

Lemma 4 *[21] Let G=(V,E) be the graph representation of a system. Two necessary conditions for G to be t-diagnosable is:*

1. $| V | = n \geq 2t + 1$*, and*

2. *each processor in G is tested by at least t other processors.*

Lemma 5 *[10] Let G=(V,E) be the graph representation of a system. Two sufficient conditions for G to be t-diagnosable is:*

1. $| V | = n \geq 2t + 1$*, and*

2. $\kappa(G) \geq t$

where $\kappa(G)$ is the connectivity of the graph G.

Hakimi and Amin presented a necessary and sufficient condition for a system to be t-diagnosable as follows:

Lemma 6 *Let G=(V,E) be the graph representation of a system with $\mid V \mid = n$. Then G is t-diagnosable if and only if*

1. $n \geq 2t+1$
2. $d_{in}(v) \geq t$, $\forall v \in V$
3. *for each integer p with $0 \leq p \leq t-1$, and each $X \subset V$ with $\mid X \mid$ = n - 2t + p, $\mid \Gamma(X) \mid > p$.*

In this paper, we will focus on undirected graph without loop, and we assume that each vertex tests the other whenever there is an edge between them. We first propose a new necessary and sufficient condition to determine whether a system is t-diagnosable. This is useful for our discussion later.

Theorem 1 *Let G=(V,E) be the graph representation of a system. We say that G is t-diagnosable if and only if for each vertex set $P \subset V$ with $\mid P \mid = p$, $0 \leq p \leq t-1$, each component C of $G-P$ satisfies $\mid V_C \mid \geq 2(t-p)+1$.*

Proof.

To prove the necessity, assume that the graph G is t-diagnosable. If the necessary condition is not true. Then there exists a set of vertex $P \subset V$ with $| P |= p, 0 \leq p \leq t-1$, such that one of the components $G - P$ has strictly less than $2(t-p)+1$ vertices. Let C be such a component with $| V_C | \leq 2(t-p)$. We can easily partition V_C into two disjoint subsets S_1 and S_2 with $| S_1 | \leq t-p$ and $| S_2 | \leq t-p$. Since there hasn't one vertex $w \in V - \{S_1 \cup S_2\}$, such that $\exists x_1 \in S_1, (w, x_1) \in E$ or $\exists x_2 \in S_2, (w, x_2) \in E$. Hence by lemma 1, $(S_1 \bigcup P, S_2 \bigcup P)$ is indistinguishable-pair. But $| S_1 \bigcup P | \leq (t-p)+p = t$ and $| S_2 \bigcup P | \leq (t-p)+p = t$. This contradicts with the assumption that G is t-diagnosable.

On the other hand, suppose that each vertex set $P \subset V$ with $| P |= p, 0 \leq p \leq t-1$, each component C of $G - P$ satisfies $| V_C | \geq 2(t-p)+1$. We take any two distinct sets of vertex S_1 and S_2, with $| S_1 | \leq t$ and $| S_2 | \leq t$. Let $P = S_1 \cap S_2$, and $0 \leq | P | \leq t-1$. Since $| S_1 | \leq t$ and $| S_2 | \leq t$. Then $| (S_1 - S_2) \bigcup (S_2 - S_1) | \leq 2(t-p)$. The number of $S_1 \Delta S_2$ can't be formed the total number of any component when we delete P from G. At least exist one vertex $w \in V - \{S_1 \bigcup S_2\}$ such that $\exists x_1 \in S_1 - P$, $(w, x_1) \in E$ or $\exists x_2 \in S_2 - P$, $(w, x_2) \in E$. Hence by lemma 1, G is t-diagnosable. This completes the proof of the theorem. □

Lemma 7 *$Q_n = (V, E)$ is n-diagnosable under the PMC model, where $n \geq 3$.*

Proof. Let $P \subset V$, and $| P |= p$, where $0 \leq p \leq n-1$. We can obtain the graph $G' = Q_n - P = (V', E')$ by deleting the vertices in P from Q_n, where $| V' |=| V | -p = 2^n - p$. Since the connectivity of Q_n is n that is presented by Saad and Schultz[22]. Hence we can know that G' is connected, and $2^n - p \geq 2(n-p)+1$. By theorem1, Q_n is n-diagnosable when $n \geq 3$. □

The following example indicated that the Q_n is not n-diagnosable when $n = 1$ or $n = 2$.

Example 2 *for $n = 1$, Q_1 as shown in figure2.4. Let $S_1 = \{v_1\}$ and $S_2 = \{v_2\}$. By lemma 1, S_1 and S_2 are indistinguishable. Hence Q_1 is not 1-diagnosable.*

for $n = 2$, Q_2 as shown in figure2.4. Let $S_1 = \{v_1, v_3\}$ and $S_2 = \{v_2, v_4\}$. By lemma 1, S_1 and S_2 are indistinguishable. Hence Q_2 is not 2-diagnosable.

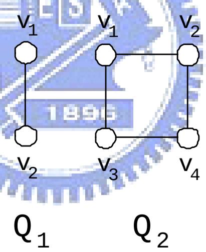

Figure 2.4: Q_1 and Q_2

Theorem 2 *Let $G_1 = (V_1, E_2)$ and $G_2 = (V_2, E_2)$ be two t-diagnosable systems with the same number of vertices, where $t \geq 2$. Then MCN $G = G_1 \bigoplus_M G_2 = (V, E)$ is (t+1)-diagnosable, where $V = V_1 \bigcup V_2$, and $E = E_1 \bigcup E_2 \bigcup M$.*

Proof. Let $S \subset V$, and $\mid S \mid = p$, $0 \leq p \leq t$. We hope to prove that the each component C of $G - S$ with $\mid V_C \mid \geq 2((t+1)-p)+1$. Let $S = S_1 \bigcup S_2$, and $S_1 \subset V_1, S_2 \subset V_2$ with

$| S_1 |= p_1, | S_2 |= p_2$. Then $p = p_1 + p_2$. We consider two cases: (1) $S_1 = \emptyset$ or $S_2 = \emptyset$, and (2) $S_1 \neq \emptyset$ and $S_2 \neq \emptyset$.

Case 1: $S_1 = \emptyset$ **or** $S_2 = \emptyset$

Without loss of generality, assume $S_1 = \emptyset$ and $S_2 = S$. Then $p_1 = 0$ and $p_2 = p$. We know that each vertex of V_2 has an adjacent neighbor in V_1, so, $G - S$ is connected. The only component C of $G - S$ is $G - S$ itself. Hence $| V_C |=| V | - | S |=| V_1 | + | V_2 | -p$. Since G_1 and G_2 are t-diagnosable. By lemma 4, $| V_1 |\geq 2t + 1$ and $| V_2 |\geq 2t + 1$. Then $| V_C |\geq 2(2t+1)-p \geq 2((t+1)-p)+1$, for $t \geq 2$. By theorem 1, G is $(t+1)$-diagnosable.

Case 2: $S_1 \neq \emptyset$ **and** $S_2 \neq \emptyset$

$S_1 \neq \emptyset$ and $S_2 \neq \emptyset$, it implies $1 \leq p_1 \leq t - 1$ and $1 \leq p_2 \leq t - 1$. Firstly, we consider any component C_1 of $G_1 - S_1$ with $| V_{C_1} |\geq 2(t - p_1) + 1$. We know that each vertex of C_1 has an adjacent neighbor w in V_2. If the vertex w is belong to S_2. We will delete it. Then at least $2(2(t - p_1) + 1) - p_2$ vertices in any component of $G - S$; likewise $2(2(t-p_1)+1)-p_2 \geq 2((t+1)-p)+1$, for $t \geq 2$. By theorem 1, G is $(t+1)$-diagnosable. Secondly, We consider any component C_2 of $G_2 - S_2$ with $| V_{C_2} |\geq 2(t - p_2) + 1$. Then each vertex of C_2 has an adjacent neighbor w in V_1. If the vertex w is belong to S_1. We will delete it. Then at least $2(2(t - p_2) + 1) - p_1$ vertices in any component of $G - S$; likewise $2(2(t - p_2) + 1) - p_1 \geq 2((t + 1) - p) + 1$, for $t \geq 2$. By theorem 1, G is $(t + 1)$-diagnosable.This completes the proof of the theorem. □

Chapter 3

Strongly t-diagnosable

In previous chapter, we explained that the Hypercube Q_n is n-diagnosable. In fact, the Crossed cube CQ_n, the Möbius cube MQ_n, and the Twisted cube TQ_n are all known as n-diagnosable but not $(n+1)$-diagnosable. In this chapter, we will presented the concept of the strongly t-diagnosable system. Besides, we will also prove that the cube family with n-dimensional are all strongly n-diagnosable for $n \geq 4$. Firstly, we take Q_3 as an example to explained that why Q_3 is not 4-diagnosable. The structure of Q_3 as shown in Figure 3.1.

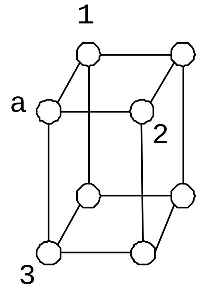

Figure 3.1: The structure of Q_3.

Let $S_1 = \{1, 2, 3\}$ and $S_2 = \{a, 1, 2, 3\}$, with $| S_1 | \leq 4$ and $| S_2 | \leq 4$. By lemma 1, S_1 and S_2 are indistinguishable-pair. Hence Q_3 is not 4-diagnosable. For each of these cubes with n-dimension, we observe that for any two distinct sets of vertex S_1 and S_2, $| S_1 | \leq n+1$ and $| S_2 | \leq n+1$, they are indistinguishable-pair implies that there exists some vertex v such that $N(v) \subset S_1 \bigcap S_2$. That is $N(v) \subset S_1$ and $N(v) \subset S_2$. We continue taking Q_4 as an example, for each vertex $v \in V(Q_4)$ and each vertex set $P \subset V(Q_4)$, $0 \leq | P | \leq 4$. $Q_4 - P$ is connected if $N(v) \nsubseteq P$. It's mean, the only component of $Q_4 - P$ is itself. Let $S_1, S_2 \subset V(Q_4)$ be two distinct sets of vertex with $| S_1 | \leq 5$, $| S_2 | \leq 5$, and $P = S_1 \bigcap S_2$. We can get that the inequality $| V(Q_4) - P | = 2^4 - | P | \geq | S_1 - P | + | S_2 - P | + 1$. Then there is at least one edge connecting $S_1 \Delta S_2$ and $V(Q_4 - (S_1 \bigcup S_2))$. By lemma 1, S_1 and S_2 are distinguishable-pair if for each $v \in V(Q_4)$, $N(v) \nsubseteq P$. Inversely, S_1 and S_2 are indistinguishable-pair, then there exists some vertex $v \in V(Q_4)$ such that $N(v) \subseteq S_1$ and $N(v) \subseteq S_2$. We observed the phenomenon and give a formally definition as follows:

Definition 9 *A system $G = (V, E)$ is strongly t-diagnosable if the following two conditions hold:*

1. *G is t-diagnosable, and*

2. *for any two distinct subsets $S_1, S_2 \subset V$ with $| S_1 | \leq t+1$ and $| S_2 | \leq t+1$,*

   either (a) (S_1, S_2) is a distinguishable pair;

or (b) (S_1, S_2) *is an indistinguishable pair and there exists a vertex* $v \in V$

such that $N(v) \subseteq F_1$ *and* $N(v) \subseteq F_2$.

By lemma 5 and definition 9, we propose a sufficient condition for a system G to be strongly t-diagnosable as follows:

Proposition 1 *Let* $G = (V, E)$ *be the graph presentation of a system with* $\mid V \mid= n$ *is strongly t-diagnosable if the following three conditions hold:*

1. $n \geq 2(t+1)+1$,
2. $\kappa(G) \geq t$, *and*
3. *for any vertex set* $P \subset V$ *with* $\mid P \mid= t$, $G - P$ *is disconnected implies that there exists a vertex* $v \in V$ *such that* $N(v) \subset P$.

Proof. To prove the proposition, we claim that condition (1) and (2) of definition 9 hold. Since condition (1) and (2), by lemma 5, G is t-diagnosable. For condition (2) of definition 9. Let S_1 and S_2 be an indistinguishable-pair, and $P = S_1 \bigcap S_2$, where $S_1 \neq S_2$, $\mid S_1 \mid \leq t+1$ and $\mid S_2 \mid \leq t+1$, then $0 \leq \mid P \mid \leq t$. If $G-P$ is connected, then there exists an edge between $S_1 \triangle S_2$ and $V - (S_1 \bigcup S_2)$. By lemma 1, S_1 and S_2 are distinguishable-pair. This is a contradiction. Hence $G - P$ is disconnected. By condition (2), $\kappa(G) \geq t$ and $0 \leq \mid P \mid \leq t$. Therefore $\mid P \mid= t$. By condition (3), there exists a vertex $v \in V$ such that $N(v) \subset P$. That is, $N(v) \subset S_1$ and $N(v) \subset S_2$. Hence condition(2) of definition 9 holds. This completes the proof of the proposition. □

Now, we propose a necessary and sufficient condition for a system to be strongly t-diagnosable as follows:

Lemma 8 *Let $G = (V, E)$ be the graph presentation of a system with $| V |= n$ is strongly t-diagnosable if and only if*

1. *$n \geq 2(t + 1) + 1$,*

2. *$\delta(G) \geq t$, and*

3. *for any indistinguishable-pair $S_1, S_2 \subset V$, $S_1 \neq S_2$, with $| S_1 |\leq t+1$ and $| S_2 |\leq t+1$ it implies that there exists a vertex $v \in V$ such that $N(v) \subset S_1$ and $N(v) \subset S_2$.*

Proof.

To prove the necessity of condition (1), we show that the assumption $n \leq 2(t+1)$ leads to a contradiction. Assume $n \leq 2(t+1)$. We can partition V into two disjoint vertex sets V_1 and V_2 with $| V_1 |\leq t + 1$ and $| V_2 |\leq t + 1$, where $V = V_1 \bigcup V_2$ and $V_1 \bigcap V_2 = \emptyset$. By lemma 1, V_1 and V_2 are indistinguishable-pair. Since G is strongly t-diagnosable. Then there exists some vertex $v \in V$ such that $N(v) \subset V_1$ and $N(v) \subset V_2$. Hence $V_1 \bigcap V_2 \neq \emptyset$. That contradicts the assumption $V_1 \bigcap V_2 = \emptyset$.

To prove the necessity of condition (2), since G is strongly t-diagnosable. By definition 9, G is also t-diagnosable. By condition(2) of lemma 4, $N(v) \geq t$ for each vertex $v \in V$. Hence $\delta(G) \geq t$.

To prove the necessity of condition (3), that is the same as condition (2) of definition 9. This completes the proof for the necessity.

On the other hand, since condition (3) of this lemma and condition(2) of definition 9 are stated the same. We need only to prove that G is t-diagnosable. Assume not, then there exists an indistinguishable-pair $S_1, S_2 \subset V$, $S_1 \neq S_2$, with $\mid S_1 \mid \leq t$ and $\mid S_2 \mid \leq t$. By condition (3), there exists a vertex $v \in V$ such that $N(v) \subset S_1$ and $N(v) \subset S_2$. By condition (2), we know that $\mid N(v) \mid \geq t$. But, $\mid S_1 \mid \leq t$ and $\mid S_2 \mid \leq t$. Hence $S_1 = S_2 = N(v)$. This contradicts the $S_1 \neq S_2$. We complete the proof of this lemma. □

The lemma given above is a method for checking whether a system is strongly t-diagnosable. Now, we propose another necessary and sufficient condition. Let $G = (V, E)$ be a strongly t-diagnosable system. If G is $(t+1)$-diagnosable. By Theorem 1, for each vertex set $P \subset V$, $\mid P \mid = p$ where $0 \leq p \leq t$, each component C of $G - P$ satisfies $\mid V_C \mid \geq 2((t+1) - p) + 1$. Otherwise, G is t-diagnosable but not $(t+1)$-diagnosable. Then there exists an indistinguishable-pair(S_1, S_2), $\mid S_1 \mid \leq t+1$ and $\mid S_2 \mid \leq t+1$. By condition(2) of Definition 9, there exists a vertex $v \in V$ such that $N(v) \subset S_1$ and $N(v) \subset S_2$, where $v \notin S_1 \bigcup S_2$. Hence {v} is a trivial component of $G - (S_1 \bigcap S_2)$. Let $P = S_1 \bigcap S_2$ and $\mid P \mid = t$, $G - P$ has a trivial component.

Theorem 3 *Let $G = (V, E)$ be the graph presentation of a system with $\mid V \mid = n$ is strongly t-diagnosable if and only if each vertex set $P \subset V$ with $\mid P \mid = p$, $0 \leq p \leq t$, the following two conditions are satisfied.*

1. *for* $0 \leq p \leq t-1$*, each component* C *of* $G-P$ *satisfies* $\mid V_C \mid \geq 2((t+1)-p)+1$*, and*

2. *for* $p = t$*, either each component* C *of* $G-P$ *satisfies* $\mid V_C \mid \geq 3$ *or else* $G-P$ *contains at least a trivial component.*

Proof.

To prove the necessity of condition (1), assume that there exists a vertex set $P \subset V$ with $\mid P \mid = p$, $0 \leq p \leq t-1$, such that $G-P$ has a component C with $\mid V_C \mid \leq 2((t+1)-p)$. We can partition V_C into two disjoint vertex sets A_1 and A_2, $A_1 \bigcup A_2 = V_C$ and $A_1 \bigcap A_2 = \emptyset$, with $\mid A_1 \mid \leq (t+1)-p$ and $\mid A_2 \mid \leq (t+1)-p$. Let $S_1 = A_1 \cup P$ and $S_2 = A_2 \cup P$. Then $\mid S_1 \mid \leq t+1$ and $\mid S_2 \mid \leq t+1$. By lemma 1, S_1 and S_2 are indistinguishable-pair. Since G is strongly t-diagnosable. By Definition 9, there exists a vertex $v \in V$ such that $N(v) \subset S_1$ and $N(v) \subset S_2$. By lemma 4 $\mid N(v) \mid \geq t$. However, $N(v) \subset S_1 \bigcap S_2 = P$ and $0 \leq p \leq t-1$, this is a contradiction.

To prove the necessity of condition (2), assume that there exists a component C of $G-P$ with $\mid V_C \mid \leq 2$. Then we have to prove that there is a trivial component in $G-P$. If $\mid V_C \mid = 1$, we are done. Assume that $\mid V_C \mid = 2$, we say $V_C = \{v_1, v_2\}$. Let $S_1 = \{v_1\} \bigcup P$ and $S_2 = \{v_2\} \bigcup P$. Then $\mid S_1 \mid = \mid S_2 \mid = t+1$, and are indistinguishable-pair. Since G is strongly t-diagnosable. By definition 9, there exists a vertex $v \in V$ such that $N(v) \subset S_1$ and $N(v) \subset S_2$. We have $P = S_1 \bigcap S_2$ and $P = N(v)$. Therefore, {v} is a trivial component in $G-P$.

On the other hand, we claim that G is strongly t-diagnosable. We have to prove that G satisfies conditions (1) and (2) of definition 9. For condition (1) of definition 9, let P be a vertex set with $\mid P \mid= p$, $0 \leq p \leq t-1$. By condition (1), each component C of $G - P$ satisfies $\mid V_C \mid \geq 2((t+1)-p)+1 \geq 2(t-p)+1$. By Theorem 1, G is t-diagnosable.

For condition (2) of definition 9, let S_1 and S_2 be an indistinguishable-pair, $S_1 \neq S_2$, with $\mid S_1 \mid \leq t+1$ and $\mid S_2 \mid \leq t+1$. Let $P = S_1 \bigcap S_2$, $\mid P \mid= p$, then $0 \leq p \leq t$. Since S_1 and S_2 are indistinguishable-pair. Hence there is no edge between $X = V - (S_1 \bigcup S_2)$ and $S_1 \Delta S_2$. Therefore, $S_1 \Delta S_2$ is disconnected from the other component in $G - P$. We observed that $\mid S_1 \Delta S_2 \mid \leq 2((t+1)-p)$. By condition(1), p is not in the range from 0 to $t-1$. So $p = t$ and $\mid S_1 \Delta S_2 \mid \leq 2((t+1)-p) = 2((t+1)-t) = 2$. By condition(2), $G - P$ must have a trivial component {v}. Hence $N(v) \subset P$. Since G is t-diagnosable, by condition(2) of lemma 4, $N(v) \geq t$. So $P = N(v)$. Then $N(v) \subset S_1$ and $N(v) \subset S_2$. Therefore, G is strongly t-diagnosable.

Thus we complete the proof of this theorem. □

Theorem 4 *Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two t-diagnosable systems with the same number of vertices, where $t \geq 2$. Then MCN $G = G_1 \bigoplus_M G_2 = (V, E)$ is strongly (t+1)-diagnosable, where $V = V_1 \bigcup V_2$ and $E = E_1 \bigcup E_2 \bigcup M$.*

Proof. Let $G = G_1 \bigoplus_M G_2 = (V, E)$ and $P \subset V$ with $\mid P \mid= p$, $0 \leq p \leq t+1$. Let $S_1 = P \bigcap V_1$ and $S_2 = P \bigcap V_2$ with $\mid S_1 \mid= p_1$ and $\mid S_2 \mid= p_2$. We will use Theorem 3 to prove this theorem. In the following proof, we consider two cases: (1) $S_1 = \emptyset$ or $S_2 = \emptyset$, and (2) $S_1 \neq \emptyset$ and $S_2 \neq \emptyset$. We shall prove that: (i) $| V_C | \geq 2((t+2)-p)+1$ for any component C of $G-P$ as $0 \leq p \leq t$, and (ii) for $p = t+1$, either any component C of $G-P$ satisfies $| V_C | \geq 3$ or else $G-P$ contains at least one trivial component.

Case 1: $S_1 = \emptyset$ or $S_2 = \emptyset$

Without loss of generality, assume $S_1 = \emptyset$ and $S_2 = P$. Since each vertex of V_2 has an adjacent neighbor in V_1. Hence $G-P$ is connected. So, $| V_C | = | V-P | = | V_1 | + | V_2 | -p$. Since G_1 and G_2 are t-diagnosable. By lemma 4, $| V_1 | \geq 2t+1$ and $| V_2 | \geq 2t+1$. Hence $| V_C | \geq 2(2t+1)-p \geq 2((t+2)-p)+1$ for $t \geq 2$ and $0 \leq p \leq t+1$.

Case 2: $S_1 \neq \emptyset$ and $S_2 \neq \emptyset$

Since $S_1 \neq \emptyset$ and $S_2 \neq \emptyset$. We know that $1 \leq p_1 \leq t$ and $1 \leq p_2 \leq t$. In this case, we divide the case into two subcases: (2.a) $1 \leq p_1 \leq t-1$ and $1 \leq p_2 \leq t-1$, and (2.b) either $p_1 = t$ or $p_2 = t$. In fact, for subcase (2.b),either $p_1 = t$ and $p_2 = 1$, or, $p_2 = t$ and $p_1 = 1$.

Subcase 2.a: $1 \leq p_1 \leq t-1$ and $1 \leq p_2 \leq t-1$

Let C_1 be the component of $G_1 - S_1$. Since G_1 is t-diagnosable. By theorem 1, $| V_{C_1} | \geq 2(t-p_1)+1$. We claim that there is at least one vertex in V_{C_1} which is connected to $V_2 - S_2$. That is $2(t-p_1)+1 \geq p_2+1$. Since $p = p_1 + p_2$, then $2(t-p_1)+1 = 2(t-p+p_2)+1 = 2p_2+2(t-p)+1$. Suppose $p \leq t$, then $| V_{C_1} | \geq 2(t-p_1)+1 \geq p_2+1$. Otherwise, $p = t+1$. With $p_1 \leq t-1$, then $p_2 \geq 2$ and $2p_2+2(t-p)+1 \geq p_2+1$. Hence $| V_{C_1} | \geq 2(t-p_1)+1 \geq p_2+1$. The claim is completed. Let C_2 be the component of $G_2 - S_2$. Since G_2 is t-diagnosable. By theorem 1, $| V_{C_2} |\geq 2(t-p_2)+1$. We let C be the component of $G - P$ such that $V_{C_1} \bigcup V_{C_2} \subset V_C$. Then $| V_C |\geq| V_{C_1} | + | V_{C_2} |\geq (2(t-p_1)+1)+(2(t-p_2)+1) = 2(2t-p+1) \geq 2((t+2)-p)+1$ for $t \geq 2$ and $0 \leq p \leq t+1$.

Subcase 2.b: either $p_1 = t$ and $p_2 = 1$, or, $p_1 = 1$ and $p_2 = t$

Without loss of generality, assume $p_2 = t$ and $p_1 = 1$. Let C_1 be a component of $G_1 - S_1$. G_1 is t-diagnosable. By theorem 1, $| V_{C_1} |\geq 2(t-p_1)+1 = 2(t-1)+1$. Since $p = t+1$ and $t \geq 2$, $| V_{C_1} |\geq 2(t-1)+1 \geq 3$. Hence the number of vertex in each component of $G - P$ has at least $2((t+2)-p)+1$ vertices.

Let C_2 be a component of $G_2 - S_2$. If V_{C_2} has some adjacent neighbor $v_1 \in V_1$ and vertex v_1 belongs to some component C_1 of $G_1 - S_1$, then the component C containing the two vertex sets V_{C_1} and V_{C_2} has at least four vertices. Otherwise, $N(V_{C_2}, V_1) \subset S_1$. With $| S_1 |= p_1 - 1$, $| N(V_{C_2}, V_1) |= 1$. That is, $| V_{C_2} |= 1$. Hence, C_2 is a trivial component.

Thus we complete the proof of this theorem. □

We will give an example to explain why the above result is not true when $t = 1$. As shown in figure3.2(a), let G_1 and G_2 are 1-diagnosable systems with vertex sets $\{v_1, v_2, v_3, v_4, v_5\}$ and $\{u_1, u_2, u_3, u_4, u_5\}$, respectively. Let $G = G_1 \bigoplus_M G_2$ be a Matching Composition Network constructed by adding a perfect matching between G_1 and G_2. By lemma 5, G is 2-diagnosable. See Figure3.2(b), let $F_1 = \{v_1, v_2, u_2\}$ and $F_2 = \{u_1, u_2, v_2\}$. By lemma 1, F_1 and F_2 are indistinguishable-pair but there doesn't exist any vertex $v \in V_1 \bigcup V_2$ such that $N(v) \subset F_1$ and $N(v) \subset F_2$. Hence G is not strongly 2-diagnosable.

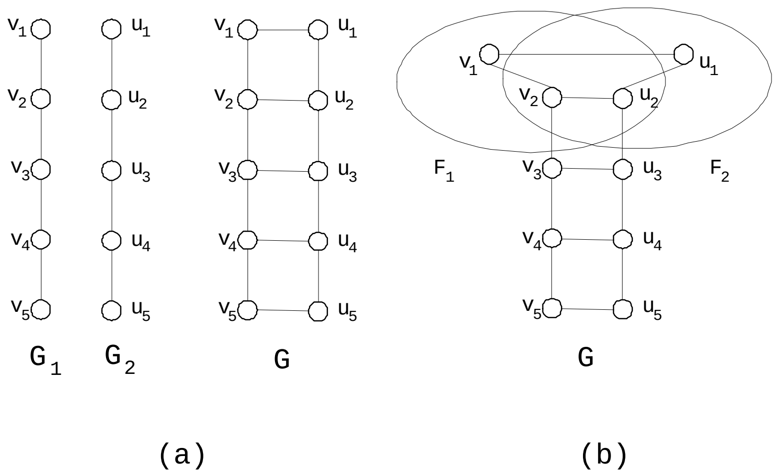

Figure 3.2: An example of non-strongly (t+1)-diagnosable as t=1.

According to theorem 4, we know that all systems of the cube family are strongly $(t+1)$-diagnosable because their subcubes are t-diagnosable for $t \geq 3$. The Hypercube Q_n, the Crossed cube CQ_n, the Twisted cube TQ_n, and the Möbius cube MQ_n are famous parts in the cube family. Hence we hold the following corollary.

Corollary 1 *The Hypercube Q_n, the Crossed cube CQ_n, the Twisted cube TQ_n, and the Möbius cube MQ_n are all strongly n-diagnosable for $n \geq 4$.*

For $n = 2$, these cubes are all a cycle of length four. They are 1-diagnosable but not 2-diagnosable. For $n = 3$, these cubes are all 3-connected, by lemma 5, they are 3-diagnosable. We now show some examples which are not strongly t-diagnosable. Let us take the 3-dimensional Hypercube Q_3 as an example. it is 3-diagnosable but not strongly 3-diagnosable from the fact that $\mid V(Q_3) \mid = 8 \leq 2(t+1)+1$ as $t = 3$, which contradicts the condition (1) of lemma 8. Let Cl_n be a cycle of length n, $n \geq 7$. It is not difficult to verify that Cl_n is 2-diagnosable, but it is not strongly 2-diagnosable. Another nontrivial example is shown in figure 3.3. This graph is 2-connected and 3-regular. We can use theorem 1 to verify that it is 3-diagnosable. As shown in figure 3.3, $S_1 = \{1, 2, 5, 6\}$ and $S_2 = \{3, 4, 5, 6\}$. (S_1, S_2) is an indistinguishable-pair, but there does not exist any vertex $v \in V(G)$ such that $N(v) \subset S_1$ and $N(v) \subset S_2$. By definition 9, this graph is not strongly 3-diagnosable.

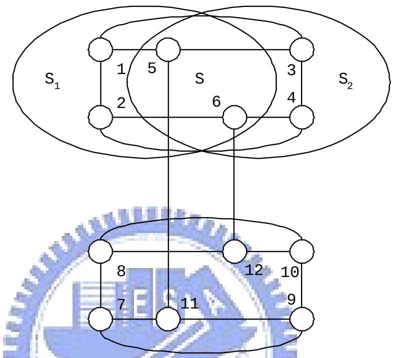

Figure 3.3: An example of non-strongly 3-diagnosable system.

Chapter 4

Conclusion

The fault diagnosis is a popular issue for interconnection network. There are some open problems which we can discuss. In recent years, researchers have considered a large number of strategies for self-diagnosis in interconnection network. The PMC model, first proposed by Preparata et al.[21], is used widely for fault diagnosis of interconnection network. In this thesis, we study the properties of fault diagnosis of the cube family. We also propose the concepts of strongly t-diagnosable systems under the PMC model. We show that the cube family with n-dimensional are all strongly n-diagnosable, where $n \geq 4$. The cube family include Hypercube, Crossed cube, Twisted cube and Möbius cube et al. There are many models which we can research except the PMC model. The Comparison model [16] that is another well-known fault diagnosis model. Hence, it is also interesting to investigate the issues of strongly t-diagnosable of a system under the Comparison model. Besides, there are two attractive problems which are worth researching. Firstly, we want to know whether the recursive interconnection networks are all strongly t-diagnosable system. Secondly, what is the diagnosability of interconnection network when we allow one good neighbor condition?

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