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# Chaos synchronization and chaos control of quantum-CNN chaotic system by variable structure control and impulse control

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### a b s t r a c t

In this paper, we derive some less stringent conditions for the exponential and asymptotic stability of impulsive control systems with impulses at fixed times. These conditions are then used to design an impulsive control law for the Quantum Cellular Neural Network chaotic system, which drives the chaotic state to zero equilibrium and synchronizes two chaotic systems. An active sliding mode control method is synchronizing two chaotic systems and controlling chaotic state to periodic motion state. And a sufficient condition is drawn for the robust stability of the error dynamics, and is applied to guiding the design of the controllers. Finally, numerical results are used to show the robustness and effectiveness of the proposed control strategy.

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## **1. Introduction**

The chaotic system exhibits unpredictable and irregular dynamics and it has been found in many engineering systems. Interestingly, chaotic models can describe complex dynamics with only few nonlinear equations without any random external inputs, and small differences in the initial state can lead to extraordinary differences in the system state. Since Ott, Grebogi, and Yorke proposed the OGY method [\[1\]](#page-8-0), a method of controlling chaos, 'controlling of chaos' is receiving increasing attention within the area of non-linear dynamics [\[2,](#page-8-1)[3\]](#page-8-2). It has many applications in various systems, while it is unfavorable in many other cases due to its irregular behavior. Therefore, both chaos utilization and elimination are important depending on the specific applications. Chaos control is an effective method for both chaos utilization and elimination and has been thoroughly studied in various fields of science.

Since the seminal work of Pecora and Carroll [\[4\]](#page-8-3), it has been an interesting and potential topic in recent years in the study of chaos synchronization in physics, mathematics and engineering community, etc., and various effective techniques and methods [\[5–11\]](#page-8-4) have been proposed over the last decade to achieve chaos synchronization. Thus, as a key technique of secret communication, chaos synchronization has become a very important goal and a subject of much on-going research.

Basically, the chaos synchronization problem means making two systems oscillate in a synchronized manner. Given a chaotic system, which is considered as the master system, and another identical system, which is considered as the slave system, the dynamical behaviors of these two systems may be identical after a transient when the slave system is driven by a control input [\[12–17\]](#page-8-5). In the aforementioned publications regarding chaos synchronization, it is often assumed that all the parameters of the chaotic systems are invariant and determinate, i.e., the chaotic models are well known.

The variable structure control technique is a discontinuous control strategy that involves, first, selecting a switching surface for the desired dynamics and, secondly, designing a discontinuous control law such that the system trajectory first reaches the surface and then stays in it forever [\[18–28\]](#page-8-6). Chaos control and the chaos synchronization chaotic system by

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impulsive control is ideal for designing digital control schemes where the control laws are generated by digital devices that are discrete in time.

As numerical examples, the recently developed Quantum Cellular Neural Network (Quantum-CNN) chaotic oscillator is used. Quantum-CNN oscillator equations are derived from a Schrödinger equation, taking into account quantum dots cellular automata structures to which, in the last decade, a wide interest has been devoted, with particular attention towards quantum computing [\[29\]](#page-8-7).

This paper is organized as follows. In Section [2,](#page-1-0) chaos synchronization and chaos control of Quantum-CNN oscillators chaotic system by variable structure control is considered. In Section [3,](#page-5-0) chaos synchronization and chaos control of Quantum-CNN oscillators chaotic system by impulse control is considered. Finally, some concluding remarks are given in Section [4.](#page-8-8)

## <span id="page-1-0"></span>**2. Chaos synchronization and chaos control of Quantum-CNN oscillators chaotic system by variable structure control**

## *2.1. Chaos synchronization*

There are two identical nonlinear dynamical systems, and the master system controls the slave system. The master system is given here

<span id="page-1-2"></span>
$$
\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{f}(\mathbf{x}) \tag{1}
$$

where  $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T \in R^n$  denotes the state vector, **A** is a  $n \times n$  coefficient matrix, and **f** is a nonlinear vector function. The slave system is given here

<span id="page-1-1"></span>
$$
\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{f}(\mathbf{y}) + \mathbf{u}(\mathbf{t})
$$
 (2)

where  $\mathbf{y}=(y_1,y_2,\ldots,y_n)^T\in R^n$  denotes a state vector and  $\mathbf{u}(\mathbf{t})=(u_1(t),u_2(t),\ldots,u_n(t))^T\in R^n$  is a control input vector. Our goal is to design a controller **u**(**t**) so that the state vector of the slave system [\(2\)](#page-1-1) asymptotically approaches the state

vector of the master system [\(1\)](#page-1-2) and finally the synchronization will be accomplished in the sense that the limit of the error vector **e**(**t**) =  $(e_1, e_2, \ldots, e_n)^\text{T}$  approaches zero:

$$
\lim_{t \to \infty} \mathbf{e} = 0 \tag{3}
$$

where

<span id="page-1-4"></span><span id="page-1-3"></span>
$$
\mathbf{e} = \mathbf{y} - \mathbf{x}.\tag{4}
$$

From Eq. [\(4\)](#page-1-3) we have

$$
\dot{\mathbf{e}} = \dot{\mathbf{y}} - \dot{\mathbf{x}} = \mathbf{A}\mathbf{e} + \mathbf{F}(\mathbf{x}, \mathbf{y}) + \mathbf{u}(\mathbf{t})
$$
\n(5)

where  $\mathbf{F}(\mathbf{x}, \mathbf{y}) = \mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x})$ .

According to the theory of active control, we can use the control input vector-function **u**(**t**) to eliminate all items that cannot be shown in the form of the error vector **e**. In this way, the vector-function **u**(**t**) can be determined.

$$
\mathbf{u}(\mathbf{t}) = \mathbf{H}(\mathbf{t}) - \mathbf{F}(\mathbf{x}, \mathbf{y}).
$$
\n<sup>(6)</sup>

And Eq. [\(5\)](#page-1-4) is rewritten as

<span id="page-1-5"></span>
$$
\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{H}(\mathbf{t}).\tag{7}
$$

Eq. [\(7\)](#page-1-5) describes the error dynamics and can be considered in terms of a control problem where the system to be controlled is a linear system with a control input **H**(**t**) as the functions of the error vector **e**. As long as these feedbacks stabilize the system, the error vector **e** converges to zero as  $t \to \infty$ . This implies that the master system [\(1\)](#page-1-2) and the slave system [\(2\)](#page-1-1) are synchronized finally. There are many possible choices for the control **H**(**t**). It is well known that the most distinguished feature of the sliding mode control technique is that, when in sliding mode, the system is robust to parametric uncertainty and external disturbances. Without loss of generality, we choose the sliding mode control law, as follows:

$$
H(t) = Kw(t) \tag{8}
$$

where  $\mathbf{K} = (k_1, k_2, \ldots, k_n)^T$  is a constant gain vector,  $w(t) \in R$  is the control input and satisfies

$$
w(t) \begin{cases} w^+(t) & s(e) > 0 \\ w^-(t) & s(e) < 0 \end{cases}
$$
 (9)

and  $s = s(e)$  is a switching surface that prescribes the desired dynamics. This results in

<span id="page-1-7"></span><span id="page-1-6"></span>
$$
\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{K}w(\mathbf{t}).\tag{10}
$$

In what follows, the appropriate sliding mode controller will be designed in terms of the sliding mode control theory. (1) Sliding surface design: Generally speaking, the sliding surface **s**(**e**) can be defined as

$$
s(\mathbf{e}) = \mathbf{C}\mathbf{e} = \text{constant} \tag{11}
$$

where  $\mathbf{C} = (c_1, c_2, \dots, c_n)$  is a constant vector.

$$
s(\mathbf{e}) = 0 \tag{12a}
$$

and

$$
\dot{s}(\mathbf{e}) = 0 \tag{12b}
$$

Substituting Eqs. [\(10\)](#page-1-6) and [\(11\)](#page-1-7) into Eq. [\(12b\),](#page-2-0) we can obtain

<span id="page-2-3"></span><span id="page-2-2"></span><span id="page-2-1"></span><span id="page-2-0"></span>∂*s*(*e*)

$$
\dot{s}(\mathbf{e}) = \frac{\partial s(\mathbf{e})}{\partial e} \dot{e} = C \left[ A e + K w(t) \right] = 0. \tag{13}
$$

Solving Eq. [\(13\)](#page-2-1) for  $w(t)$  yields the equivalent control  $w_{eq}(t)$ 

$$
w_{eq}(t) = -(CK)^{-1}CAe \tag{14}
$$

where the existence of  $(CK)^{-1}$  is a necessary condition.

Putting Eq. [\(14\)](#page-2-2) into Eq. [\(10\),](#page-1-6) the state equation in the sliding mode is given as follows:

$$
\dot{\mathbf{e}} = \left[ \mathbf{I} - \mathbf{K} (\mathbf{C} \mathbf{K})^{-1} \mathbf{C} \right] \mathbf{A} \mathbf{e}.
$$
 (15)

Suppose that the vector **K** is selected such that (**A**, **K**) is controllable.

(2) Design of the sliding mode controller: Assume that the constant plus proportional rate reaching law is applied. The reaching law can be chosen such that

$$
\dot{s} = -q \cdot \text{sgn}(s) - rs \tag{16}
$$

where sgn( $\cdot$ ) denotes the sign function, and the gains  $q > 0$  and  $r > 0$  are determined such that the sliding condition is satisfied and sliding mode motion will occur.

From Eqs. [\(10\)](#page-1-6) and [\(11\),](#page-1-7) it can be found that

$$
\dot{s}(e) = I[Ae + Kw(t)].
$$
\n(17)

By Eq.  $(16)$ , we have

$$
w_{eq}(t) = -(\mathbf{C}\mathbf{K})^{-1} [\mathbf{C}(r\mathbf{I} + \mathbf{A})\mathbf{e} + q \cdot \text{sgn}(s)]. \tag{18}
$$

(3) Robust stability analysis: In order to check the stability of the above controlled system, we can construct the following Lyapunov function

<span id="page-2-5"></span><span id="page-2-4"></span>
$$
\mathbf{V} = \frac{1}{2}s^2\tag{19}
$$

and then, differentiation of the above expression Eq. [\(19\)](#page-2-4) with respect to time yields

<span id="page-2-6"></span>
$$
\dot{\mathbf{V}} = \dot{s}s = \frac{\partial s}{\partial e}\dot{e}s = C\dot{e}s.
$$
\n(20)

Substituting Eqs. [\(10\)](#page-1-6) and [\(18\)](#page-2-5) into Eq. [\(20\),](#page-2-6) then we can obtain

$$
\dot{\mathbf{V}} = s\mathbf{C}[\mathbf{A}\mathbf{e} + \mathbf{K}w(\mathbf{t})] = s\mathbf{C} \{ \mathbf{A}\mathbf{e} - \mathbf{K}(\mathbf{C}\mathbf{K})^{-1} [\mathbf{C}(r\mathbf{I} + \mathbf{A})\mathbf{e} + q \cdot \text{sgn}(s)] \}
$$
  
=  $-rs^2 - sq \cdot \text{sgn}(s).$  (21)

Since the expression  $-gq \cdot sgn(s)$  is always negative when  $e \neq 0$ , the inequality  $\dot{V} = \dot{s} s < 0$ .

As an example, let us consider Quantum-CNN system. For a two-cell Quantum-CNN, the following differential equations are obtained [\[29\]](#page-8-7):

The master system is described by:

<span id="page-2-7"></span>
$$
\begin{cases}\n\dot{x}_1 = -2a_1\sqrt{1 - x_1^2} \sin x_2 \\
\dot{x}_2 = -\omega_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1 - x_1^2}} \cos x_2 \\
\dot{x}_3 = -2a_2\sqrt{1 - x_3^2} \sin x_4 \\
\dot{x}_4 = -\omega_2(x_3 - x_1) + 2a_2 \frac{x_3}{\sqrt{1 - x_3^2}} \cos x_4\n\end{cases}
$$
\n(22)

where  $x_1$ ,  $x_3$  are polarizations,  $x_2$ ,  $x_4$  are quantum phase displacements,  $a_1$  and  $a_2$  are proportional to the inter-dot energy inside each cell and  $\omega_1$  and  $\omega_2$  are parameters that weigh effects on the cell of the difference of the polarization of neighboring cells, like the cloning templates in traditional CNNs. Let  $a_1 = a_2 = 4.9$ ,  $\omega_1 = 1.13$ ,  $\omega_2 = 0.85$  and the gain vector  $K = [0, 1, 0, 1]^T$ . The initial values of Quantum-CNN systems are taken as  $x_1(0) = 0.8$ ,  $x_2(0) = -0.77$ ,  $x_3(0) = -0.72$ ,  $x_4(0) = 0.97, y_1(0) = -0.98, y_2(0) = 0.87, y_3(0) = 0.92$ , and  $y_4(0) = -0.93$  respectively.

<span id="page-3-0"></span>

**Fig. 1.** Time histories of states, state errors.

The slave system is described by:

$$
\begin{cases}\n\dot{y}_1 = -2a_1\sqrt{1 - y_1^2} \sin y_2 \\
\dot{y}_2 = -\omega_1(y_1 - y_3) + 2a_1 \frac{y_1}{\sqrt{1 - y_1^2}} \cos y_2 \\
\dot{y}_3 = -2a_2\sqrt{1 - y_3^2} \sin y_4 \\
\dot{y}_4 = -\omega_2(y_3 - y_1) + 2a_2 \frac{y_3}{\sqrt{1 - y_3^2}} \cos y_4\n\end{cases}
$$
\n(23a)

<span id="page-4-0"></span>

**Fig. 2.** Phase portrait of Quantum-CNN by variable structure control.



The result is shown in [Fig. 1.](#page-3-0)

#### *2.2. Chaos control*

Assume that the aim is to control system  $\dot{x} = Ax + f(x) + u(t)$  tracking a given desired state vector  $y = [y_1 y_2 \dots y_n]^T$ , where  $\dot{y} = Ay$ . Let  $e = x - y$  be the tracking error vector.

The tracking error dynamics is:

$$
\dot{e} = \dot{x} - \dot{y} = Ae + f(x) - u(t)
$$
\n(24)

where  $f(x)$  is nonlinear item vector. The controller is designed as  $u(t) = H(t) + f(x)$  in which  $H(t) = Kw(t)$ . The newly defined control signal w(*t*) is determined through the sliding mode approach,

$$
w(t) \begin{cases} w^+(t) & s(e) > 0 \\ w^-(t) & s(e) < 0 \end{cases}
$$
 (25)

*s*(*e*) is the switching surface and is considered as

$$
s(e) = Ce.\tag{26}
$$

The reaching law assumed to be  $\dot{s} = -q \cdot \text{sgn}(s) - rs$ . This design results in the following control signal.

$$
w(t) = -(CK)^{-1} [C(rI + A)e + q \cdot sgn(s)].
$$
\n(27)

It can be shown that the closed loop system will be stable for positive *r* and *q* parameters.

<span id="page-5-1"></span>

**Fig. 3.** Time histories of states by impulse control.

Finally, let us consider our dynamic system Quantum-CNN system, Eq. [\(22\).](#page-2-7) The result is shown in [Fig. 2](#page-4-0) for 1-T periodic motion.

#### <span id="page-5-0"></span>**3. Chaos control and chaos synchronization of Quantum-CNN oscillators chaotic system by impulse control**

## *3.1. Chaos control*

A technique for suppressing chaos is to apply a periodic impulse input to the system [\[3\]](#page-8-2). Consider the system of the form [\(22\)](#page-2-7) and assume that the system is controlled by a periodic impulse input

$$
u = \rho \sum_{i=0}^{\infty} \delta(\tau - iT_d) \tag{28}
$$

where  $\rho$  is a constant impulse intensity,  $T_d$  is the periodic between two consecutive impulses, and  $\delta$  is the standard delta function. With different values of  $\rho$  and  $T_d$  the controlled system can be stabilized at different periodic orbits or fix points.

Finally, let us consider our dynamic system Quantum-CNN system with periodic impulse of linear feedback. The equation considered is

$$
\begin{cases}\n\dot{x}_1 = -2a_1\sqrt{1 - x_1^2} \sin x_2 - u_1x_1, \\
\dot{x}_2 = -\omega_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1 - x_1^2}} \cos x_2 - u_2x_2, \\
\dot{x}_3 = -2a_2\sqrt{1 - x_3^2} \sin x_4 - u_3x_3, \\
\dot{x}_4 = -\omega_2(x_3 - x_1) + 2a_2 \frac{x_3}{\sqrt{1 - x_3^2}} \cos x_4 - u_4x_4.\n\end{cases}
$$
\n(29)

Let  $a_1 = a_2 = 2.47$ ,  $\omega_1 = 1$ ,  $\omega_2 = 1$ ,  $\rho_1 = 89$ ,  $\rho_2 = 87$ ,  $\rho_3 = 91$ ,  $\rho_4 = 87$  and  $T_d = 0.01$ . The initial values of Quantum-CNN systems are taken as  $x_1(0) = 0.8$ ,  $x_2(0) = -0.77$ ,  $x_3(0) = -0.72$ , and  $x_4(0) = 0.57$  respectively.

The result is shown in [Fig. 3.](#page-5-1)

#### *3.2. Chaos synchronization*

The synchronization by unidirectional/bi-directional linear coupling with periodic impulse is studied. Two chaos systems using bi-directional/unidirectional linear coupling with periodic impulse can be written as

$$
\dot{x} = Ax + h(x) + u_1(y - x) \tag{30}
$$

and

$$
\dot{y} = Ay + h(y) + u_2(x - y) \tag{31}
$$

where  $x, y \in R^n$  represent the state vectors of the chaotic systems,  $A \in R^{n \times n}$  is a constant matrix,  $h \in R^{n \times n}$  is a continuous nonlinear function of *x*, *y*, *u*<sub>1</sub> and *u*<sub>2</sub> are constant gains of periodic impulse signals which represent the coupled parameters. If  $u_1$  is equal zero then the systems call unidirectional linear coupling synchronization by periodic impulse. If  $u_1$  and  $u_2$  are nonzero, the systems call bi-directional linear coupling synchronization by periodic impulse.

<span id="page-6-0"></span>

**Fig. 4.** Time histories of states, state errors for uni-direction linear couple.

As an example, we study unidirectional/bi-directional linear coupling synchronization by periodic impulse for Quantum-CNN chaotic system. The equations considered are

$$
\begin{cases}\n\dot{x}_1 = -2a_1\sqrt{1 - x_1^2} \sin x_2 + u_{11}(y_1 - x_1) \\
\dot{x}_2 = -\omega_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1 - x_1^2}} \cos x_2 + u_{12}(y_2 - x_2) \\
\dot{x}_3 = -2a_2\sqrt{1 - x_3^2} \sin x_4 + u_{13}(y_3 - x_3) \\
\dot{x}_4 = -\omega_2(x_3 - x_1) + 2a_2 \frac{x_3}{\sqrt{1 - x_3^2}} \cos x_4 + u_{14}(y_4 - x_4)\n\end{cases}
$$
\n(32)

<span id="page-7-0"></span>

**Fig. 5.** Time histories of states, state errors for bi-direction linear couple.

and

$$
\begin{cases}\n\dot{y}_1 = -2a_1\sqrt{1 - y_1^2}\sin y_2 + u_{21}(x_1 - y_1) \\
\dot{y}_2 = -\omega_1(y_1 - y_3) + 2a_1\frac{y_1}{\sqrt{1 - y_1^2}}\cos y_2 + u_{22}(x_2 - y_2) \\
\dot{y}_3 = -2a_2\sqrt{1 - y_3^2}\sin y_4 + u_{23}(x_3 - y_3) \\
\dot{y}_4 = -\omega_2(y_3 - y_1) + 2a_2\frac{y_3}{\sqrt{1 - y_3^2}}\cos y_4 + u_{24}(x_4 - y_4).\n\end{cases}
$$
\n(33)

The initial values of Quantum-CNN systems are taken as  $y_1(0) = -0.2$ ,  $y_2(0) = 0.41$ ,  $y_3(0) = 0.25$ , and  $y_4(0) = -0.81$ respectively. Let unidirectional linear coupling gain parameters  $\rho_{11} = 0$ ,  $\rho_{12} = 0$ ,  $\rho_{13} = 0$ ,  $\rho_{14} = 0$ ,  $\rho_{21} = 64$ ,  $\rho_{22} = 54$ ,  $\rho_{23} = 98$ ,  $\rho_{24} = 62$  and bi-directional linear coupling gain parameters  $\rho_{11} = 32$ ,  $\rho_{12} = 27$ ,  $\rho_{13} = 49$ ,  $\rho_{14} = 31$ ,  $\rho_{21} = 32$ ,  $\rho_{22} = 27, \rho_{23} = 49, \rho_{24} = 31.$ 

The result is shown in [Figs. 4](#page-6-0)[–5](#page-7-0) for unidirectional linear coupling and bi-directional linear coupling, respectively.

#### <span id="page-8-8"></span>**4. Conclusions**

Two chaotic Quantum-CNN systems are synchronized by three methods: unidirectional linear coupling by impulse control, bi-directional linear coupling by impulse control and variable structure control. The chaos controls of a Quantum-CNN system are also studied. The impulse control, and variable structure control are used to suppress chaos to fixed point or regulation motion. Numerical simulations are used to verify the effectiveness of the proposed controllers. These chaos synchronization and control methods can be also used for other chaotic systems.

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