

國立交通大學

統計學研究所

碩士論文

利用 GARCH-NIG Model 對台指選擇權定價

Pricing the TAIEX option with GARCH-NIG Model



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中華民國九十三年六月

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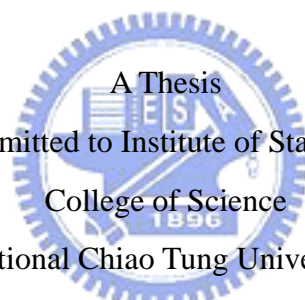
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誌 謝

即將揮別研究所的生涯，在這期間能夠完成我的碩士論文，除了增加對自我能力的肯定之外，也讓更融會貫通將書本上的知識來應用在這篇論文上。在這之中，首要感謝的是我的指導老師李昭勝教授細心、以及專業的指導，讓我在這一路上更有信心來往前邁進，再來是博士班的學長牛維方耐心不厭其煩的指導我，讓論文可以順利完成，在這背後，我的父母給我精神上的支柱也是一大力量。

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葉宇青 謹誌于

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摘要

此篇論文的重點在於對台指的選擇權來做評價，利用 Barndorff-Nielsen 在 1997 年提出的 Normal Inverse Gaussian 的分配再加上變異數服從 GARCH 的過程形成 GARCH-NIG 的模型。

利用最大概似估計法來估計模型中之參數，再利用 Monte Carlo 的方法來預測選擇權價格。接著與 Black-Schole 模型預測出的價格做比較。

Pricing the TAIEX option with GARCH-NIG Model

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ABSTRACT

In this paper , we focus on pricing option of TAIEX using GARCH-NIG model formed by Normal Inverse Gaussian(NIG) distribution proposed by Barndorff-Nielsen in 1997 combined with the variance of NIG distribution following GARCH process.

Using maximum likelihood estimation method to estimate parameters within model . After that, we forecast option price using Monte Carlo method and compare the option price forecasted by Black-Schole model.

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Introduction:

Numerous recent studies on option pricing have acknowledged the fact that volatility changes overtime in time series of asset returns as well as in the empirical variances implied from option prices through the Black & Scholes (1973) model. Many of these studies focused on modeling the asset return dynamics through stochastic volatility (SV) models. Due to analytically intractable likelihood functions and hence the lack of available efficient estimation procedures, SV models were until recently viewed as an unattractive class of stochastic process compared to other time-varying volatility processes, such as ARCH/GARCH models. Moreover, to calculate option prices based on SV models we need, besides parameter estimates, a representation of the unobserved historical volatility, which is again far from being straightforward to obtain. Therefore, while the SV generalization of option pricing has, thanks to advances in econometric estimation techniques, recently been shown to improve over the Black-Scholes model in terms of the explanatory power for asset-return dynamics, its empirical implications on option pricing itself have not yet been adequately tested due to the aforementioned lack of a representation of the unobserved volatility. Can the SV generalization of the option pricing model help resolve the well-known systematic empirical biases associated with the Black-Scholes model, such as the volatility "smile" (e.g. Rubinstein (1985)), asymmetry of such "smile" or "smirk" (e.g. Stein (1989))? How substantial is the gain, if any, from such generalization compared to relatively simpler models? The purpose of this thesis is to answer the above questions by studying the empirical performance of SV models in pricing options on the TAIEX, and investigating the respective effect of stochastic interest rates, stochastic volatility, and asymmetric asset returns on option prices

in a multivariate SV model framework.

The NIG (Normal Inverse Gaussian) distribution is well-known in financial field. So, in this thesis we use NIG model to model returns. Using daily data from Taiwan's market, we would like to show that NIG model is a good model for stock returns compared with other option pricing models. For more details about Normal Inverse Gaussian (NIG), see Barndorff-Nielsen (1997).

The seminal ARCH paper by Engle (1982) triggered one of the most active and fruitful areas of research in econometrics over the past two decades. The success of the ARCH/GARCH class of models at capturing volatility clustering in financial markets is well documented (see, for example, Bollerslev, Chou and Kroner, 1992). Meanwhile, the inability of the ARCH/GARCH models coupled with the auxiliary assumption of conditionally normally distributed errors to fully account for all of the mass in the tails of the distributions of say, daily returns, is also well recognized. Indeed, several alternative error distributions were proposed in the early ARCH literature to better account for the deviation from the normality in the conditional distributions of the returns, including the t-distribution of Bollerslev (1987), the General Error Distribution (GED) of Nelson (1991), and more recently, the Normal Inverse Gaussian (NIG) distribution of Barndorff - Nielsen (1997), Anderson (2001) and Jensen and Lunde (2001). The motivation behind these alternative error distributions has been almost exclusively empirical and pragmatic in nature. In the present thesis, building on the Mixture of Distribution Hypothesis (MDH) (Clark, 1973) along with the recent idea of realized volatility (RV), we provide a sound empirical foundation for the distributional assumptions underlying the GARCH-NIG model.

Be consistent with the absence of arbitrage and a time-changing Brownian motion, the MDH postulates that the distribution of returns is normal, but with a stochastic variance. In the original formulation in Clark (1973), the variance is assumed to be i.i.d. lognormally distributed, resulting in a lognormal-normal mixture distribution for the returns. Importantly, to explicitly account for the volatility clustering effect Taylor(1982, 1986)proposed an extension of the MDH setup by making the latent logarithmic variance to follow a Gaussian autoregression, resulting in the lognormal Stochastic Volatility(SV) model; see also Anderson(1996). Since the joint distribution of the returns in the SV model is not known in closed form, both estimation and inference for these types of models are considerably more complicated than for the ARCH/GARCH class of models.

In contrast to the existing SV literature, which treats the mixing variable as latent, we have shown that by measuring the daily variance by the corresponding realized volatility constructed from the sum of intraday high-frequency returns, the daily return standardized by the realized volatility is approximately normally distributed. Therefore, even though the realized volatilities are subject to measurement error vis-à-vis the true daily latent volatilities, the normality of the standardized returns is consistent with the basic tenets of the MDH and the use of the realized volatility as the underlying mixing variable. Moreover, we find that the distribution of the realized volatility conditional on the past squared daily returns (as well as the unconditional distribution of the realized volatility) is closely approximated by an Inverse Gaussian (IG) distribution. Taken together, these results imply that in practical modeling situations where the high-frequency data are not actually available, the daily returns should be well described by a NIG model.

Section 1 describes the Normal Inverse Gaussian distribution and

GARCH-NIG option pricing model. In section 2, we introduce the method to estimate parameters in GARCH-NIG option pricing model. In section 3, we describe empirical analysis, including market description and data description. Section 4 presents the results and some discussions. Section 5 concludes.



1. The Model

1.1 Normal Inverse Gaussian distribution

The Mixture-of-Distributions Hypothesis (MDH) starts from the premise that the distribution of discretely sampled returns, conditional on some latent information arrival process, is Gaussian. This assumption is justified theoretically if the underlying price process follows continuous sample path diffusion (see, for instance, the discussion in Andersen *et al.*, 2002, and Barndorff- Nielsen and Shephard, 2001). However, the integrated volatility process that serves as the mixture variable in this situation is not directly observable. As noted above, this has spurred numerous empirical investigations into the use of alternative volatility proxies and/or mixture variables. Meanwhile, in the diffusion setting the integrated volatility may, in theory, be estimated arbitrarily well by the summation of finely sampled squared high-frequency returns, or so-called realized volatilities. This suggests the following empirically testable starting point for the MDH,

$$r_t | RV_t \sim N(0, RV_t) \quad , \quad (1)$$

where r_t refers to the discretely sampled one-period returns from time $t-1$ to t , and RV_t denotes the corresponding realized volatility proxy measured over the same time interval.

$$RV_t \sim IG(h, \alpha) \quad , \quad (2)$$

where the density function for the IG distribution may be expressed in standardized form as,

$$IG(RV_t; h, \alpha) = \frac{\left(\frac{1}{\alpha h}\right)^{\frac{1}{2}} (RV_t)^{\frac{-3}{2}}}{(2\pi)^{\frac{1}{2}}} \exp\left\{\alpha - \frac{1}{2}\left(\frac{\alpha h}{RV_t} + \frac{\alpha(RV_t)}{h}\right)\right\}, \quad (3)$$

Now, combining the distributional assumptions in (1) and (2) the implied unconditional distribution for the returns should be Normal Inverse Gaussian (NIG),

$$f(r_t) = \int_0^{\infty} f(r_t | RV_t) f(RV_t) dRV_t = NIG(h, 0, 0, \alpha), \quad (4)$$

with the following closed-form density function,

$$NIG(z; h, 0, 0, \alpha) = \frac{\alpha^{\frac{1}{2}}}{\pi h^{\frac{1}{2}}} \exp(\alpha) q\left(\frac{z}{(h\alpha)^{\frac{1}{2}}}\right)^{-1} K_1\left(\alpha q\left(\frac{z}{(h\alpha)^{\frac{1}{2}}}\right)\right),$$

where $q(x) = \sqrt{1+x^2}$ and $K_1(z)$ denotes the modified Bessel function of third kind and order one. The NIG distribution was first used for modeling speculative returns in Barndorff-Nielsen (1997). It may be viewed as a special case of the Generalized Hyperbolic Distribution in Barndorff-Nielsen (1978).

Although the NIG distribution in equation (3) may adequately capture the fat-tailed unconditional return distributions, it does not account for the well-documented volatility clustering, or ARCH, effects. In order to incorporate conditional heteroscedasticity in the return process within the MDH framework, define the I_{t-1} information set generated by the past daily returns,

$$I_{t-1} = h\{r_{t-1}, r_{t-2}, \dots\},$$

$$f(RV_t | I_{t-1}) = IG(h_t, \alpha),$$

where h_t depends exclusively on the coarser I_{t-1} information set. Specifically,

for the results reported below, the conditional mean in the IG distribution is assumed to follow the recursive GARCH(1,1) like structure,

$$h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 h_{t-1}. \quad (5)$$

The GARCH(1,1)-NIG model now arises naturally by combining equation (1) augmented with the I_{t-1} information set, $r_t | \mathbf{RV}_t, I_{t-1} \sim N(0, \mathbf{RV}_t)$ with the conditional distribution for \mathbf{RV}_t in equations (4) and (5),

$$f(r_t | I_{t-1}) = \int_0^{\infty} f(r_t | \mathbf{RV}_t, I_{t-1}) f(\mathbf{RV}_t | I_{t-1}) d\mathbf{RV}_t = \text{NIG}(h_t, \mathbf{0}, \mathbf{0}, \alpha). \quad (6)$$

1.2 The GARCH-NIG option pricing model

Consider a discrete-time economy and let X_t be the asset price at time t . Its one-period log rate of return is assumed to be conditionally normal inverse Gaussian distributed under probability measure P . That is,

$$\ln \frac{X_t}{X_{t-1}} = r + \lambda \sqrt{h_t} - \frac{1}{2} h_t + \varepsilon_t, \quad (7)$$

where ε_t has mean zero and conditional variance h_t under measure P ; r is the constant one-period risk-free rate of return and λ the constant unit risk premium. We further assume that ε_t follows a GARCH (1,1) process of Bollerslev (1986) under measure P . Formally, $\varepsilon_t | \phi_{t-1} \sim \text{NIG}(\alpha, \mathbf{0}, \mathbf{0}, h_t)$ under measure P ,

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \quad (8)$$

where ϕ_t is the information set of all information up to and including time; $\alpha_1 + \beta_1 < 1$; $\alpha_0 \geq 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$. In other words, the conditional variance is

a linear function of the past squared disturbances and the past conditional variances.

Definition : A pricing measure Q is said to satisfy the locally risk-neutral valuation relationship(LRNVR) if measure is mutually absolute continuous with respect to

measure P , $\ln\left(\frac{X_t}{X_{t-1}} \mid \phi_{t-1}\right)$ follows Normal Inverse Gaussian distribution(under Q),

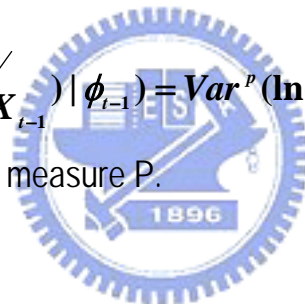
$$E^Q\left(\frac{X_t}{X_{t-1}} \mid \phi_{t-1}\right) = e^r$$

where r stands for interest rate,

and,

$$Var^Q\left(\ln\left(\frac{X_t}{X_{t-1}}\right) \mid \phi_{t-1}\right) = Var^P\left(\ln\left(\frac{X_t}{X_{t-1}}\right) \mid \phi_{t-1}\right)$$

almost surely with respect to measure P .



Theorem: The LRNVR implies that, under pricing measure Q ,

$$\ln\left(\frac{X_t}{X_{t-1}}\right) = r - \frac{1}{2}h_t + \xi_t,$$

where

$$\xi_t \mid \phi_{t-1} \sim NIG(\alpha, \mathbf{0}, \mathbf{0}, h_t)$$

and

$$h_t = \alpha_0 + \alpha_1(\xi_{t-1} - \lambda\sqrt{h_{t-1}})^2 + \beta_1 h_{t-1}.$$

Pricing contingent payoffs requires temporally aggregating one-period asset returns to arrive at a random terminal asset price at some future point in time. The terminal asset price is derived in the following corollary:

Corollary:

$$X_T = X_t \exp \left[(T-t)r - \frac{1}{2} \sum_{s=t+1}^T h_s + \sum_{s=t+1}^T \xi_s \right]$$

From above corollary, we get the following time-t value of the European call option with exercise price k maturing at time T ,

$$C = e^{-(T-t)r} E^Q[\max(X_T - k, 0) | \phi_t].$$



2. Estimation of the model

Estimation of the parameters of the different models is straightforward using the maximum likelihood method. The log likelihood of the sample for the GARCH-NIG models is given by

$$l(h_t, \alpha)$$

$$= \sum_{i=1}^n \left(\frac{1}{2} \ln \alpha - \ln \pi \right) - \sum_{i=1}^n \left(\frac{1}{2} \ln h_t - \alpha \right) - \sum_{i=1}^n \frac{1}{2} \ln \left(1 + \frac{X_i^2}{h_t \alpha} \right) + \sum_{i=1}^n \ln K_1 \left(\alpha \left(1 + \frac{X_i^2}{h_t \alpha} \right)^{\frac{1}{2}} \right)$$

where n is the number of observations and h_t is given by the model chosen.



3. Empirical analysis

The empirical analysis starts with a description of the options data. It proceeds to estimate the GARCH-NIG model with time series data on index returns and with options data.

3.1 Market description

TSEC, Taiwan Stock Exchange Corporation, maintains stock price indices, to allow investors to grab both overall market movement and different industrial sectors' performances conveniently. The indices may be grouped into market value indices and price average indices. The former are similar to the Standard & Poor's Index, weighted by the number of outstanding shares, and the latter are similar to the Dow Jones Industrial Average and the Nikkei Stock Average. The Taiwan Stock Exchange Capitalization Weighted Stock Index ("TAIEX") is the most widely quoted of all TSEC indices. The base year value as of 1966 was set at 100. TAIEX is adjusted in the event of new listing, de-listing and new shares offering to offset the influence on TAIEX owing to non-trading activities. We summarize other details and rule of TAIEX and TXO into the following table.

Table 3.1.1

Item	Description
Underlying Index	Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX)
Ticker Symbol	TXO
Exercise Style	European

Multiplier	NT\$ 50 (per index point)
Expiration Months	Spot month, the next two calendar months followed by two additional months from the March quarterly Cycle (March, June, September, and December)
Strike Price Interval	100 index points in spot month, the next two calendar months 200 index points in the additional two months from the March quarterly Cycle
Strike (Exercise) Price	<p>When listing series of new expiration months, one series with at-the-money strike price is listed based on the previous day's closing price of the underlying index rounded to the nearest multiples of 100.</p> <p>1. For the spot month and the next two calendar months: fifth other series each with in-the-money and out-of-the-money strike prices with price interval of 100 points are listed.</p> <p>2. For the next two quarter-months: Three other series each with in-the-money and out-of-the-money strike prices with price interval of 200 points are listed.</p> <p>Up to the 5th business days before expiration,</p> <p>1. For the spot month, and the next two calendar months: additional series are added when the underlying trades through the fifth highest or</p>

	<p>lowest strike price available, to maintain at least 5 in- and 5 out-of-the-money strike prices</p> <p>2. For the next two quarter-months: additional series are added when the underlying trades through the third highest or lowest strike price available, to maintain at least 3 in- and 3 out-of-the-money strike prices</p>
Premium Quotation	<p>< 10 points: 0.1 point (NT\$5)</p> <p>>=10 points, <50 points: 0.5 point (NT\$ 25)</p> <p>>=50 points, <500 points: 1 point (NT\$ 50)</p> <p>>=500 points, <1,000 points: 5 point (NT\$ 250)</p> <p>>=1,000 points: 10 point (NT\$ 500)</p>
Daily Price Limit	<p>+/- 7% of previous day's closing price of the underlying index</p>
Position Limit	<p>Individuals: 8,000 contracts on either side of the market.</p> <p>Institutional Investors: 16,000 contracts on either side of the market.</p> <p>Institutional investors may apply for an exemption from the above limit on trading accounts for hedging purpose.</p> <p>Exemptions are allowed for Future Proprietary Firms.</p>

Trading Hours	08:45AM - 1:45 PM Taiwan time Monday through Friday of the regular Taiwan Stock Exchange business days
Last Trading Day	The third Wednesday of the delivery month
Expiration Date	The first business day following the last trading day
Final Settlement Price	The final settlement price for each contract is computed from the first fifteen-minute volume-weighted average of each component stock's prices in that index on the final settlement day. For those component stocks that are not traded during the beginning fifteen-minute interval on the final settlement day, their last closing prices would be applied instead
Settlement	Cash settlement. An option that is in-the-money and has not been liquidated or exercised on the expiration day shall, in the absence of contrary instructions delivered to the Exchange by the Clearing Member representing the option buyer, be exercised automatically

3.2 Description of data

We use daily data on TAIEX from January 1, 2002 to March 20, 2003 to estimate GARCH-NIG parameters. Then we use GARCH (1,1)-NIG option pricing model to forecast option price from March 21 to April 10. Each day, we chose

different strike prices from 4000 to 5100 and select one contract with the highest volume of trade for each strike price on that day to do call option pricing. We exclude the option data from 9:00 to 9:30 and the data after 13:25, as well. Moreover, we eliminate all transactions taking place during the last week before expiration (to avoid the expiration-related price effects). We summarize all the data we chose into the following table, respectively. The patterns of TAIEX and returns are shown in Figures 3.2.1 and 3.2.2 and results of estimation are given in Table 3.2.1.

Table 3.2.1. Estimation results.

$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}$	$\hat{\lambda}$	Loglikelihood
0.0000005	0.017708	0.96425	6.0901	0.2085	3750.09

Table 3.2.2. Data summaries.

	Moneyness (K/S)	Average price	Std	Number of observations
In the money	<0.95	372.59	96.92	29
At the money	0.95~1.05	121.28	63.15	57
Out of the money	>1.05	18.33	13.38	46
All calls		143.64	146.30	132

Figure 3.2.1 The price pattern of TAIEX.

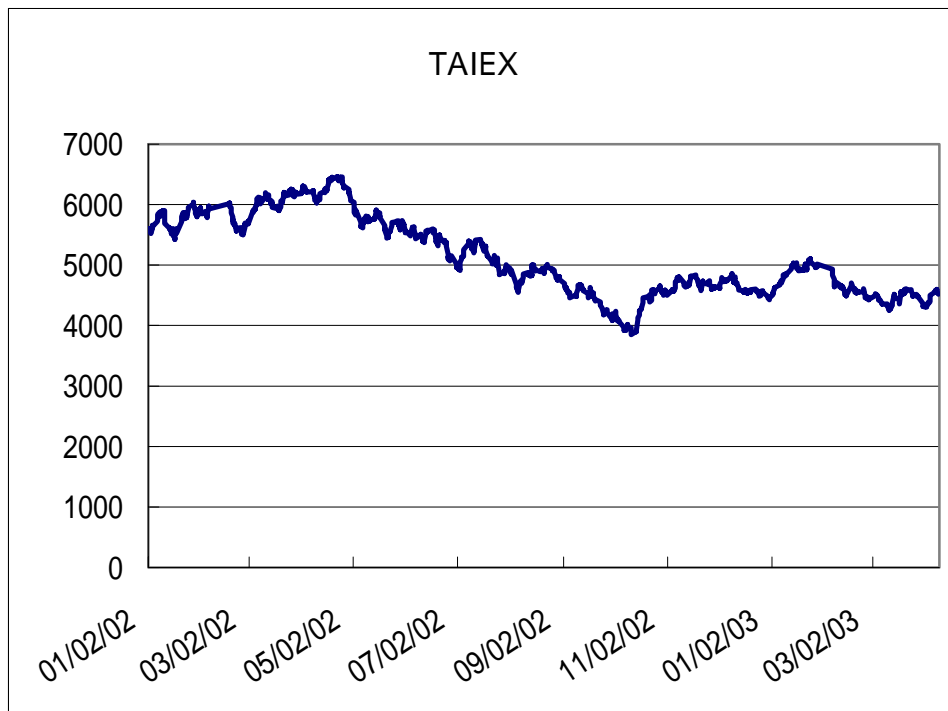
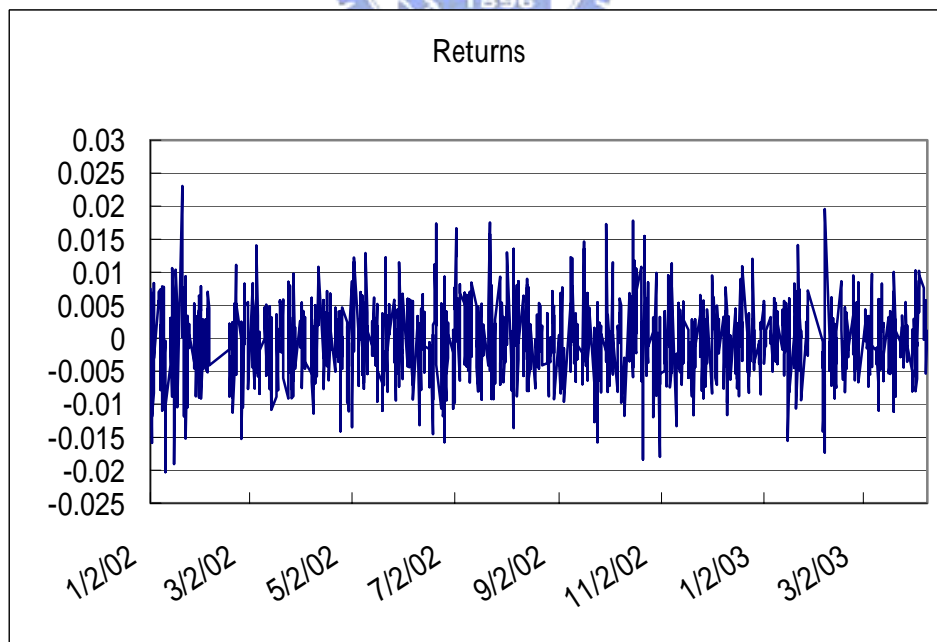


Figure 3.2.2 The log return of TAIEX.



3.3 Comparison with other option pricing models

In this subsection, we compare the Black-Scholes with GARCH (1,1)-NIG option pricing model. Since the GARCH (1, 1) model is the most commonly used GARCH process, our discussion for the remainder of this thesis will be restricted to the GARCH (1,1)-NIG model. We have introduced GARCH (1,1)-NIG option pricing model, so now we review Black-Scholes option pricing models.

The Black-Scholes option pricing model is presented as follows.

The call option price formula can be written as,

$$c = X_t N(d_1) - ke^{-r(T-t)} N(d_2),$$

where $d_1 = \frac{\ln(X_t/k) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{(T-t)}}$,

$$d_2 = \frac{\ln(X_t/k) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{(T-t)}} = d_1 - \sigma\sqrt{(T-t)}$$


$N(x)$ is a cumulative standard normal distribution, X_t is a spot price, k is the strike price, $(T-t)$ is time to mature, r stands for interest rate, and σ represents the volatility of return.

4. Numerical results and discussion

Monte Carlo simulation is used in the computation of the GARCH-NIG option price. The Monte Carlo method can be traced back to Boyle (1977). It is a convenient method for the GARCH-NIG option pricing model because the distribution for the temporally aggregated asset return cannot be derived analytically.

The general characteristics of the GARCH-NIG option pricing model compared with Black-Scholes formula are presented below for discussion. We divide our results into four parts, overall, in-the-money, at-the-money, and out-of-the-money.

Table 4. The relative absolute errors of GARCH-NIG and B-S model.



Model \ Relative absolute errors	GARCH-NIG	B-S
Overall	23.87	28.72
In the money	1.38	1.40
At the money	3.40	4.13
Out of the money	18.99	23.06

Overall:

From Table 4 and Figure 4.1, we can see that the relative absolute errors of call option price of GARCH-NIG model is smaller than the B-S model. So the performance of GARCH-NIG model is better than the B-S model. We also see that the higher option price we have, the larger error we get.

In the money:

We pick up the samples by $\frac{k}{s} < 0.95$, where k is a strike price and s is index. Then we sum all the relative absolute errors of the samples we select and get the following table. From table 4 and figure 4.2, we see that the dispersion of the errors of GARCH-NIG is larger than B-S, but the relative absolute errors of GARCH-NIG is smaller than B-S. So even the “in-the-money” case, the performance of GARCH-NIG is better than B-S, but there are no significant differences.

Figure 4.1. Errors of option price on GARCH-NIG compared with the B-S model. The errors of GARCH-NIG are drawn by square dots and triangular dots represent the errors of B-S model.

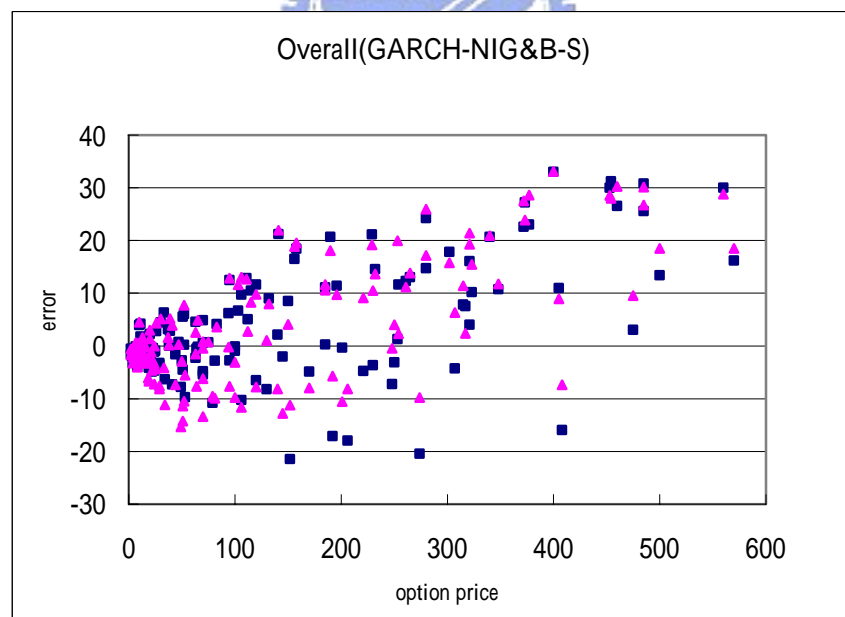
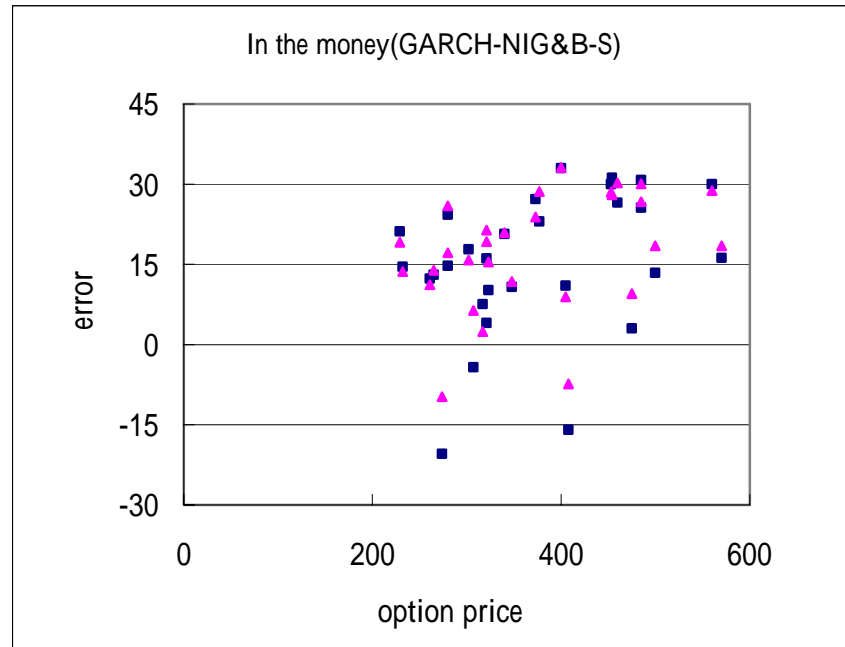


Figure 4.2. Errors of option price on GARCH-NIG compared with B-S model. The errors of GARCH-NIG are drawn by square dots and triangular dots represent the errors of B-S model.



At the money:

We select the samples by $0.95 \leq \frac{k}{s} < 1.05$, then sum all the relative absolute errors. The results are also shown in Table 4 and Figure 4.3.

Out of the money:

We select the samples by $1.05 \leq \frac{k}{s}$, then sum all the relative absolute errors. The results are shown in Table 4 and Figure 4.4.

Figure 4.3. Errors of option price on GARCH-NIG compared with B-S model. The errors of GARCH-NIG are drawn by square dots and triangular dots represent the errors of B-S model.

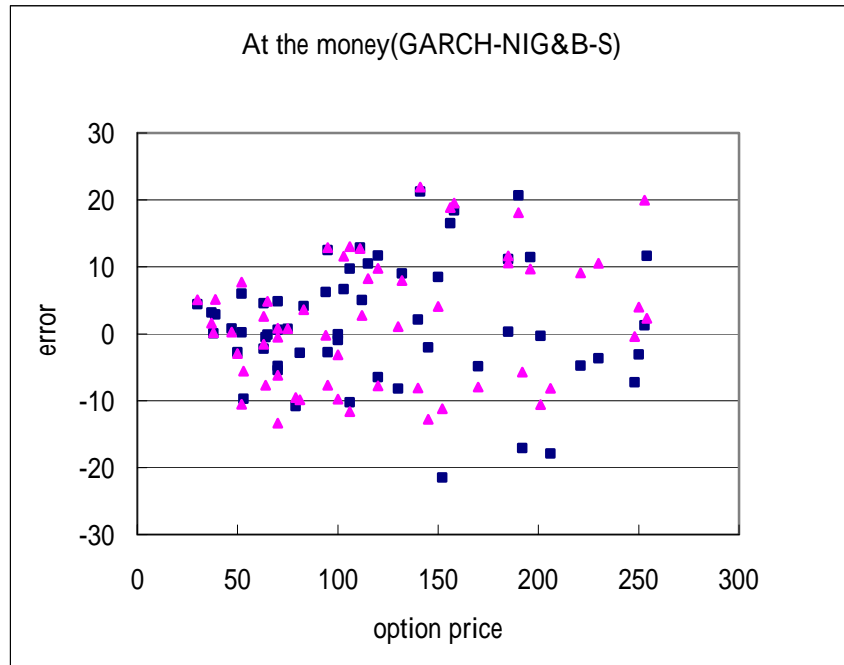


Figure 4.4. Errors of option price on GARCH-NIG compared with B-S model. The errors of GARCH-NIG are drawn by square dots and triangular dots represent the errors of B-S model.

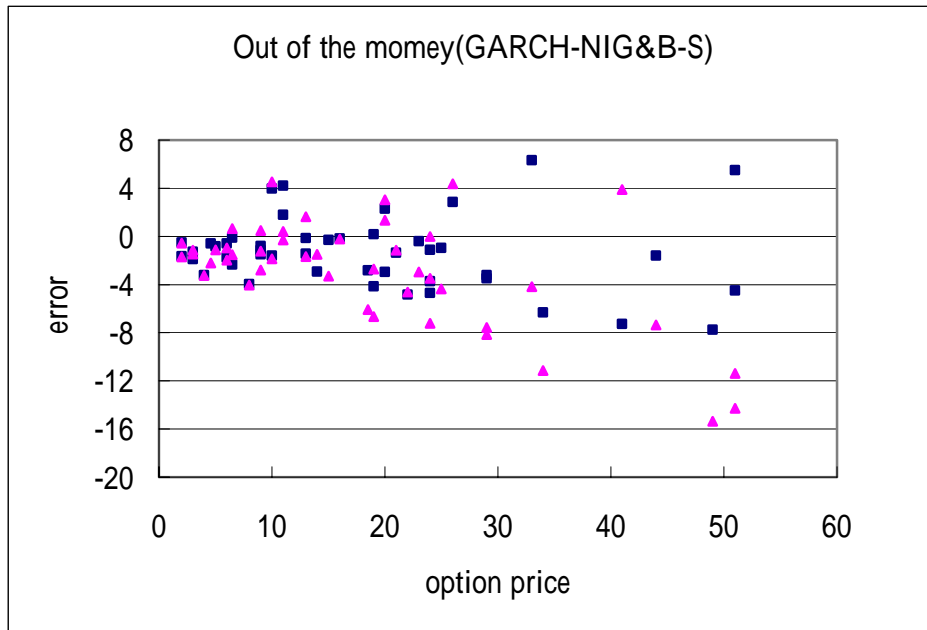


Figure 4.5. The relative absolute errors in different situations.



From above figures and tables, we can see that there are no significant differences in "in-the-money" situation. But in other situations, like "at-the-money" or "out-of-the-money", the GARCH-NIG model makes significant improvement in option pricing. Also from Figure 4.5 we can see the relative absolute errors of the BS are higher than GARCH-NIG in different situations. Hence, according to this

research, GARCH-NIG model gets better prediction than B-S model. It means that if we regard the volatility of the return as a constant like B-S model, we may get worse prediction. In other words, if we consider the volatility of the return as a time-variant value, then the error of estimation can be reduced.



5. Conclusions

This paper presents empirical analysis for the price of European call option on the TAIEX using GARCH-NIG model. The GARCH-NIG option pricing model has a number of desirable features and presents a real possibility of correcting the pricing biases associated with the Black-Scholes model.



Appendices:

1. Derive the log likelihood function of GARCH-NIG model.

The probability density function of NIG distribution is given by,

$$f(z; h, \alpha) = \frac{\alpha^{\frac{1}{2}}}{\pi h^{\frac{1}{2}}} \exp(\alpha) q\left(\frac{z}{h\alpha}\right)^{-1} K_1\left(\alpha q\left(\frac{z}{h\alpha}\right)\right)$$

where $q(x) = \sqrt{1+x^2}$ and $K_1(z)$ denotes the modified Bessel function of third order and index one. The log likelihood function is given in the following equation,

$$L_t(h_t, \alpha; z_i) = \prod_{i=1}^n f(z_i)$$

$$\ln L_t = \sum_{i=1}^n (\ln f(z_i))$$

$$= \sum_{i=1}^n \left(\frac{1}{2} \ln \alpha - \ln \pi (h_t)^{\frac{1}{2}} + \alpha - \ln \left(q\left(\frac{z_i}{h_t \alpha}\right) \right) + \ln \left(K_1\left(\alpha q\left(\frac{z_i}{h_t \alpha}\right)\right) \right) \right)$$

$$= \sum_{i=1}^n \left(\frac{1}{2} \ln \alpha - \ln \pi \right) - \sum_{i=1}^n \left(\frac{1}{2} \ln h_t - \alpha \right) - \sum_{i=1}^n \frac{1}{2} \ln \left(1 + \frac{z_i^2}{h_t \alpha} \right)$$

$$+ \sum_{i=1}^n \ln K_1 \left(\alpha \left(1 + \frac{z_i^2}{h_t \alpha} \right)^{\frac{1}{2}} \right)$$

2. Simulation of normal inverse Gaussian random variables.

We give a way of simulating NIG random variables using the fact that the distribution can be written as a normal variance-mean mixture with mixing function IG. The algorithm looks like this:

(1) Sample \mathbf{RV}_t from $\mathbf{IG}(\delta^2, \alpha^2 - \beta^2)$ and let $\sigma^2 = \mathbf{RV}_t$,

(2) Sample Y from $N(\mathbf{0}, \mathbf{1})$,

(3) Return $\mathbf{Z} = \mu + \beta\sigma^2 + \sigma Y$.

The remaining problem is to simulate inverse Gaussian variables. If we write the inverse Gaussian density function as

$$f_z(z) = \sqrt{\frac{\chi}{2\pi z^3}} \exp\left(-\frac{1}{2} \left(\frac{\chi(z-\mu)^2}{\mu^2 z}\right)\right)$$

where $\mu = \sqrt{\frac{\chi}{\psi}}$, then $V = \frac{\chi(Z-\mu)^2}{\mu^2 Z} \sim \chi^2(1)$. Now, let v_0 be a realization

of V and solve the above quadratic equation for Z . We obtain two solutions

$$z_1 = \mu + \frac{\mu^2 v_0}{2\chi} - \frac{\mu}{2\chi} \sqrt{4\mu\chi v_0 + \mu^2 v_0^2}, \text{ and } z_2 = \frac{\mu^2}{z_1}, \text{ where } z_1 \text{ should be}$$

chosen with probability $\frac{\mu}{\mu + z_1}$ and z_2 with probability $\frac{z_1}{\mu + z_1}$.

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