國立交通大學

統計學研究所

碩士論文

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Estimation of Dynamic Origin-Destination by Gaussian State-Space model with Travel Time

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碩 士 論 文

A Thesis
Submitted to Institute of Statistics
College of Science
National Chiao Tung University
in Partial Fulfillment of the Requirements
for the Degree of Master
in
Statistics
June 2004
Hsinchu, Taiwan, Republic of China

中華民國九十三年六月

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中文摘要

動態旅次起迄推估長久以來為運輸管理之核心，經由路網中偵測器所收集的資料，可以進行路網相關交通狀態的估計或預測，並根據預測結果進而模擬短時間內的交通狀況擬定適當的交通控制與管理方式，維持交通順暢。本研究為對高速公路旅次起迄流量之估計採用狀態空間模型並考慮兩地之間的旅行時間，配合上統計理論上的卡門濾波模式(Kalman filter)與吉柏司樣本法(Gibbs sampler)去構築本研究之模型，並比較傳統沒有考量旅行時間的模型之差異性。

Estimation of Dynamic Origin-Destination by Gaussian State-Space model with Travel Time

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ABSTRACT

Estimation of dynamic estimation of the O-D flow is the kernel of the traffic management for a long time. Through the data collected by the detectors in the network, we can estimate and predict the traffic condition about the network. According to the prediction results we can simulate the traffic condition and draft the associated appropriate traffic control and management to keep the free traffic. In this thesis, using the state-space model with travel time, estimation of the O-D flow of the freeway is considered. Using the Kalman filter and Gibbs sampler to complete the revised model, we compare the results with these from the traditional model without considering the travel time.

誌謝

在這裡，我要先感謝對我這兩年付出最多心力的周幼珍老師，在這兩年的指導過程中，不管是課業或生活態度上給予了我很多正面的建議。在這過程中，我逐漸堅定了要走的方向，也期望在未來的旅程中能與周幼珍老師有更多的互動。接著要感謝的是運管所卓訓榮老師與博士班的黃銘崇學長在論文的方法討論與資料的收集上給予我最大的幫忙，還有運管所一起參與的同學與學弟們，對我來說，這是一段美好的回憶。最後，感謝陪我在課餘時間一起生活兩年的研究所學長與同學：牛維方、陳泰賓與吳自強學長給予我在課業上的幫助，也讓我培養出了打羽球的興趣，還有大頭超、大胖輝、蘇董、李董、廢祥、柯董、怡君、淑珍，最後是巧慧，感謝妳們在我的碩士生活的最初與最後留下了美好的回憶。還有410 室的Ming、Ken、葉氏夫妻、三寶文、黑色細肩帶楊華勝，雖然大家各分東西，但天下無不散的筵席，該走的留不住，就在這裡為下次的再見面開個頭吧。紙短情長，就讓我們這群不務正業的學生們，青山不改，綠水長流，咱們後會有期啦。

這兩年，給予我最大鼓勵與無條件支持的，是我的父母我的家人，在此僅以此篇論文獻給在我背後的她們。

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中華民國九十三年六月

Contents

1 Introduction

The origin-destination (O-D) flow plays an important role of most traffic operational analyses. The O-D flow has been used on the traffic assignment and the traffic flow simulation. Traditionally the O-D data collection is based mainly on field surveys, which not only costs a lot of budget, labor, and time, but also causes missing values easily when the traffic is heavy. Traffic engineers have been looking for statistical methods to estimate the O-D flow from less expensive data. Recent researchers estimate the O-D flow using Gaussian state-space model with an unknown transition matrix and Kalman filter but without considering the time factor. As the development of ITS (intelligent transportation system) changing with each passing day, the traditional traffic information becomes useless in the ATMS (advanced traffic management system). In order to achieve the aim of traffic control, the real time forecasts and estimates of the traffic condition in the future are desirable since the information can be used not only for ramp metering control but also for informing drivers the relevant information through the VMS (variable message sign) or HAR (highway advisory radio) to avoid the jammed traffic and save the travel time. Traditionally, researchers predict the O-D information ignoring the travel time which will produce erroneous results, especially when the travel time is long. In this thesis, we will investigate estimation of the O-D matrix with the travel time factor being taken into consideration in the dynamic system.

Here we briefly state that the state-space model consists of the state equation, representing the transition of the state variables, and the observation equation, describing the relationship between the state variables and the present and time lagged observations. An adaptive Monte Carlo technique known as the Gibbs sampler will be used to estimate the unknown transition matrix and more importantly, the state variables in the state equation of the state space model. The classical state-space model was as follows:

$x_{t+1} = Fx_t + v_{t+1}$, called the state equation,

$y_t = H'x_t + w_t$, called the observation equation,

where v_t and w_t are errors and if they process Gaussian distribution we called the model Gaussian state-space model. The Kalman filter algorithm is used to predict the stat variables since it gives the optimal predicted values when the model is Gaussian. To included the time factor, the model is modified as follows:

$x_{t+1} = Fx_t + v_{t+1}$: the state equation,

$y_t = \sum_{i=0}^{d} H_i' x_{t-i} + w_t$: the observation equation with time lagged,

and we will investigate in more detail in the next sections.

In the traditional Kalman filter, matrices F and H are assumed to be known in advance. But in fact we can only find the matrix H from the network. And under the assumptions, which are outside the system, F can easily cause the model behave not well enough when the traffic flow is unstable. Here we propose using the Gibbs sampler to solve this problem. In our model, we initialize matrix F as an identity matrix and then use the Gibbs sampler to estimate F from the full conditional distributions and the observations. The general procedure to estimate the state variable (the O-D flows) is as follows: First, use the speed data collected by sensors on the freeway to calculate the travel time. Second, we use the calculated travel times to determine whether the vehicles arrive in the specific time interval or not and this will help us to find out the incidence matrix H exactly. Third, conditional on the transition matrix F obtained in the previous stage, the state variables are filtered by the Kalman filter. Fourth, after the filter is finished, we use the Gibbs sampler to estimate the matrix F and then repeat these steps till the output converges.

2 Literature Review

Estimation of dynamic O-D matrices from traffic counts in a transportation network has received increasing attention over decades. Traditionally, the O-D flow matrices are considered only for a certain time period of interest, and thus are estimated with the average traffic count data of that period. A comprehensive review of research along this line can be found in Nugyen (1984) and Cascetta and Nugyen (1988). Such methods are static in nature, relying on the prior O-D information. Vardi (1996) considered the problem of estimating the traffic intensity between all "source-destination pairs" of nodes of a communication network from repeated measurements by maximum likelihood estimation. Tebaldi and West (1998) addressed the network count inference problem from a Bayesian aspect. In their framework of inference, the previous information on the O-D matrices are required, which is unrealistic. Their approach is also static and not suitable for predicting future O-D matrices. Since the number of relations between traffic counts and O-D pairs is usually far less than the O-D pairs, difficulties arise and some additional assumptions are needed.

To extend the O-D estimation methods in a dynamic system environment, researchers (e.g., Nihan and Davis {1989} and Cremer and Keller {1987}) proposed the use of time series traffic counts to formulate the relationships and applied to small networks by the methods of least squares and maximum likelihood. Okutani (1987) proposed using the Kalman filtering procedures for forecasting the O-D flows in a dynamic system, but gave no description with respect to the dynamic traffic assignment proportions on which the model was based. All the results mentioned above were obtained by a system ignoring the travel time and therefore of limited applicability. Recently in order to fit the requirement of the ATIS (advanced traveler information system) and ATMS (advanced transportation management system) of ITS (intelligent transportation system), researchers paid lots of effect on estimation and predicting of travel time. Dailey (1999) used the state-space model to describe the relationship between measurements from single loop detectors, Chen and Chien (2001) predicted the travel time using probe vehicles data, Coifman (2002) estimated the travel time by data obtained from dual loop detectors, and Petty et al. (1998) treated the cumulative upstream and downstream arrival vehicles as a stochastic process and estimated the probability density function of the travel time.

In this thesis, the state-space model will be combined with the concept of the travel time. Since it is relatively manageable in a highway system (the route choice issue is not involved), we will apply the proposed model to the data of National Highway No. 3. This thesis is organized as follows. The model specification and the algorithms used will be presented in Section 3. The empirical results are reported in Section 4. Conclusions and possible further enhancements are summarized in Section 5.

3 Model Specifications and Methodology

3.1 Problem Description and Notations

Suppose that a section of the highway system is divided into N segments so that each segment has only a pair of on- and off-ramps. Therefore, including the beginning and the ending segments, there are $N+1$ nodes, N links and N origins and destinations, where the i^{th} origin and the $(i-1)^{th}$ destination are the same ramp in the highway for $i=2,\cdots,N-1$. The notations used in the model are listed as follows:

$O_{i,t}$: the number of vehicles starting from the i^{th} origin at time interval t for $i=1,\cdots,N$;

$D_{j,t}$: the number of vehicles exiting the j^{th} destination at time interval t for $j=1,\cdots,N$;

$T_{ij,t}$: the number of vehicles entering the highway from i^{th} origin to j^{th} destination at time interval t for $1\le i\le j\le N$.

The O_i 's and D_j 's can be easily obtained from the detectors fitted in the highway system, T_{ij} 's are the unobserved variables of fundamental importance for the transportation planning and management purpose. The problem to be solved is to use the time series of link traffic flow { $O_{i,t}$ } and { $D_{j,t}$ } to estimate the time-varying O-D flows { $T_{ij,t}$ }. The state-space model is a natural alternative to specify the relationship between the variables. The observations { $O_{i,t}$ } and { $D_{j,t}$ } will be stacked up to form an observation vector, i.e.,

$$y_t = (O_{1,t}, O_{2,t}, \cdots, O_{N,t}, D_{1,t}, D_{2,t} \cdots, D_{N,t})',$$

$$x_t = (T_{11,t}, T_{12,t}, \cdots, T_{1N,t}, T_{22,t}, T_{23,t}, \cdots T_{2N,t}, \cdots T_{N-1,N-1,t}, T_{N-1,N,t}, T_{NN,t})'$$

$$= (x_{1,t}, x_{2,t}, \cdots, x_{p,t})', \text{ where } p = N(N+1)/2.$$

Observe that $O_{i,t} = \sum_{j=i}^{N} T_{ij,t}$, and that $D_{j,t} = \sum_{i=j}^{N} T_{ij,t}$ if the travel time could be ignored. Therefore, the basic model consists of a state equation indicating the transition of the state vector,

$$x_t = F x_{t-1} + v_t,$$

and an observation equation indicating the relationship between x_t and y_t,

$$y_t = H' x_t + w_t, \text{ where } H \text{ is a } p \times q \text{ incidence matrix with } q = 2N.$$

3.2 The State-Space Model with Known Transition Matrix

3.2.1 State-Space Model

Let y_t denote a $q \times 1$ vector of variables observed at time t, y_t can be described in terms of an unobserved $p \times 1$ vector x_t called the state vector. The state-space model is given by the following system of equations:

$$x_t = F x_{t-1} + v_t, \qquad t = 1, 2, \cdots, n \tag{1}$$

$$y_t = H' x_t + w_t, \qquad t = 1, 2, \cdots, n. \tag{2}$$

The development of the system over time is determined by x_t's according to equation (1), but because x_t's are not observed the analysis must be based on the observations y_t's. The matrices F and H' are matrices of parameters of dimensions $p \times p$ and $q \times p$, respectively. Equation (1) is known as the state equation and equation (2) the observation equation. The initial state variable x_0 is assumed to be $N_p(\mu_0, P_0)$ and independent of $v_1,\ldots,v_n$ and $w_1,\ldots,w_n$, where μ_0 and P_0 are assumed to be known. The $p \times 1$ vectors v_t's and the $q \times 1$ vectors w_t's are independent white noises such that $E(v_t) = 0$, $E(w_t) = 0$, $Cov(v_t) = \Sigma$, $Cov(w_t) = \Gamma$, and $Cov(v_t, w_t) = 0$, for $t = 1,2,\cdots,n$, where Σ and Γ are $p \times p$ and $q \times q$ positive definite covariance matrices, respectively.

Typically we assume $v_t \overset{i.i.d.}{\sim} N_p(0, \Sigma)$ and $w_t \overset{i.i.d.}{\sim} N_q(0, \Gamma)$, v_t's and w_t's are independent, and the matrices Σ and Γ are known and positive definite. Equation (2) has the structure of the linear regression model and equation (1) represents a vector autoregressive model, where the Markovian nature accounts for many of the properties of the state-space model. And the assumptions enable updating of estimates via the Kalman filter easily.

3.2.2 Kalman Filter

Let $\hat{x}_{t+1|t} = E(x_{t+1} \mid y_1, \cdots, y_t, x_1, \cdots, x_t)$ be the forecast of x_{t+1} based on the information up to time t and abbreviate as the form $E(x_{t+1} \mid y_t)$. The Kalman filter calculates these forecasts recursively, generating $\hat{x}_{1|0}, \hat{x}_{2|1}, \cdots, \hat{x}_{t|t-1}$ in succession.

The mean squared error (MSE) of the forecasts $\hat{x}_{t+1|t}$ is represented by the following $p \times p$ matrix :

$$P_{t+1|t} \equiv E[(x_{t+1} - \hat{x}_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})'] \tag{3}$$

From (1) we have

$$\hat{x}_{t+1|t} \equiv E(x_{t+1} \mid y_t) = E(Fx_t \mid y_t) = FE(x_t \mid y_t) \equiv F\hat{x}_{t|t} .$$

Substituting it into equation (3), we have

$$P_{t+1|t} = FP_{t|t}F' + \Sigma ,$$

(4)

where $P_{t|t} \equiv E[(x_t - E(x_t \mid y_t))(x_t - E(x_t \mid y_t))']$ denotes the MSE of updating of x_t when y_t becomes available. Next consider forecasting the value of y_t. Let $\hat{y}_{t|t-1} \equiv E(y_t \mid x_{t-1})$. From equation (2),

$$\hat{y}_{t|t-1} = E(y_t \mid x_{t-1}) = H'E(x_t \mid x_{t-1}) = H'\hat{x}_{t|t-1} . \tag{5}$$

From equation (3) the MSE of this forecast is

$$M_{t|t-1} \equiv E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})'] = H'P_{t|t-1}H + \Gamma .$$

(6)

After predicting the value of y_t, we can update the current value of x_t on the basis of the observation of y_t to produce $\hat{x}_{t|t} = \hat{E}(x_t \mid y_t)$. This can be evaluated using the formula for updating a linear projection:

$$\begin{aligned}\hat{x}_{t|t} = \hat{x}_{t|t-1} &+ \{E[(x_t - \hat{x}_{t|t-1})(y_t - \hat{y}_{t|t-1})']\} \\ &\times \{E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})']\}^{-1} \times (y_t - \hat{y}_{t|t-1})\end{aligned}$$

(7)

then we have

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + P_{t|t-1}HM_{t|t-1}^{-1}(y_t - H'\hat{x}_{t|t-1}) . \tag{8}$$

From $P_{t|t} \equiv E(x_t - E(x_t))(x_t - E(x_t))'$ we have

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}HM_{t|t-1}^{-1}H'P_{t|t-1} . \tag{9}$$

Therefore, the recursion for predicting x_{t+1} when y_t comes in can be expressed as follows:

$$\begin{aligned}\hat{x}_{t+1|t} &= F\hat{x}_{t|t} + FP_{t|t-1}HM_{t|t-1}^{-1}(y_t - H'\hat{x}_{t|t-1}) \\ &\equiv F\hat{x}_{t|t} + K_t(y_t - H'\hat{x}_{t|t-1}),\end{aligned} \tag{10}$$

where K_t is the Kalman gain matrix.

The complete procedure of the Kalman filter is summarized below.

Step 1 Initialize by giving the initial values $\hat{x}_0 = E(x_0) = \mu_0 = 0_{p\times1}$, $Cov(x_0) = P_0$ and let the predicted value of x_1 ($\hat{x}_{1|0} = \mu_{1|0}$) with no observations be $F\mu_0$, and therefore has MSE $P_{1|0}$ from equation (4).

Step 2 Predict the state variable x_t with information up to time $t-1$

$$E(x_t \mid y_{t-1}) \equiv \mu_{t|t-1} = F\mu_{t-1},$$

$$Cov(x_t \mid y_{t-1}) = P_{t|t-1} = FP_{t-1|t-1}F' + \Sigma.$$

Step 3 Predict y_t

$$E(y_t \mid y_{t-1}) = \hat{y}_{t|t-1} = H'\mu_{t|t-1},$$

$$Cov(y_t \mid y_{t-1}) = M_{t|t-1} = H'P_{t|t-1}H + \Gamma.$$

Step 4 Update the parameters

$$\mu_t \equiv \mu_{t|t} = \mu_{t|t-1} + P_{t|t-1}HM_{t|t-1}^{-1}(y_t - \hat{y}_{t|t-1}),$$

$$P_t \equiv P_{t|t} = P_{t|t-1} - P_{t|t-1}HM_{t|t-1}^{-1}H'P_{t|t-1},$$

$$t = t+1.$$

Step 6 If $t = n$, then stop.

Step 7 Go to step 2.

3.3 Gibbs sampler

The Gibbs sampler is a statistical method for generating random variables from a joint distribution via conditional distributions without deriving the actual density. Given an arbitrary starting values $(Z_1^{(0)}, Z_2^{(0)}, \cdots, Z_k^{(0)})$, we start the sampling as follows:

$$Z_1^{(1)} \sim \left[Z_1 \mid Z_2^{(0)}, Z_3^{(0)}, \cdots, Z_k^{(0)}\right] ,$$

$$Z_2^{(1)} \sim \left[Z_2 \mid Z_1^{(0)}, Z_3^{(0)}, \cdots, Z_k^{(0)}\right] ,$$

$$\vdots$$

$$Z_k^{(1)} \sim \left[Z_k \mid Z_1^{(0)}, Z_2^{(0)}, \cdots, z_{K-1}^{(0)}\right] .$$

Then we have a new set of values $\left(Z_1^{(1)}, Z_2^{(1)}, \cdots, Z_k^{(1)}\right)$ to continue the iteration. After m such iterations we have $\left(Z_1^{(m)}, Z_2^{(m)}, \cdots, Z_k^{(m)}\right)$. The paper of Geman and Geman (1984) showed that the following results hold under mild conditions.

1. $\left(Z_1^{(m)}, Z_2^{(m)}, \cdots, Z_k^{(m)}\right) \to (Z_1, Z_2, \cdots, Z_k)$ and so that for each i, $Z_i^{(m)} \xrightarrow{d} Z_i$ as $m \to \infty$.

2. For any Borel measurable function T of $(Z_1, Z_2, \cdots, Z_k)$ whose expectation exists,

$$\lim_{m\to\infty} \frac{1}{m} \sum_{j=1}^{m} T\left(Z_1^{(j)}, Z_2^{(j)}, \cdots, Z_k^{(j)}\right) \xrightarrow{a.s.} E\left(T\left(Z_1, Z_2, \cdots, Z_k\right)\right).$$

3.4 State-Space Model with unknown Transition matrix

3.4.1 Estimation of F by Gibbs Sampler

From state-space model, equation (1) can be written as

$$v_t = x_t - Fx_{t-1}, \quad v_t \sim N_p(0,\Sigma) \quad t = 1,2,\cdots,n.$$

The joint distribution of $V = (v_1, v_2, \cdots, v_n)'$ is

$$p(V \mid F,\Sigma) \propto |\Sigma|^{-n/2} \exp(-\frac{1}{2}\sum_{i=1}^{n} v_i'\Sigma^{-1}v_i). \qquad (11)$$

Denote $S(F)$ to be the $p \times p$ symmetric matrix $S(F) = \{S_{ij}(F_i, F_j)\}$ with

$$S_{ij}(F_i, F_j) = \sum_{u=1}^{n} v_{ui} \cdot v_{uj} = \sum_{u=1}^{n} (x_{ui} - F_i x_{u-1})(x_{uj} - F_j x_{u-1}), \quad i, j = 1,2,\cdots,p, \qquad (12)$$

where F_i denote the i^{th} row of F.

Then the exponent in (11) can be expressed as

$$\exp(-\frac{1}{2} trS(F)\Sigma^{-1}) \qquad (13)$$

From the transition equation of (1) we have $x_t' = x_{t-1}'F' + v_t', \quad t = 1,2,\cdots,n$

Let $X_n' = \begin{bmatrix} x_1' \\ \vdots \\ x_n' \end{bmatrix}$, $X_{n-1}' = \begin{bmatrix} x_0' \\ \vdots \\ x_{n-1}' \end{bmatrix}$, $V' = \begin{bmatrix} v_1' \\ \vdots \\ v_n' \end{bmatrix}$, then the transition equation has the form of a linear model:

$$X_n' = X_{n-1}'F' + V' \qquad (14)$$

and equation (12) can be expressed as

$$S_{ij}(F_i, F_j) = (X_{n(i)}' - X_{n-1}'F_i')'(X_{n(j)}' - X_{n-1}'F_j'), \qquad (15)$$

where $X'_{n(i)}$ denotes the i^{th} column of X'_n.

Let $\hat{F}_i' = (X_{n-1}X'_{n-1})^{-1}X_{n-1}X'_{n(i)}$ denote the least square estimator of F_i', so that equation (16) can further be rewritten as

$$S_{ij}(F_i, F_j) = (X'_{n(i)} - X'_{n-1}\hat{F}_i')'(X'_{n(j)} - X'_{n-1}\hat{F}_j') + (F_i' - \hat{F}_i')'X_{n-1}X'_{n-1}(F_j' - \hat{F}_j') \tag{16}$$

Consequently, $S(F) = A + (F' - \hat{F}')'X_{n-1}X'_{n-1}(F' - \hat{F}')$, $\quad (17)$

where $A = \{a_{ij}\}$ is a $p \times p$ matrix with

$$a_{ij} = (X_{n(i)} - X_{n-1}\hat{F}')'(X_{n(j)} - X_{n-1}\hat{F}')$$

For the prior distribution of the parameters (F, Σ), we shall first assume that F and Σ are approximately independent so that

$$p(F, \Sigma) \approx p(F) \cdot p(\Sigma)$$

and it is appropriate to take F as locally uniform, i.e. $p(F) \propto constant$. Then the joint posterior distribution of (F, Σ) is

$$p(F, \Sigma \mid X_n, X_{n-1}) \propto |\Sigma|^{-\frac{1}{2}(n+p+1)} \exp[-\frac{1}{2}tr\Sigma^{-1}S(F)], \tag{18}$$

and by changing of variable we have

$$p(F, \Sigma^{-1} \mid X_n, X_{n-1}) \propto |\Sigma^{-1}|^{\frac{1}{2}(n-p-1)} \exp[-\frac{1}{2}tr\Sigma^{-1}S(F)], \tag{19}$$

is the Wishart distribution. Then the marginal posterior distribution of F has the following form

$$p(F \mid X_n, X_{n-1}) \propto |S(F')|^{-\frac{n}{2}}, \tag{20}$$

from equation (15) we have

$$p(F \mid X_n, X_{n-1}) \propto |A + (F' - \hat{F}')'X_{n-1}X'_{n-1}(F' - \hat{F}')|^{-\frac{n}{2}} \tag{21}$$

is the kernel of the $matric - t$ distribution.

Theorem 1. [Johnson and Kotz (1972)]

Let T be the random $p \times q$ matrix, $T = (U'^{\frac{1}{2}})^{-1} X$,

where $U \sim W_p(P, m-q)$, $m > p+q-1$, independently of X, the distribution of the transpose of each row of X, x_i', is denoted by $x_i' \sim N_q(0, Q)$; P and Q are positive definite. The joint density function of T is

$$p(T) = [k(m,p,q)]^{-1} \times |Q|^{\frac{1}{2}(m-p)} |P|^{\frac{1}{2}q} |Q + T'PT|^{-\frac{1}{2}m}$$

These results were obtained by Dickey (1967)

According to the above theorem, the marginal distribution of F with corresponding parameters, $p = q$ and $m = n$, $n > 2p-1$, can be written as

$$p(F') \propto |A|^{\frac{1}{2}(n-p)} |X_{n-1}X'_{n-1}|^{\frac{1}{2}p} |A + (F' - \hat{F}')' X_{n-1}X'_{n-1}F|^{-\frac{n}{2}}$$

Sampling scheme: generate x_t and F from the conditional distribution.

Step 1 $x_t \mid F, x_{t-1}, \Sigma \sim N_p(Fx_{t-1}, \Sigma)$

Step 2 $F' \mid X_n, X_{n-1}, \Sigma \sim p(F' \mid X_n, X_{n-1})$

1 Generate $W \sim W_p(n-p, X_{n-1}X'_{n-1})$

2 Generate $Z = (z_1, z_2, \cdots, z_p)'$ and $z_i \sim N_p(0, A)$

3 Let $F' = [(W^{\frac{1}{2}})']^{-1} Z$

3.4.2 Gaussian state space without considering travel time

In Section 3.2.2 we use Kalman filter to update the parameter vector after observing the new observation y_t. In a traditional state-space model, the transition matrix F is known before we start to filter the stat vector by Kalman filter. But in reality F is usually unknown, one way to solve this problem is to combine Gibbs sampler with Kalman filter. The complete algorithm is summarized as follows:

Algorithm 1 (Gaussian state space model without considering travel time)

Step 1 (Initialization)

1. given the initial value of $F^{(0)} = I_{p\times p}$
2. Given Σ and Γ
3. Given μ_0, P_0 to generate $x_0^{(g)}$ from $N_p(\mu_0, P_0)$
4. $g = 0$

Step 2 (Generate $x_t^{(g)}, t = 0,1,2,\cdots,n$)

1. Generate $x_t^{(g)}$ from $N_p(\mu_t, P_t)$
2. Generate $x_{t|t-1}^{(g)}$ from $x_t \mid x_{t-1}^{(g)}, F^{(g)} \sim N_p(F^{(g)} x_{t-1}^{(g)}, \Sigma)$
3. Use the Kalman filter to filter $x_t^{(g)}$ (here we update μ_t and P_t)

   3.1 In Step 3 of the Kalman filter the part of predicting of y_t we use

   $\hat{y}_t = H'\hat{x}_{t|t-1}$ to predict y_t
4. Repeat 2,3 for $t = 2,3,\cdots,n$

Step 3 (Generate $F'^{(g)}$)

1. Calculate $X_{n-1}^{(g)} X_{n-1}'^{(g)}$
2. Calculate $A^{(g)} = \{a_{ij}^{(g)}\},\ a_{ij}^{(g)} = (X_{n(i)}'^{(g)} - X_{n-1}'^{(g)} \hat{F}_i'^{(g)})'(X_{n(j)}'^{(g)} - X_{n-1}'^{(g)} \hat{F}_j'^{(g)})$

   where $\hat{F}_i'^{(g)} = (X_{n-1}^{(g)} X_{n-1}'^{(g)})^{-1} X_{n-1}'^{(g)} X_{n(i)}'^{(g)}$
3. Generate $w \sim W_p(X_{n-1}^{(g)} X_{n-1}'^{(g)}, n-p)$

4. Generate $Z = (z_1', z_2', \cdots, z_p')', \; z_k \overset{iid}{\sim} N_p(0, A^{(g)})$

5. Generate $F'^{(g)} = ((w^{\frac{1}{2}})')^{-1} Z$

6. $g = g+1$, if $g < m$ go to step2

Step 4 (Prediction)

Then we have $\{X^{(1)}, \cdots, X^{(m)}\}$. At last, we will have the estimator

$$\hat{X} = \frac{1}{k} \sum_{g=m-k}^{m} X^{(g)} \quad \text{and} \quad \hat{F}' = \frac{1}{k} \sum_{g=m-k}^{m} F'^{(g)} \quad (m > k)$$

After having $\hat{F}$, the predict of the state vector can be obtained according to (10) with F replaced by $\hat{F}$.

3.4.3 Gaussian state space model with considering travel time

In modeling the interrelation between O-D flow matrix and link flow in a freeway network, it is unrealistic to ignore the travel time from one place to the destination. In this thesis, we propose a more adequate state-space model for dynamic O-D estimation by taking into account the travel time required by each trip. The incident matrix in the observation equation H' is a matrix with elements ones and zeros depending on whether or not the elements of x_t the path flow will or will not contribute to affect the observation y_{ti} at the same t, i.e. the travel time is ignored. By considering the travel time required form on place to another, y_{ti}, the link flow obtained at time interval t is accumulated by the path flows possibly occurred from several time intervals before. Therefore, the interrelationship between y_t and x_t can be formulated by the following model:

$$x_t = Fx_{t-1} + v_t,$$

$$y_t = \sum_{i=0}^{d} H_i' x_{t-i} + w_t, \quad d < n, \quad t = 1,2,\cdots,n. \qquad (22)$$

where $x_t = (x_{t1}, x_{t2}, \cdots, x_{tp})'$ is the state vector at time t, $t = 1,2,\cdots,n$ and x_{tj} denotes the number of vehicles traveled at time t of O-D pair j, $j = 1,2,\cdots,p$, the observed vector at time interval t, $y_t = (y_{t1}, y_{t2}, \cdots, y_{tq})'$, and y_{ti} is the counts of vehicles entering or exiting the i^{th} node, $i = 1,2,\cdots,q$.

$H_i'(i = 0,1,\cdots,d)$ is the $q \times p$ O-D pair incident matrices with zero-one as its elements, and $\sum_{i=0}^{d} H_i' = H'$ in the equation (2) of the standard state-space model. The number d is the maximum number of time periods needed for a path flow contributing the corresponding link flow.

We will use the same notation as in the previous derivation $\hat{x}_{t+1|t}$ representing the forecast of x_{t+1} based on y_t. The MSE of the predictor $\hat{x}_{t+1|t} = E(x_{t+1} \mid y_t)$, $P_{t+1|t} = E[(x_{t+1} - \hat{x}_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})']$ can be expressed as $P_{t+1|t} = FP_{t|t}F' + \Sigma$. The next stage of predicting the value of $y_t : \hat{y}_{t|t-1} = E(y_t \mid x_{t-1}, x_{t-2}, \cdots, x_{t-m})$ from equation (18) we have $E(y_t \mid x_t) = \sum_{i=0}^{d} H_i' x_{t-i}$ and then

$$\hat{y}_{t|t-1} = E(y_t \mid x_{t-1}) = \sum_{i=0}^{d} H_i' E(x_{t-i} \mid x_{t-1}) = H_0' \hat{x}_{t|t-1} + H_1' \mu_{t-1|t-1} + \sum_{i=2}^{d} H_i' x_{t-i} \qquad (23)$$

where the $\mu_{t-1|t-1} = E(x_{t-1} \mid x_{t-1})$. The MSE of the predictor $\hat{y}_{t|t-1}$ is as follows:

$$M_{t|t-1} = [H_0'(x_t - \hat{x}_{t|t-1}) + H_1'(x_{t-1} - \mu_{t-1|t-1})][H_0'(x_t - \hat{x}_{t|t-1}) + H_1'(x_{t-1} - \mu_{t-1|t-1})]'$$

$$= H_0' P_{t|t-1} H_0 + H_1' P_{t-1|t-1} (H_0' F)' + (H_0' F) P_{t-1|t-1} H_1 + H_1' P_{t-1|t-1} H_1 + \Gamma \qquad (24)$$

This can be evaluated using the formula for updating a linear projection:

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + \{E[(x_t - \hat{x}_{t|t-1})(y_t - \hat{y}_{t|t-1})']\} \times \{E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})']\}^{-1} \times (y_t - \hat{y}_{t|t-1}) \tag{25}$$

then we have

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + (P_{t|t-1}H_0 + FP_{t-1}H_1)M_{t|t-1}^{-1}(y_t - \hat{y}_{t|t-1}) \tag{26}$$

and from $P_{t|t} \equiv E(x_t - E(x_t))(x_t - E(x_t))'$ we have

$$P_{t|t} = P_{t|t-1} - (P_{t|t-1}H_0 + FP_{t-1}H_1)M_{t|t-1}^{-1}(P_{t|t-1}H_0 + FP_{t-1}H_1)' \tag{27}$$

so we can modify the original Kalman filter as bellow:

Step 1 Initialize the algorithm by giving the initial values $\hat{x}_0 = E(x_0) = \mu_0$, $Var(x_0) = P_0$ and let the predict value of x_1 ($\hat{x}_{1|0} = \mu_{1|0}$) with no observations be $F\mu_0$, and therefore has MSE $P_{1|0}$.

and start with $t=1$

Step 2 estimating the next stage:

$$E(x_t \mid y_{t-1}) = \mu_{t|t-1} = F\mu_t$$

$$Var(x_t \mid y_{t-1}) = P_{t|t-1} = FP_{t|t}F' + \Sigma$$

Step 3 prediction of y_t

$$E(y_t \mid y_{t-1}) = \hat{y}_{t|t-1} = H_0'\mu_{t|t-1} + H_1'\mu_{t-1} + \sum_{i=2}^{d} H_i'x_{t-i} \quad \text{(equation (23))}$$

$$Var(y_t \mid y_{t-1}) = M_{t|t-1} \quad \text{(equation (24))}$$

$$= H_0'P_{t|t-1}H_0 + H_1'P_{t-1|t-1}(H_0'F)' + (H_0'F)P_{t-1|t-1}H_1 + H_1'P_{t-1|t-1}H_1 + \Gamma$$

Step 4 update the parameters

$$\mu_t = \mu_{t|t} = \mu_{t|t-1} + (P_{t|t-1}H_0 + FP_{t-1}H_1)M_{t|t-1}^{-1}(y_t - \hat{y}_{t|t-1}) \quad \text{(equation (26))}$$

$$P_{t|t} = P_{t|t-1} - (P_{t|t-1}H_0 + FP_{t-1}H_1)M_{t|t-1}^{-1}(P_{t|t-1}H_0 + FP_{t-1}H_1)' \text{(equation } (27))$$

$$t = t+1$$

Step 5 if $t = n$ stop

Step 6 go to step 2

And we can update the Algorithm 1 as:

Algorithm 2 (Gaussian state space model with travel time)

Step 1 (Initialization)

1. give the initial value of $F^{(0)} = I_{p\times p}$
2. Given Σ and Γ
3. Given μ_0, P_0 to generate the prior state-matrix $X_0^{(g)} = (x_0^{(g)}, x_{0-1}^{(g)}, \cdots, x_{0-d}^{(g)})$,

   where $x_i^{(g)} \sim N_p(\mu_0, P_0)$ for $i = 0,-1,\cdots,-d$
4. $g = 0$

Step 2 (Generate $x_t^{(g)}, t = 0,1,2,\cdots,n$)

1. Generate $x_t^{(g)} \sim N_p(\mu_t, P_t)$
2. Generate $x_{t|t-1}^{(g)}$ from $x_t \mid x_{t-1}^{(g)}, F^{(g)} \sim N_p(F^{(g)}x_{t-1}^{(g)}, \Sigma)$
3. Use the Kalman filter to filter $x_t^{(g)}$

   The updated Kalman filter the part of predicting of y_t we use

   $\hat{y}_t = H_0' x_{t|t-1}^{(g)} + H_1' \mu_{t-1}^{(g)} + \sum_{i=2}^{d} H_i' x_{t-i}^{(g)}$ to predict y_t and change the

   matrix $X_t^{(g)} = (x_{t-0}^{(g)}, x_{t-1}^{(g)}, \cdots, x_{t-d}^{(g)})$ to continue the filter.
4. Repeat 2,3 for $t = 2,3,\cdots,n$

Step 3 (Generate $F'^{(g)}$)

1. Calculate $X_{n-1}^{(g)} X_{n-1}'^{(g)}$

2. Calculate $A^{(g)} = \{a_{ij}^{(g)}\}$, $a_{ij}^{(g)} = (X_{n(i)}'^{(g)} - X_{n-1}'^{(g)} \hat{F}_i'^{(g)})'(X_{n(j)}'^{(g)} - X_{n-1}'^{(g)} \hat{F}_j'^{(g)})$

   where $\hat{F}_i'^{(g)} = (X_{n-1}^{(g)} X_{n-1}'^{(g)})^{-1} X_{n-1}'^{(g)} X_{n(i)}'^{(g)}$

3. Generate $w \sim W_p(X_{n-1}^{(g)} X_{n-1}'^{(g)}, n-p)$

4. Generate $Z = (z_1', z_2', \cdots, z_p')'$, $z_k \overset{iid}{\sim} N_p(0, A^{(g)})$

5. Generate $F'^{(g)} = ((w^{\frac{1}{2}})')^{-1} Z$

6. $g = g+1$, if $g < m$ go to step 2.

Step 4 (Prediction)

Then we have $\{X^{(1)}, \cdots, X^{(m)}\}$. At last, we will have the estimator

$$\hat{X} = \frac{1}{k} \sum_{g=m-k}^{m} X^{(g)} \quad \text{and} \quad \hat{F}' = \frac{1}{k} \sum_{g=m-k}^{m} F'^{(g)} \quad (m > k)$$

4 Numerical example

4.1 Data Collection

The data we analyze are collected from Tuchen (土城) to Jhulin (竹林) of northbound of National Highway No.3, a section of 60 kilometer-long, on Feb. 12, 2002 from 6:00 AM to 13:00 PM to study the traffic of morning peak hour and its effect. There are eight intersections, each with detectors on both the mainline and the on-and off-ramps recording the speed of vehicles, counts, number of vehicles that pass over the detector, and occupancy, the fraction of time that some vehicle is detected. See Figure 1 for the map of the Northern Area Network and Facilities Layout of National Highway.

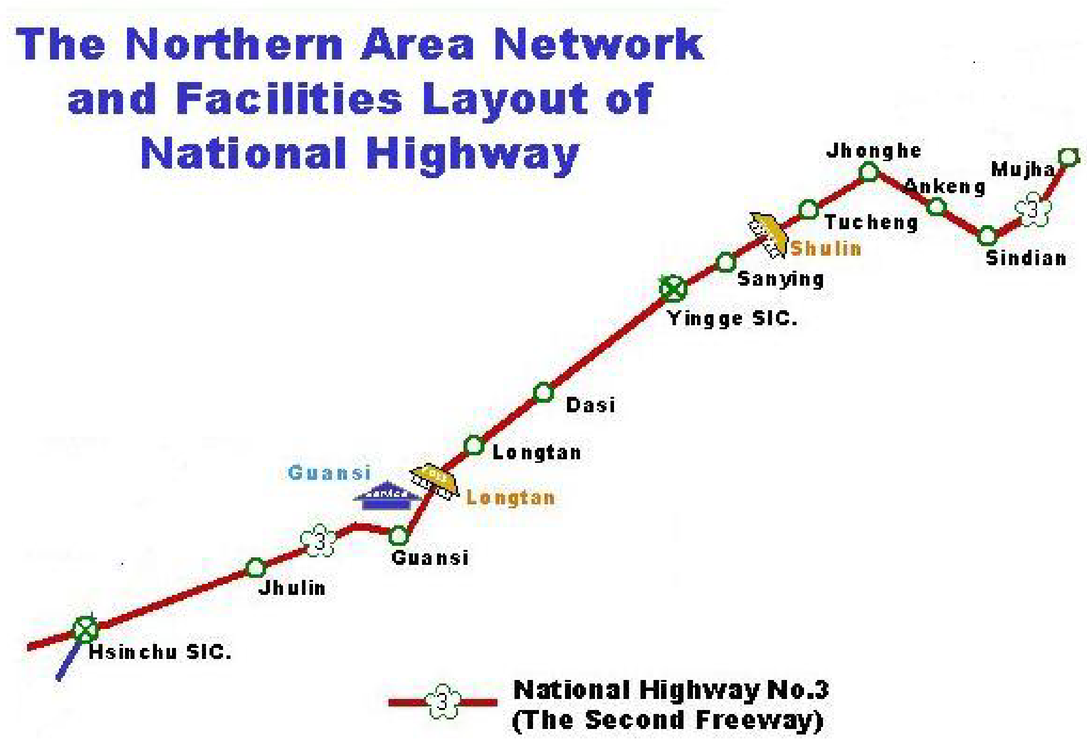

Figure 1: Part of the Northern Area Network of National Highway

Figure 2 shows the schematic map of the study area, the vertical lines indicate the sensor location (ramp location and mainstream location). And the black and gray arrows are the on-ramps (origin) and the off-ramps (destination), respectively, in this area.

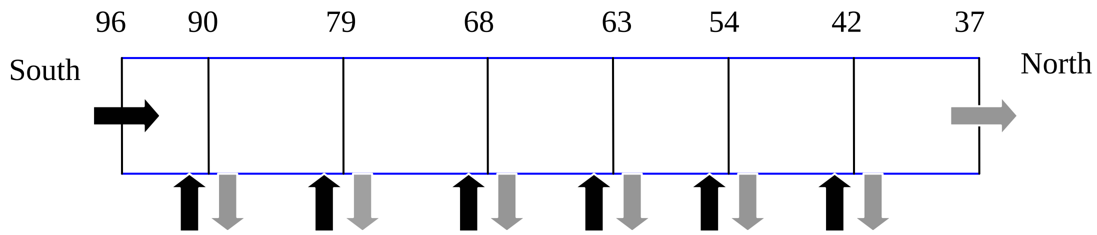

Figure 2: Dynamic system of data source.

Therefore, as shown in Figure 2, there are seven subsections separated by the milepost 90 (Jhulin,竹林), 79 (Guansi,關西), 68 (Longtan,龍潭), 63 (Dasi,大溪), 54

(Yingge,鶯歌), 42 (Tuchen,土城). To make the study area a closed system, we add the vehicles entering the freeway at 96 kilometer location as O_1 and the vehicles exiting the freeway at 37 kilometer as D_7. The detectors are of double-loop type with twenty-second time resolution, but the data we have are aggregated to five minute averages by Taiwan Area National Freeway Bureau (TANFB). The counts obtained from an on-ramp represents the number of vehicles start from that location, denoted by O_i, the counts obtained from an off-ramp is the number of vehicles go to that location, denoted by D_j. O_i's and D_j's are the observations we will use.

Table 1 is the data types we get from the mainstream and we choose the speed data to calculate the travel time and choose the flow at 37 kilometer as the downstream flow and the flow at 96 kilometer as the upstream flow to balance the system. Table 2 is the data types we get from the ramps and we choose the flow data as our observed O_i, D_j

Table 1: The data recorded by the detectors on the mainstream.

Date	Location	Lane	Time	occupancy	interval	Speed	flow
212	2N37	4	00:00	4	201	89	144
212	2N37	4	00:05	3	263	89	54
212	2N37	4	00:10	5	233	87	129
212	2N37	4	00:15	4	231	87	126
212	2N37	4	00:20	4	226	91	131

Table 2: The data recorded by the detectors at ramps.

Date	Location	Time	Lane	Interval	Speed	Flow
212	IC28-2	00:00	1	402	68	19
212	IC28-2	00:05	1	426	72	6
212	IC28-2	00:10	1	433	67	13
212	IC28-2	00:15	1	308	75	21
212	IC28-2	00:20	1	526	77	13

Table 3: The relationship of T_{ij} and O_i, D_j

O \ D	D_1	D_2	D_3	D_4	D_5	D_6	D_7
O_1	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}	T_{16}	T_{17}
O_2	0	T_{22}	T_{23}	T_{24}	T_{25}	T_{26}	T_{27}
O_3	0	0	T_{33}	T_{34}	T_{35}	T_{36}	T_{37}
O_4	0	0	0	T_{44}	T_{45}	T_{46}	T_{47}
O_5	0	0	0	0	T_{55}	T_{56}	T_{57}
O_6	0	0	0	0	0	T_{66}	T_{67}
O_7	0	0	0	0	0	0	T_{77}

4.2 Data Analysis

We denote the flow from the i^{th} origin and to the j^{th} destination as O_i and D_j, respectively, for $i, j = 1,2,\cdots,7$ and T_{ij} as the flow between the i^{th} Origin and the j^{th} destination. Since it is northbound data, the O-D flow matrix is upper triangular, table 3 describes the complete relationship between these notations. And here we get the state variable x_t and observed variable y_t as follows:

$$y_t = (O_{1,t}, O_{2,t}, \cdots, O_{7,t}, D_{1,t}, D_{2,t}, \cdots, D_{7,t})'$$

$$x_t = (T_{11,t}, T_{12,t}, \cdots, T_{17,t}, T_{22,t}, T_{23,t}, \cdots, T_{27,t}, \cdots, T_{66,t}, T_{67,t}, T_{77,t})'$$

And we deal with the speed data to evaluate the travel time in each section of freeway. Suppose we measured the speed at each sensor $s_1, s_2, \cdots s_k$ and the distance between each sensor $l_1, l_2, \cdots, l_n$ then we have the average travel time $= \sum_{i=1}^{k} \frac{l_i}{s_i}$. Then we have the average travel time at each section of freeway, so that we can use these information to divide the matrix H into $H_k = \{h_{ij}^k\}, (k = 1,2,\cdots,d)$ with elements $\mathrm{h}_{\mathrm{ij}}^{\mathrm{k}} = \begin{cases} 1, \text{ if } \mathrm{T}_{\mathrm{ij}} \text{ affect } \mathrm{O}_{\mathrm{i}} \text{ or } \mathrm{D}_{\mathrm{j}} \text{ in time k} \\ 0, \text{ otherwise} \end{cases}$, where d is the maximum number of time periods that T_{ij} still affect O_i and D_j. And thus we have all the data to start our computation.

4.3 Flow Prediction

After using the specified model to predict the O-D flow, we denote $\hat{X}_{simple}$ and $\hat{X}_{updated}$ the prediction of traditional and the revised model, respectively. Since the O-D flow matrix is unobservable, we can check the appropriateness only through comparing the predicted O_i, D_j with the observation.

Let $y_t = (O_{1,t}, O_{2,t}, \cdots, O_{7,t}, D_{1,t}, D_{2,t}, \cdots, D_{7,t})'$, then

$$y_{t,simple} = H' x_{t,simple} \quad \text{for } t = 1,2,\cdots,n,$$

$$y_{t,updated} = \sum_{i=0}^{d} H_i' x_{t-i,updated} \quad \text{for } t = 1,2,\cdots,n,$$

where $x_{t,simple}$ and $x_{t,updated}$ are the t^{th} column of $\hat{X}_{simple}$ and $\hat{X}_{updated}$.

In the following there are fourteen figures, each of which describes the patterns of the two predicted flows of both of the models. As we can see that the ups and downs of the observations are in general followed pretty well by both predicted series and one series performs better than the other for different cases, but neither of them is better for all the nodes. For example, in Figure 3 for the first origin, the patterns of the predicted series are similar as that of the observations, although the simple model gives closer predicted values. But for the other origins, the more sophisticated model outperforms the simple one in both the predicted values and the patterns. As to destination nodes, both models show more variation, in some figures the traditional model even shows better patterns like Figure 14 and Figure 16. Note that in the Figure 16, there has an outlier in the observation exceeding 800 vehicles in a five minutes interval, the traditional model faithfully predicts the outlier value and the revised model shows the similar pattern but not the similar value of observation pattern at the price of underestimate at all other time intervals. So it seems that the revised model performs better on the average and the simple model performs better when the outlier happens.

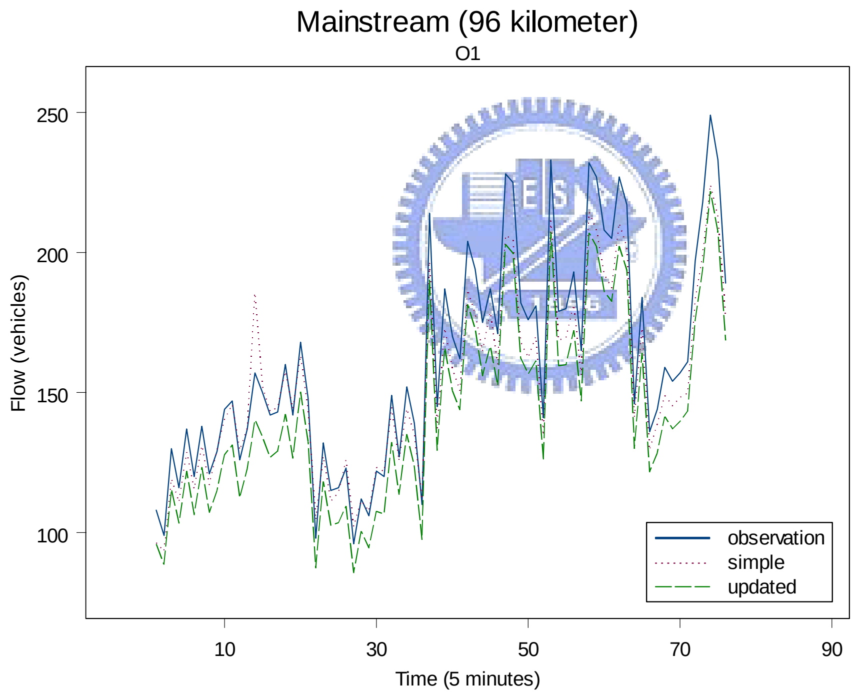

Figure 3: Observed and predicted flow entering the 1^{th} origin.

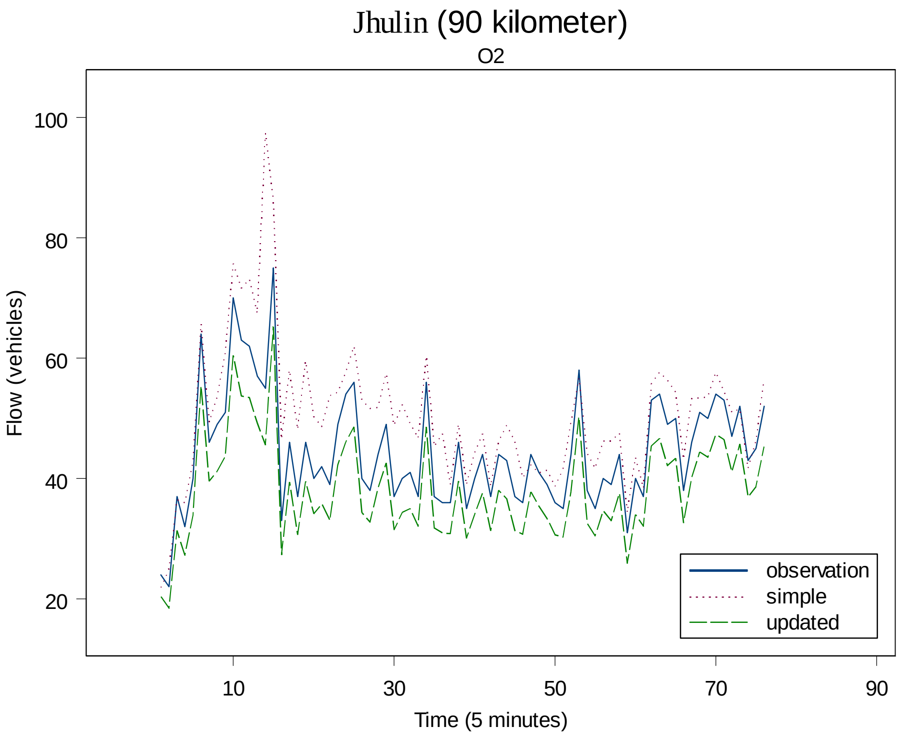

Figure 4: Observed and predicted flow entering the 2^{th} origin.

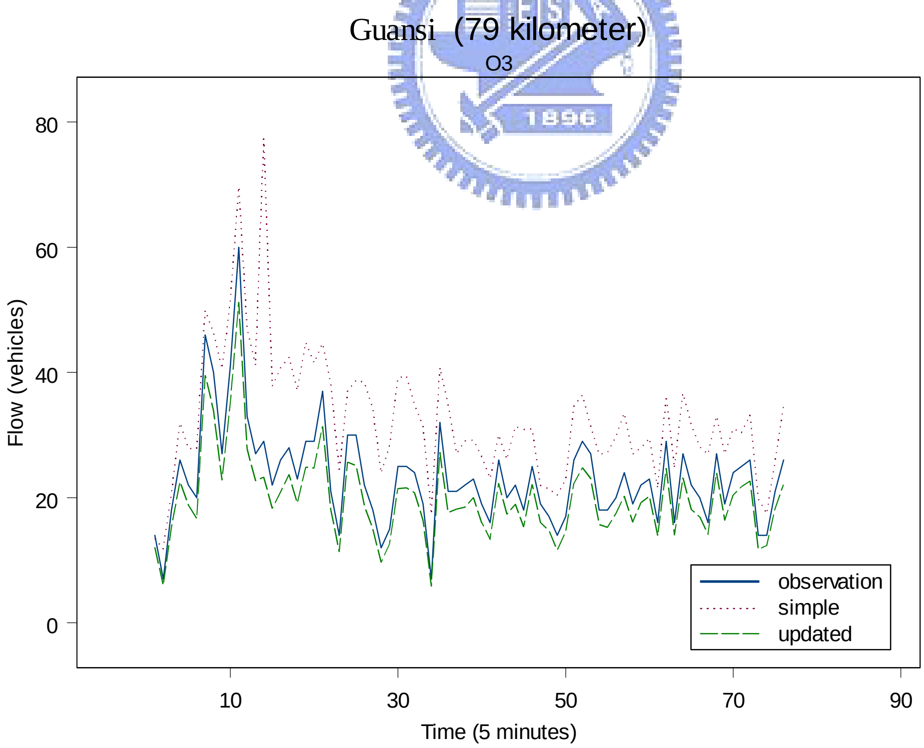

Figure 5: Observed and predicted flow entering the 3^{th} origin.

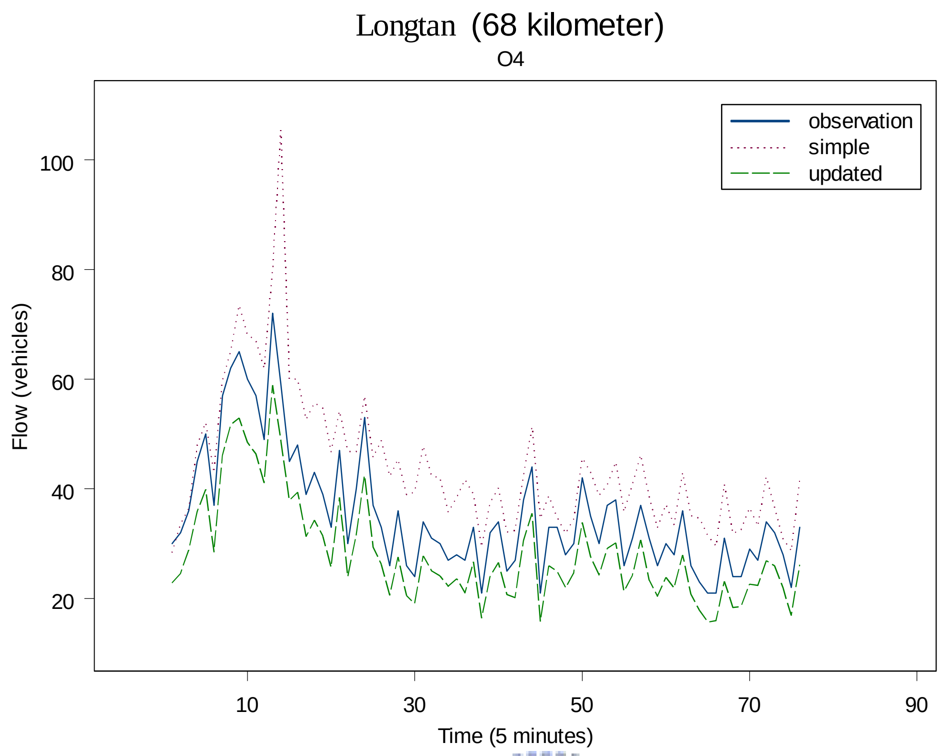

Figure 6: Observed and predicted flow entering the 4^{th} origin.

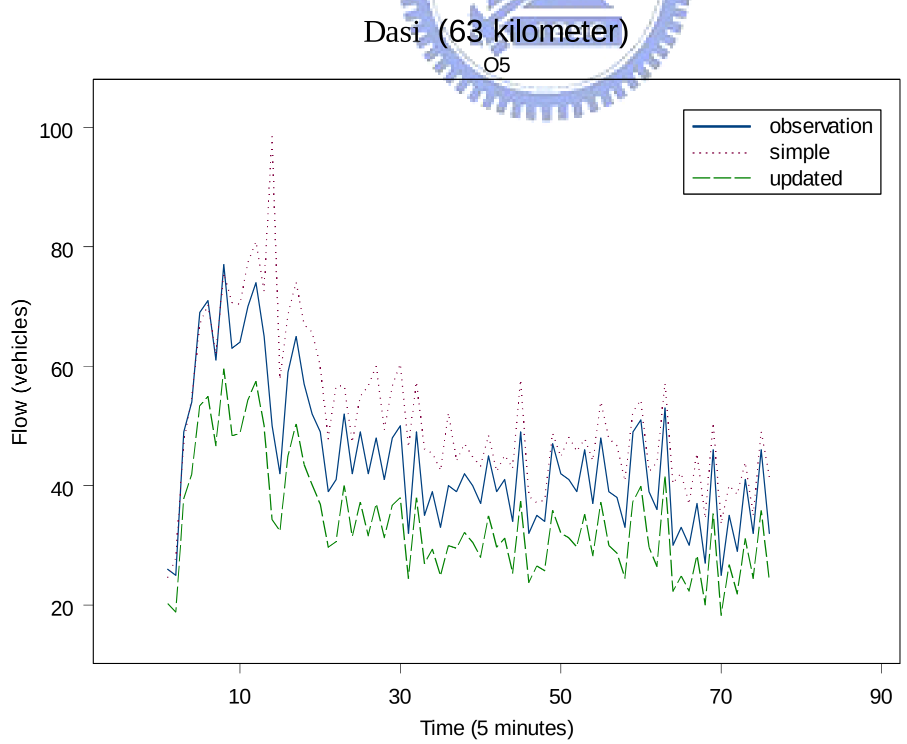

Figure 7: Observed and predicted flow entering the 5^{th} origin.

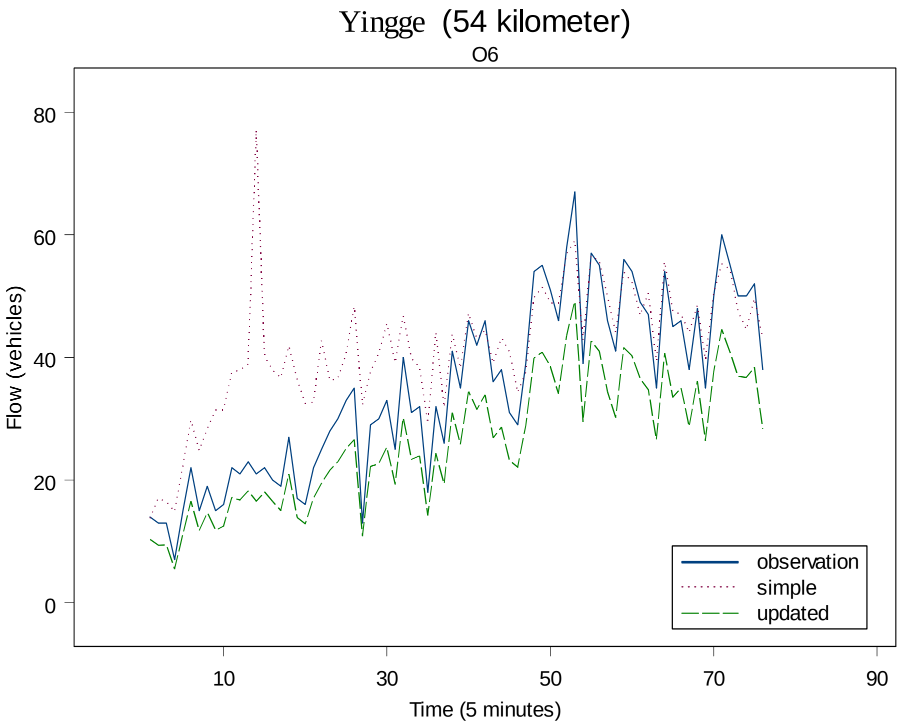

Figure 8: Observed and predicted flow entering the 6^{th} origin.

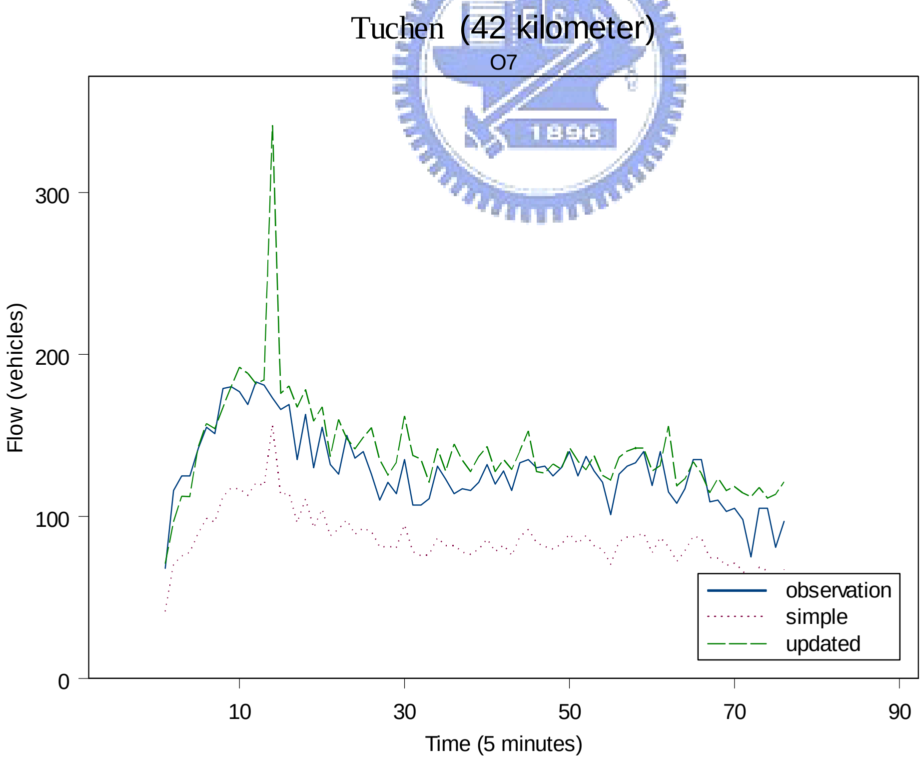

Figure 9: Observed and predicted flow entering the 7^{th} origin.

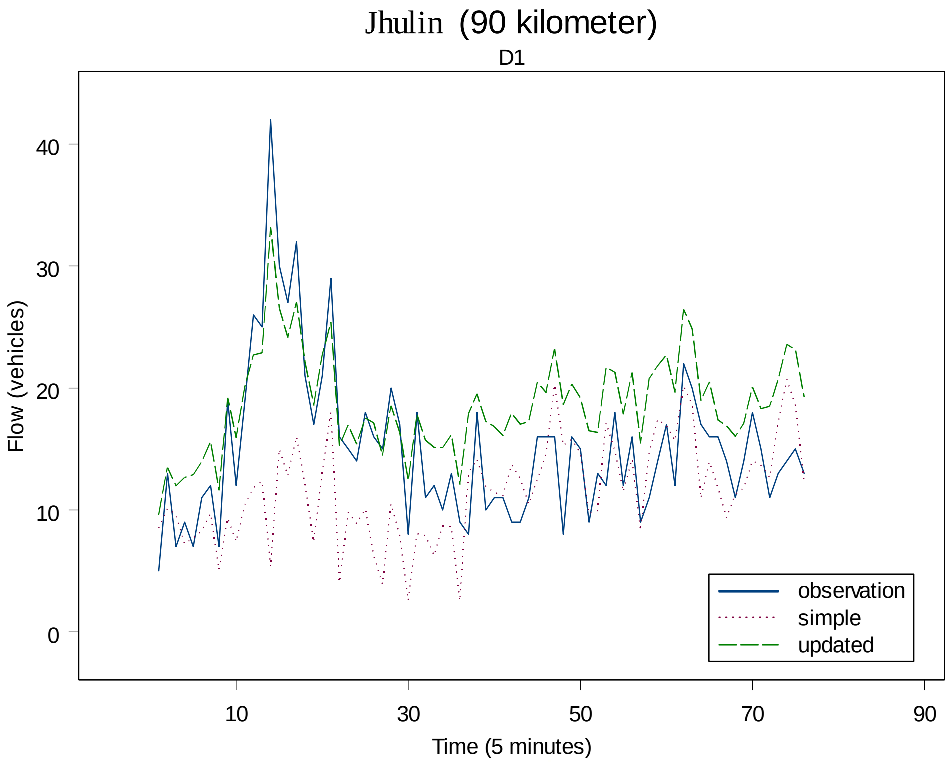

Figure 10: Observed and predicted flow exiting the 1^{th} destination.

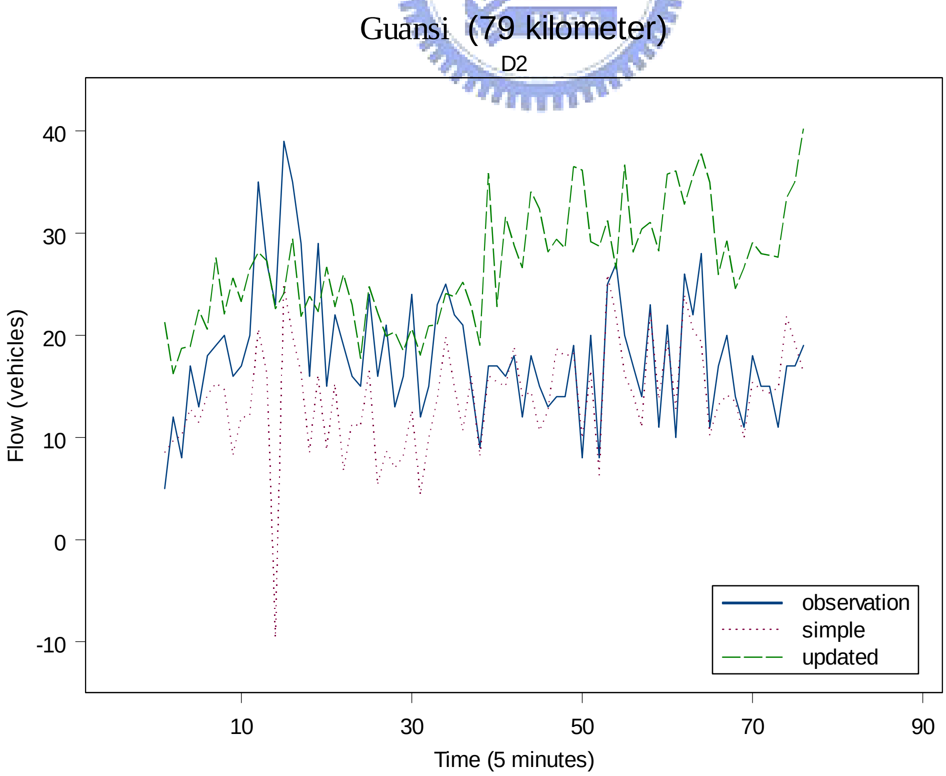

Figure 11: Observed and predicted flow exiting the 2^{th} destination.

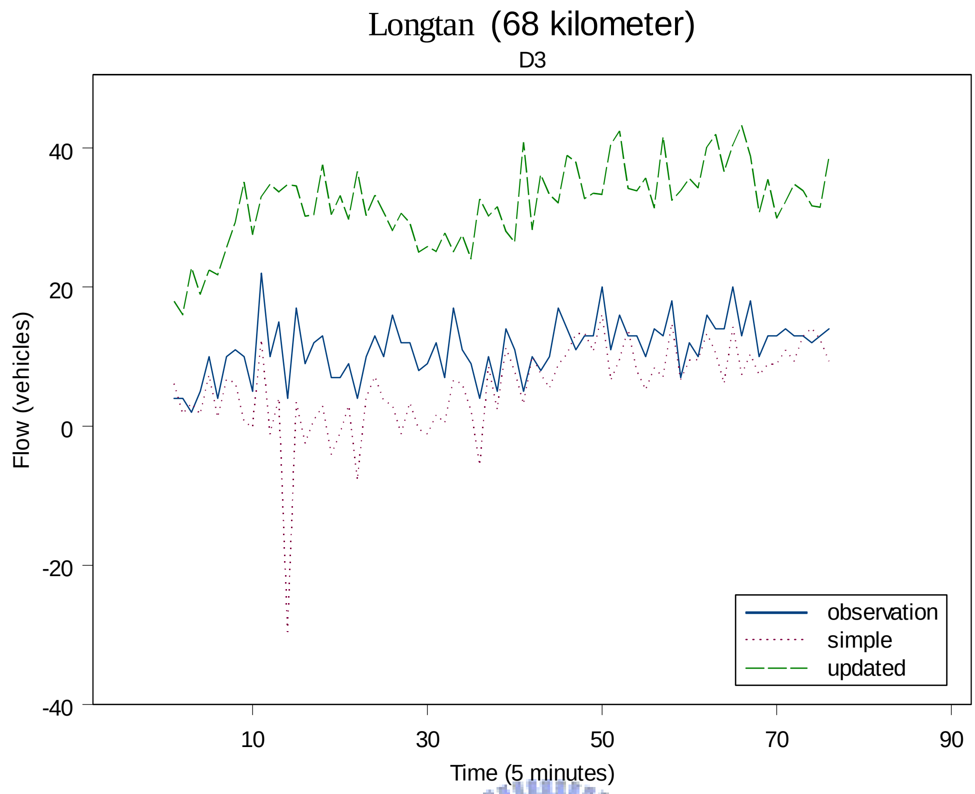

Figure 12: Observed and predicted flow exiting the 3^{th} destination.

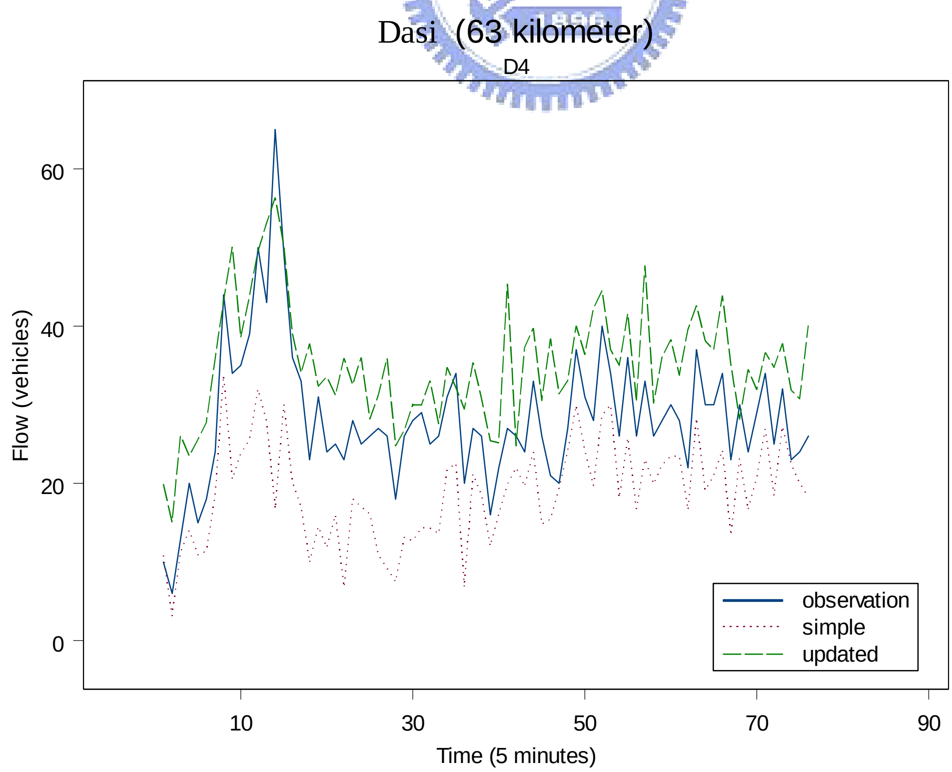

Figure 13: Observed and predicted flow exiting the 4^{th} destination.

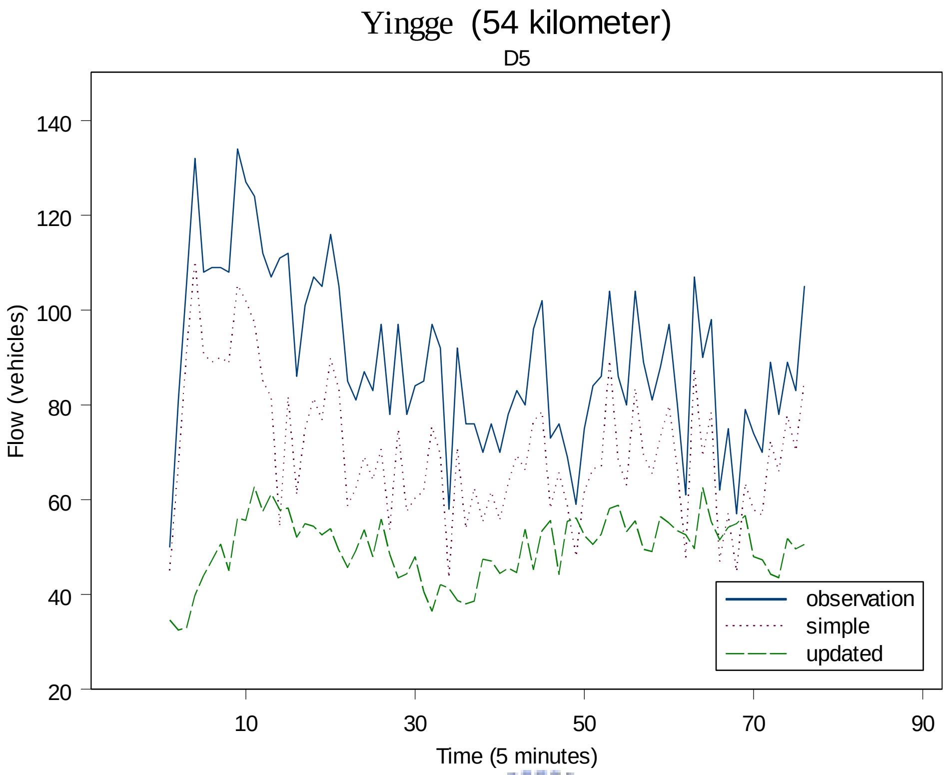

Figure 14: Observed and predicted flow exiting the 5^{th} destination.

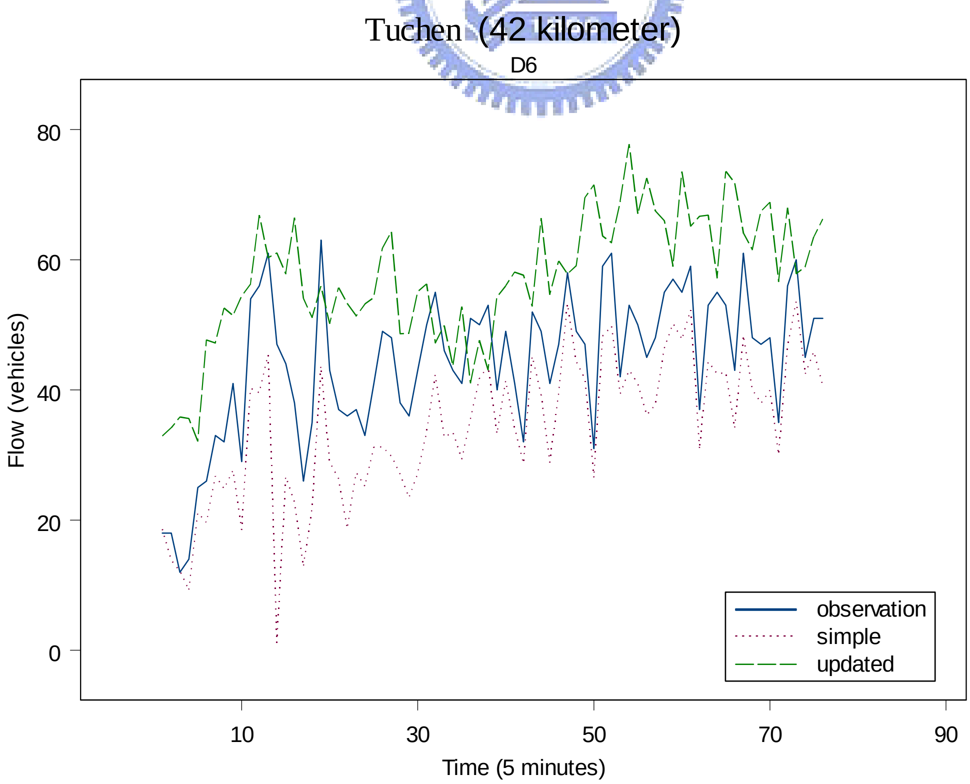

Figure 15: Observed and predicted flow exiting the 6^{th} destination.

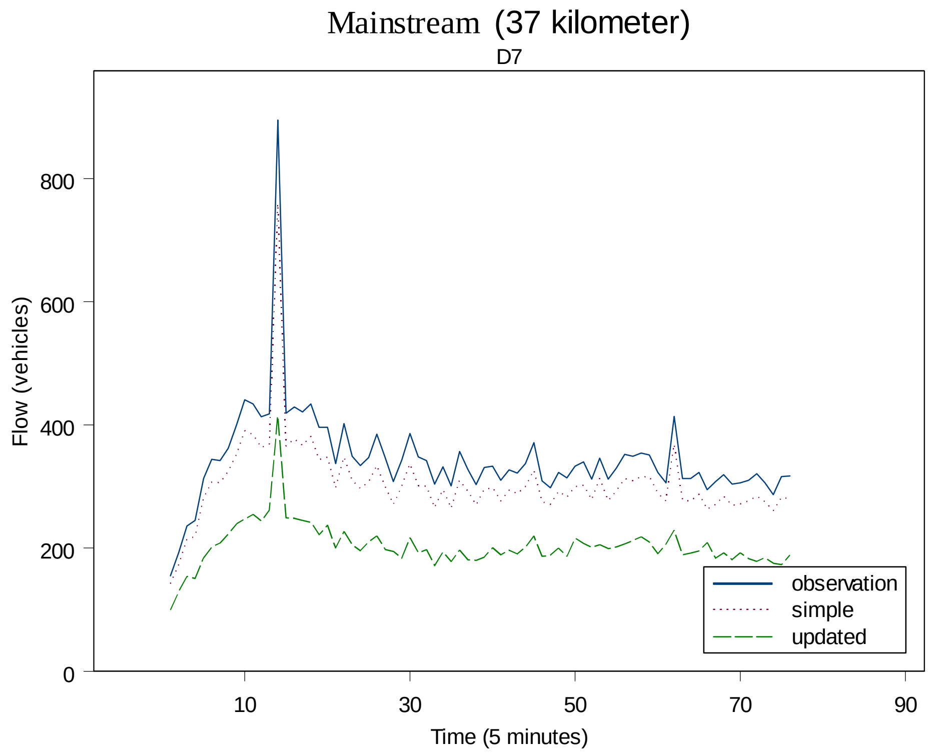

Figure 16: Observed and predicted flow exiting the 7^{th} destination.

Let $T_{ij} = \left(T_{ij}^1, T_{ij}^2, \cdots T_{ij}^t, \cdots, T_{ij}^n\right)$ be the flow entering the i^{th} origin and exiting j^{th} destination during each time period. We will compare the two patterns $T_{ij,simple} = (T_{ij,simple}^1, \cdots, T_{ij,simple}^n)$ and $T_{ij,updated} = (T_{ij,updated}^1, \cdots, T_{ij,updated}^n)$ which are the same row of the $\hat{X}_{simple}$ and $\hat{X}_{updated}$. There are totally twenty-eight O-D pairs. Here we choose some more interesting figures meaningful to discuss.

Most figures show that both patterns behave similarly as in Figures 17 and 18. But when the flows are low, less than 20, say, the simple method behaves badly as to give large negative numbers, and the updated method seldom to give unreasonable results of this kind, see Figure 19 and 20. In the first O_1 and the last D_7, Figure 21 and Figure 22 show the large difference between the two models. The simple model gives big spikes without any signs in the data, the only reason we can give is that the nodes for these two figures are the boundaries of the study area and all the vehicles entering or exiting the system were assigned to O_1 or D_7, respectively.

Comparing from the figures of O_i, D_j and T_{ij} we find that most figures have similar pattern for these three series. But from the figures of T_{ij}, the simple pattern have too many negative points. Since the flow is always positive, we suggest that the model considering the travel time is better than the simple model.

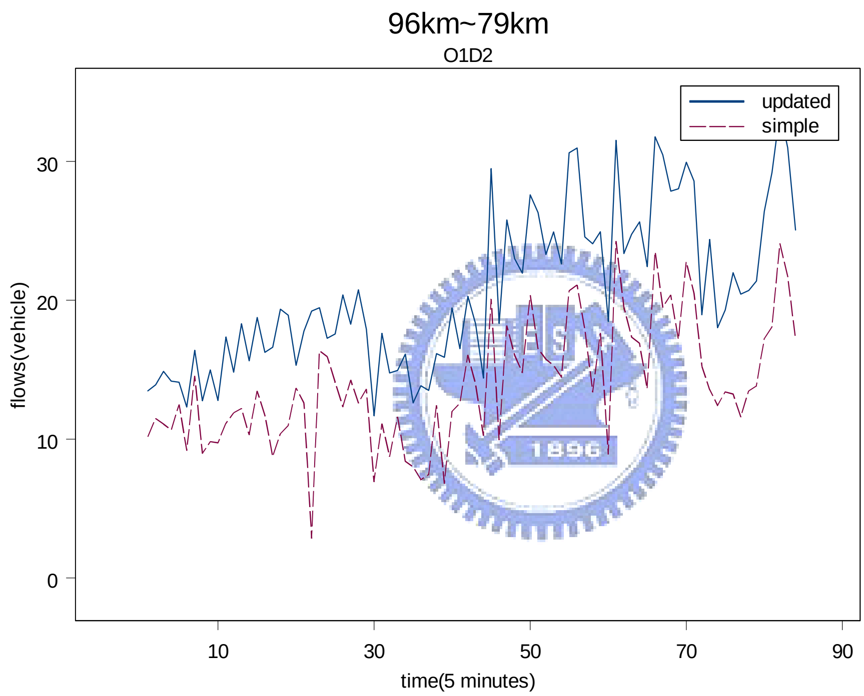

Figure 17: Predicted flow between the upstream and Gaunsi.

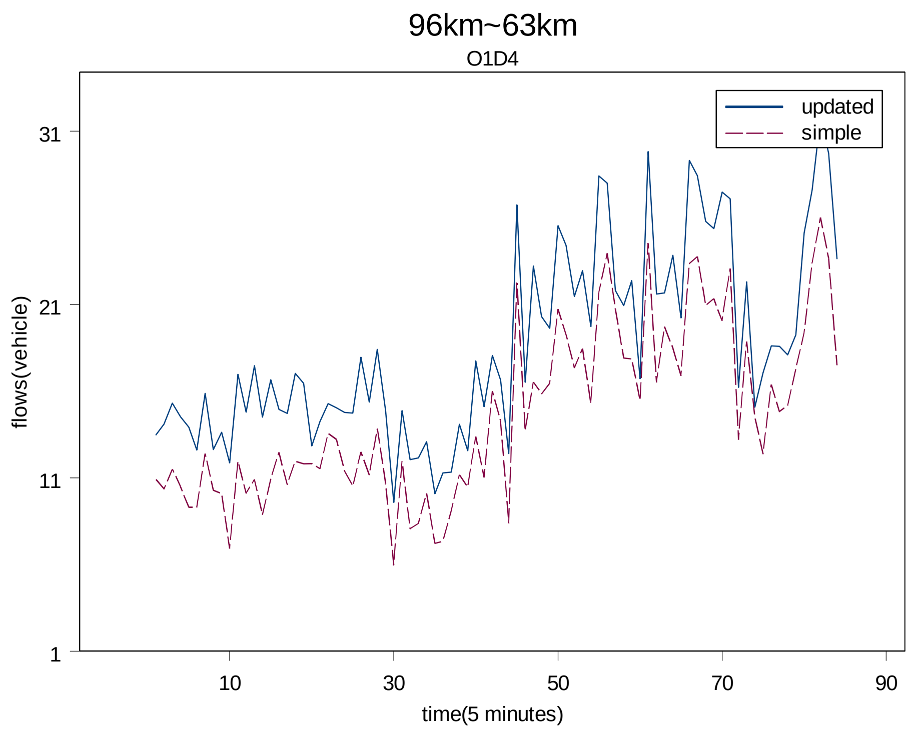

Figure 18: Predicted flow between the upstream and Dasi.

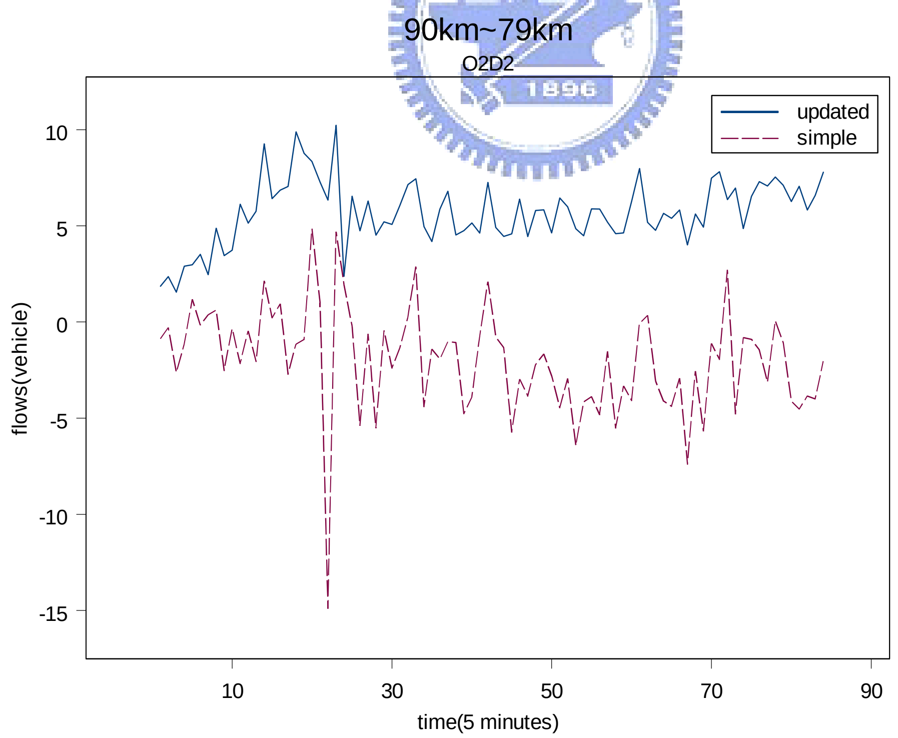

Figure 19: Predicted flow between Jhulin and Gaunsi.

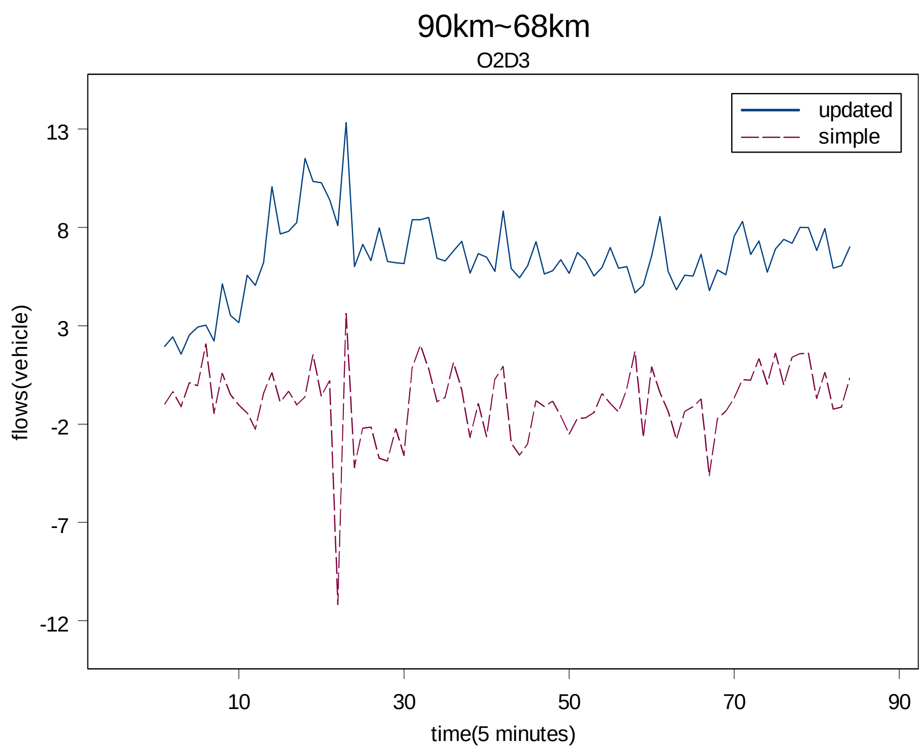

Figure 20: Predicted flow between Jhulin and Dasi destination.

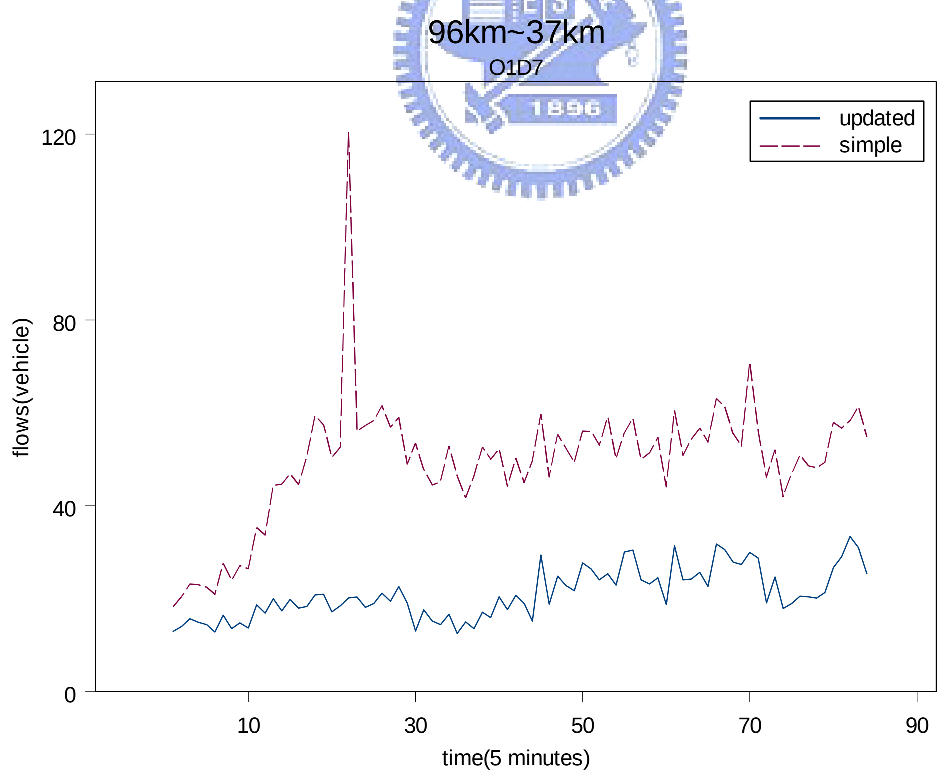

Figure 21: Predicted flow between the upstream and downsteram.

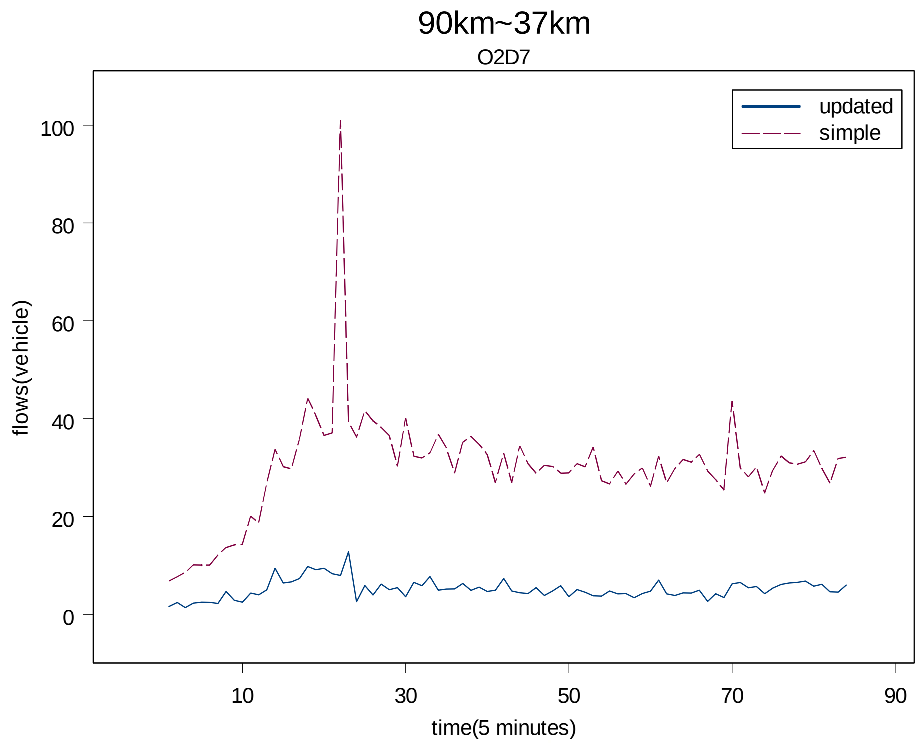

Figure 22: Predicted flow between Gaunsi and downstream.

5 Conclusion

In this thesis we propose to incorporate the effect of travel time into the Gaussian state space model and developed an algorithm to estimate the unknown transition matrix and forecast the O-D flow matrix simultaneously. By doing so the performance of the model improved in different aspects. In the most figures of flow prediction, we find that the trends of flows have much different behavior in the different time interval. This means that we should assume that the matrix F varies with time. In other words, when using this model to estimator O-D flow, the total studying time period should not be too long in order not to let the pattern of O-D flow be unreasonable. In the future work, we could extend matrix F to vary with time or depend on some exogenous factors to improve the ability in describing the reality of the model. Non-Gaussian distribution is also an alternative for the distribution of the error terms.

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