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Reliability formulation for composite laminates subjected to first-ply failure

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Methods formulated on the basis of the concept of first-ply failure and the structural reliability theory are presented for the reliability analysis of laminated composite plates. In the reliability formulation, an appropriate phenomenological failure criterion is used to establish the limit state equation of the laminated composite plates, and different numerical techniques are employed to evaluate the reliability of the plates. Experimental investigations of lamina strengths and first-ply failure loads of laminated composite plates were performed. Baseline probability distributions of lamina strength parameters constructed from the test data are used to study the reliability of the laminated plates. The accuracy of the proposed models in reliability assessment of the laminated plates are verified by the experimental results on first-ply failure load distributions. © 1997 Elsevier Science Ltd.

INTRODUCTION

Laminated composite plates are important components in the construction of aircrafts, automobiles, and mechanical and marine structures. In general, these structures are operated in severe environments and subjected to complex loadings. To ensure no sudden catastrophe for the structures, the reliability of the structures must be thoroughly investigated before use. In order to have a meaningful reliability evaluation, realistic reliability models must be adopted in the reliability analysis of the laminated composite structures. Recently, a number of researchers have proposed different methods for studying the reliability of composite laminates [1-6]. For instance, Kam *et al.* [5] presented a load space formulation technique for the reliability analysis of laminated composite plates. In the previous reliability studies of laminated composite plates, however, only the theoretical aspect was considered and no experimental data were presented to verify the accuracy of their proposed methods.

In this paper, methods formulated on the basis of the first-ply failure concept are presented for the reliability analysis of laminated composite plates with random strength parameters. Phenomenological failure criteria are used to construct the limit state equation of the laminated composite plates. Different numerical techniques are adopted to derive the reliability of the laminated composite plates from the probability distributions of the lamina strength parameters. The feasibility and accuracy of the proposed methods are validated by the experimental data.

FAILURE ANALYSIS OF LAMINATED COMPOSITE PLATES

Stress analysis of a laminated composite plate is accomplished via the finite-element method, which is constructed on the basis of the first-order shear-deformation theory [7]. The element contains five degrees of freedom (three displacements and two shear rotations) per node. In the evalution of the element stiffness matrix, a nine-node Lagrangian element with reduced integration using the 2×2 Gauss rule is adopted. Stresses at node points of an element are determined from those at the integration points via the extrapolation method. Five independent stress components at any point in the laminated plate are considered in the finite-element analysis. The first-ply failure load of the laminated composite plate is defined as the strength of the plate. The first-ply failure analysis of the laminated plate is performed via the use of a phenomenological failure criterion. Currently, there are a number of phenomenological failure criteria available for the failure analysis of composite laminates [8]. In general, the failure criteria can be classified into two categories, namely, independent and dependent failure criteria. For instance, maximum stress and Tsai–Wu failure criteria belong to the categories of independent and dependent failure criteria, respectively. Herein, maximum stress and Tsai–Wu criteria are adopted in the first-ply failure analysis of laminated composite plates. The maximum stress criterion states that the ratios of stresses in the principal material directions to the respective strengths must be less than 1, otherwise failure is said to have occurred, that is

$$R_i = \frac{\sigma_i}{X_i} < 1 \ (i = 1, 2, 4, 5, 6) \qquad (1)$$

where R_i are stress ratios; σ_1 and σ_2 are normal stress components; σ_4, σ_5 and σ_6 are shear stress components; X_1 and X_2, are the lamina normal strengths in the 1, 2 directions; and X_4, X_5 and X_6 are the shear strengths in the 23, 13 and 12 planes, respectively; $X_5 = X_6$. When σ_1, σ_2 are of a compressive nature they should be compared with X_{1C}, X_{2C} which are normal strengths in compression along the 1, 2 directions, respectively. The Tsai–Wu criterion can be expressed as

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j \geq 1 \qquad (2)$$

with

$$F_1 = \frac{1}{X_T} - \frac{1}{X_C}; \quad F_2 = \frac{1}{Y_T} - \frac{1}{Y_C}$$

$$F_{11} = \frac{1}{X_T X_C}; \quad F_{22} = \frac{1}{Y_T Y_C}$$

$$F_{44} = \frac{1}{R^2}; \quad F_{55} = \frac{1}{S^2}$$

$$F_{12} = \frac{1}{2\sqrt{X_T X_C Y_T Y_C}} \qquad (3)$$

where $X_T = X_1$, $Y_T = X_2$, $R = X_4$, $S = X_5$, $X_C = X_{1C}$ and $Y_C = Y_{2C}$.

RELIABILITY FORMULATION

In the reliability formulation for laminated composite plates strength parameters of the constituent composite laminae are treated as independent baseline random variables. The constituent laminae of the laminated composite plates are assumed to possess the same material properties and strength parameters. Herein, the reliability model for composite laminates with random strength parameters subjected to first-ply failure is formulated on the basis of the structural reliability theory. The failure probability, P_f, of a composite laminate is expressed as

$$P_f = \int \cdots \int_{g>0} f_{X_i}(x_i) \cdots f_{X_q}(x_q)\, dx_i \cdots dx_q \qquad (4)$$

where g is the limit state equation of the laminated plate; X_i are independent random strength parameters; $f_{X_i}(x_i)$ are baseline probability density functions; and the integration is performed over the failure region, $g > 0$. In general, the limit state equation represents a surface in the strength space where the surface separates the survival and failure regions. Figure 1 shows the limit state curve, $g(x_1, x_2) = 0$, in the strength plane. Herein, the limit state equation of the plate is constructed using either the maximum stress or the Tsai–Wu failure criteria.

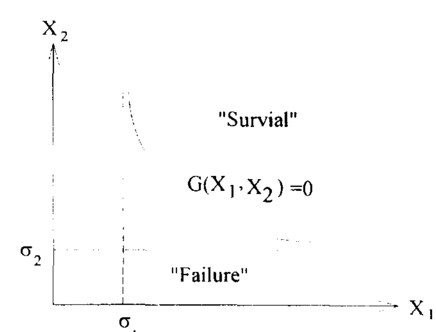

Fig. 1. Two-dimensional limit state curve constructed on the basis of dependent criteria.

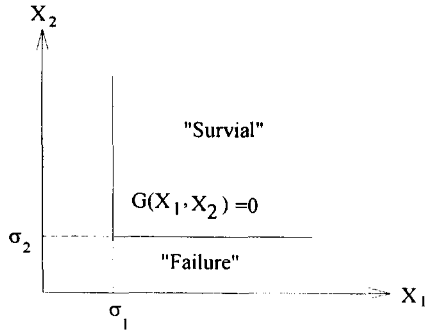

Fig. 2. Two-dimensional limit state curve constructed on the basis of maximum stress criterion.

Maximum stress criterion (independent)

In view of eqn (1) the limit state equation is expressed as

$$g = \prod_{i=1, i \neq 3}^{6} (R_i - 1) = 0 \tag{5}$$

where $\prod$ is the notation of multiplication. For the two-dimensional case, the limit state curve in the strength plane is shown in Fig. 2. In view of eqn (4), the reliability of the laminated plate, P_S, is written as

$$P_S = \prod_{i=1, i \neq 3}^{6} \int_{\sigma_i}^{\infty} f_{x_i}(x_i)\,\mathrm{d}x_i \tag{6}$$

where σ_i are the largest stress components in the laminated plate. Equation (6) can be solved easily via the use of numerical integration.

Tsai–Wu criterion (dependent)

In this model, the stress state of the most critical point in the laminated plate is used to construct the limit state equation of the plate. In view of eqn (2), the limit state equation is expressed as

$$g = F_i\sigma_i + F_{ij}\sigma_i\sigma_j - 1 = 0 \tag{7}$$

Unlike the independent failure criteria, the solution of eqn (4) will be difficult if not untractable for dependent failure criteria. Herein, the modified β-method [9] is used to evaluate the reliability of the plate.

Table 1. Statistics of strength parameters

Strength parameter	Mean value (MPa)	Coefficient of variation (CV) (%)
$X_1 = X_T$	1537.2	2.1
$X_2 = Y_T$	42.7	6.3
$X_4 = R$	79.67	5.7
$X_5 = T$	102.42	5.7
$X_6 = S$	102.42	5.7
$X_{1C} = X_C$	1722.1	2.1
$X_{2C} = Y_C$	213.95	6.3

EXPERIMENTAL VERIFICATION

Experiments of centrally loaded laminated composite square plates of length $a = 100$ mm and ply thickness $h_i = 0.121$ mm were performed to verify the accuracy of the proposed reliability models. The laminated composite plates under consideration were made of graphite–epoxy (Q-1115) perpreg tapes supplied by the Toho Co., Japan. The properties of the composite material were determined from experiments conducted in accordance with the relevant ASTM standards [10] and their mean values are given as: $E_1 = 139.4$ GPa, $E_2 = 7.65$ GPa, $G_{12} = G_{13} = 4.35$ GPa, $G_{23} = 1.02$ GPa and $\nu_{12} = 0.29$.

The statistics of the lamina strength parameters are listed in Table 1spherical head and a fixture for clamping the specimen. The fixture

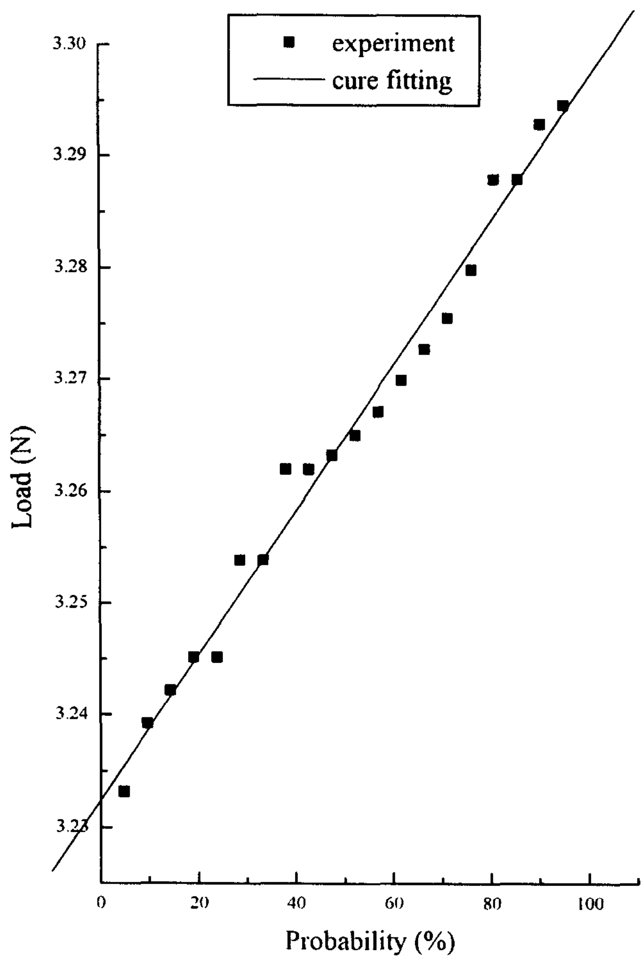

Fig. 3. First-ply failure load data of the $[0°/90°_2/0°_9]$ plate fitted by lognormal distribution.

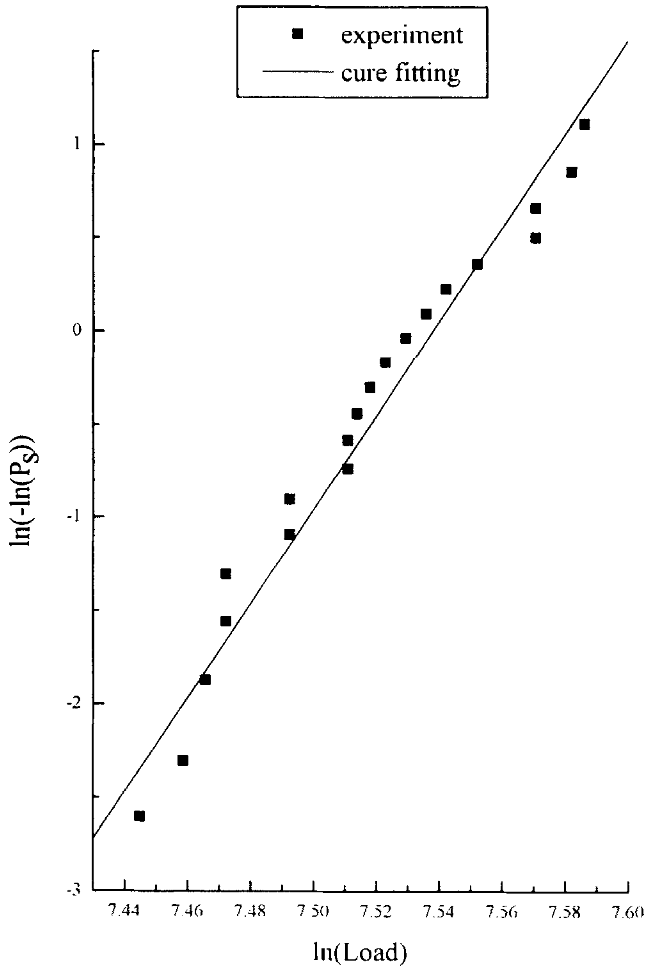

Fig. 4. First-ply failure load data of the $[0°/90°_2/0°_9]$ plate fitted by Weibull distribution.

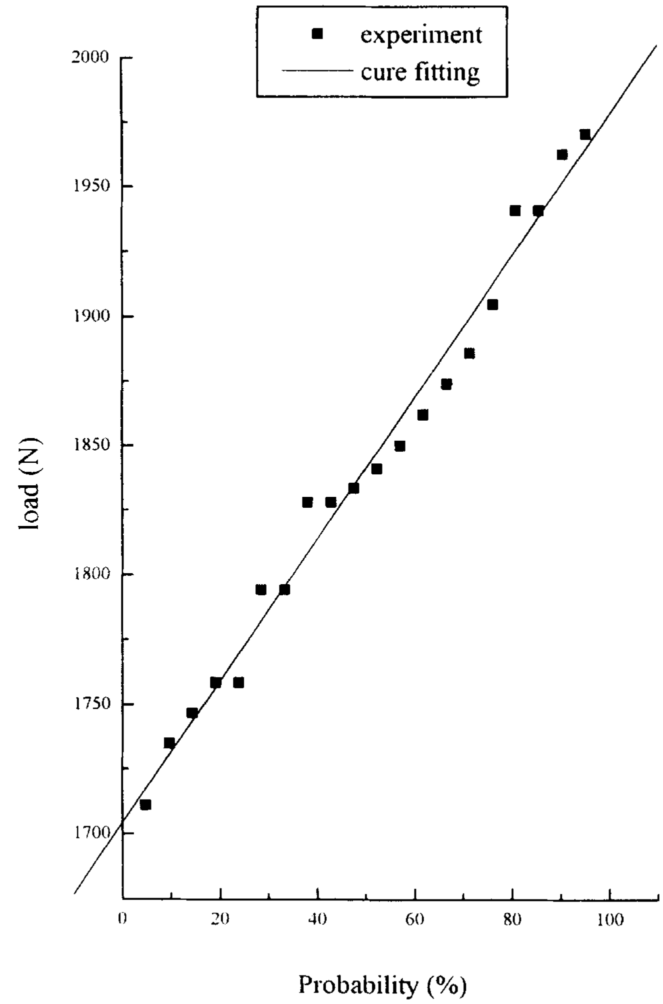

Fig. 5. First-ply failure load data of the $[0°/90°_2/0°_9]$ plate fitted by normal distribution.

Table 2. Statistical parameters of experimental probability distributions of first-ply failure load

Plate	Normal		Weibull		Lognormal	
	Mean $\bar{P}$	σ_P	Scale parameter	Shape parameter	$E[\ln P]$	$\sigma_{\ln P}$
$[0°/90°_2/0°_6]_S$	1841.1	75.7	1878.14	25.1877	3.26	0.0178
$[0°_6/90°_6]_S$	1207.7	65.6	1240.53	18.493	3.08	0.0237
$[45°/-45°_2/45°_6]_S$	2091.6	117.88	2148.21	18.4856	3.32	0.0248
$[45°_6/-45°_6]_S$	1296.3	72.93	1331.58	18.3931	3.11	0.0246

Note: σ = standard deviation; $E[\ ]$ = expected value.

Table 3. Experimental plate reliabilities derived from various probability distributions

Plate	$P(N)$	Reliability		
		Normal	Weibull	Lognormal
$[0°/90°_2/0°_6]_S$	1725	0.9241	0.8914	0.9376
$[0°_6/90°_6]_S$	1125	0.9306	0.8992	0.9344
$[45°/-45°_2/45°_6]_S$	1900	0.9529	0.9018	0.9639
$[45°_6/-45°_6]_S$	1175	0.9638	0.9047	0.9762

Table 4. Theoretical plate reliabilities derived from various baseline probability density distributions

Plate	$P(N)$	Independent reliability model			Dependent reliability model		
		Normal	Weibull	Lognormal	Normal	Weibull	Lognormal
$[0°/90°_2/0°_6]_S$	1725	0.876	0.8232	0.8495	0.851	0.817	0.827
$[0°_6/90°_6]_S$	1125	0.9233	0.865	0.904	0.904	0.843	0.871
$[45°/-45°_2/45°_6]_S$	1900	0.9362	0.8758	0.9173	0.9333	0.8747	0.9095
$[45°_6/-45°_6]_S$	1175	0.9493	0.889	0.933	0.9207	0.862	0.8945

was made up of two square steel frames. During testing the laminated plate was clamped using the two steel frames, which were connected together by four bolts. A stroke control approach was adopted in constructing the load–deflection relation for the laminated plate. The loading rate was slow enough for inertia effects to be neglected. During loading, two acoustic emission sensors were used to measure the stress waves released at the AE sources in the laminated plate. The measured acoustic emissions were converted by the AMS3 (AE) system to a set of signal describers such as peak amplitude, energy, rise time and duration, which were then used to identify the first-ply failure load of the laminated plate [7].

RESULTS AND DISCUSSION

First-ply failure load data of laminated composite plates with different lamination arrangements, namely $[0°/90°_2/0°_9]_S$, $[0°_6/90°_6]_S$ $[45°/-45°_2/45°_9]_S$ and $[45°_6/-45°_6]_S$, obtained from experiments are fitted by various probability distributions via the probability papers. For instance, Figs 3–5 show the first-ply failure load data of the $[0°/90°_2/0°_9]_S$ plate fitted by normal, Weibull and lognormal distributions, respectively. It is noted that Weibull distribution can yield the most conservation values for plate reliability when small failure probability, e.g. $P_f<0.1$, is considered. The statistical parameters of the experimental probability distributions of the first-ply failure loads for the laminated plates are listed in Table 2. The experimental reliability of the laminated composite plates, which are subjected to a center point load, P, of different magnitudes, derived from various probability distributions are listed in Table 3. It should be noted that the $[45°/-45°_2/45°_9]_S$ plate, which has been optimally designed, yields the highest reliability. The reliability of the laminated composite plates is also determined using the aforementioned reliability models and baseline probability density functions. Table 4 lists the theoretically predicted reliabilities for the plates with different lamination arrangements. The differences between the experimental and theoretical plate reliabilities are given in Table 5. It is noted that in general the differences between the experimental and theoretical results are small (less than 12%) irrespective to the types

Table 5. The difference between theoretical and experimental reliabilities of composite plates[1]

Plate			Independent reliability model			Dependent reliability model		
Lay-up	$P(N)$	Experiment distribution	Normal	Weibull	Lognormal	Normal	Weibull	Lognormal
$[0°/90°_2/0°_6]_S$	1725	Normal	5.2	10.9	8.1	7.9	11.6	10.5
		Weibull	1.7	7.6	4.7	4.5	8.3	7.2
		Lognormal	6.5	12.2	9.4	9.2	12.8	11.8
$[0°_6/90°_6]_S$	1125	Normal	0.8	7.0	2.8	2.8	9.4	6.4
		Weibull	2.7	3.8	0.5	0.5	6.3	3.1
		Lognormal	1.2	7.4	3.2	3.2	9.8	6.8
$[45°/-45°_2/45°_6]_S$	1900	Normal	1.7	8.1	3.7	2.0	8.2	4.5
		Weibull	3.8	2.9	1.7	3.5	3.0	0.8
		Lognormal	2.8	9.1	4.8	3.1	9.2	5.6
$[45°_6/-45°_6]_S$	1175	Normal	1.5	7.8	3.2	4.5	10.6	7.2
		Weibull	4.9	1.7	3.1	1.7	4.7	1.1
		Lognormal	2.7	8.9	4.4	5.7	11.7	8.3

[1]Difference $= \left(\dfrac{\text{experiment} - \text{theory}}{\text{experiment}}\right)\%$.

of probability distributions used in modeling the distributions of the lamina strength parameters and first-ply failure load. In particular, when the baseline probability density functions are normal and the distribution of first-ply failure load is Weibull, the differences can be less than 5% for the laminated composite plates irrespective to the method used in the reliability analysis.

CONCLUSIONS

Different methods were presented for the reliability assessment of laminated composite plates. The methods were constructed on the basis of the concept of first-ply failure and the structural reliability theory. The feasibility and accuracy of the present methods were validated by the experimental distributions of first-ply failure loads of laminated composite plates with different lamination arrangements. The effects of different types of baseline probability density functions on the system failure probability of laminated composite plates were studied. It was found that the use of Weibull distribution in the reliability design of laminated composite plates could yield more conservative results. The use of normal distribution for modeling the lamina strength parameters in the reliability analysis could yield very accurate results for the plates when their first-ply failure loads were modeled as Weibull variates. Both analytical methods are suitable for the reliability analysis of laminated composite plates.

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