Refractive-index measurement based on the effects of total internal reflection and the uses of heterodyne interferometry

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A new method for measuring the refractive index is presented. First, the phase difference between s and p polarizations that is due to the total internal reflection is measured by heterodyne interferometry. Then, substituting this phase difference into the Fresnel equations, we can obtain the refractive index of the test medium.

1. Introduction

Refractive index is an important characteristic constant of optical materials. Although there are some techniques[1–12] that have been proposed for measuring refractive index, almost all of them are related to the measurement of light intensity variations. However, the stability of a light source, the scattering light, the internal reflection, and other factors influence the accuracy of measurements and decrease the resolution of results.

In this research a new method for measuring the refractive index is proposed. First, the phase difference between s and p polarizations at the total internal reflection is measured by a heterodyne interferometric technique. Then it is substituted into Fresnel's equations, and the refractive index of the test medium is obtained. It has several merits, including simple optical setup, easy operation, high stability, high measurement accuracy, and rapid measurement. We demonstrate its feasibility.

2. Principle

A. Phase Difference Resulting from Total Internal Reflection

A ray of light in air is incident at θ_i on one side surface of an isosceles prism with refractive index n_1 and base angle θ_p, as shown in Fig. 1. The light ray is refracted into the prism and it propagates toward the base surface of the prism. At the base surface of the prism there is a boundary between the prism and the test medium of refractive index n_2 where $n_1 > n_2$. If the angle of incidence at the boundary is θ_1, we have

$$\theta_1 = \theta_p + \sin^{-1}\left(\frac{\sin \theta_i}{n_1}\right). \quad (1)$$

Here the signs of θ_1 and θ_i are defined as positive if they are measured clockwise from a surface normal. If θ_1 is larger than the critical angle, the light is totally reflected at the boundary. According to Fresnel's equations, the amplitude reflection coefficients of s and p polarizations can be expressed as[13]

$$r_s = \frac{\cos \theta_1 - i(\sin^2 \theta_1 - n^2)^{1/2}}{\cos \theta_1 + i(\sin^2 \theta_1 - n^2)^{1/2}} = \exp(i\delta_s), \quad (2)$$

$$r_p = -\frac{n^2 \cos \theta_1 - i(\sin^2 \theta_1 - n^2)^{1/2}}{n^2 \cos \theta_1 + i(\sin^2 \theta_1 - n^2)^{1/2}} = \exp(i\delta_p), \quad (3)$$

respectively, where $n = n_2/n_1$. Hence the phase difference of s polarization relative to p polarization is

$$\phi = \delta_s - \delta_p = 2 \tan^{-1}\left[\frac{(\sin^2 \theta_1 - n^2)^{1/2}}{\tan \theta_1 \sin \theta_1}\right]. \quad (4)$$

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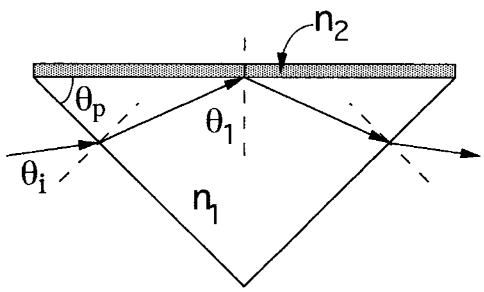

Fig. 1. Total internal reflection at the boundary between a prism and a test medium.

Substituting Eq. (1) into Eq. (4), we have

$$\phi(n, \theta_i) = 2 \tan^{-1}\left(\frac{\{\sin^2[\theta_p + \sin^{-1}(\sin \theta_i/n_1)] - n^2\}^{1/2}}{\tan[\theta_p + \sin^{-1}(\sin \theta_i/n_1)]\sin[\theta_p + \sin^{-1}(\sin \theta_i/n_1)]}\right), \tag{5}$$

which can be rewritten as

$$n = \sin[\theta_p + \sin^{-1}(\sin \theta_i/n_1)] \times \left\{1 - \tan^2\left(\frac{\phi}{2}\right)\tan^2[\theta_p + \sin^{-1}(\sin \theta_i/n_1)]\right\}^{1/2}. \tag{6}$$

It is obvious from Eq. (6) that n_2 can be calculated with the measurement of phase difference ϕ under the experimental conditions in which θ_i, θ_p, and n_1 are specified.

B. Phase Difference Measurements with a Heterodyne Interferometric Technique

Shyu *et al.*[14] proposed a method for measuring the phase retardation of a wave plate by using a heterodyne interferometric technique. The schematic diagram of the optical arrangement of our method, which is based on similar considerations, is designed and shown in Fig. 2(a). The linearly polarized light passing through an electro-optic modulator EO is incident on a beam splitter BS and divided into two parts: the reference beam and the test beam. The reference beam is reflected from BS and passes through an analyzer AN_r, then enters the photodetector D_r. If the amplitude of light detected by D_r is E_r, then the intensity measured by D_r is $I_r = |E_r|^2$. Here I_r is the reference signal. On the other hand the test beam is transmitted through BS and enters a prism. It is totally reflected at the boundary between the prism and the test medium; then it propagates out of the prism. Finally it passes through an analyzer AN_t and is detected by another photodetector D_t. If the amplitude of the test beam is E_t, then D_t measures the intensity $I_t = |E_t|^2$. And I_t is the test signal.

For convenience the $+z$ axis is in the propagation direction and the y axis is in the vertical direction. Let the incident light be linearly polarized at 45° with respect to the x axis, the fast axis EO under an applied electric field be in the horizontal direction, and both the transmission axes AN_r and AN_t be at 45° with respect to the x axis. If a sawtooth signal of an angular frequency w and an amplitude $V_{\lambda/2}$, the half-wave voltage of the EO, is applied to the electro-optic modulator, then by using the Jones calculus we can get

$$I_r = 1/2[1 + \cos(wt + \phi_r)], \tag{7}$$

$$I_t = \frac{I_0}{2}[1 + \cos(wt + \phi)], \tag{8}$$

which are similar to the derivations obtained by Shyu *et al.*[14] In Eqs. (7) and (8), ϕ_r and ϕ are the phase differences between s and p polarizations owing to the reflection at BS and the total internal reflection in the prism, respectively; and I_0 is the intensity of the test beam relative to the reference beam. These two sinusoidal signals are sent to the phase meter as shown in Fig. 2(a). The phase difference between the ref-

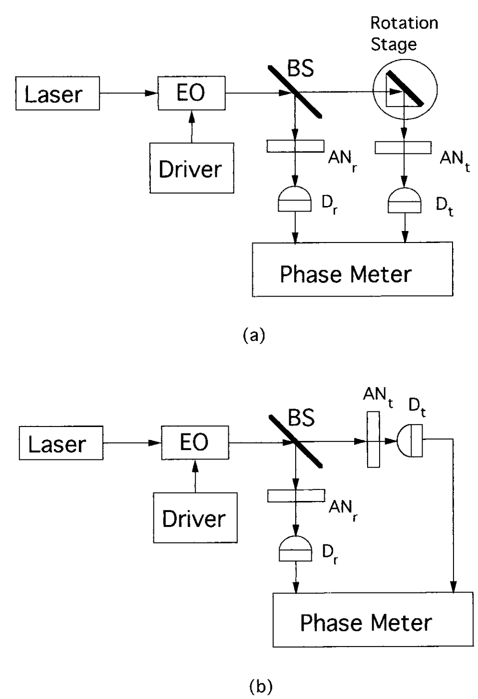

Fig. 2. Schematic diagrams for measuring the phase difference owing to (a) the total internal reflection and (b) the reflection at BS: EO, electro-optic modulator; BS, beam splitter; AN, analyzer; D, photodetector.

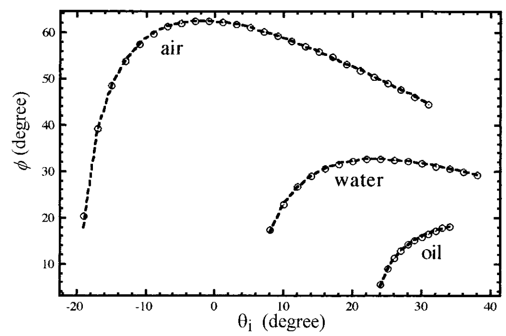

Fig. 3. Theoretical and experimental curves of ϕ versus θ_i for air, water, and index-matching oil.

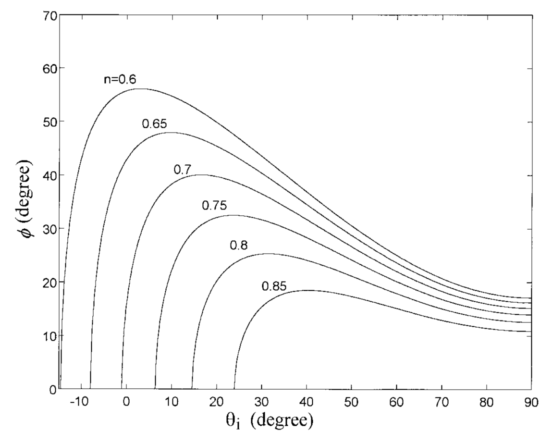

Fig. 4. Relation curves of ϕ versus θ_i for several different n.

erence signal and test signal,

$$\phi' = \phi - \phi_r, \tag{9}$$

can be obtained. In the second measurement let the test beam enter the photodetector D_t without passing through the total internal reflection in the prism, as shown in Fig. 2(b). The test signal still has the form of Eq. (8) but this time with $\phi = 0$. Therefore the phase meter in Fig. 2(b) represents $-\phi_r$. Substituting $-\phi_r$ into Eq. (9) we can obtain the phase difference ϕ. Finally, by using Eq. (6), we can evaluate the refractive index of the test medium with high accuracy.

3. Experiments and Results

To show the feasibility of this technique, the refractive indices of air, pure water, and index-matching oil were measured. A He–Ne laser with a 632.8-nm wavelength and an electro-optic modulator (Model PC200/2, manufactured by England Electro-Optics Developments Ltd.) with a half-wave voltage of 170 V were used in this test. The frequency of the sawtooth signal applied to the electro-optic modulator was 2 kHz. A high-precision rotation stage (PS-θ-90) with the angular resolution of 0.005° manufactured by Japan Chuo Precision Industrial Company Ltd. was used to mount the prism. For easier operation a right-angle prism made of SF11 glass with a 1.77862 refractive index was used, and $\theta_i = -2°$, 14°, and 30° were chosen for air, pure water, and index-matching oil, respectively. In these conditions the phase differences of air, pure water, and index-matching oil were measured to be 62.58°, 29.00°, and 16.00°, respectively. After introducing these data into Eq. (6) we obtained the refractive indices of these samples as 1.0003, 1.332, and 1.508, respectively.

Moreover the theoretical and the experimental curves of ϕ versus θ_i for these three media are shown in Fig. 3. In the figure the dotted curves represent the theoretical values that we obtained by introducing the above refractive indices into Eq. (5), and the circles represent the experimental results. It is clear that these three curves show good correspondence.

4. Discussion

The measurable range of this technique is limited by the refractive index of the prism. To expand the measurable range it is better to use a prism with a high refractive index. Figure 4 shows the relation curves of ϕ versus θ_i for several different relative refractive indices n from Eq. (5). From these curves it is obvious that the resolution of this technique depends on not only the angular resolution of the phase meter but also the angle of incidence θ_i and the relative refractive index n. To avoid the influence of the measurement error of θ_i, it is better to operate this technique in the condition $d\phi/d\theta_i = 0$ where the slope of the curve is at horizontal, i.e., ϕ is at maximum. In this condition from Eq. (6) we have

$$\Delta n \cong \frac{dn}{d\phi} \times \Delta\phi, \tag{10}$$

and

$$\frac{dn}{d\phi} = \frac{-1}{2\left(1 - \tan^2\left(\frac{\phi}{2}\right)\tan^2\theta_1\right)^{1/2}} \times \tan\left(\frac{\phi}{2}\right)\sec^2\left(\frac{\phi}{2}\right)\sin\theta_1\tan^2\theta_1, \tag{11}$$

where Δn and $\Delta\phi$ are the errors in the relative refractive index and the phase difference, respectively. In fact the main source of error comes from the deviation angle θ_R between the polarization direction of p polarization of the incident beam and the incidence plane of the total internal reflection in the prism. And it introduces an error in the phase difference,[15]

$$\Delta\phi = \frac{\tan\phi(\sec 2\theta_R - 1)}{1 + \sec 2\theta_R \tan^2\phi}, \tag{12}$$

into ϕ. Combining Eqs. (5), (10), (11), and (12) we obtain the relation curves of Δn versus n for several different θ_R as shown in Fig. 5. It can be eliminated by improving the optical alignment as follows. First, let the test medium be air, and its refractive

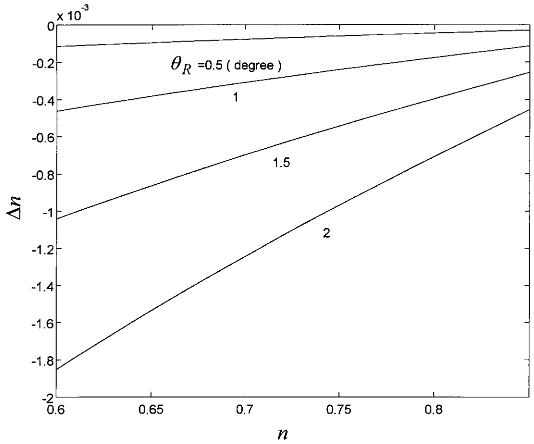

Fig. 5. Relation curves of Δn versus n for several different θ_R.

index can be evaluated with the Edlén equation.[16] Next, substituting the experimental and theoretical data into Eq. (12) we can calculate θ_R. Finally, a half-wave plate is inserted into the optical setup to correct the polarization angle of the input light. In our experiments, when a phase meter with 0.01° resolution is used to measure these media in ideal experimental conditions, the resolutions of our results are better than 1.0×10^{-3}.

Owing to its common path configuration our system has high stability against air turbulence and environmental vibrations. Because it measures the phase difference instead of the light intensity, it is free of the influences from the instability of a light source and the scattered light. Furthermore it has a better resolution.

5. Conclusion

A new method for measuring the refractive index is proposed. First, the phase difference between s and p polarizations at the total internal reflection is measured by the heterodyne interferometric technique. Then it is substituted into Fresnel's equations, and the refractive index of the test medium is obtained. The method has several merits such as simple optical setup, easy operation, high stability, high measurement accuracy, and rapid measurement. Its feasibility has been demonstrated.

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