Effect of nonparabolicity on transverse magnetoresistance of semiconductors in strong magnetic fields*

Chhi-Chong Wu and Anna Chen
Department of Applied Mathematics, National Chiao Tung University, Hsinchu, Taiwan, China

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The effect of nonparabolic band structure on the transverse magnetoresistance in a semiconductor such as n-type InSb in the presence of a dc magnetic field is studied taking into account the inelasticities in the electron-phonon scattering. We discuss this effect of nonparabolicity in semiconductors for both deformation-potential coupling and piezoelectric coupling to acoustic phonons. Results show that the numerical values of the transverse magnetoresistance for the case of the piezoelectric coupling are much smaller than those for the case of the deformation-potential coupling. Therefore, the deformation-potential coupling mechanism plays the dominant role for the transverse magnetoresistance in strong magnetic fields in n-type InSb. We also found that the nonparabolicity of the energy-band structure will change the effect of the temperature on the transverse magnetoresistance besides the enhancement of its magnitude. Our numerical results are found to be in qualitative agreement with experimental results in the quantum limit.

I. INTRODUCTION

The quantum effect of a dc magnetic field on the scattering of electrons in semiconductors has been studied by many authors.[1-19] This quantum effect is being investigated intensively in such semiconductors as indium antimonide and indium arsenide because of the high carrier mobilities and small effective masses in these compounds. The longitudinal and transverse magnetoresistance are the two most investigated properties in which the effect of the dc magnetic field is exhibited. These magnetoresistances depend on the nature of energy-band structures of the material, the carrier mobilities, the strength of the dc magnetic field, and the temperature. It has been found that the nonparabolicity of the energy bands in nondegenerate semiconductors can lead to a nonzero longitudinal magnetoresistance even when the energy and magnetic field dependences of the relaxation time are neglected in strong magnetic fields.[8] This result is contrary to the zero magnetoresistance predicted by the usual Boltzmann theory which assumes that the collisions are unaffected by the dc magnetic field. Askerov *et al.*[20,21] have pointed out that in the case of degenerate semiconductors, the band nonparabolicity and the scattering inelasticity have a strong influence on the field dependences of the longitudinal and transverse magnetoresistance. The transverse magnetoresistance for nondegenerate semiconductors with isotropic parabolic energy bands has been studied for the case where acoustic phonons are the dominant scattering mechanism,[11] where it was shown that the transverse magnetoresistance increases with the dc magnetic field in the quantum limit. Arora[19] found that the transverse magnetoresistance changes dramatically with inelasticity, while the longitudinal magnetoresistance remains essentially unchanged. Consequently, inelasticity may be expected to play an active role and should be included for electronic transport in the transverse configuration.

In the case of a nonparabolic band structure, one has to consider the effect of the band shape not only in the density of states, but also in the scattering probability. It is the purpose of the present paper to study the effect of the nonparabolicity of the conduction band in nondegenerate semiconductors on the transverse magnetoresistance, taking into account the inelasticity of the acoustic phonons. We investigate this effect for the inelastic scattering of acoustic phonons from the deformation-potential and piezoelectric coupling. The scattering is treated in the Born approximation for strong magnetic fields. In Sec. II, we calculate the transverse magnetoresistance for the deformation-potential coupling and piezoelectric coupling to acoustic phonons. It is assumed here that the inelasticity is the dominant mechanism in resolving the divergence which occurs for the strong-field transverse magnetoresistance. In Sec. III, we present numerical results and give a brief discussion.

II. THEORETICAL DEVELOPMENT

In the nonparabolic model, the energy eigenvalue equation for electrons in a uniform dc magnetic field $\vec{B}$ directed along the z axis is[22]

$$H_0\left(1+\frac{H_0}{E_g}\right)\Psi_{\vec{k}n} \equiv \frac{1}{2m^*}\left[p_x^2+\left(p_y-\frac{eBx}{c}\right)^2+p_z^2\right]\Psi_{\vec{k}n} = E_{\vec{k}n}\left(1+\frac{E_{\vec{k}n}}{E_g}\right)\Psi_{\vec{k}n}, \quad (1)$$

where E_g is the energy gap between the conduction bands, m^* is the effective mass of electrons at the minimum of the conduction band, and $E_{\vec{k}n}$ is the true energy of the system, defined by $H_0\Psi_{\vec{k}n} = E_{\vec{k}n}\Psi_{\vec{k}n}$. The eigenfunctions and eigenvalues for Eq. (1) are given by

$$\Psi_{\vec{k}n} = \exp(ik_y y + ik_z z)\,\Phi_n[x - (\hbar c/eB)k_y] \tag{2}$$

and

$$E_{\vec{k}n} = -\tfrac{1}{2}E_g\left\{1 - \left[1 + \frac{4}{E_g}\left((n+\tfrac{1}{2})\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m^*}\right)\right]^{1/2}\right\}, \tag{3}$$

where k_y and k_z are the y and z component of the electron wave vector $\vec{k}$, $\Phi_n(x)$ is the harmonic-oscillator wave function, and $\omega_c = |e|B/m^*c$ is the cyclotron frequency of electrons. Since $(\hbar k_{z_{\max}})^2/2m^*E_g \simeq k_BT/E_g \ll 1$ at the low temperatures in which we are interested ($T < 120$ °K), Eq. (3) can be expanded as

$$E_{\vec{k}n} \simeq \tfrac{1}{4}E_g(b_n - b_n^{-1}) + \hbar^2 k_z^2/2m^* b_n\,, \tag{4}$$

where

$$b_n = 1 + 2(n+\tfrac{1}{2})\hbar\omega_c/E_g\,.$$

When $(n+\frac{1}{2})\hbar\omega_c + \hbar^2k_z^2/2m^* \ll E_g$, the energy eigenvalues reduce to those obtained using the parabolic model for the band structure. However, when the dc magnetic fields come into the high-field region, the energy levels of electrons are quite different from those that would be predicted by using the parabolic model.

From the nondegenerate statistics, we have the Boltzmann distribution in the form of

$$f_{\vec{k}n} = \left(\frac{\sqrt{2}\,\pi^{3/2}\hbar^2 n_0}{m^{*3/2}\,\omega_c(k_BT)^{1/2}}\right) \times \exp[-E_g(b_n - b_n^{-1})/4k_BT - \hbar^2k_z^2/2m^*k_BTb_n] \times\left[\sum_{n=0}^{\infty} b_n^{1/2}\exp\left(\frac{-E_g(b_n - b_n^{-1})}{4k_BT}\right)\right]^{-1}, \tag{5}$$

with the carrier density n_0.

For the scattering due to acoustic phonons, the dissipative current lying in the direction of the total electric field is given by[11,23-25]

$$J_d = \frac{|e|L^2 v_H\hbar}{k_BT} \times \sum_{\vec{k}n,\vec{k}'n'} \frac{(k_y - k_y')^2}{2} f_{\vec{k}n}(1 - f_{\vec{k}'n'})\,W_{\vec{k}n,\vec{k}'n'}\,, \tag{6}$$

where $L = (\hbar/m^*\omega_c)^{1/2}$ is the classical radius of the lowest Landau level, $\vec{v}_H = c(\vec{E}\times\vec{B})/B^2$ is the Hall velocity with the applied electric field $\vec{E}$, and $W_{\vec{k}n,\vec{k}'n'}$ is the transition probability in the Born approximation between the Landau states $\vec{k}n$ and $\vec{k}'n'$. For acoustic-phonon scattering, the transition probability is given by

$$W_{\vec{k}n,\vec{k}'n'} = \frac{2\pi}{\hbar}\sum_q |C(q)|^2 \times[\,|M_{\vec{k}n,\vec{k}'n'}(\vec{q})|^2(N_q+1) \times\delta(E_{\vec{k}n} - E_{\vec{k}'n'} - \hbar\omega_q) + |M_{\vec{k}n,\vec{k}'n'}(-\vec{q})|^2 \times N_q\,\delta(E_{\vec{k}n} - E_{\vec{k}'n'} + \hbar\omega_q)]\,, \tag{7}$$

where $q = (i, \vec{q})$ denotes collectively the branch and wave vector for the phonon mode with the energy $\hbar\omega_q$, $N_q = [\exp(\hbar\omega_q/k_BT) - 1]^{-1}$ is the Planck distribution function for the phonons in thermal equilibrium, $C(q)$ is the electron-phonon coupling constant, and $\delta(x)$ is the Dirac δ function. The two terms in Eq. (7) give the contributions to the scattering rate of the phonon emission and absorption processes, respectively. Now, the matrix element $|M_{\vec{k}n,\vec{k}'n'}(\vec{q})|^2$ in Eq. (7) can be expressed as[11,24]

$$|M_{\vec{k}n,\vec{k}'n'}(\vec{q})|^2 = |\langle\vec{k}n|\exp(i\vec{q}\cdot\vec{r})|\vec{k}'n'\rangle|^2 = \left(\frac{n!}{n'!}\right)\exp(-\tfrac{1}{2}L^2q_\perp^2)\left(\frac{L^2q_\perp^2}{2}\right)^{n'-n}\left[L_n^{n'-n}\left(\frac{L^2q_\perp^2}{2}\right)\right]^2 \delta(k_y - q_y - k_y')\,\delta(k_z - q_z - k_z') \quad \text{for } n' \geq n \tag{8a}$$

$$|M_{\vec{k}n,\vec{k}'n'}(\vec{q})|^2 = |\langle\vec{k}n|\exp(-i\vec{q}\cdot\vec{r})|\vec{k}'n'\rangle|^2 = \left(\frac{n'!}{n!}\right)\exp(-\tfrac{1}{2}L^2q_\perp^2)\left(\frac{L^2q_\perp^2}{2}\right)^{n-n'}\left[L_{n'}^{n-n'}\left(\frac{L^2q_\perp^2}{2}\right)\right]^2 \delta(k_y + q_y - k_y')\,\delta(k_z + q_z - k_z') \quad \text{for } n' \leq n\,, \tag{8b}$$

where $L_n^m(x)$ is the associated Laguerre polynomial,[26] and q_z, $q_\perp$, and q_y are the components of the phonon wave vector directed parallel to the dc magnetic field, normal to the dc magnetic field, and in the $\vec{B}\times\vec{E}$ direction, respectively. $\delta(E_{\vec{k}n} - E_{\vec{k}'n'} - \hbar\omega_q)$ and $\delta(E_{\vec{k}n} - E_{\vec{k}'n'} + \hbar\omega_q)$ in Eq. (7) are given by

$$\delta(E_{\vec{k}n} - E_{\vec{k}'n'} - \hbar\omega_q) = (m^*/\hbar^2 q_{n'n})\,b_n b_{n'}(b_{n'} - b_n)^{-1}[\delta(k_z + q_z b_n/(b_{n'} - b_n) + q_{n'n}) + \delta(k_z + q_z b_n/(b_{n'} - b_n) - q_{n'n})] \quad \text{for } n' > n \tag{9a}$$

and

$$\delta(E_{\vec{k}n}-E_{\vec{k}'n'}+\hbar\omega_q)=(m^*/\hbar^2 q_{nn'})\,b_n b_{n'}(b_n-b_{n'})^{-1}[\delta(k_z+q_z b_n/(b_n-b_{n'})+q_{nn'})+\delta(k_z+q_z b_n/(b_n-b_{n'})-q_{nn'})] \quad \text{for } n'<n, \tag{9b}$$

with

$$q_{n'n}\equiv\left(\frac{m^*E_g}{2\hbar^2}\right)^{1/2}\left(1+b_{n'}b_n+\frac{4\omega_q\hbar b_{n'}b_n}{E_g(b_{n'}-b_n)}+\frac{2q_z^2\hbar^2 b_{n'}b_n}{m^*E_g(b_{n'}-b_n)^2}\right)^{1/2}.$$

Substituting Eqs. (5), (7), (8a), (8b), (9a), and (9b) into (6), one can obtain

$$J_d=n_0|e|v_H E_g^{1/2}\left[4\pi^{5/2}(k_BT)^{3/2}m^*\omega_c^2\sum_{n=0}^{\infty}b_n^{1/2}\exp\left(\frac{-E_g(b_n-b_n^{-1})}{4k_BT}\right)\right]^{-1}$$
$$\times\sum_{m=-\infty}^{\infty}{}'\sum_{l=0}^{\infty}\int d^3q\,|C(q)|^2q_y^2N_q\left(\frac{l!}{m(m+l)!}\right)b_{m+l}b_l(1+b_{m+l}b_ld_m)^{-1/2}\exp(-\tfrac{1}{2}q_\perp^2L^2)\left(\frac{q_\perp^2L^2}{2}\right)^m\left[L_l^m\left(\frac{q_\perp^2L^2}{2}\right)\right]^2$$
$$\times\exp\left[-\frac{1}{k_BT}\left(\tfrac{1}{2}E_gb_l+\frac{q_z^2E_g^2}{4m^2\omega_c^2m^*}+\frac{lq_z^2\hbar E_g}{2m^2\omega_cm^*}+\frac{(m+1)q_z^2\hbar E_g}{4m^2\omega_cm^*}+\tfrac{1}{2}m\hbar\omega_c+\frac{\omega_qE_g}{2m\omega_c}+\frac{(l+1/2)\omega_q\hbar}{m}\right)\right]$$
$$\times\cosh\left(\frac{q_zE_g^{3/2}}{2\sqrt{2}\,m\omega_ck_BT(m^*)^{1/2}}(1+b_{m+l}b_ld_m)^{1/2}\right), \tag{10}$$

where $d_m\equiv1+2\omega_q/m\omega_c+q_z^2E_g/2m^2\omega_c^2m^*$, and $\sum'$ indicates a summation over all m except $m=0$. The term in which $m=0$ is vanished, because the second exponential function in the integrand contains a negative value of $1/m^2$. This means that no contribution of the dissipative current for the case of the transition $m=0$. Since the value of $q_zE_g^{3/2}/m\omega_ck_BT(m^*)^{1/2}$ is small in the high-field region, one can expand $\cosh(x)$ up to the second order in x and obtain

$$J_d=n_0|e|v_HE_g^{1/2}\exp(-E_g/2k_BT)\left[32\sqrt{2}\pi^{5/2}(k_BT)^{3/2}m^*\omega_c^2\sum_{n=0}^{\infty}b_n^{1/2}\exp\left(\frac{-E_g(b_n-b_n^{-1})}{4k_BT}\right)\right]^{-1}$$
$$\times\sum_{m=-\infty}^{\infty}{}'\int d^3q\,|C(q)|^2q_y^2N_q\exp\left\{-\frac{1}{k_BT}\left(\frac{q_z^2E_g^2}{4m^2\omega_c^2m^*}+\frac{\omega_qE_g}{2m\omega_c}-\tfrac{1}{2}\omega_q\hbar\right)-\tfrac{1}{2}q_\perp^2L^2\coth\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]\right\}$$
$$\times\left(P(m,\omega_c,T;\omega_q,\vec{q})\,I_m\left\{\frac{q_\perp^2L^2}{2}\operatorname{csch}\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]\right\}\right.$$
$$\left.+Q(m,\omega_c,T;\omega_q,\vec{q})\,I_{m+1}\left\{\frac{q_\perp^2L^2}{2}\operatorname{csch}\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]\right\}\right), \tag{11}$$

where we have used the relation[27]

$$\sum_{l=0}^{\infty}\frac{l!}{(l+m)!}[L_l^m(x)]^2y^l=\frac{(x^2y)^{-m/2}}{1-y}\exp\left(-\frac{2xy}{1-y}\right)I_m\left(\frac{2xy^{1/2}}{1-y}\right), \tag{12}$$

and $I_m(x)$ is the modified Bessel function of the first kind. Functions $P(m,\omega_c,T;\omega_q,\vec{q})$ and $Q(m,\omega_c,T;\omega_q,\vec{q})$ are given by

$$P(m,\omega_c,T;\omega_q,\vec{q})=\left[\frac{q_z^2E_g^3}{2\sqrt{2}\,m^3\omega_c^2m^*k_B^2T^2}\left(1+\frac{3(m+1)\hbar\omega_c}{E_g}\right)(d_m+1)^{1/2}\right.$$
$$+\left(\frac{4\sqrt{2}}{m}+\frac{4\sqrt{2}(m+1)\hbar\omega_c}{mE_g}-\frac{(m+1)q_z^2\hbar E_g^2}{2\sqrt{2}m^3\omega_cm^*k_B^2T^2}\right)(d_m+1)^{-1/2}$$
$$\left.+\frac{4\sqrt{2}(m+1)\hbar\omega_c}{mE_g}(d_m+1)^{-3/2}\right]\operatorname{csch}\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]$$
$$+\left[\frac{3q_z^2\hbar E_g^2}{4\sqrt{2}m^3\omega_cm^*k_B^2T^2}(d_m+1)^{1/2}+\left(\frac{2\sqrt{2}\hbar\omega_c}{mE_g}-\frac{q_z^2\hbar E_g^2}{4\sqrt{2}m^3\omega_cm^*k_B^2T^2}\right)(d_m+1)^{-1/2}+\frac{2\sqrt{2}\hbar\omega_c}{mE_g}(d_m+1)^{-3/2}\right]$$
$$\times\left\{m+1-q_\perp^2L^2-(m+1)\exp\left[-\frac{\hbar\omega_c}{k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]\right\}\operatorname{csch}^3\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right] \tag{13}$$

and

$$Q(m,\omega_c,T;\omega_q,\vec{q})=q_\perp^2L^2\left[\frac{3q_z^2\hbar E_g^2}{4\sqrt{2}m^3\omega_c m^* k_B^2T^2}(d_m+1)^{1/2}+\left(\frac{2\sqrt{2}\hbar\omega_c}{mE_g}-\frac{q_z^2\hbar E_g}{4\sqrt{2}m^3\omega_c m^* k_B^2T^2}\right)(d_m+1)^{-1/2}\right.$$
$$\left.+\frac{2\sqrt{2}\hbar\omega_c}{mE_g}(d_m+1)^{-3/2}\right]\coth\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]\operatorname{csch}^2\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]. \tag{14}$$

For acoustic-phonon scattering in semiconductors via the deformation-potential-coupling mechanism, the electron-phonon coupling constant is given by[23,24]

$$|C(q)|^2=E_1^2q\hbar/2\rho v_s, \tag{15}$$

where E_1 is the deformation potential constant, ρ is the mass density of the crystal, and v_s is the sound velocity. For the high-temperature approximation, the energies of the phonons involved in the scattering processes can be neglected.[11,24] Then, we have $N_q\simeq k_BT/\hbar\omega_q=k_BT/\hbar v_s|\vec{q}|$. In strong magnetic fields, the transverse magnetoresistance can be approximated by[11]

$$\rho_\perp=EJ_d/(n_0|e|v_H)^2. \tag{16}$$

The expression for the resistivity in the absence of a dc magnetic field due to the deformation-potential coupling is[24]

$$\rho_0=[3(2m^*k_BT)^{3/2}/8\pi^{1/2}\hbar^4](m^*/n_0e^2)(E_1^2/\rho v_s^2). \tag{17}$$

Using Eqs. (11) and (15)–(17), the transverse magnetoresistance due to the deformation-potential coupling is obtained,

$$\left(\frac{\rho_\perp}{\rho_0}\right)_D=\hbar^4E_g^{1/2}\exp\left(-\frac{E_g}{2k_BT}\right)\left[96\pi^2(k_BT)^2(m^*)^{5/2}\omega_c\sum_{n=0}^{\infty}b_n^{1/2}\exp\left(-\frac{E_g(b_n-b_n^{-1})}{4k_BT}\right)\right]^{-1}$$
$$\times\sum_{m=-\infty}^{\infty}{}'\int d^3q\,q_y^2\exp\left\{-\frac{1}{k_BT}\left(\frac{q_z^2E_g^2}{4m^2\omega_c^2m^*}+\frac{\omega_qE_g}{2m\omega_c}-\tfrac{1}{2}\omega_q\hbar\right)-\tfrac{1}{2}q_\perp^2L^2\coth\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]\right\}$$
$$\times\left(P(m,\omega_c,T;\omega_q,\vec{q})I_m\left\{\frac{q_\perp^2L^2}{2}\operatorname{csch}\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]\right\}\right.$$
$$\left.+Q(m,\omega_c,T;\omega_q,\vec{q})I_{m+1}\left\{\frac{q_\perp^2L^2}{2}\operatorname{csch}\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]\right\}\right). \tag{18}$$

For the acoustic-phonon scattering due to the piezoelectric interaction, the electron-phonon coupling constant is given by[23,24]

$$|C(q)|^2=(P^2\hbar/2\rho v_s)q/(q^2+q_s^2), \tag{19}$$

where P is the piezoelectric coupling constant, and the Fermi-Thomas screening wave number q_s is given by $q_s^2\simeq4\pi e^2n_0/\kappa k_BT$ (κ is the dielectric constant). The resistivity in the absence of a dc magnetic field due to the piezoelectric coupling is[24]

$$\rho_0=[3(2m^*k_BT)^{1/2}G(2k_BT)/32\pi^{1/2}\hbar^2](m^*/n_0e^2)(P^2/\rho v_s^2), \tag{20}$$

where

$$G(x)=1-(q_s^2\hbar^2/8m^*x)\ln(1+8m^*x/q_s^2\hbar^2). \tag{21}$$

Using Eqs. (11), (16), (19), and (20), the transverse magnetoresistance due to the piezoelectric coupling is obtained,

$$\left(\frac{\rho_\perp}{\rho_0}\right)_P=\hbar^2E_g^{1/2}\exp\left(-\frac{E_g}{2k_BT}\right)\left[12\pi^2k_BT(m^*)^{3/2}G(2k_BT)\sum_{n=0}^{\infty}b_n^{1/2}\exp\left(-\frac{E_g(b_n-b_n^{-1})}{4k_BT}\right)\right]$$
$$\times\sum_{m=-\infty}^{\infty}{}'\int d^3q\left(\frac{q_y^2}{q^2+q_s^2}\right)\exp\left\{-\frac{1}{k_BT}\left(\frac{q_z^2E_g^2}{4m^2\omega_c^2m^*}+\frac{\omega_qE_g}{2m\omega_c}-\tfrac{1}{2}\omega_q\hbar\right)-\tfrac{1}{2}q_\perp^2L^2\coth\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]\right\}$$
$$\times\left(P(m,\omega_c,T;\omega_q,\vec{q})I_m\left\{\frac{q_\perp^2L^2}{2}\operatorname{csch}\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]\right\}\right.$$
$$\left.+Q(m,\omega_c,T;\omega_q,\vec{q})I_{m+1}\left\{\frac{q_\perp^2L^2}{2}\operatorname{csch}\left[\frac{\hbar\omega_c}{2k_BT}\left(d_m-\frac{\omega_q}{m\omega_c}\right)\right]\right\}\right). \tag{22}$$

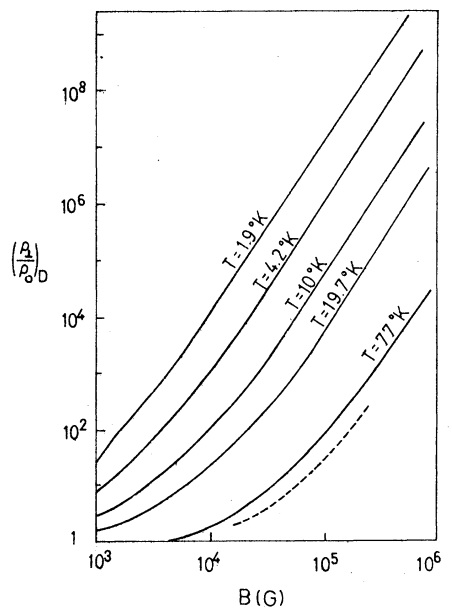

FIG. 1. Transverse magnetoresistance $(\rho_\perp/\rho_0)_D$ as a function of dc magnetic field B at different temperatures. The dashed curve indicates experimental results.

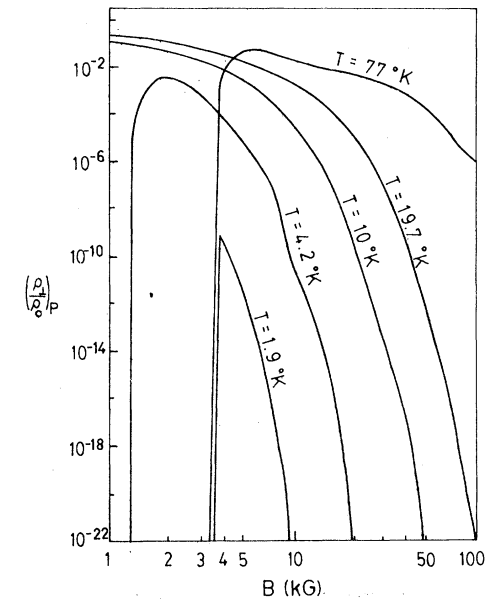

FIG. 3. Transverse magnetoresistance $(\rho_\perp/\rho_0)_P$ as a function of dc magnetic field B at different temperatures.

III. NUMERICAL RESULTS AND DISCUSSION

The expressions in Eqs. (18) and (22) can be approximated by making use of the conditions for the strong magnetic-field region[11,19,23,24] $\hbar\omega_q \simeq \hbar v_s q_\perp \gg m^* v_s^2$, $\hbar\omega_c = \hbar^2/m^*L^2 \gg m^* v_s^2$, and $m^* v_s^2/E_g \simeq q_z^2/q_\perp^2 \ll 1$. As a numerical example,[28] we consider the

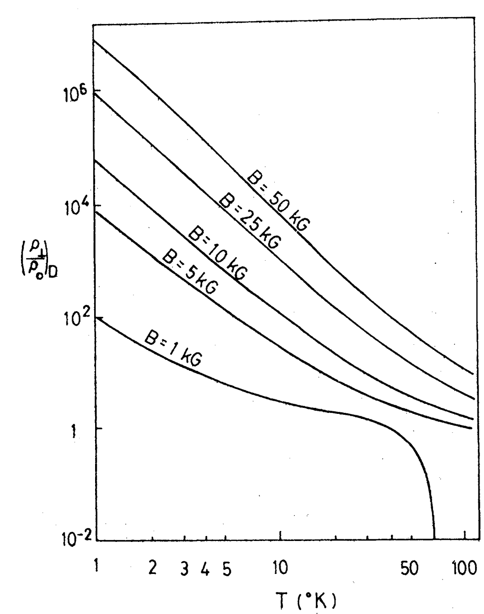

FIG. 2. Transverse magnetoresistance $(\rho_\perp/\rho_0)_D$ as a function of temperature T at different magnetic fields.

transverse magnetoresistance for both cases of the deformation-potential coupling and piezoelectric coupling in n-type InSb. The relevant values of physical parameters for this material are $n_0 = 1.75 \times 10^{14}\text{cm}^{-3}$, $m^* = 0.013 m_0$, $\kappa = 18$, $E_g = 0.2$ eV, and $v_s = 4\times 10^5$ cm/sec. In Fig. 1, it is shown that the transverse magnetoresistance for the case of the deformation-potential coupling to acoustic phonons increases with the dc magnetic field. The dashed curve represents the experimental results with $n_0 = 2.3\times 10^{14}$ cm^{-3} and $T = 77.4$ °K.[14] The numerical results shown here for the nonparabolic band structure are enhanced over those for the parabolic band structure.[11] However, this transverse magnetoresistance will also change with the temperature, owing to the factor $E_g/k_B T$. We plot the transverse magnetoresistance as a function of the absolute temperature for the case of the deformation-potential coupling at some dc magnetic fields as shown in Fig. 2. It can be seen that the transverse magnetoresistance decreases with increasing the temperature. When the dc magnetic field is at $B = 1$ kG, the transverse magnetoresistance will drop to zero in a neighborhood of $T = 74$ °K. This abnormal phenomenon indicates that our present method used here will be invalid at the high-temperature region with low magnetic fields. For the piezoelectric coupling the transverse magnetoresistance changing with the dc magnetic field at some values of temperature is shown in Fig. 3. It shows that the numerical values of the transverse magnetoresistance for the case of the piezoelectric

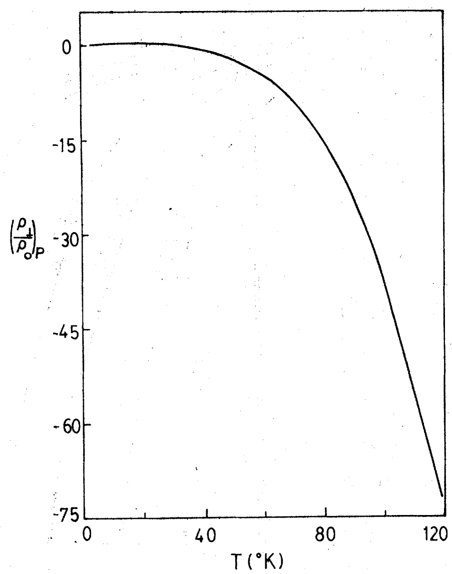

FIG. 4. Transverse magnetoresistance $(\rho_\perp/\rho_0)_P$ as a function of temperature T at $B=1$ kG.

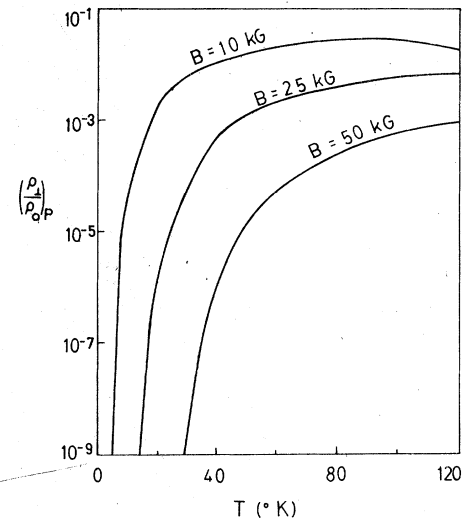

FIG. 6. Transverse magnetoresistance $(\rho_\perp/\rho_0)_P$ as a function of temperature T at $B=10$, 25, and 50 kG.

coupling is much smaller than those for the case of the deformation-potential coupling. We also plot the transverse magnetoresistance as a function of the absolute temperature for the piezoelectric coupling as shown in Figs. 4–6. In Fig. 4, it shows that the transverse magnetoresistance decreases with increasing the temperature at lower dc magnetic fields. When the dc magnetic field is at

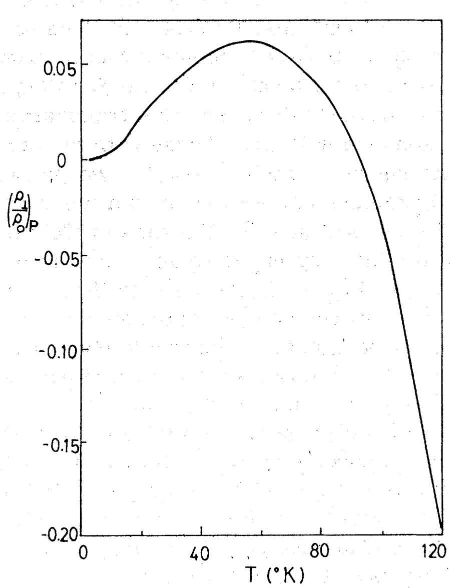

FIG. 5. Transverse magnetoresistance $(\rho_\perp/\rho_0)_P$ as a function of temperature T at $B=5$ kG.

$B=5$ kG, the transverse magnetoresistance increases with the temperature up to $T=52$ °K and then decreases with the temperature. However, when the dc magnetic field is larger than $B=5$ kG, the transverse magnetoresistance increases with the temperature.

We have investigated the transverse magnetoresistance for the case where acoustic phonons are the dominant scattering mechanism. It has been shown that the acoustic-phonon scattering due to the piezoelectric coupling contributes very insignificantly to the transverse magnetoresistance in our present numerical analysis. The most important and dominant contribution to the transverse magnetoresistance is the acoustic-phonon scattering due to the deformation-potential coupling. We compare our numerical results for the case of the deformation-potential coupling with experiments performed by Aliev *et al.*[14] It shows that our numerical results are in qualitative agreement with experimental results in the quantum limit. We also have shown that the nonparabolicity of energy-band structure will change the effect of the temperature on the transverse magnetoresistance besides the enhancement of its magnitude. This means that the factor E_g/k_BT will play an important role in the transverse magnetoresistance for the nonparabolic band structure in nondegenerate semiconductors. Therefore, in the high-temperature region, the effect of the nonparabolicity on nondegenerate semiconductors will be diminished, and the numerical results of the transverse magnetoresistance we obtained would be the same as those for the parabolic band structure.

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[28]We integrate Eqs. (18) and (22) with respect to q_z and $q_\perp$, and sum over all n and m using the computer in Computer Center of National Chiao Tung University, Taiwan, China.