In contrast to the screens considered by Lang [3], those forming our complementary structures must have resistivities and conductivities satisfying (10) or (15) at all pairs of corresponding points. Resistive sheets having (normalized) resistivities up to about 3 are readily available and have found useful application for cross section reduction purposes. A purely conductive sheet would be more difficult to realize, but it is possible that this could be done over a limited frequency range at least.

Though it would be natural to seek a Babinet principle for a combination sheet (which includes an impedance sheet as a particular case), it seems unlikely that such a principle exists. The resulting scattered fields no longer have the symmetry properties that characterize individual electric and magnetic current sheets, and as regards the type of proof presented here, the procedure fails in the deduction of $E^{(1)}$ and $H^{(1)}$ from their values in the half space $z > 0$. This is not surprising since an impedance sheet is opaque whereas resistive and conductive ones are not.

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The Pulsed Field due to an Electric Dipole in the Presence of a Perfectly Conducting Wedge

KUAN-KIN CHAN AND LEOPOLD B. FELSEN, FELLOW, IEEE

Abstract—**The transient dyadic Green's function for a perfectly conducting wedge is expressed in terms of differentiation and integration operations on the scalar Dirichlet and Neumann Green's functions. For the special case of a half plane, this general form is shown to reduce to a previously derived elementary result.**

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K. K. Chan was with the Polytechnic Institute of New York, Farmingdale, NY. He is now with the National Chiao-Tung University, Hsinchu, Taiwan, Republic of China.

L. B. Felsen is with the Polytechnic Institute of New York, Farmingdale, NY 11735.

I. INTRODUCTION

The problem of diffraction of the pulsed field from an arbitrarily oriented electric or magnetic dipole by a perfectly conducting wedge has recently been solved [1]. The solution, constructed by an image representation in an infinitely extended angular space, was obtained in closed form in terms of elementary functions similar to those appearing in the result for diffraction of pulsed scalar point source fields. In the following, we show that from this solution, it is possible to derive a representation for the diffracted pulsed vector field in terms of the Dirichlet and Neumann Green's functions characterizing the scalar pulse problems. The axial Hertz potential generated by an electric dipole oriented parallel to the edge of the wedge is proportional to the scalar Dirichlet Green's function. However, a dipole directed transverse to the edge generates a transverse Hertz vector which is obtainable from the scalar Green's functions only by operations involving both differentiations and integrations with respect to certain space and time coordinates. A compact operator form of the solution is presented.

When the general solution is specialized to the case of a half plane, it is possible to reduce the result so that each component of the transverse Hertz vector is proportional to a scalar Green's function plus an explicit remainder term devoid of integrations. The remainder terms are required to satisfy the edge condition for the vector field, as noted by Mohsen and Senior [2] who derived the half plane result directly. Our form of the wedge solution may be regarded as a generalization of the half plane solution of Mohsen and Senior, whereby the dyadic Green's function is expressed in terms of the scalar Green's functions, albeit via differentiations and integrations. We have note been able to eliminate the integral operations in the general case.

While we present only the electric dyadic Green's function (electric dipole excitation), the same procedure can be followed for the magnetic dyadic Green's function (magnetic dipole excitation). Moreover, because of the trivial scalarizability of the longitudinal dipole problem [3], we deal only with transverse dipole orientation.

In Section II, the previously obtained Hertz vector expression [1] is manipulated so as to exhibit explicitly its dependence upon the scalar Green's functions. The reduction of the general formulas for the case of a half plane is presented in Section III.

II. GENERAL SOLUTION

The solution for the time-dependent transverse Hertz vector generated by a transverse electric dipole with impulsive dipole moment $p_0\delta(t)$ is given as follows [1]:

$$\pi(r, r'; t) = \pi^g(r, r'; t) + \pi^d(r, r'; t), \qquad r' = (\rho', \phi', 0) \quad (1)$$

where $\pi^g(r, r'; t)$ is the geometric-optical part

$$\pi^g(r, r'; t) = \frac{1}{\epsilon}\sum_n p_n \frac{\delta\left(t - \dfrac{|r - r_n|}{c}\right)}{4\pi|r - r_n|} U(\pi - |\phi - \phi_n|) + \frac{1}{\epsilon}\sum_n \bar{p}_n \frac{\delta\left(t - \dfrac{|r - \bar{r}_n|}{c}\right)}{4\pi|r - \bar{r}_n|} U(\pi - |\phi - \bar{\phi}_n|) \quad (2)$$

with $U(x)$ denoting the Heaviside unit function and $\bar{r}_n = (\rho', \bar{\phi}_n, 0)$. The diffracted part π^d is given by

$$\pi_{x,y}{}^d(r, r'; t) = -\frac{c}{4\pi^2\epsilon} \frac{\operatorname{Re} A_{x,y}(\phi, \phi'; i\beta)}{\rho\rho' \sinh\beta} U\left(t - \frac{l}{c}\right) \quad (3)$$

where $l = [(\rho + \rho')^2 + z^2]^{1/2}$ and

$$A_x = -\frac{\pi}{2\alpha}\{[Q_1(\phi - \phi') - Q_1(\phi + \phi')] \cdot \cos\nu - [Q_2(\phi - \phi') + Q_2(\phi + \phi')] \sin\nu\} \quad (4a)$$

$$A_y = -\frac{\pi}{2\alpha}\{[Q_2(\phi - \phi') - Q_2(\phi + \phi')] \cdot \cos\nu + [Q_1(\phi - \phi') + Q_1(\phi + \phi')] \sin\nu\} \quad (4b)$$

$$\beta = \cosh^{-1}\left[\frac{(ct)^2 - \rho^2 - \rho'^2 - z^2}{2\rho\rho'}\right] \quad (4c)$$

$$Q_1(\varphi) = \cos(\varphi - i\beta - \pi) \cot\frac{\varphi - i\beta - \pi}{2\alpha/\pi} - \cos(\varphi + i\beta + \pi) \cot\frac{\varphi + i\beta + \pi}{2\alpha/\pi} \quad (5a)$$

$$Q_2(\varphi) = \sin(\varphi - i\beta - \pi) \cot\frac{\varphi - i\beta - \pi}{2\alpha/\pi} - \sin(\varphi + i\beta + \pi) \cot\frac{\varphi + i\beta + \pi}{2\alpha/\pi}. \quad (5b)$$

The dipole with unit strength moment $\boldsymbol{p}_0$ lies in the $z = 0$ plane at the point $\boldsymbol{r}' = (\rho', \phi', 0)$ in the presence of a perfectly conducting wedge composed of two intersecting half planes at $\phi = 0$ and $\phi = \alpha$ in a (ρ, ϕ, z) cylindrical coordinate system (Fig. 1). $\boldsymbol{p}_n$ and $\bar{\boldsymbol{p}}_n$ denote image sources which lie on the circle $\rho = \rho'$ and decompose into two sets with angular coordinates $\phi_n = 2n\alpha + \phi'$ and $\bar{\phi}_n = 2n\alpha - \phi'$, respectively, where $n = 0, \pm 1, \pm 2, \cdots$ and $0 \leqslant \phi' \leqslant \alpha$. An image at ϕ_n has the orientation

$$\boldsymbol{p}_n = \boldsymbol{x}_0 \cos(2n\alpha + \nu) + \boldsymbol{y}_0 \sin(2n\alpha + \nu) \quad (6a)$$

while an image at $\bar{\phi}_n$ has the orientation

$$\bar{\boldsymbol{p}}_n = -\boldsymbol{x}_0 \cos(2n\alpha - \nu) - \boldsymbol{y}_0 \sin(2n\alpha - \nu) \quad (6b)$$

where ϵ, and subsequently μ, are the permittivity and permeability in vacuum.

We shall now turn our attention to the diffracted field. It can be shown from (5) that

$$\frac{\pi}{2\alpha}[\operatorname{Re} Q_2(\varphi) \sin\varphi + \operatorname{Re} Q_1(\varphi) \cos\varphi] = \cosh\beta \operatorname{Re} B(\varphi; i\beta) \quad (7a)$$

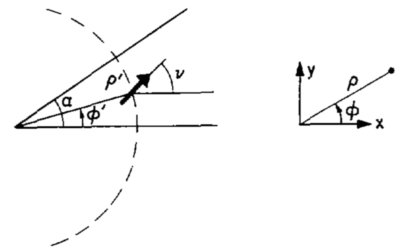

Fig. 1. Wedge configuration with point dipole excitation.

$$\frac{\pi}{2\alpha}[-\operatorname{Re} Q_2(\varphi) \cos\varphi + \operatorname{Re} Q_1(\varphi) \sin\varphi] = -\sinh\beta \frac{\partial}{\partial\beta}\int_0^{\varphi} dx \operatorname{Re} B(x; i\beta) \quad (7b)$$

where

$$\operatorname{Re} B(\varphi; i\beta) = -\frac{\pi}{2\alpha}\left[\frac{\sin\frac{\pi}{\alpha}(\varphi - \pi)}{\cosh\frac{\pi\beta}{\alpha} - \cos\frac{\pi}{\alpha}(\varphi - \pi)} - \frac{\sin\frac{\pi}{\alpha}(\varphi + \pi)}{\cosh\frac{\pi\beta}{\alpha} - \cos\frac{\pi}{\alpha}(\varphi + \pi)}\right]. \quad (8)$$

Substituting Re Q_1 and Re Q_2 from (7) into (4a) and (4b), one obtains,

$$\begin{aligned}\operatorname{Re} A_x(\phi, \phi'; i\beta) &= \cosh\beta[-\cos(\phi - \phi' + \nu) \operatorname{Re} B(\phi - \phi'; i\beta) \\ &\quad + \cos(\phi + \phi' - \nu) \operatorname{Re} B(\phi + \phi'; i\beta)] \\ &\quad + \sinh\beta\left[\sin(\phi - \phi' + \nu)\frac{\partial}{\partial\beta}\int_0^{\phi - \phi'} dx \operatorname{Re} B(x; i\beta)\right. \\ &\quad \left. - \sin(\phi + \phi' - \nu)\frac{\partial}{\partial\beta}\int_0^{\phi + \phi'} dx \operatorname{Re} B(x; i\beta)\right] \end{aligned} \quad (9a)$$

$$\begin{aligned}\operatorname{Re} A_y(\phi, \phi'; i\beta) &= \cosh\beta[-\sin(\phi - \phi' + \nu) \operatorname{Re} B(\phi - \phi'; i\beta) \\ &\quad + \sin(\phi + \phi' - \nu) \operatorname{Re} B(\phi + \phi'; i\beta)] \\ &\quad + \sinh\beta\left[-\cos(\phi - \phi' + \nu)\frac{\partial}{\partial\beta}\int_0^{\phi - \phi'} dx \operatorname{Re} B(x; i\beta)\right. \\ &\quad \left. + \cos(\phi + \phi' - \nu)\frac{\partial}{\partial\beta}\int_0^{\phi + \phi'} dx \operatorname{Re} B(x; i\beta)\right]. \end{aligned} \quad (9b)$$

Recognizing that $\text{Re}\, B(\varphi; i\beta)$ is an even function of φ, one may write

$$\begin{aligned}\text{Re}\, A_x\,(\phi, \phi'; i\beta) &= -\cos(\phi' - \nu)\cos\phi\cosh\beta B'(\phi,\phi';\beta)\\ &- \sin(\phi' - \nu)\sin\phi\cosh\beta B''(\phi,\phi';\beta)\\ &+ \cos(\phi' - \nu)\sin\phi\sinh\beta\frac{\partial}{\partial\beta}\int_0^{\phi} d\phi\, B'(\phi,\phi';\beta)\\ &- \sin(\phi' - \nu)\cos\phi\sinh\beta\frac{\partial}{\partial\beta}\int_0^{\phi} d\phi\, B''(\phi,\phi';\beta)\\ &-2\cos(\phi' - \nu)\sin\phi\sinh\beta\frac{\partial}{\partial\beta}\int_0^{\phi'} d\phi\, \text{Re}\, B(\phi; i\beta)\end{aligned} \tag{10a}$$

where

$$B'(\phi,\phi';\beta) = \text{Re}\, B(\phi - \phi'; i\beta) - \text{Re}\, B(\phi + \phi'; i\beta) \tag{10b}$$

$$B''(\phi,\phi';\beta) = \text{Re}\, B(\phi - \phi'; i\beta) + \text{Re}\, B(\phi + \phi'; i\beta). \tag{10c}$$

A similar expression is obtained for $\text{Re}\, A_y$. From (3) and (10a) the transverse Hertz vector can then be written as

$$\begin{aligned}-\epsilon\pi_x{}^d(r, r'; t) &= \Bigg[\cos(\phi' - \nu)\Bigg\{\left(\cos\phi\cosh\beta - \sin\phi\frac{\partial}{\partial\beta}\int_0^{\phi} d\phi\sinh\beta\right) G_d{}'(\phi,\phi')\\ &+ \sin\phi\frac{\partial}{\partial\beta}\int_0^{\phi'} d\phi\sinh\beta\, G_d{}''(\phi, 0)\Bigg\} + \sin(\phi' - \nu)\\ &\cdot\left(\sin\phi\cosh\beta + \cos\phi\frac{\partial}{\partial\beta}\int_0^{\phi} d\phi\sinh\beta\right)\\ &\cdot G_d{}''(\phi,\phi')\Bigg] U\left(t - \frac{l}{c}\right)\end{aligned} \tag{11}$$

and similarly for $\pi_y{}^d$. Here,

$$G_d{}'(\phi,\phi') = -\frac{c}{4\pi^2}\frac{1}{\rho\rho'\sinh\beta}B'(\phi,\phi';\beta) \tag{12a}$$

$$G_d{}''(\phi,\phi') = -\frac{c}{4\pi^2}\frac{1}{\rho\rho'\sinh\beta}B''(\phi,\phi';\beta) \tag{12b}$$

are the diffracted parts of the scalar Green's functions for the Dirichlet and Neumann boundary conditions, respectively, [3, p. 668]. The dependence on ρ, ρ', z, z', and t has not been explicitly indicated in the arguments of $G_d{}'$ and $G_d{}''$.

Introducing the operators

$$L_{11} = \cos\phi\cosh\beta - \sin\phi\frac{\partial}{\partial\beta}\int_0^{\phi} d\phi\sinh\beta \tag{13a}$$

$$L_{22} = \sin\phi\cosh\beta + \cos\phi\frac{\partial}{\partial\beta}\int_0^{\phi} d\phi\sinh\beta \tag{13b}$$

$$L_{12} = \sin\phi\frac{\partial}{\partial\beta}\int_0^{\phi'} d\phi\sinh\beta \tag{13c}$$

$$L_{21} = -\cos\phi\frac{\partial}{\partial\beta}\int_0^{\phi'} d\phi\sinh\beta \tag{13d}$$

and also

$$\cos(\phi' - \nu) = \boldsymbol{a}_0\cdot\boldsymbol{\rho}_0 = \boldsymbol{a}_0\cdot[\boldsymbol{x}_0\cos\phi' + \boldsymbol{y}_0\sin\phi'] \tag{14a}$$

$$\sin(\phi' - \nu) = \boldsymbol{a}_0\cdot(-\boldsymbol{\phi}_0) = \boldsymbol{a}_0\cdot[\boldsymbol{x}_0\sin\phi' - \boldsymbol{y}_0\cos\phi'] \tag{14b}$$

where $\boldsymbol{a}_0$ is the unit vector along the direction of the dipole while $\boldsymbol{\rho}_0$, $\boldsymbol{\phi}_0$ and $\boldsymbol{x}_0$, $\boldsymbol{y}_0$ are unit vectors along the Cartesian and cylindrical coordinates, respectively, one reduces (11) and its counterpart for $\pi_y{}^d$ to the dyadic form

$$\pi^d(r, r'; t) = \boldsymbol{a}_0\cdot\bar{P}^d(r, r'; t)U\left(t - \frac{l}{c}\right) \tag{15}$$

where

$$\begin{aligned}-\epsilon\bar{P}^d(r, r'; t) &= [(\boldsymbol{x}_0\cos\phi' + \boldsymbol{y}_0\sin\phi')(\boldsymbol{x}_0L_{11} + \boldsymbol{y}_0L_{22})G_d{}'(\phi,\phi')\\ &+ (\boldsymbol{x}_0\cos\phi' + \boldsymbol{y}_0\sin\phi')(\boldsymbol{x}_0L_{12} + \boldsymbol{y}_0L_{21})G_d{}''(\phi, 0)\\ &+ (\boldsymbol{x}_0\sin\phi' - \boldsymbol{y}_0\cos\phi')(\boldsymbol{x}_0L_{22} - \boldsymbol{y}_0L_{11})G_d{}''(\phi,\phi')].\end{aligned} \tag{15a}$$

This is the desired representation which expresses the Hertz vector in terms of the scalar Green's functions. While the elementary formula in (3) is more convenient for actual computation of the field, the result in (15) shows how the dyadic Green's function can be obtained from the scalar Green's functions. When dyadic Green's functions are formulated directly, such relations between the fundamental scalar and vector problems are usually obscured.

The vector electromagnetic fields (and hence the dyadic Green's functions) are derived from the Hertz potential in (1) by the conventional equations

$$E(r, r'; t) = \nabla\nabla\cdot\pi(r, r'; t) - \mu\epsilon\frac{\partial^2}{\partial t^2}\pi(r, r'; t) \tag{16a}$$

$$H(r, r'; t) = \epsilon\nabla\times\frac{\partial}{\partial t}\pi(r, r'; t) \tag{16b}$$

and the potential itself satisfies the scalar wave equation

$$\left(\nabla^2 - \mu\epsilon\frac{\partial^2}{\partial t^2}\right)\pi(r, r'; t) = -\frac{1}{\epsilon}\boldsymbol{p}_0\,\delta(r - r')\delta(t). \tag{17}$$

III. SPECIAL CASE OF A HALF PLANE

When the wedge degenerates into the half plane $\alpha = 2\pi$, the expressions in (15) can be simplified. First,

$$\mathrm{Re}\, B(\varphi; i\beta) = \frac{1}{2} \frac{\cosh\frac{\beta}{2}\cos\frac{\varphi}{2}}{\cosh^2\frac{\beta}{2} - \sin^2\frac{\varphi}{2}} \tag{18}$$

whence

$$\frac{\partial}{\partial\beta}\int_0^{\phi} d\phi\, \mathrm{Re}\, B(\phi \pm \phi'; i\beta)$$

$$= \frac{\partial}{\partial\beta}\left[\frac{1}{2}\ln\frac{\cosh\frac{\beta}{2} + \sin\frac{\phi\pm\phi'}{2}}{\cosh\frac{\beta}{2} - \sin\frac{\phi\pm\phi'}{2}}\Bigg|_0^{\phi}\right]$$

$$= \frac{1}{4}\sinh\frac{\beta}{2}\left[\left(\frac{1}{\cosh\frac{\beta}{2} + \sin\frac{\phi\pm\phi'}{2}} - \frac{1}{\cosh\frac{\beta}{2} - \sin\frac{\phi\pm\phi'}{2}}\right)\right.$$
$$\left. - \left(\frac{1}{\cosh\frac{\beta}{2} \mp \sin\frac{\phi'}{2}} - \frac{1}{\cosh\frac{\beta}{2} \pm \sin\frac{\phi'}{2}}\right)\right]. \tag{19}$$

When these are now substituted into (15), one obtains after straightforward but tedious manipulations,

$$\pi_x{}^d = \left\{\frac{1}{\epsilon}\cos\nu G_d{}'(\phi, \phi') + \frac{c}{2\pi^2\epsilon\rho\rho'}\frac{\cosh\frac{\beta}{2}}{\sinh\beta}\left[\sin\frac{\phi}{2}\sin\frac{\phi'}{2}\cos\nu\right.\right.$$
$$\left.\left. - \sin\frac{\phi}{2}\cos\frac{\phi'}{2}\sin\nu\right]\right\} U\left(t - \frac{l}{c}\right) \tag{20a}$$

$$\pi_y{}^d = \left\{\frac{1}{\epsilon}\sin\nu G_d{}''(\phi, \phi') + \frac{c}{2\pi^2\epsilon\rho\rho'}\frac{\cosh\frac{\beta}{2}}{\sinh\beta}\left[\cos\frac{\phi}{2}\cos\frac{\phi'}{2}\sin\nu\right.\right.$$
$$\left.\left. - \cos\frac{\phi}{2}\sin\frac{\phi'}{2}\cos\nu\right]\right\} U\left(t - \frac{l}{c}\right). \tag{20b}$$

This agrees with the result of Mohsen and Senior [2].

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