Effective interaction between nucleons in the lower and upper *sd* shell*

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A shell model calculation is performed for the *sd* shell nuclei with mass numbers $18 \leq A \leq 20$ and $36 \leq A \leq 38$. The energy spectra of the lower and upper *sd* shell regions can be well reproduced simultaneously using a mass-dependent single effective interaction.

[NUCLEAR STRUCTURE Calculated energy levels and effective interactions, shell model.]

Numerous shell model calculations have been applied with considerable success to the study of properties of the nuclei in either the lower end ($18 \leq A \leq 20$) (Refs. 1–6) or the upper end ($36 \leq A \leq 38$) (Refs. 7–9) of *sd* shell. Different effective interactions are obtained for each of these regions. The purpose of this work is to investigate the possibility that the energy spectra of both lower and upper *sd* shell nuclei can be explained simultaneously by a single effective interaction.

There are essentially two approaches to the calculation of the energy spectra by least-squares fit. The first treats of two-body matrix elements of the residual interaction as free parameters which are varied until a best fit to the spectra is obtained. However, there are 63 two-body matrix elements for the *sd* shell and the experimental values are not very abundant making the fitting rather difficult. The second approach is to assume a specific form for the effective interaction and the single-particle wave functions and consider the interaction strengths to be free parameters. Although the fitting obtained is not as good as that obtained in the first approach, the number of parameters is much fewer in this case. Thus, we adopt the second approach in the present calculation.

For the nuclei in lower *sd* shell ($18 \leq A \leq 20$), an inert ^{16}O core is assumed and the active particles are distributed in the $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ orbits, but the number of particles in the $d_{3/2}$ orbit is always restricted to be 0, 1, or 2. For the upper *sd* shell nuclei ($36 \leq A \leq 38$), an inert ^{40}Ca core is assumed and the $d_{3/2}$, $s_{1/2}$, and $d_{5/2}$ orbits are included in the active model space, but the number of holes in $d_{5/2}$ orbit is restricted to be 0, 1, or 2.

The effective interaction is assumed to be a two-range central-plus-tensor potential which is proposed by Schiffer and True.[10] In a rather extensive analysis of the data over the entire nuclear chart, Schiffer and True have found an interesting similarity exists, in different multiplets, between the angular distributions and the multipole coefficients of the effective two-body interactions. This remarkable discovery suggests the universality of the residual interaction. In the present calculations, the radial dependence of the potential is taken to be the Yukawa type with the interaction ranges $r_0 = 1.415$ fm and $r_1 = 2.0$ fm similar to that used by Schiffer and True. The harmonic oscillator wave functions are employed with the oscillator constant, $\nu = 0.96A^{-1/3}$ fm^{-2}, where $A = 19$ for the lower *sd* shell and $A = 37$ for the upper region. In this way, the effective interaction between the nucleons has the same form for the lower and upper *sd* shell, but the mass dependence is taken into account through the oscillator constant ν. With the above prescription, the interaction strengths consist of the $T = 0$, singlet-odd (CSO), triplet-even (CTE), and $T = 1$, singlet-even (CSE), triplet-odd (CTO), components of both ranges for central force, and the triplet-even (TTE), triplet-odd (TTO) components of short range for the tensor force. These 10 interaction strengths and 4 single-particle energies [i.e., $\epsilon(s_{1/2})$ and $\epsilon(d_{3/2})$ referring to $d_{5/2}$ orbit for lower *sd* shell nuclei, $\epsilon(s_{1/2})$ and $\epsilon(d_{5/2})$ referring to $d_{3/2}$ orbit for upper end] are treated as free parameters in the least-squares fit.

We use for the experimental data essentially all known reliable J^π energy levels in the $18 \leq A \leq 20$ and $36 \leq A \leq 38$ regions in the least-squares calculation. The total number of levels used is 59, in which 32 levels are chosen from the lower end and 27 levels from the upper region. Only those levels which clearly seem to arise from configurations outside the model space are omitted. Examples of this latter group are the 1^+ state at 1.7 MeV in ^{18}F, $\frac{1}{2}^+$ state at 3.23 MeV in ^{19}O, 0^+ state at 4.45 MeV in ^{20}O, 2^+ state at 7.83 MeV in ^{20}Ne, 0^+ state at

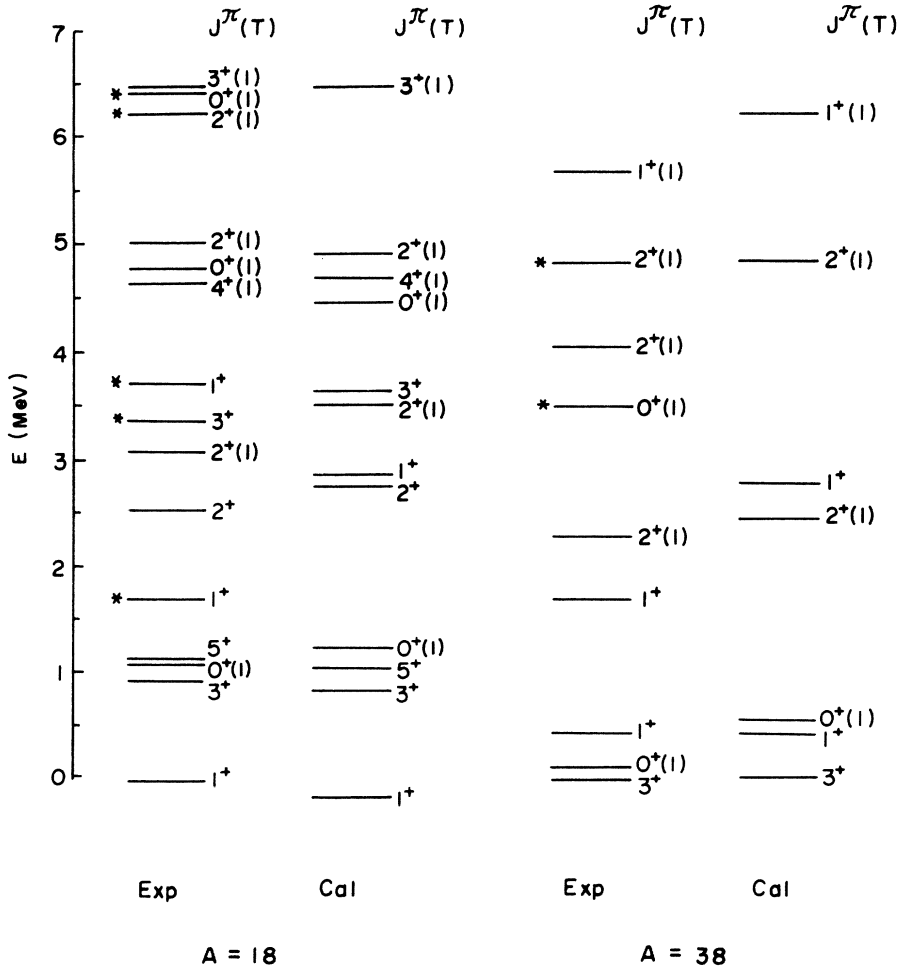

FIG. 1. Experimental and calculated energy spectra for $A = 18$ and 38. The states marked with asterisks are excluded in the least-squares fit.

3.38 MeV, and 2^+ state at 4.70 MeV in ^{38}Ar. These levels cannot be reproduced if one omits the core excitations.[2,4,8,11,12]

The calculation is divided into three different parts. The 10 strengths and 4 single-particle energies are first adjusted to give best fit to the total 59 levels of known J^π in ^{18,19,20}O, ^{18,19}F, ^{20}Ne, ^{38,37}K, ^{38,36}Ar, ^{37,36}Cl, and ^{36}S. Then these parameters are varied to fit to the 32 levels of the lower *sd* region, and finally, they are varied to get best fit to the 27 levels of the upper region. We find that the results of these three calculations are almost identical. This result strongly indicates that the effective interaction we obtained is very reasonable and the spectra of both lower and upper ends can be explained by this effective interaction.

The results of our least-squares fit to the energy level data are shown in Figs. 1–3. The general agreement is quite good except for very few levels. The root-mean-square deviation between the calculated and the observed energies is 0.38 MeV. The single-particle energies we obtained for the lower end are $\epsilon(s_{1/2}) = 0.87$ MeV and $\epsilon(d_{3/2}) = 5.08$ MeV which are exactly identical to the experimental values. Our best fits of the single hole $d_{3/2}$-$s_{1/2}$ and $d_{3/2}$-$d_{5/2}$ energy differences are 2.76 and 9.31 MeV which are also similar to those obtained in previous works.[8,9]

In the case of $A = 18$ and 38, the calculated energy levels are in reasonable agreement with experiment. It should be mentioned that the second 1^+ state of ^{18}F, the third 0^+ and 2^+ states of ^{18}O, and the second 0^+ and the third 2^+ levels of ^{38}Ar cannot be accounted for in our ^{16}O or ^{40}Ca core model and, therefore, they are not included in the least-squares fit. This is consistent with other calculations using various approaches. When core excitation is included, the above mentioned levels in ^{18}F

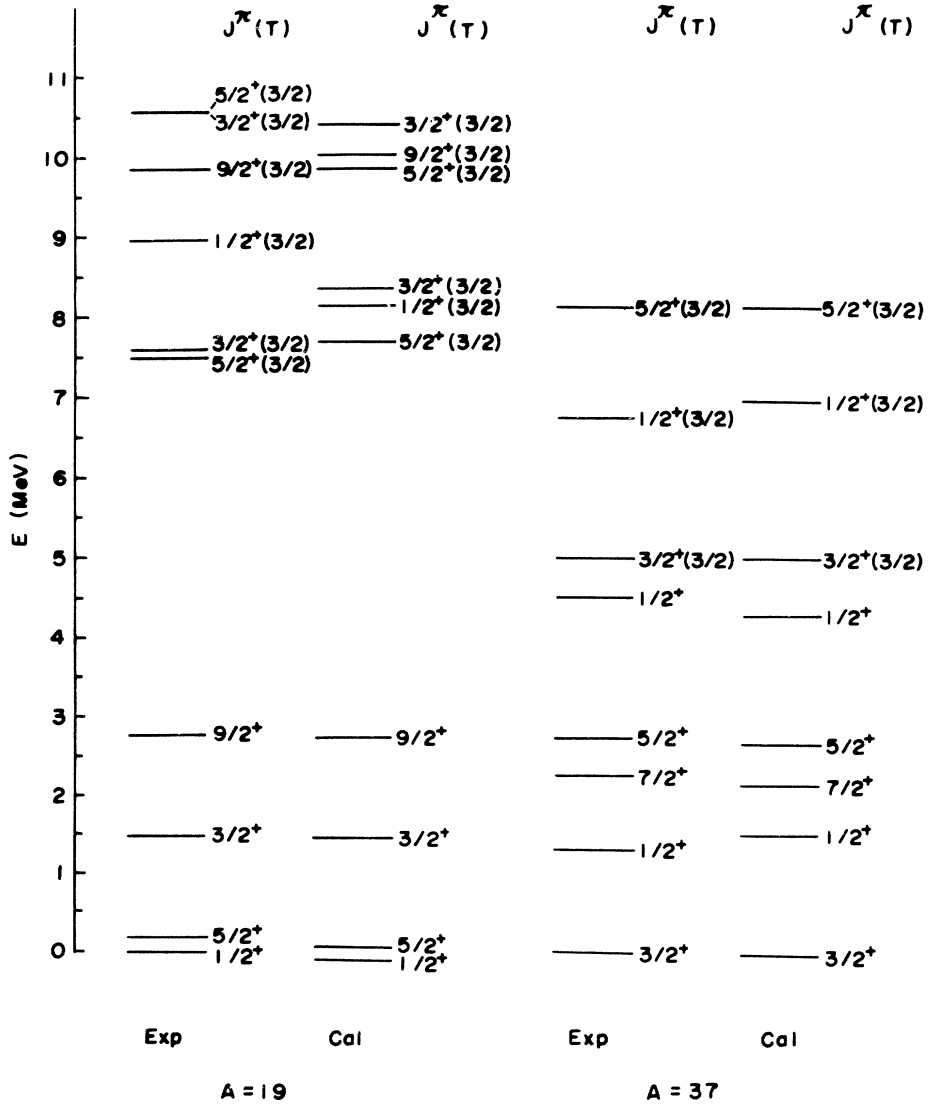

FIG. 2. Experimental and calculated energy spectra for $A = 19$ and 37. The states marked with asterisks are excluded in the least-squares fit.

and ^{18}O can be reproduced as by McGrory and Wildenthal using ^{12}C as an inert core,[4] and the two levels of ^{38}Ar are obtained by Gray *et al.*[12] All the low-lying states are well reproduced for $A = 18$ and 38 except the second 1^+ state in ^{38}K. The calculated excitation energy for this level is 2.27 MeV which is displaced about 1.06 MeV from the observed value. This is the worst agreement of our calculated energies including in the least-squares fitting. For $A = 19$ and 37, the agreement with experiment is quite satisfactory. The ground state band structure in ^{19}F starting from $K = \frac{1}{2}^+$ can be reproduced by our truncated shell model calculation. In particular, the second $\frac{1}{2}^+$ level of ^{37}K is only displaced about 0.2 MeV from the experimental value, while in other calculations using various interactions,[8,9] the displacement is at least 0.7 MeV. For $A = 20$ and 36, the second 0^+ state of ^{20}O and the third 2^+ state in ^{20}Ne are not included in the least-squares fit because the core excitation is very important for these states.[4] In addition, the second 0^+ excited state of ^{20}Ne is also excluded in the calculation since this state can be categorized as a state which results from the weak coupling of the first excited 0^+ state of ^{16}O (the 4p-4h state) to the ground state of ^{20}Ne.[13] The third 0^+ excited state is explained very well in our calculation but the result for the first 0^+ excited state is not so

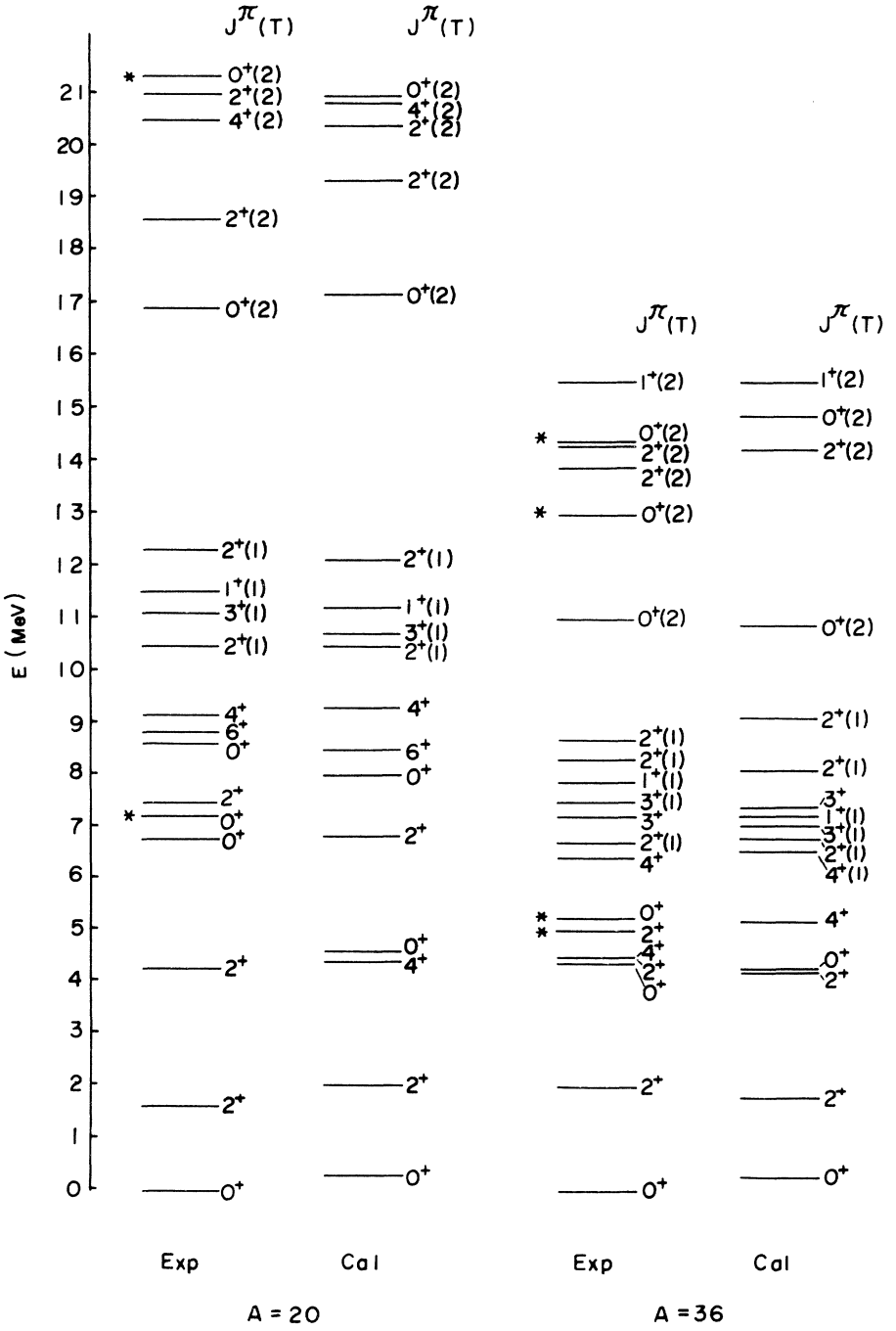

FIG. 3. Experimental and calculated energy spectra for $A = 20$ and 36. The states marked with asterisks are excluded in the least-squares fit.

satisfactory. Nevertheless, the interesting rotational structure of $K = 0^+$ in ^{20}Ne is reproduced. In the case of ^{36}S, we find that the second 0^+ state cannot fit. This state is also missing in the previous calculation with modified Tabakin potential.[8]

The interaction strengths we obtained and the results of Schiffer and True are shown in columns 3 and 4 of Table I, respectively. The magnitudes and signs of the strengths of these two works are very similar to each other except for CTE of both ranges. The strength CTE of long range has negative sign in our case. This is related to the fact

TABLE I. Interaction strengths (in MeV) compared with those of Schiffer and True (ST).

	Ours	ST
	Short range	
CSO	153.86	125.53
CTE	−20.63	−118.09
CSE	−53.45	−49.32
CTO	−125.74	−155.82
TTE	−67.86	−42.52
TTO	−25.48	−6.10
	Long range	
CSO	−59.87	−44.37
CTE	−13.04	27.27
CSE	13.78	15.47
CTO	56.14	62.06

TABLE II. Two-body matrix elements (in MeV) of the effective interaction for the lower and upper *sd* shell compared with those obtained by Kuo.

Configuration				T	J	Lower	Upper	Kuo
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	−2.53	−1.96	−1.95
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	1	0	−3.20	−2.44	−2.44
				1	2	−0.85	−0.52	−1.04
				1	4	−0.02	0.13	−0.05
$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	1	0	−2.51	−1.84	−0.81
				1	2	−0.23	−0.04	0.08
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1	−4.58	−3.65	−3.18
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	0	1	−1.73	−1.57	−1.03
				0	3	−2.02	−1.66	−0.86
				0	5	−3.94	−3.13	−3.66
$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	0	1	−1.73	−1.45	−0.47
				0	3	−3.04	−2.47	−2.59

that the strength of our CTE of short range is much less attractive than that of Schiffer and True.

The diagonal two-body matrix elements are also calculated as shown in Table II. A comparison is made with Kuo's values.[14] Generally speaking, the matrix elements agree with one another qualitatively. It is found that the matrix elements for the lower end are more attractive than those for upper end.

Using a single effective interaction of a two-range central-plus-tensor potential, the energy spectra of the lower and upper *sd* shell nuclei are simultaneously calculated with good agreement with experiment provided that the mass dependence in the two-body matrix elements is suitably taken into account through the oscillator constant. This result suggests that the nucleon-nucleon interaction for any orbit may be described by a single effective interaction if enough model spaces are considered. We believe that the resulting effective interaction in this work can be used in other calculations on *sd* shell nuclei. The extension of this effective interaction to the region $21 \leq A \leq 35$ will be reported later.

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[1]E. C. Halbert, J. B. McGrory, B. H. Wildenthal, and S. P. Pandya, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1969), Vol. 4.

[2]T. Y. Lee, S. T. Hsieh, and C. M. Yang, Phys. Rev. C 5, 2013 (1972).

[3]A. Arima, S. Cohen, R. D. Lawson, and M. H. Macfarlane, Nucl. Phys. A108, 94 (1968).

[4]J. B. McGrory and B. H. Wildenthal, Phys. Rev. C 7, 974 (1973).

[5]P. J. Ellis and T. Engeland, Nucl. Phys. A144, 161 (1970).

[6]A. K. Deka and Mahouta, Phys. Rev. C 13, 2044 (1976).

[7]E. C. Halbert, J. B. McGrory, and B. H. Wildenthal, Phys. Rev. Lett. 20, 112 (1968).

[8]A. E. L. Dieperink and P. J. Brussaard, Nucl. Phys. A128, 34 (1969).

[9]H. C. Chiang, S. T. Hsieh, T. Y. Lee, and M. C. Wang, Phys. Rev. C 5, 2016 (1972).

[10]J. P. Schiffer and W. W. True, Rev. Mod. Phys. 48, 191 (1976).

[11]G. A. P. Engelbertink and P. W. M. Glaudemans, Nucl. Phys. A123, 225 (1969).

[12]W. S. Gray, P. J. Ellis, T. Wei, R. M. Polichar, and J. Janeecke, Nucl. Phys. A140, 494 (1970).

[13]J. B. McGrory, Phys. Lett. 31B, 339 (1970).

[14]T. T. S. Kuo, Nucl. Phys. A103, 71 (1967).