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碩士論文

台灣地區金融市場波動性實證研究

Empirical Performance of Financial Market

Volatility in Taiwan

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謝國文 博士

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Abstract

Forecasting volatility plays an important role in financial market, because volatility predictions are crucial for the successful implementation of risk management. The use of high frequency data approximately renders volatility from a latent to an observable quantity, and opens the new field of visions to forecast future volatilities. Use of realized volatility constructed from high-frequency intraday returns, in contrast, permits the use of traditional time-series methods for modeling and forecasting. Main goals of this thesis are to find general and powerful forecasting procedures for volatilities based on Taiwan high frequency data, to evaluate the predictive potential of volatility forecasts for the true latent volatility, to analyze the impact of more reliable volatility predictions on the quality of three widely used risk measures, and to test for parameter changes in the GARCH(1,1) models. For that purpose, this thesis explores some statistical models for predicting the daily volatility of financial time series.

Keywords: change point, GARCH (1,1) models, realized volatility.

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摘要

預測金融市場的波動性(volatility)非常重要,因為能否準確的預測波動對於風險管理的成敗扮演著關鍵性的角色。波動性是無法由金融市場所提供的資訊得知,然而我們可使用頻繁交易的資料來表示此一無法觀測的值,這也開啟了預測未來波動性的新視野。我們利用以日內報酬所得到的 realized volatility 當作真實波動以與時間序列的方法做出的模型做比較。本論文主要的目的是以台灣的資料找出能有效預測波動性之模型、計算真實的波動性以用來代表無法直接觀測到的波動性、以三種不同的方式分析出最可靠的波動預測模型、並測試出何時 GARCH(1,1)模型需要重新估計參數。為此,本文利用一些統計中的方法和時間序列模型去預測金融市場的日波動。

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很高興能有機會到交大來唸書，光陰似箭，轉眼間兩年的碩士班將告一個段落，在研究生涯將結束的前夕，不能免俗的，要感謝很多人。

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1. Introduction

The explosive growth of applications of econometrics to finance is due primarily to the increased availability of financial data, increased computer power and the greater interest in the performance of financial markets in current economic discussions. Asset returns volatility is a central feature of many financial market problems such as risk management and option pricing. However, it is still an ambiguous term because of different concepts and definitions.

Up to now, many literatures have focused on the parametric approach considering volatility as an unobservable variable and using a fully specified functional model for the ex-ante expected volatility. Modeling the volatility was one of the most common topics in the financial literature by using all ARCH types models and stochastic volatility models. With respect to financial econometrics, Bollerslev (2001) declares that the development of ARCH has been one of the two most important developments in this field over the past two decades.

However, realized volatility affords the empirical measurement of the latent notional volatility on the discrete time interval $[t-h, t]$. Similar to the instantaneous volatility measures, realized volatilities may be classified according to whether the estimation of the notional volatility only uses returns observations falling in the interval $[t-h, t]$, which we call “local”, or also incorporates returns outside $[t-h, t]$. The most obvious local measure for daily volatility is the daily absolute return. More recently a series of papers (Andersen, Bollerslev, Diebold and Labys 2001a and Barndoff-Nielsen and Shepard 2002a, b, c) has formalized and generalized this intuition by applying the quadratic variation theory to the broad class of special (finite mean) semi-martingales.

To make clear the purpose and main theme from the very beginning, the main

goal of this thesis is to find general and powerful forecasting models for volatilities based on high frequency data. Particularly, we will use Taiwan high frequency data to fit in-sample volatility, and to find more reliable volatility predictions. The problem of testing for a parameter change has been an important issue in statistics. It started in the quality control context and then moved rapidly to other fields. Finally, we will check whether the results (our expected volatilities) are consistent with the actual volatilities (realized volatility). This will be a powerful method to examine our well-done forecasting models.

The change point issue has attracted much attention from lots of researches in time series analysis. In the thesis, we applied some tests to a real data set. Since the data set suffered from a period of SARS, it may detect structural changes owing to changes of events or policy. Ignoring it can lead to a false prediction.

The rest of the thesis proceeds as follows: In Chapter 2, we describe data types. Chapter 3 establishes every volatility notation and illustrates some models we used in this thesis. Particularly, there are some special methods to detect structural change. Chapter 4 reports some definitions of evaluating volatility measures and uses these measures to assess performance of each model theoretically. In Chapter 5, we try to summarize empirical analysis, and propose some interesting issues. Chapter 6 concludes the discussion.

2. Data

The data used in this thesis are indices of the Taiwan Stock Exchange (TAIEX)¹ and the Taiwan Stock Exchange option (TXO) obtained for the period from Jan. 2, 2003 to Aug. 29, 2003, and 163 days in the aggregate. We chose this period because it was the period of initial and terminal stages when epidemic SARS spread globally. Figure 2.1 shows the daily probable amount of SARS cases happened in Taiwan from Feb. 25, 2003 to Sep. 4, 2003. According to Figure 2.1, epidemic SARS reached the high summit during the period from the middle of April to the middle of May. It means that high level infection of SARS coronaviruses lasted for more than one month.

2.1 TAIEX

TAIEX is the comprehensive index of all companies entered the market in Taiwan Stock Exchange Corporation, and the method to calculate depends on market value of weighted-average. Taiwan Stock Exchange Corporation provided data banks of the index TAIEX since Aug. 8, 1998.

There are three types of indices in calculating methods: price-weighted, value-weighted, and equal-weighted. TAIEX belongs to type two and is common in global financial markets. Value-weighted is calculated by *total market value in the current period (outstanding shares* closed stock price)/total market value of base period*. Therefore, at the same level of advance-decline, companies with great market value have much more influence than that with small ones.

¹ At present, the most commonly used index is Taiwan weighted index. It includes more than six hundreds stocks of listed companies in Taiwan, and it is also the most famous index in Republic of China. The code of Taiwan weighted index in quote systems is TWII.

The plot of stock prices from Feb. 25, 2003 to Sep. 4, 2003 is given in Figure 2.2.

2.2 TXO

Option of Taiwan Stock Exchange Capitalization Weighted Stock Index (TXO) is a financial derivative started on Dec. 24, 2001. This option is European style, thus there is no possibility of early exercise. Trading hours are 08:45AM - 1:45 PM Taiwan time Monday through Friday of the regular Taiwan Stock Exchange business days. This option product belonged to the new financial derivative, so people were not familiar with it at beginning. However, many stock trading companies tried to promote TXO. Therefore, it is quite popular today. We can see the detailed description in Table 2.1.



3. Volatility Notations and Illustration of Models

A special feature of stock volatility is the fact that it is not directly observable. For example, consider the daily logarithmic rate of returns of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX). The daily volatility is not directly observable from the returns because there is only one observation in a trading day. If intraday data of the stock, such as 5-minute returns, are available, then one can estimate the daily volatility by realized volatility. The accuracy of such an estimate deserves a careful study. Throughout this thesis, we try to do research in volatility.

We will first start with theoretical illustration of GARCH model that has been popular in financial time series analysis.

3.1 General Autoregressive Conditional Heteroscedastic (GARCH) Model



We assume there is an asset guaranteeing an instantaneously risk-free rate of interest. We have the following characterization of the logarithmic asset price, p_t , where $0 \leq t \leq T$. The continuously compounded return over the time interval $[t-h, t]$ is

$$r_{t,h} = p_t - p_{t-h}, \quad 0 \leq h \leq t \leq T \quad (3.1)$$

or is directly written as

$$r_{t,h} = \log\left(\frac{p_t}{p_{t-h}}\right). \quad (3.2)$$

The discrete-time models, at a minimum, assume that the correct specification of the h -step ahead conditional mean and variances are known up to a low-dimensional parameter vector.

The general autoregressive conditional heteroscedastic model, in brief, GARCH(m,s) model (Bollerslev, 1986), can be expressed as follows,

$$\begin{aligned}\sigma_{t,h}^2 &= \alpha_0 + \sum_{j=1}^m \alpha_j [r_{t-j,h} - \mu_{t-j,h}]^2 + \sum_{i=1}^s \beta_i \sigma_{t-i,h}^2, \\ &\equiv \alpha_0 + \alpha(L,h)[r_{t,h} - \mu_{t,h}]^2 + \beta(L,h)\sigma_{t,h}^2,\end{aligned}\quad (3.3)$$

where $\alpha_0 > 0, \alpha_i \geq 0$, for $i > 0$, $\beta_0 \geq 0$, $\sum_{i=1}^{\max(m,s)} (\alpha_i + \beta_i) < 1$, and L stands for lag operator. For example, $L(r_t) = r_{t-1}$.

In most applications, only lower order GARCH models are used frequently, such as GARCH(1,1), GARCH(2,1), and GARCH(1,2) models. We will use the GARCH(1,1) process of Duan(1995) for daily returns in this thesis. The GARCH(1,1) model lists as follows,

$$\begin{aligned}r_t &= r + \lambda\sqrt{h_t} - \frac{1}{2}h_t + a_t, \quad a_t | F_{t-1} \sim N(0, h_t), \\ h_t &= \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2,\end{aligned}\quad (3.4)$$

where $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0$. We use the maximum likelihood (ML) method to estimate the parameters $\alpha_0, \alpha_1, \beta_1, \lambda$, where λ is the constant unit risk premium. (Under conditional lognormality, one plus the conditionally expected rate of return equals $\exp(r + \lambda\sqrt{h_t})$).

3.2 Realized Volatility

Andersen et al.(2001a, b) propose the sum of squared returns daily realized volatility estimator which sums the squares of intraday returns.

Let $r_{t,i}$, where $0 \leq i \leq n$, represent a set of n+1 intraday returns for day t, and when $i = 1$ represents the five minutes commencing at the open, and concluding with

the five minutes at the end when $i = n$.


The realized volatility for trading day t , from the close on day $t-1$ to the close on day t , is defined by

$$\hat{\sigma}_t^2 = \sum_{i=0}^n r_{t,i}^2, \quad (3.5)$$

where $0 \leq i \leq n$. The realized volatility is simply the second sample moment of the log return process over a fixed interval.

We use a sampling frequency of 5-minute returns, which is high enough such that our daily realized volatilities are largely free of measurement error. We show in Figure 3.1.

3.3 Change Point Detection with Methods in Quality Control



In 2003, the most influential event globally is SARS, especially in Asia. The peak period of the high infectious disease is from April to June. Consequently, we must pay attention to the event. However, if we ignore structural change on purpose, in other words, do not readjust the estimation of all parameters in time series models, it is not a reasonable forecasting at all. For this reason, we hope that methods in quality control can help detect some change points. We can then cut a complete period into several subperiods in accordance with these change points. In order to ensure the candidates of change points to be accepted statistically, we will utilize the likelihood ratio test for this purpose in the next section. Once the change points are identified, we estimate parameters and calculate volatility for each subperiod.

Quality control plays an important role in industries of manufacturing. Quality control is a process that measures output relative to a standard, and take action when

output does not meet the standard.

A control chart is essentially a picture of a sampling distribution. That is, it consists of a series of sample values or “statistics” which, if they were gathered together instead of being plotted in sequence, would form a distribution. A natural pattern has three characteristics simultaneously. They can be summarized as follows:

1. Most of the points are near the solid centerline.
2. A few of the points spread out and approach the control limits.
3. None of the points (or at least only a very rare and occasional point) exceeds the control limits.

If any characteristic stated above is missing, the pattern will look unnatural. A control chart may indicate out of statistical control either when one or more points fall beyond the control limits or when the plotted points exhibit some nonrandom patterns of behavior. The Western Electric Handbook (1956) suggests several decision rules for detecting nonrandom points on control charts. It shows that the unnatural consecutive pattern is out of control if there is any one listed below:

1. One point plots outside the 3 sigma control limits;
 2. Two out of three consecutive points plot between a distance of 2 sigma and 3 sigma;
 3. Ten out of eleven consecutive points plot on one side of the centerline;
 4. A run² of five consecutive points plots on one side of the centerline;
- or
5. Five consecutive points are in a rising or falling trend.

²A run is a sequence of observations with a certain characteristic.

Under models with Markov properties, each innovation of a financial time series can be treated as independent. Therefore, it is natural to assume that quality control may be a good tool to find nonrandom and unnatural points. We can use it to find candidates of change points, which will be the subject of the next section.

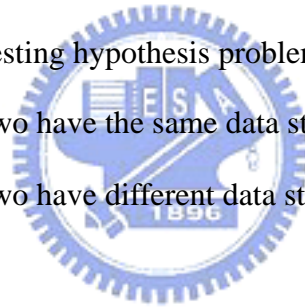
3.4 Likelihood Ratio Test (LRT)

After using several methods of quality control, we can find some candidates of change points. If the X th day is a candidate, the interval between the first day and the $(X-1)$ st day is period one and interval between the X th day and the n th day is period two.

Consider the following testing hypothesis problem.

H_0 : period one and period two have the same data structure,

H_1 : period one and period two have different data structures.



For a GARCH(1,1) model, the likelihood function (L) is given by

$$L = \prod_{t=1}^T \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left[-\frac{(r_t - \mu_t)^2}{2\sigma_t^2}\right]. \quad (3.6)$$

Reject H_0 if

$$\lambda = -2 \log \left[\frac{L(\text{period1} + \text{period2})}{L(\text{period1})L(\text{period2})} \right] > \chi_{4,1-0.05}^2 = 9.488, \quad (3.7)$$

where $\chi_{4,1-0.05}^2$ is the upper 100(1-0.05)% point of the chi-square distribution with 4 degrees of freedom³.

³ The degree of freedom is the amount of difference between parameters under H_0 and H_1 . For instance, there are 4 parameters under H_0 and 8 parameters under H_1 in the GARCH(1,1) model. Thus, the degree of freedom here equals to 4.

When it comes to the end of hypothesis testing, we can conclude whether the candidates are change points or not from the LRT. Once the change points are established, we estimate the parameters of the GARCH(1,1) model in each subperiod.



4. Evaluating Volatility Measures

It is almost impossible to describe an evaluation criterion that is commonly acceptable. However, given our objective of measuring the general degree of predictability of the volatility theoretically and practically, we will avoid using very sophisticated evaluation criteria. Hence, we will stay with the statistical tradition of reporting summary statistics based directly on the differences between forecasts and realizations.

Standard accuracy measures

Three criteria are used here to evaluate the accuracy of the forecasts: root mean squared error (RMSE), mean absolute error (MAE) and logarithmic error (LE).

Mean squared error provides a quadratic loss function which disproportionately weights large forecast errors more heavily relative to mean absolute error, and hence the latter (MAE) may be particularly useful in forecasting situations when large forecast errors are disproportionately more serious than small errors. Finally, logarithmic error was employed in Pagan and Schwert (1990) and Andersen et al. (1999).

Root mean squared error:

$$RMSE = \left[\frac{1}{n} \sum (\hat{\sigma}_{GARCH} - \sigma_{RV})^2 \right]^{\frac{1}{2}}. \quad (4.1)$$

Mean absolute error:

$$MAE = \frac{1}{n} \sum |\hat{\sigma}_{GARCH} - \sigma_{RV}|. \quad (4.2)$$

Logarithmic error:

$$LE = \frac{1}{n} \sum \left[\log \left(\frac{\sigma_{RV}}{\hat{\sigma}_{GARCH}} \right) \right]^2. \quad (4.3)$$

In measures of (4.1), (4.2), and (4.3), $\hat{\sigma}_{GARCH}$ are the volatilities estimated by the GARCH(1,1) model, and σ_{RV} are realized volatilities.

With the accuracy measures of RMSE, MAE, and LE, it is easier to judge which model is more consistent with “true” volatility. The next chapter will provide results of empirical analysis for each criterion under the GARCH model.



5. Empirical Analysis

Estimation of the parameters and volatilities in a GARCH(1,1) model could be based on data from the entire period. However, if we do not consider some extraordinary events occurring during the period while estimating the parameters, the estimates so obtained may not be meaningful. Hence, we want to detect some change points and find several such events according to historical incidents. Consequently, we will divide 163 days into several subperiods. We then estimate the parameters of the GARCH(1,1) model for each subperiod. At the same time, we hope that volatilities estimated with change points can have better performance than those ignoring such changes. That is, volatilities with change points will yield smaller values of accuracy measures.

5.1 Empirical Volatility Measurement



By using methods in the Western Electric Handbook (1956) (see methods 1 to 5 in section 3.3), we find some candidates of change points.

Since there is no point exceeding the 3-sigma limit from Figure 5.1, we say that the whole pattern is random by method 1 of Western Electric rules. In Table 5.1, there are two candidates of change points by using method 2 but the LRT values are less than 9.488. That means the candidates of change points can not change data structure. By method 3, as indicated in Table 5.2, there is only one candidate with the LRT value less than the critical value. Absence of change points detected by method 5 (Five consecutive points are in a rising or falling trend) produces a natural pattern. It is much similar to the result of method 1 above.

We can observe from Table 5.3 that when period 1 (Jan. 2, 2003 to May 14, 2003)

and period 2 (May 15, 2003 to Aug. 29, 2003) had been decided, the value of the likelihood ratio test is bigger than 9.488.

$$\lambda = -2 \log \left[\frac{L(\text{period1} + \text{period2})}{L(\text{period1})L(\text{period2})} \right] > \chi_{4,1-0.05}^2 = 9.488$$

According to the LRT, the only change point occurred on May 14, 2003. The LRT statistic yields a value bigger than the critical value of 9.488. However, other candidate points are not change points according to the LRT. Our finding coincides with the news announcement on May 14, 2003 that the epidemic SARS had reached the countryside of Mainland China.⁴ Hence, we can separate 163 days into two subperiods.

From Figure 5.2, it is clear and observable that the 87th point is a possible change point of events. Before the 87th day, the returns had more substantial variations than the remainder. The epidemic SARS has since subsided and consequently the stock market became more stable.

After making sure of the change point, we can rewrite separate formulae for separate subperiods as follows:

$$\begin{aligned} r_t &= 0.014 - 1.79319\sqrt{h_t} - \frac{1}{2}h_t + a_t, \\ h_t = \sigma_t^2 &= 0.0003956 + 0.0044637a_{t-1}^2 + 0.726661\sigma_{t-1}^2, \end{aligned} \quad (5.1)$$

for the GARCH(1,1) model of period 1, and

$$\begin{aligned} r_t &= 0.014 - 1.50269\sqrt{h_t} - \frac{1}{2}h_t + a_t, \\ h_t = \sigma_t^2 &= 0.00015201 + 0.0173133a_{t-1}^2 + 0.13366\sigma_{t-1}^2, \end{aligned} \quad (5.2)$$

for the GARCH(1,1) model of period 2. These formulae indicate that the parameters change significantly. Meanwhile, if we ignore the change and fit the GARCH(1,1)

⁴ Data source: <http://sars.health.gov.tw/article.asp?channelid=H&serial=189&click=>

model for the entire period, we have

$$r_t = 0.014 - 1.57103\sqrt{h_t} - \frac{1}{2}h_t + a_t,$$

$$h_t = \sigma_t^2 = 0.0006657 + 0.0043164a_{t-1}^2 + 0.480879\sigma_{t-1}^2, \quad (5.3)$$

which is quite different from either (5.1) or (5.2). The estimates of parameters for different models are shown in Table 5.4.

For an GARCH model, the standardized shocks

$$\tilde{a}_t = \frac{a_t}{\hat{\sigma}_t}$$

are iid and $N(0,1)$. And the Ljung-Box (1978) Q-statistic is expressed as below

$$Q(m) = T(T+2) \sum_{l=1}^m \frac{\hat{\rho}_l^2}{T-l},$$

where $\hat{\rho}_l$ is the lag- l sample autocorrelation of \tilde{a}_t or \tilde{a}_t^2 . Table 5.7 shows the value of the Ljung-Box Q-statistic for the squares of the standardized shocks. It is also noted that the Q-statistic has also been computed for the standardized shocks themselves and the adequacy of the models has been established as well. We can check this table and find the fitted models are all adequate.

Table 5.5 reports the in-sample forecasting accuracy criteria RMSE, MAE, and LE, respectively. It compares the performance of the fitted models. The RMSE criterion is not very robust, and in practice easily influenced by a few large values. The MAE criterion is less susceptible to these values.

Under the accuracy measures for the GARCH(1,1) model, we find higher values for RMSE, MAE and LE when no change point was detected. For example, RMSE for the GARCH(1,1) model with change point is 0.0190417 versus 0.0234972 without change point. Thus, detecting structural change is meaningful in fitting the GARCH(1,1) models to financial data.

5.2 Some Issues of Interest

There are some interesting issues derived from the empirical results. We will illustrate them as follows.

Besides estimating volatility, GARCH types of models have some interesting and useful properties. If we standardized the return,

$$z_t = \frac{r_t}{\hat{\sigma}_t}, \quad (5.4)$$

then z_t will be distributed as $N(0,1)$ when the normality assumption is reasonable for r_t . We can verify this by calculating some statistics, such as mean, standard deviation, skewness and excess kurtosis, as given in Table 5.6. Skewness characterizes the degree of asymmetry of a distribution around its mean. Positive (Negative) skewness indicates a distribution with an asymmetric tail extending towards more positive (negative) values. The skewness of normal distribution is zero. A significant degree of asymmetry (skewness regardless of sign) is bigger than 2 standard errors of skewness (*ses*). The *ses* can be estimated by using the following formula (Tabachnick & Fidell, 1996):

$$ses = \sqrt{\frac{6}{N}},$$

where N is the number of observations.

The excess kurtosis of a standard normal random variable is zero. A distribution with positive excess kurtosis is said to have heavy tails. It implies that the distribution has more mass on the tails than a normal distribution. A significant degree of mass tail (kurtosis regardless of sign) is bigger than 2 standard errors of kurtosis (*sek*). The *sek* can be estimated roughly using the following formula (Tabachnick & Fidell, 1996):

$$sek = \sqrt{\frac{24}{N}},$$

where N is the number of observations.

From Table 5.6, it is clear that the GARCH(1,1) models have heavy tails due to positive excess kurtosis. Hence, the GARCH(1,1) models are not reasonable for modeling volatilities. Since no proper detection of change point is done, the GARCH(1,1) model without change points can be affected by the oscillation of prices coming from some unusual events. In order to get more insight, we combine Table 5.6, Figure 5.3.1 and Figure 5.3.2. Figure 5.3.1 and Figure 5.3.2 are the quantile-quantile (q-q) plots. The q-q plot is a graphical technique for determining if the data have come from a certain distribution. Realized volatility may be fitted with standard normal distribution as shown in Figure 5.3.2. However, it will provide only daily reference and will have no capability of prediction. Meanwhile, it seems that the GARCH(1,1) models are fitted somewhat poorly with the standard normal distribution as shown in Figure 5.3.1. From Table 5.6, we see that the GARCH(1,1) models with or without change points have positive skewness. That means that standardized returns under the GARCH(1,1) models are slightly skewed to the right. The values of Jaque-Bera (JB) test⁵ in Table 5.6 show that the GARCH(1,1) model with change points is fitted better with standard normal distribution than the GARCH(1,1) model without change points. Furthermore, the volatilities under the GARCH(1,1) models are overvalued because their variances of standardized returns are less than 1, as shown in Table 5.6. In short, the GARCH(1,1) models with change points will be a useful model for our purpose, even though it is not perfect.

⁵ Jarque and Bera (1980,1987) showed the Jaque-Bera (JB) test of normality:

$$JB = n \left[\frac{S^2}{6} + \frac{K^2}{24} \right] \sim \chi_2^2$$

, where S = skewness coefficient and K = excess kurtosis coefficient. For a normally distributed variable, S = 0 and K = 0. Therefore, the JB test of normality is a test of the joint hypothesis that S and K are 0 and 0, respectively.

6 Conclusion

Throughout the thesis, the period of the data is from Jan. 2, 2003 to Aug. 29, 2003. It is the period of initial and terminal stages when epidemic SARS spread globally. During this period, many events related to SARS deserve some discussions.

In chapter 5, the LRT was applied to the TAIEX data and detected several change points. We deeply hope that ignoring the change points will lead to a wrong conclusion. As a matter of fact, it is important to detect change points. Overall, we believe our test constitutes a functional tool for testing for a parameter change in the GARCH(1,1) models. We anticipate that methods in the Western Electric Handbook that combined with the LRT can be extended to other types of time series models.

Finally, empirical results are consistent with our expectation. We should attach importance to structural change in time series models which will lead to better prediction. However, overvalued-volatility is a defect of GARCH models. Perhaps, realized volatility mixed with GARCH model can be a useful model. We leave the task of extension to other types of GARCH models for future research.

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Tables

Table 2.1 TAIEX option

Item	Description
Underlying Index	Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX)
Ticker Symbol	TXO
Exercise Style	European
Multiplier	NT\$ 50 (per index point)
Expiration Months	Spot month, the next two calendar months followed by two additional months from the March quarterly Cycle (March, June, September, and December)
Strike Price Interval	100 index points in spot month, the next two calendar months 200 index points in the additional two months from the March quarterly Cycle
Strike (Exercise) Price	<p>When listing series of new expiration months, one series with at-the-money strike price is listed based on the previous day's closing price of the underlying index rounded down to the nearest multiples of 100.</p> <ol style="list-style-type: none"> 1. For the spot month and the next two calendar months: Three other series each with in-the-money and out-of-the-money strike prices with price interval of 100 points are listed. 2. For the next two quarter-months: Two other series each with in-the-money and out-of-the-money strike prices with price interval of 200 points are listed. <p>Up to the 5th business days before expiration,</p> <ol style="list-style-type: none"> 1. For the spot month, and the next two calendar months: additional series are added when the underlying trades through the third highest or lowest strike prices available, to maintain at least 3 in- and 3 out-of-the-money strike prices 2. For the next two quarter-months: additional series are

	added when the underlying trades through the second highest or lowest strike price available, to maintain at least 2 in-and 2 out-of-the-money strike prices
Premium Quotation	<p>< 10 points: 0.1 point (NT\$5)</p> <p>>=10 points, <50 points: 0.5 point (NT\$ 25)</p> <p>>=50 points, <500 points: 1 point (NT\$ 50)</p> <p>>=500 points, <1,000 points: 5 point (NT\$ 250)</p> <p>>=1,000 points: 10 point (NT\$ 500)</p>
Daily Price Limit	+/- 7% of previous day's closing price of the underlying index
Position Limit	<p>Individuals: 8,000 contracts on either side of the market.</p> <p>Institutional Investors: 16,000 contracts on either side of the market.</p> <p>Institutional investors may apply for an exemption from the above limit on trading accounts for hedging purpose.</p> <p>Exemptions are allowed for Future Proprietary Firms.</p>
Trading Hours	08:45AM - 1:45 PM Taiwan time, Monday through Friday of the regular Taiwan Stock Exchange business days
Last Trading Day	The third Wednesday of the delivery month
Expiration Date	The first business day following the last trading day
Final Settlement Price	The final settlement price for each contract is computed from the first fifteen-minute volume-weighted average of each component stock's prices in that index on the final settlement day. For those component stocks that are not traded during the beginning fifteen-minute interval on the final settlement day, their last closing prices would be applied instead
Settlement	Cash settlement. An option that is in-the-money and has not been liquidated or exercised on the expiration day shall, in the absence of contrary instructions delivered to the Exchange by the Clearing Member representing the option buyer, be exercised automatically

Data source: <http://www.taifex.com.tw/>

Table 5.1 Candidates of change points by method 2

(Two out of three consecutive points plot between a distance of 2 sigma and 3 sigma)

Candidates	Date	LRT value	Testing
Change point= 72	Apr. 22, 2003	LRT=4.51952<9.488	Accept H_0
Change point= 73	Apr. 23, 2003	LRT=5.19142<9.488	Accept H_0

Table 5.2 Candidates of change points by method 3

(Ten out of eleven consecutive points plot on one side of the centerline)

Candidates	Date	LRT value	Testing
Change point= 101	Jun. 3, 2003	LRT=7.5551<9.488	Accept H_0

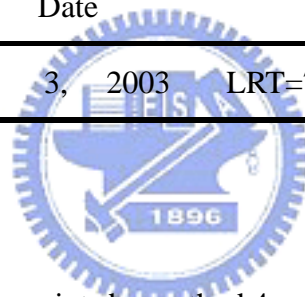


Table 5.3 Candidates of change points by method 4

(A run of five consecutive points of returns plot on one side of the centerline)

Candidates	Date	LRT value	Testing
Change point= 36	Mar. 3, 2003	LRT=3.31969<9.488	Accept H_0
Change point= 37	Mar. 4, 2003	LRT=3.21291<9.488	Accept H_0
Change point= 87	May 14, 2003	LRT=10.75551>9.488	Reject H_0
Change point=106	Jun. 11, 2003	LRT=5.44082<9.488	Accept H_0
Change point=107	Jun. 12, 2003	LRT=5.24297<9.488	Accept H_0
Change point=119	Jun. 30, 2003	LRT=2.92873<9.488	Accept H_0
Change point=120	Jul. 1, 2003	LRT=2.83138<9.488	Accept H_0

Table 5.4 Parameter estimates of GARCH (1,1) model

Parameters	Alpha0	Alpha1	Beta1
GARCH(1,1) model of period 1	0.00039566 (0.0000432084)	0.00446372 (0.002081046)	0.72666151 (0.0004901274)
GARCH(1,1) model of period 2	0.00015201 (0.0000383383)	1.01731336 (0.0029162786)	2.13366277 (0.0688668976)
GARCH(1,1) model of period 1+2	0.00066570 (0.0016216986)	1.00431641 (0.0208480670)	2.48087884 (1.1261979466)

The numbers in parentheses are the standard errors of the parameters.

Table 5.5 The calculations of accuracy measures in every model

	Volatility of GARCH(1,1) model	Volatility of GARCH(1,1) model
	with change point	without change point
RMSE	0.01904166569	0.02349722250
MAE	0.01433519575	0.02296283234
LE	0.81310630801	1.254419791781

Table 5.6 Some statistics of standardized returns

	GARCH(1,1) with c. p.*	GARCH(1,1) without c. p.	RV
Mean	0.130611159	0.05201497	0.041427136
Standard deviation	0.745337197	0.450746075	1.180303863
Skewness	0.662977222	0.454234493	-0.318000015
Excess kurtosis	1.399504106	2.020160196	0.542824948
JB test	25.24304206	33.32238285	4.748427444

* c.p. means change point

Table 5.7 The Ljung-Box Q-Statistic

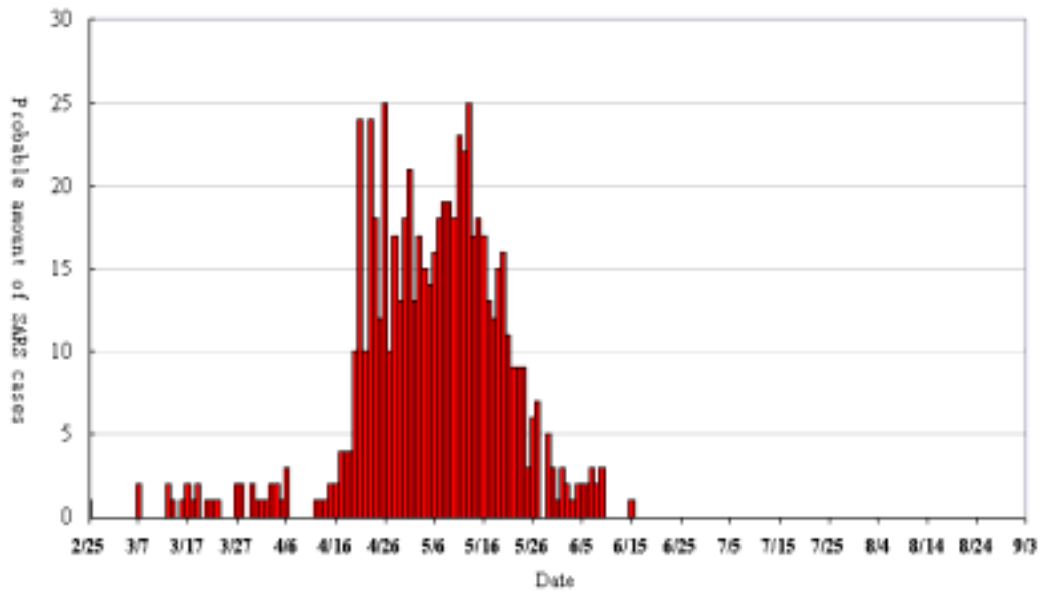
Ljung-Box Q-Statistic	Q(10)	Q(20)
GARCH(1,1) model of period 1	5.5473 (0.85175957)	13.6110 (0.84966122)
GARCH(1,1) model of period 2	11.7482 (0.30225856)	19.5059 (0.48919712)
GARCH(1,1) model of period 1+2	6.8301 (0.74137724)	11.6695 (0.92698101)

The numbers in parentheses are p value of the test statistic. Thus, these models appear to be adequate.



Figures

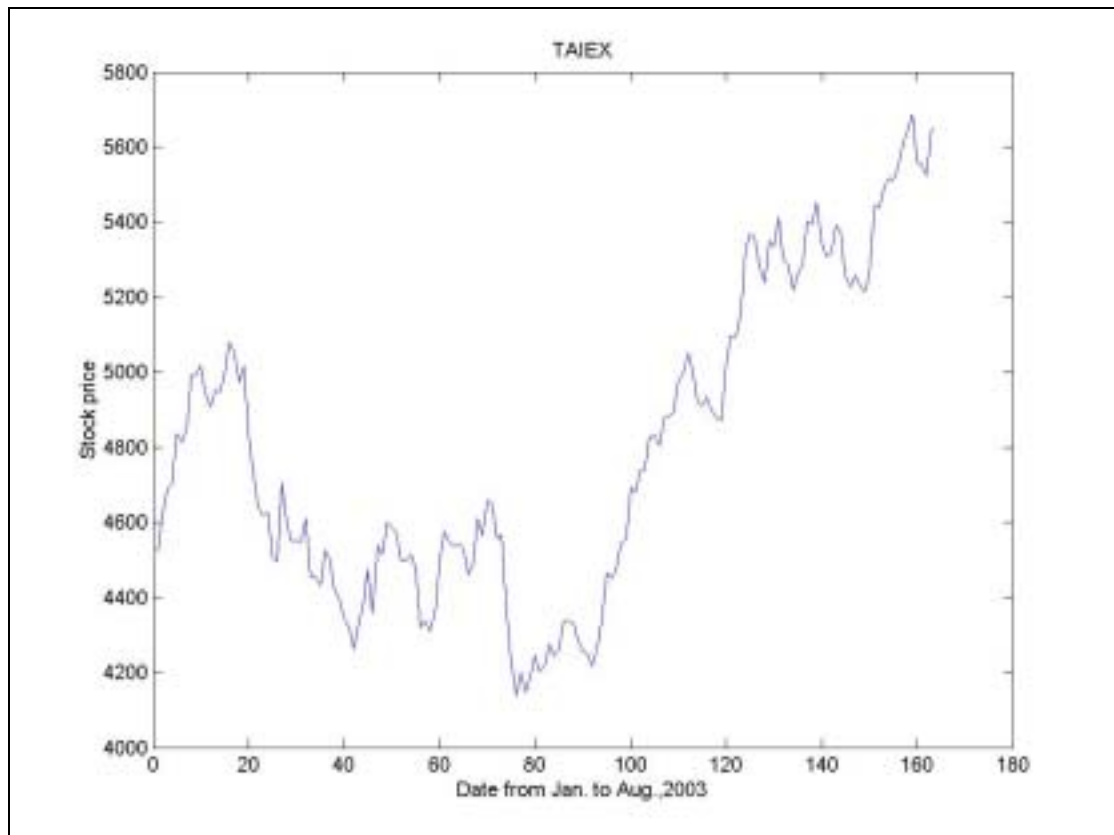
Figure 2.1 The SARS daily probable cases happened in Taiwan from Feb. 25, 2003 to Sep. 4, 2003.



Data source: <http://sars.health.gov.tw/>



Figure 2.2 Stock prices



Data source : Taiwan Stock Exchange Capitalization Weighted Stock Index from 2003.1.2 to 2003.8.29, 162 days in the aggregate. It comes from Taiwan Stock Exchange Corporation. Website: <http://www.tse.com.tw/>

Figure 3.1 Realized volatility of TAIEX during Jan.2, 2003 to Aug. 29, 2003 with 5-min intraday returns.

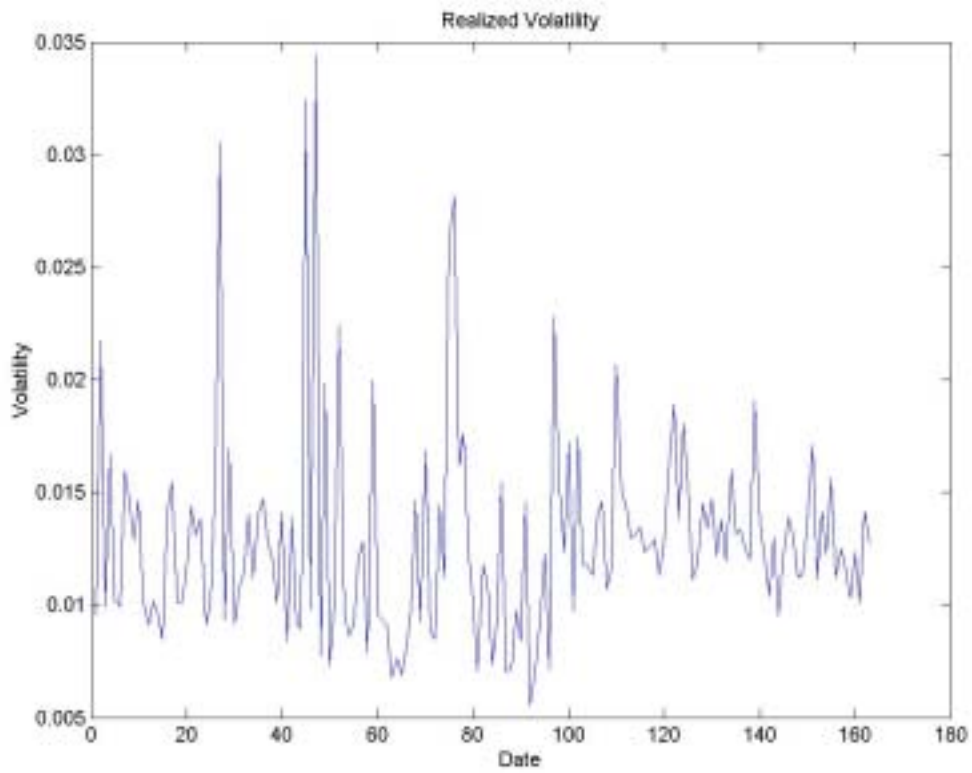


Figure 5.1 Log returns under 6 sigma criterion

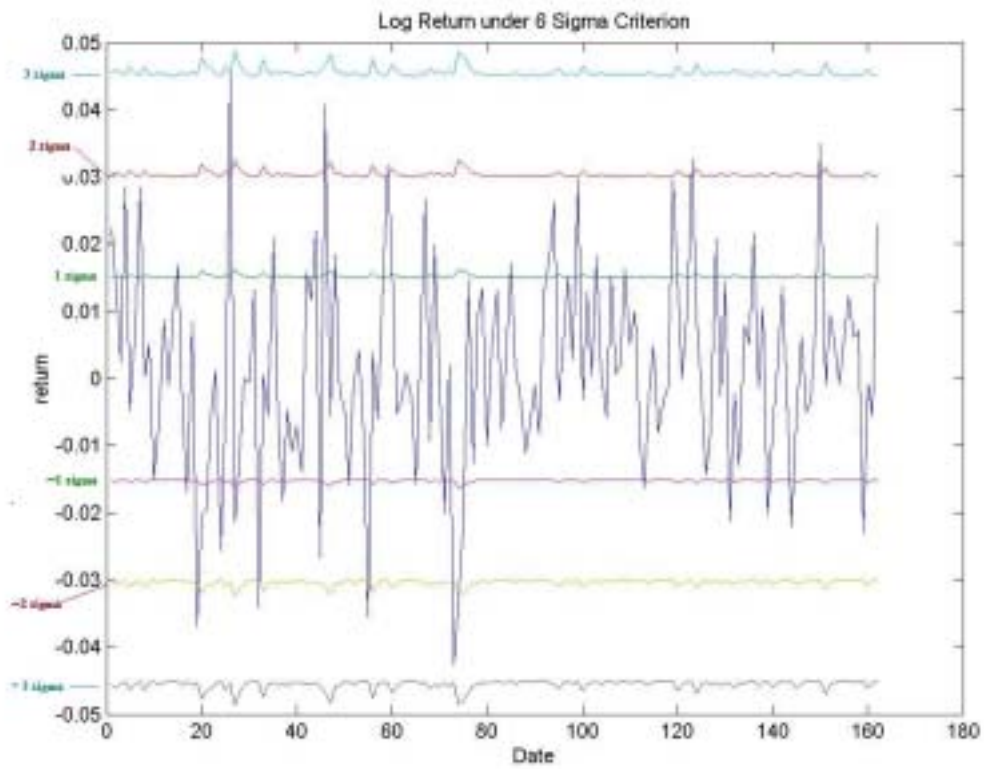


Figure 5.2 Log returns from Jan. 2, 2003 to Aug. 29, 2003

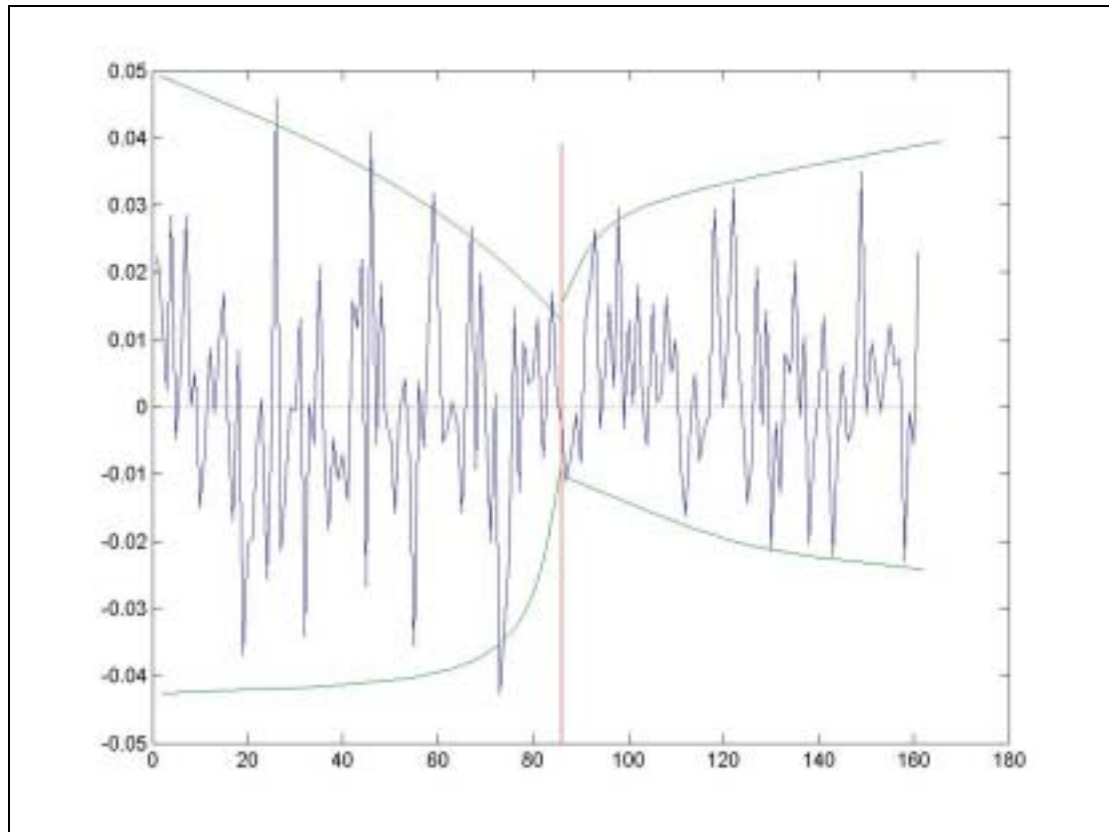
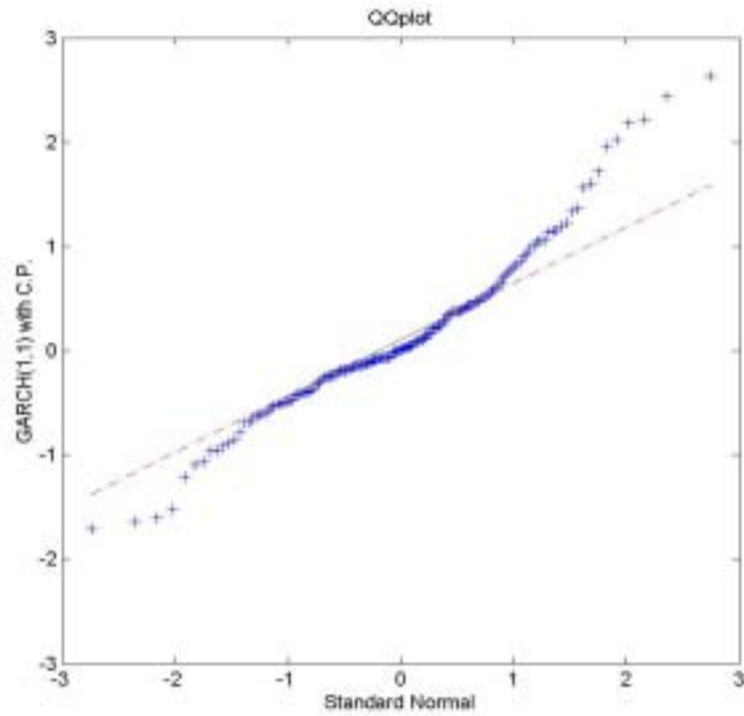


Figure 5.3.1 QQ-plots of the GARCH(1,1) models versus standard normal distribution

(A)



(B)

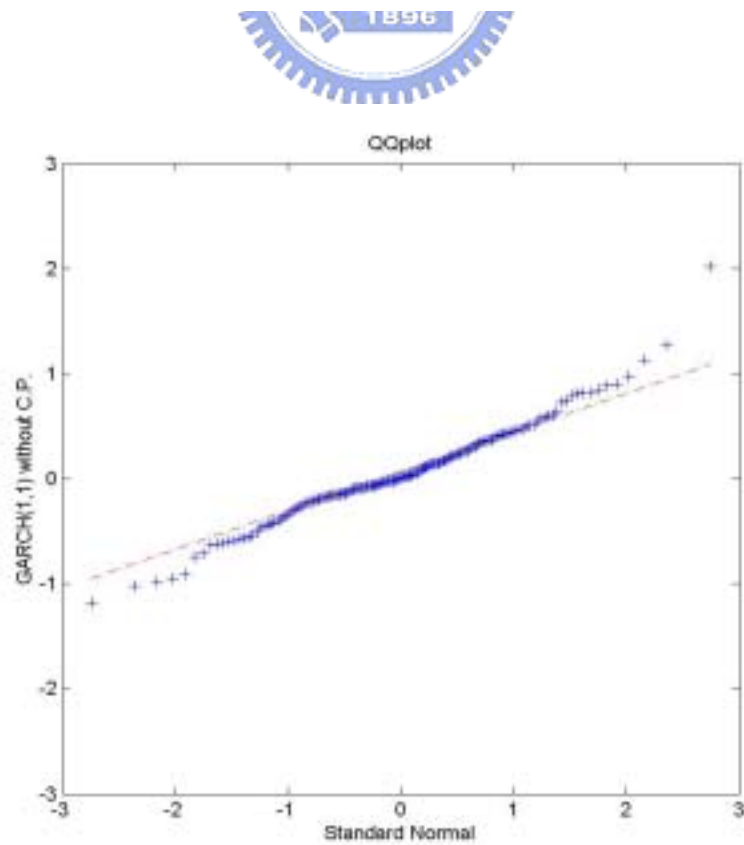


Figure 5.3.2 QQ-plot of realized volatility versus standard normal distribution

