# 國立交通大學

## 管理科學研究所

## 碩 士 論 文

# 在不同免疫策略下
# 臺灣地區債券投資組合績效比較

# The Comparison of Performance for Bond Portfolio in Taiwan under Different Immunization Strategies


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## Abstract

The objective of immunization strategy is to maintain the terminal value of bond portfolio above or close to the target value of portfolio no matter how high or low the interest rate is. The empirical researches in Taiwan concerning immunization strategies and term structure don't consider the removal of the coupon effect and the variation in two estimated coefficients.

Thus, this study uses Nelson & Siegel (1987) model with Gauss-Newton method to determine term structure as well as variate two estimated coefficients simultaneously. The major findings are as follows: (1) M-Vector strategies(M3, M4, and M5) perform quite well in both historical data and the two simulated data. (2) Traditional duration strategies and M-Square strategies can't protect against the change in the risk of interest rate , especially when interest rate changes by a large scale. (3) Nelson & Siegel (1987) model does a good job in fitting the term structure in Taiwan which is demonstrated from high R-Square.

Keywords: immunization strategy, term structure, Nelson & Siegel model.

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## 摘要

免疫策略的目標是無論利率如何變化，維持債券投資組合的期末價值高於或儘可能接近期初目標投資組合價值。在 Nelson & Siegel (1987)的利率期間結構模式的基礎下，過去台灣在此方面的實證研究上，無考量到同時變動利率期間結構變動之二因子，此外，大部份也沒有考慮到票息效果的影響。

因此，本研究利用高斯-牛頓法估計利率期間結構的係數，並考量同時模擬變動任意兩係數下，債券投資組合績效之變化。實證結果有以下三點發現：(1) M-Vector 策略中的 M3, M4,及 M5 無論在模擬資料及實際資料中，皆能一致性地排名於前。(2) 傳統的存續期間免疫策略及 M-Square 策略並不能保護債券投資組合免除利率風險，尤其在利率大幅度變動下。(3) Nelson & Siegel (1987) 模型能相當有效地配適台灣地區實際利率期間結構曲線。

關鍵字:免疫策略、利率期間結構、Nelson & Siegel 模式.

# 誌謝

兩年的研究所生涯，在一陣兵慌馬亂中，即將譜上休止符了。這段日子以來，生活上，學業上，都受到許多人的扶持與照顧，才能順利完成學業。

在論文上，由於起步較慢，打擾指導教授林建榮老師的次數也與日劇增，老師卻不曾有過慍色，在論文的最後關頭成為學生的一大支柱，真不知該如何報答老師。此外，也相當感謝李經遠老師，口試委員林哲群老師及岳夢蘭老師的批評及建議。

在日常生活上，好在，有一群比馬戲團還具娛樂效果的同學(3pway、annie、章魚慶、小鳳)，英文讀書會的成員：雅雯及姿羽，球友建輝，還有軒宸、caca、明慧、小黑(承志)、小師父、耀中、小緯、信助、逸鴻及許許多多一時想不起的朋友們，你們的相伴，讓我揮灑出一片多采多姿的研究所雲彩。

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# 1. Introduction

Capital market is composed of two separate market: stock market and bond market. The former provides a channel for corporations to obtain capital from issuing equity; on the contrary, the latter allows companies to raise capital by issuing debt. In developed countries, these two markets play important roles on the intermediary of capital raising; however, in Taiwan, bond market is still premature compared to stock market. The following Table and Figure show the comparison between bond and stock market in Taiwan.

TABLE 1. 1

The Trading Amounts of Stock Market and Bond Market in Taiwan from 1996-2003

| Year \ Market | Stock Market | Bond Market |
|---|---|---|
| **1996** | 12,907,562 | 28,297,525 |
| **1997** | 37,241,148 | 40,391,963 |
| **1998** | 29,618,969 | 54,957,730 |
| **1999** | 29,291,525 | 52,432,572 |
| **2000** | 30,526,568 | 68,843,106 |
| **2001** | 18,354,935 | 118,992,507 |
| **2002** | 21,873,952 | 134,399,037 |
| **2003** | 20,333,237 | 203,623,979 |

Note: 1. The unit of trading amounts is one million New Taiwan Dollars.

Data Source: Taiwan Central Bank(2004 Feb)

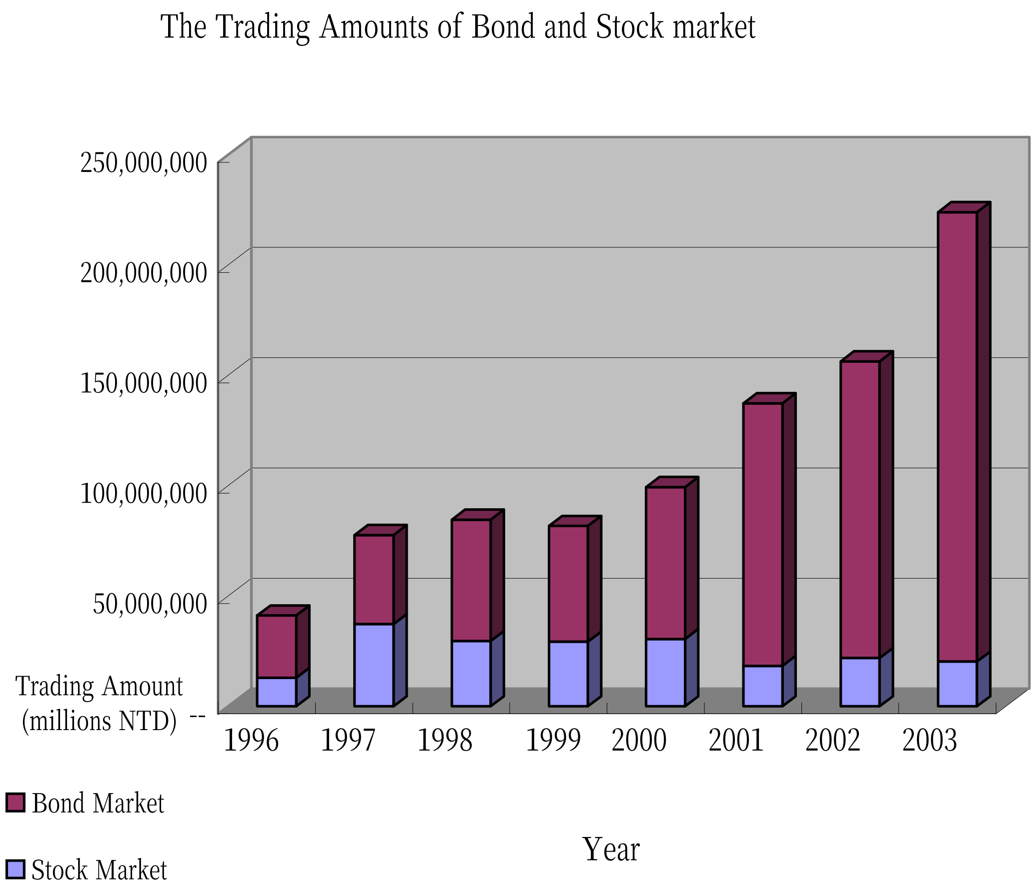




FIGURE 1. 1

The Trading Amounts of Stock Market and Bond Market in Taiwan from 1996-2003

Data Source: Taiwan Central Bank(2004 Feb)

From Figure 1.1, it is clear that since 2001 the trading amounts for bond market has boomed many times compared to those of stock market as a result of the concerns by authorities who started to take actions to create a robust capital market.

With the economic recession, governments around the world such as United States, issued a great number of various bonds to raise sufficient fund in attempt to process more upcoming activities. As a result, the more government debts the financial institutions held, the higher interest rate risk they encountered. To avoid such risk, immunization strategies are used for eliminating it. Immunization strategy is defined as a set of bond management procedures that aims at protecting investors against changes in interest rates. In other words, the objective of immunization strategies is to maintain the terminal value of bond portfolio above the original portfolio value no matter how high or low the interest rate is.

Term structure is another key factor that we should pay attention to, besides immunization strategies. Under different specification of term structure model, we can derive entirely different discount function. That is to say, the fundamental fairness of the pricing of interest-rate related commodity or derivatives depends on the set of term structure model. A number of term structure models are developed for the past few decades, which can be divided into three categories, including theoretical models-equilibrium model, arbitrage-free model and curve fitting method.

Market efficiency depends on the fair pricing of all financial instruments. Accordingly, term structure plays an important role in the pricing of interest-rate-related commodities. A recent document from Bank for International Settlements (BIS)[1] provides an overview of estimation methods used by some of the most important central banks.

I. Nelson & Siegel model: This model is used by the Central Bank such as Belgium, Finland, France, Italy, Spain, etc.

II. Svensson model: This model is used by the Central Bank such as Germany, Norway, Spain, Sweden, UK, etc.

III. Smoothing Splines model: This model is used by the Central Bank such as Japan, USA, etc.

Among these models, I select Nelson & Siegel model, so-called parsimonious model,

[1] Excerpt from, Damir Filipovic, "Consistency Problems for Heath-Jarrow-Morton Interest Rate Models" *lecture notes in mathematics, Springer, 2001.*

also, the prototypical model of Svensson as the main valuation standard. The reasons to select this term structure are 1. outstanding capability of portraying the curve of term structure, so, the Central Bank all over the world use it to determine fairly valuing standard. 2. few empirical studies in Taiwan focused on this model. 3. the easy accessibility and simplicity.

The contributions of this study are that an excellent model, which is called Nelson & Siegel model, is used to portray the curve of term structure of interest rate; in addition, by means of variating two parameters in Nelson & Siegel model, the impact of variation on immunization strategies is measured.

The remainder of the thesis is organized as follows. In Section 2, several term structure models and various immunization strategies are mentioned. Section 3 express the procedures in research design on detail. Empirical result is shown in Section 4. Section 5 concludes.

# 2. Literature Review

## 2.1 Term structure models

As mentioned earlier, term structure models may be divided into three categories, they are general equilibrium model, no-arbitrage model and curve fitting, respectively. Subsequently, I describe them concisely in order.

### 2.1.1 General Equilibrium Model

Vasicek (1977) observed that the economy tends toward some equilibrium based on such fundamental factors as the productivity of capital, long-term monetary policy, and so on, and short-term rates will be characterized by mean reversion. His model is written as $dr = k(\theta - r)dt + \sigma dw$, the constant $\theta$ denotes the long term value and the positive constant $k$ denotes the speed of mean reversion. $\sigma dw$ describes the volatility of short rate with zero mean. Generally speaking, the random walk is a unstable process. To improve this shortcoming, this model uses Ornstein-Uhlenbeck process, a stable process, so that the interest rate will reverse to the mean value of long-term rate, but not to infinite value to the last.

### 2.1.2 No-Arbitrage Model

The van to no-arbitrage model falls down on Ho and Lee (1986). They take the term structure as given and derive the feasible subsequent term structure movements, which must satisfy certain constraints to ensure that they are consistent with an equilibrium framework. Through binomial lattice, the bond price must equal the price determined by the initial discount function, or there exists an opportunity for riskless arbitrage. The model of extreme form is

$$dr = \theta(t)dt + \sigma dz .$$

As mentioned earlier, Ho and Lee also build a binomial lattice to derive bond price. The binomial lattice is demonstrated as follows

$$
\begin{array}{ccl}
 & & P_{i+1}^{(n+1)}(\cdot) \quad \text{upstate} \\
 & \nearrow & \\
P_i^{(n)}(\cdot) & & \\
 & \searrow & \\
 & & P_i^{(n+1)}(\cdot) \quad \text{downstate}
\end{array}
$$

$P_i^{(n)}(\cdot)$ is the discount function, and the superscript denotes the time, the subscript denotes the state. Namely , it is a stochastic process for the variation of interest rate, thus, the variation of bond price either goes up or goes down with path-dependence. The drawbacks, however, are that the interest rate will tend toward infinite to the last.

### 2.1.3 Curve Fitting

McCulloch (1971) developed a piecewise quadratic polynomial function to fit a smooth curve from observations on prices of securities, and, derived a regression equation for bond pricing formula, consequently, estimating the regressions by weighted least-square method. Finally, the discount function and term structure are infered from the above regressors.

Owing to finding that exponential spline and spline methods are subject to the same shortcomings that the resulting yield function tends to bend sharply toward the end of the maturity range observed in the sample, Nelson & Siegel (1987) introduces a parametrically parsimonious model for yield curves that has the ability to improve the above shortcomings and that is flexible enough to represent the shapes generally associated with yield curves: monotonic, humped, and S shaped. Further, the model explain 96% of the variation in bill yields across maturities during the period 1981-1983, showing its excellent ability to predict the price of the long-term Treasury bond. Nelson and Siegel (1987) model is derived from a instantaneous forward rate, which is the solution to a second-order differential equation with real and unequal roots:

$$r(m) = \beta_0 + \beta_1 \times \exp(-m/\tau_1) + \beta_2 \times \exp(-m/\tau_2) \qquad (2.1)$$

where $\tau_1$ and $\tau_2$ are time invariants associated with the equation, and $\beta_0$, $\beta_1$, and $\beta_2$ are determined by initial conditions. Integrate (2.1) from zero to m and divides by m, and the resulting function is

$$R(m) = \beta_0 + \beta_1 (\tau/m)\left[1 - \exp(-\tau/m)\right] + \beta_2 (\tau/m)\left[1 - \exp(-m/\tau)(m/\tau + 1)\right]. \qquad (2.2)$$

where R(m) denotes the yield to maturity on a bond at m point in time; $\beta_0$ is the long-term component, which causes the yield curve to shift by the same amount; $\beta_1$ denotes the short-term component, measured by the change of yield curve's slope; $\beta_2$ denotes the medium-term component, which describes the variation in curvature of yield curve; $\tau$ is a time component, which determines the rate at which the regressors decay to zero. Small values of $\tau$ correspond rapid decay in the regressors and therefore will able to fit curvature at low maturities well while being unable to fit excessive curvature over longer maturity ranges and vice versa.

Kuo (1998) estimates the term structure of interest rate proposed by Nelson and Siegel (1987) model. He minimizes the difference between real yield to maturity and estimated yield to maturity to derive the estimated coefficients of term structure. However, the definition of term structure is the relationship between spot rate and time to maturity for bond so that the term structure in his study is not the real term structure but just a yield curve.

## 2.2 Immunization Strategies

In 1971, Fisher & Weil expanded the concept of duration to apply the immunization strategies to empirical study so as to prevent portfolio value of bonds from falling below the target value at the end of horizon. Under the assumption that the movement of term structure should be shifted by the same amount, an excellent result was shown in terms of deviation from target rate of return. The performance of duration strategies could avoid much more interest rate risk than could the traditional naïve and maturity strategies, where duration strategy is defined as equating duration of portfolio with investment horizon. It got criticism, however, for its unreasonable assumption about term structure.

Bierwag and Kaufman(1977), Khang(1979) and others have postulated alternative models of interest rate behaviors, developed a corresponding different measure of duration, however, the drawback is that the value of portfolio is protected only against the particular type of interest rate change assumed.

See that Fisher & Weil's unreasonable assumption, Fong & Vasicek(1984) also raised so-called M-square assuming the variation of the term structure was randomly, which is the function of time. Otherwise, they proved this strategy provided a lower limit on the change in the end-of-horizon value of an immunized portfolio for an interest rate change. The lower limit, $\Delta I_H / I_H \geq (-1/2)KM^2$, is the product of two terms, of which one is a function of the interest rate change only, and the other depends only on the structure of the portfolio. The first term, denoted by –1/2K, is with respect to interest rate outside the control of investor. Therefore, minimize the second term, $M^2$, the exposure of the portfolio to any interest rate change is the lowest.

Only assuming that the term structure of continuously compounded interest rates can be expressed as a polynomial, Chambers and Carleton(1981) demonstrated hat the finite and noninstantaneous return of a default-free bond can be expressed as a linear combination of a duration vector and a shift vector. They derive immunization strategies from the Chambers and Carleton model and test them. The result of the portfolio tests indicates that the duration vector approach is successful in improving immunization performance, relative to the traditional maturity and naïve strategies.

Not only the shifted movement of term structure but also the change of slope and curvature on term structure affect the efficiency of immunization strategies, the M-absolute model, proposed by Nawalkha and Chambers(1996), is claimed to prevent against the following two factors—variation of slope or curvature. The advantages of this model is that it doesn't specify particular type of interest rate change and it's considerably simple and valid relative to all the other models.

Nawalkha and Chambers(1997) developed M-vector model, like the M-square model, not imposing strong assumptions on the particular type of interest rate change. The M-vector also provides a more generalized approach to interest rate risk hedging than the term structure functional form-based traditional duration vector model given by Cooper(1977), Chambers, Carleton, and McEnally(1988). In addition, setting the order of M-vector to 1 reduces it to the traditional duration model, while setting the order equaling to 2 reduces it to the alternative approach to the M-square model give by Fong and Fabozzi(1985). In the case of high-order setting, it provides sufficiently protection against the change of slope and curvature in the term structure.

Kuo(1998) measured the performance of bond portfolio by M-Vector strategies, proposed by Nawalkha & Chambers(1997). In addition to historical data, he also tested the M-Vector strategies under the variation of simulated term structure. His result shows that under the historical data, M5 strategies can stand against more interest rate risk than all the other M-Vector strategies. Under the simulated term structure of interest rate, the result is shown that M1 strategy dominates other strategies in terms of parallel shift of term structure; in the case of variation of slope of term structure, M2 strategy prevails over all the other immunization strategies; in terms of variation of curvature of term structure, M2 and M3 strategies beat other strategies. At last, he drew the conclusions that if the variation on slope or curvature of term structure is larger than that on parallel shift of term structure, M-Vector

Wu(1998) tested the traditional immunization strategies and multi-factor immunization strategies under historical data and simulated data over Taiwan market range from 1993 to 1997. In terms of historical data, M-Square and M-Absolute dominate the traditional immunization strategies; in contrast to simulated data, M-Square, M-Absolute, and M-Vector strategies beat other strategies apparently.

Based on above empirical results, the immunization strategies can be classified into four categories according to its characteristics of immunization

I. Variation of term structure on parallel shift: M1 and traditional duration strategies. (bullet and barbell strategies)

II. Variation of term structure on slope: M2 strategy.

III. Variation of term structure on curvature: M2 and M3 strategies.

IV. Variation of term structure on slope and curvature: M-Absolute strategy.

# 3. Research Design

## 3.1 Data Source

The data is collected from Taiwan GreTai Securities Market during 1999-2004. The bond samples sums up at most 75 samples in 2004, at least 19 samples in 1999. Because of the imperfect bond market and simplicity, I try to modify the collected data as follows

1. The bond sample is collected at the end of April and October during the sample period. If there is no transaction at that day for the $i^{th}$ underlying bond, I collect the trading data the day before that specific day as sample. Even if there is no transaction within two weeks on that month, the virtual price is created by term structure of interest rate.
2. For the sake of simplicity, bonds with twice coupon payment per year are deleted[2].

Empirical results cover the historical data ranging from 1999 to 2004 and simulated data ranging from 2001 to 2004. Thus for historical data, I set the total investment horizon to 5 years; for simulated data, it is set to 3 years. Besides, the performance of simulated data is evaluated every half a year; that is to say, the frequency of reinvestment for simulated data is 6 months different from that of historical data.

## 3.2 Research Assumption

To simplify the empirical work, but not to lose the reality of market, this research makes the following five assumptions

1. The purchasing amount of bonds should be an integer; that is, investors could just

[2] Since 1996, all bonds were issued with once coupon payment per year. Accordingly, this procedure has little bit of impact on the process of analysis.

take an integer position in an investment.

2. There is no personal income tax.
3. Unlimited short sales are allowed.
4. The initial investment amount is 100000 New Taiwan Dollars (NTD), and the face value for each bonds is 100 dollars.
5. There is no transaction cost, in other words, there is no cost of buying or selling any bond.

## 3.3 Immunization strategies

The main objective of this thesis is to compare the performance between several immunization strategies, ranging from traditional methods to the latest multi-factor methods, on the basis of a specific term structure. In the following section, I introduce the immunization strategies, then, the term structure used to value bonds.

1. Bullet strategy: This strategy is the simplest immunization strategy restricted to the condition that investment horizon equals to duration of portfolio and is to minimize the time interval between duration and investment horizon. Construction of bullet strategy is as follows

$$\text{Min} \quad \sum_{i=1}^{n} n_i p_i (D_i - H)^2$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \frac{n_i p_i D_i}{I_0} = H$$

$$\sum_{i=1}^{n} \mathrm{n}_i \mathrm{p}_i = \mathrm{I}_0 \ .$$

$n_i$ denotes the holding quantity of $i^{th}$ bond; $p_i$ denotes the current price of $i^{th}$ bond; $D_i$ denotes the $i^{th}$ bond's Macaulay Duration; H is the target investment horizon; $I_0 = \sum_{j=1}^{m} c_j p_o(s_j)$ denotes the initial portfolio value and $c_j$ is the cash inflow at time j, where $p_0(t)=\exp\left(-\int_0^t i(t)dt\right)$ is the current discount function and i(t) is the current forward rate of term t..

2. Barbell strategy: This strategy is to maximize the time interval between duration and investment, under the constraint of equating the investment horizon to the duration of portfolio. Construction of barbell strategy is as follows

$$\text{Max} \quad \sum_{i=1}^{n} n_i p_i (D_i - H)^2$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \frac{n_i p_i D_i}{I_0} = H$$

$$\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{n_i p_i} = \mathrm{I_0} \ .$$

3. M-Square strategy: This strategy is to select a portfolio which makes its M-Square value($M_i^2$) the minimum, and modify it half a year. Construction of M-Square strategy is as follows

$$\text{Min} \quad \sum_{i=1}^{n} n_i p_i M_i^{\ 2}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \frac{n_i p_i D_i}{I_0} = H$$

$$\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{n_i p_i} = \mathrm{I_0}$$

$$\text{where } \mathrm{M_i}^2 = \sum_{j=1}^{m} \frac{(s_j - H)^2 c_j p_o(s_j)}{I_0}$$

4. M-Absolute strategy: This strategy is to select a portfolio which makes its M-Absolute value($M_i^A$) the minimum, and modify it half a year. Construction of M-Absolute strategy is as follows

$$\text{Min} \quad \sum_{i=1}^{n} n_i p_i M_i^{\ A}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \frac{n_i p_i D_i}{I_0} = H$$

$$\sum_{i=1}^{n} n_i p_i = I_0$$

where $M_i^A = \sum_{j=1}^{m} \frac{|s_j - H| c_j p_o(s_j)}{I_0}$.

5. M-Vector strategy: This strategy is to select a portfolio which makes its M-Vector value($M_i^m$) the minimum, and modify it half a year. Because the superscript, m, in $M_i^m$ denotes the power, there are five immunization strategies by means of changing the value of m from 1 to 5[3]. Thus, according to m value, these five strategies can be denoted by M1, M2, M3, M4, and M5, respectively. Construction of M-Vector is as follows

$$\text{Min} \quad \sum_{i=1}^{n} (n_i p_i)^2$$

$$\text{s.t.} \quad \sum_{i=1}^{n} n_i p_i M_i^m = 0 \quad \forall m = 1,2,3,4,5$$

$$\sum_{i=1}^{n} n_i p_i = I_0$$

where $M_i^m = \sum_{j=1}^{m} \frac{(s_j - H)^m c_j p_o(s_j)}{I_0}$.

Excellent model makes it easier to predict the terminal value of a set of bonds. In the light of the empirical studies in Taiwan which don't remove the "coupon effect[4]" from term structure, this research uses a well-known and widely used term structure – Nelson & Siegel (1987) model, as the valuing model.

## 3.4 Term Structure Model

I select Nelson & Siegel (1987) model as the term structure model for its

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3 The value is limited to 5 due to that the high value for m denotes the change of interest rate by large range; however, it's reasonable to assume that the interest rate in Taiwan doesn't change by large range.

4 The impact of coupon level on the yield-to-maturity of coupon bonds with the same maturity is called coupon effect. More generally, yields across fairly priced securities of the same maturity vary with the cash flow structure of the securities.

excellent fitting ability. The model is shown as follows

$$R(m) = \beta_0 + \beta_1(\tau/m)\left[1 - \exp(-\tau/m)\right] + \beta_2(\tau/m)\left[1 - \exp(-m/\tau)(m/\tau + 1)\right] \qquad (3.1)$$

where R(m) denotes the yield to maturity on a bond in time m; $\beta_0$ is the long-term component, which causes the yield curve to shift by the same amount; $\beta_1$ denotes the short-term component, measured by the change of yield curve's slope; $\beta_2$ denotes the medium-term component, which describe the variation in curvature of yield curve; $\tau$ is a time component, which determines the rate at which the least-squares estimators decay to zero. A small values of $\tau$ corresponds to rapid decay in the least-squares estimators and therefore will be able to fit the curvature at low maturities well while it is unable to fit excessive curvature over longer ranges in maturity, and vice versa.

After realizing the meaning of each term contained in Nelson-Siegel's model, nevertheless, I encounter the problem that R(m), the spot rate for different maturity, may not be collected easily from bond market due to few zero-coupon bonds. The solution to this problem is to denote $P_i$ the market price of $i^{th}$ bond which has been observed. The following equation shows the relationship between the market price and the value of the bonds

$$P_i = B_i + \varepsilon_i, \quad 1 \le i \le n \qquad (3.2)$$

$$B_i = \sum_{i=1}^{n} \frac{CF_{im}}{\left[1 + R(m)\right]^{t_{im}}}, 1 \le i \le n. \qquad (3.3)$$

Where $P_i$ denotes the current market price of $i^{th}$ bond; $B_i$ denotes the theoretical value of $i^{th}$ bond; i means $i^{th}$ bond; n denotes the number of bonds; $CF_{im}$ is the cash flow of $i^{th}$ bond in time m; R(m) denotes the spot rate m periods from now on; $t_{im}$ is the time interval for $i^{th}$ bond at t away from the next coupon-paying date.

Thus, rearrange (3.1) (3.2) (3.3), I get the following result

$$P_i = \sum_{m=t}^{T} \frac{CF_{im}}{\left\{1+\beta_0+\beta_1(\tau/m)(1-\exp(-m/\tau))+\beta_2(\tau/m)(1-\exp(-m/\tau)(m/\tau+1))\right\}^{t_{im}}} + \varepsilon_i, 1 \le i \le n$$

(3.4)

Equation (3.4) is a nonlinear regression model, so we can adopt the Gauss-Newton method to approximate the nonlinear regression model and, then, employs ordinary least-squares to estimate the parameters. Specifically, the least-squares estimate method to estimate the coefficients is expressed as follows

$$\underset{\beta_0\beta_1\beta_2\tau}{Min} \sum_{i=1}^{n} (P_i - B_i)^2 = \sum_{i=1}^{n} \left\{ P_i - \sum_{m=t}^{T} \frac{CF_{im}}{[1+R(m)]^{t_{im}}} \right\}^2 . \tag{3.5}$$

Where $R(m) = \hat{\beta}_0 + \hat{\beta}_1(\tau/m)\left[1-\exp(-\tau/m)\right] + \hat{\beta}_2(\tau/m)\left[1-\exp(-m/\tau)(m/\tau+1)\right]$. There are four parameters in this model. So, it's rather difficult to partially differentiate at $\tau$. According to Chang(2000), for simplicity, it's better to set a range to $\tau$. Here, following Chang's suggestion, this study set $\tau \in S$, $S = \{0.1 \times k \,|\, 1 \le k \le 100\}$. Given $\tau$, (3.5) can be transformed as

$$\underset{\beta_0(\tau)\beta_1(\tau)\beta_2(\tau)}{Min} \sum_{i=1}^{n} (P_i - B_i)^2 . \tag{3.6}$$

Thus, the parameters which need to be estimated are reduced to three parameters. As far as Gauss-Newton method concerned, the choice of initial starting values is very essential because a poor choice may result in slow convergence, or even divergence. However, the choice of initial starting values is not the main issue in this study, so just leave it in appendix.

After determining the initial starting values for $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$, the next step is to use Gauss-Newton method to calculate the terminal value until converging. A Taylor series expansion on (3.6) is performed to approximate the non-linear regression

$$Q_i \cong f(M_i, N_i, g^{(0)}) + \left[\frac{\partial f(M_i, N_i, \beta_k)}{\partial \beta_k}\right](\beta_k - g_k^{(0)}) \tag{3.7}$$

$$\frac{\partial f(M_i, N_i, \beta_k)}{\partial \beta_0} = \sum_{m=t}^{T}(-m)\times(CF_{im})\times(1+R(m))^{-t_{im}}$$

$$\frac{\partial f(M_i, N_i, \beta_k)}{\partial \beta_1} = \sum_{m=t}^{T}(-m)\times(CF_{im})\times(1+R(m))^{-t_{im}}\times\left[(\frac{\tau}{m})(1-\exp(\frac{-m}{\tau}))\right]$$

$$\frac{\partial f(M_i, N_i, \beta_k)}{\partial \beta_2} = \sum_{m=t}^{T}(-m)\times(CF_{im})\times(1+R(m))^{-t_{im}}\times\left[(\frac{\tau}{m})(1-(\frac{m}{\tau}+1)\exp(\frac{-m}{\tau}))\right]$$

where

$$\underset{p\times 1}{g}^{(0)} = \begin{bmatrix} g_0^{(0)} \\ g_1^{(0)} \\ \vdots \\ g_{p-1}^{(0)} \end{bmatrix} \tag{3.8}$$

Note that $g^{(0)}$ is the vector of the parameters' starting values and, thus, $\beta_k = g_k^0$, where $k = 0,1,2$. To simplify the notation as follows

$$f_i^{(0)} = f(M_i, N_i, g^{(0)}) \tag{3.9}$$

$$\alpha_k^{(0)} = \beta_k - g_k^{(0)} \tag{3.10}$$

$$D_{ik}^{(0)} = \left[\frac{\partial f(M_i, N_i, \beta_k)}{\partial \beta_k}\right]_{\beta = g^{(0)}} \tag{3.11}$$

$$P_i \approx f_i^{(0)} + \sum_{k=0}^{p-1} D_{ik}^{(0)} \alpha_k^{(0)} \quad . \tag{3.12}$$

Denote $P_i - f_i^{(0)}$ by $Q_i^{(0)}$, the linear regression model approximation can be written as

$$Q_i^{(0)} \approx \sum_{k=0}^{p-1} D_{ik}^{(0)} \alpha_k^{(0)} + \varepsilon_i \ . \tag{3.13}$$

Then, express the linear regression model approximation (3.12) in matrix form as follows

$$Q^{(0)} \approx D^{(0)} \alpha^{(0)} + \varepsilon \, . \tag{3.14}$$

where

$$\underset{n\times 1}{Q^{(0)}} = \begin{bmatrix} P_1 - f_1^{(0)} \\ \vdots \\ P_n - f_n^{(0)} \end{bmatrix} \tag{3.15}$$

$$\underset{n\times p}{D^{(0)}} = \begin{bmatrix} D_{1,0}^{(0)} & \cdots & D_{1,p-1}^{(0)} \\ \vdots & & \vdots \\ D_{n,0}^{(0)} & \cdots & D_{n,p-1}^{(0)} \end{bmatrix}$$

$$\underset{p\times 1}{\alpha^{(0)}} = \begin{bmatrix} \alpha_0^{(0)} \\ \vdots \\ \alpha_{p-1}^{(0)} \end{bmatrix}$$

$$\underset{n\times 1}{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

The parameters $\underset{3\times 1}{\alpha^{(0)}}$ can be estimated by least-squares

$$b^{(0)} = (D^{(0)'} D^{(0)})^{-1} D^{(0)'} Q^{(0)}. \tag{3.16}$$

where $b^{(0)}$ is the vector of the least-squares coefficients. Then, it's easy to use these least-squares estimates to obtain revised estimated regression coefficients $g_k^{(1)}$ by means of (3.10)

$$g_k^{(1)} = g_k^{(0)} - b_k^{(0)}. \tag{3.17}$$

where $g_k^{(1)}$ denotes the revised estimate of $\beta_k$ at the end of the first iteration. Express (3.17) in matrix form

$$g^{(1)} = g^{(0)} - b^{(0)}. \tag{3.18}$$

To examine whether the revised regression coefficients represent adjustments in the proper direction, residual sum of square is employed

$$SSE^{(0)} = \sum_{i=1}^{n} [P_i - f(M_i, N_i, g^{(0)})]^2 = \sum_{i=1}^{n} (P_i - f_i^{(0)})^2. \tag{3.19}$$

The revised estimated regression coefficients after the first iteration are $g^{(1)}$, and the corresponding residual sum of square is updated as $SSE^{(1)}$

$$SSE^{(1)} = \sum_{i=1}^{n} [P_i - f(M_i, N_i, g^{(1)})]^2 = \sum_{i=1}^{n} (P_i - f_i^{(1)})^2 . \tag{3.20}$$

If the Gauss-Newton method is working effectively in the first iteration, $SSE^{(1)}$ should be smaller than $SSE^{(0)}$ since the revised estimated regression coefficients $g^{(1)}$ should be better estimates. The Gauss-Newton method therefore repeats the procedure just described, with $g^{(1)}$ now used for the new starting values. The iterative process continues until the differences between successive least-squares criterion measures $SSE^{(s+1)} - SSE^{(s)}$ become negligible.

### 3.4.1 Term Structure Model-Simulated Part

According to Lai(1996), the key factors with relatively great effect on the variation of term structure are the estimated coefficient, $\hat{\beta}_0$ and $\hat{\beta}_2$. Most of the empirical studies, however, related to simulated data over past few years focused on only one factor's variation while treating others as given. To improve the empirical study, my study is based on the combinations of any two factors from the three estimated coefficients; that is, there are total three combinations of simulated data to be examined in this study.

I. Variation of $\hat{\beta}_0$ and $\hat{\beta}_1$

In this scenario, $\hat{\beta}_2$ is controlled unchangeably for observing the impact of the other two factors change on performance of bond portfolio. At every modifying date, the value of $\hat{\beta}_0$ and $\hat{\beta}_1$ is either subtracted or added by 0.01 until the last modifying date.

II. Variation of $\hat{\beta}_0$ and $\hat{\beta}_2$

In this scenario, $\hat{\beta}_1$ is controlled unchangeably for observing the impact of the other two factors change on performance of bond portfolio. At every modifying date, the value of $\hat{\beta}_0$ and $\hat{\beta}_2$ is subtracted by 0.01 until the last modifying date.

III. Variation of $\hat{\beta}_1$ and $\hat{\beta}_2$

In this scenario, $\hat{\beta}_0$ is controlled unchangeably for observing the impact of the other two factors change on performance of bond portfolio. At every modifying date, the value of $\hat{\beta}_1$ and $\hat{\beta}_2$ is subtracted by 0.01 until the last modifying date.

Consequently, the input value of total investment amounts at the beginning of every modifying period is calculated as follows

$$I_{Pt} = \sum_{i=1}^{n} P_{i,t} \times N_{i,t-1} + INT_{i,t} \times N_{i,t-1} \tag{3.20}$$

$I_{Pt}$ is the total investment amount of portfolio at date t, $P_{i,t}$ denotes the market value of $i^{th}$ bond at date t, and $INT_{i,t}$ denotes the interest of $i^{th}$ bond at date t. According to the above procedure, the investment amount at the beginning of modifying period could be determined. However, once there are lacking for quotes on several bonds, it is necessary to determine the virtual value of bonds lacking quotes by term structure of interest rate.

To measure the derivation between realized return and target return, the difference between the realized return and target return is used as the indicator to measure immunization risk

$$\text{DIF}=(R_r - R_t) \tag{3.21}$$

$R_r$ denotes the realized return at the end of investment horizon, and $R_t$ denotes the target return determined at the beginning of investment horizon.

Finally, following is the procedures for simulated data.

1. Determine the virtual bond price at the beginning of each investment period:

   Input the parameters of term structure to each investment period so that virtual price for each bond can be determined. Then, implement the criteria of immunization strategies to select optimal bond portfolio.

2. Determine the virtual bond price at the end of each investment period:

   In order to obtain the terminal amounts of bond portfolio as initial investment of next period, it's necessary to subtract the time to maturity of each bond, determine the terminal virtual price of each bond so as to have terminal amounts of bond portfolio from (3.10).

3. After finishing step 1 & 2, the subsequent procedures are the same as those of historical data.

# 4. Empirical Result

Table 4.1 shows the summary of statistics for all investment horizon. There are only 19 quotes of bonds on April 1999 because of the immature bond market. As time goes by, as the authorities started to be concerned about taking valid actions on the bond market, the number of bonds shown in Table 4.1 have doubled many times as of April 1999. In addition, the trend of yield to maturity went down over past five years.

In order to evaluate fitness of the term structure of interest rate by Nelson & Siegel (1987) model, the coefficient of determination is used as a measure to examine the efficiency of this model. The definition of coefficient of determination is as follows

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(P_i - \hat{B}_i)^2}{\sum_{i=1}^{n}(P_i - \overline{P})^2}. \tag{4.1}$$

$P_i$ denotes the current market price of i[th] bond, $\hat{B}_i$ is the theoretical price derived from Nelson & Siegel (1987) model and $\overline{P}$ is the average price of all bonds at a specific date. The definition of this indicator is similar to the coefficient of determination used in linear regression.

Table 4.2 describes the empirical results gained by Gauss-Newton method including the date of tests, the optimal value of $\tau$,[5] number of iterations, coefficient of determination, and the optimal regression coefficients. From table 4.2, $R^2$ is at least 0.92 during the whole period, showing Nelson & Siegel (1987) model does portray the term structure in Taiwan precisely. The estimated regression coefficients $\hat{\beta}_1$ and

5 $\tau$ is the speed of decay, due to the maturity of most bonds is within 20 years, it is reasonable to set maximum value of $\tau$ to 20.

TABLE 4. 1 Summary of Statistics for All Investment Horizons

| Attribution | Date | 1999/04 | 2000/4 | 2001/4 | 2001/10 | 2002/04 | 2002/10 | 2003/04 | 2003/10 | 2004/04 |
|---|---|---|---|---|---|---|---|---|---|---|
| Coupon Rate (unit:%) | Average | 6.51 | 6.3 | 6.11 | 5.82 | 5.57 | 5.32 | 5.02 | 4.81 | 4.55 |
| | Max | 7.3 | 7.3 | 7.3 | 7.3 | 7.3 | 7.3 | 7.3 | 7.3 | 7.3 |
| | Min | 5.13 | 5.13 | 4.63 | 3.5 | 2.25 | 2.25 | 1.38 | 1 | 1 |
| Maturity | Average | 9.83 | 10.95 | 8.82 | 9.87 | 9.52 | 9.2 | 8.67 | 8.09 | 7.85 |
| | Max | 19.98 | 20.1 | 19.33 | 29.71 | 29.25 | 28.71 | 28.25 | 27.71 | 27.25 |
| | Min | 3.56 | 2.59 | 1.1 | 1.06 | 0.6 | 0.63 | 0.17 | 0.05 | 0.12 |
| Market Price | Average | 105.18 | 103.06 | 108.83 | 106.82 | 105.87 | 105.44 | 104.39 | 104.41 | 113.27 |
| | Max | 113.04 | 110.44 | 120.25 | 119.9 | 120.27 | 120.45 | 120.78 | 121.05 | 141.41 |
| | Min | 89.86 | 92.44 | 99.27 | 86.11 | 86.43 | 86.79 | 72.73 | 73.51 | 93.11 |
| Duration | Average | 8.3 | 8.63 | 6.62 | 7.14 | 6.78 | 6.79 | 6.66 | 6.23 | 6.17 |
| | Max | 12.42 | 14.85 | 12.38 | 17.31 | 16.17 | 17.3 | 17.24 | 17.55 | 17.22 |
| | Min | 4.47 | 2.42 | 1.44 | 0.97 | 0.55 | 0.62 | 0.17 | 0.05 | 0.11 |
| Yield to Maturity (unit:%) | Average | 5.56 | 5.76 | 4.64 | 3.49 | 4 | 3.16 | 2.11 | 2.31 | 2.15 |
| | Max | 6.16 | 5.94 | 5.25 | 4.35 | 4.62 | 4.2 | 3.46 | 3.24 | 3.12 |
| | Min | 5.14 | 5.07 | 4.16 | 2.5 | 2.6 | 1.85 | 1 | 0.7 | 0.7 |
| Samples | | 19 | 26 | 45 | 52 | 58 | 61 | 61 | 70 | 75 |
| Original Samples | | 24 | 33 | 52 | 58 | 64 | 66 | 66 | 74 | 79 |

Note: 1. Original samples are samples that haven’t been filtered out by research assumptions.

TABLE 4. 2

Empirical Results by Gauss-Newton Method

| Date | $\tau$ | Number of Gauss-Newton Iterations | $R^2$ | Regression Coefficients | | |
|---|---|---|---|---|---|---|
| | | | | $\hat{\beta}_0$ | $\hat{\beta}_1$ | $\hat{\beta}_2$ |
| 1999/04 | 20 | 140 | 0.9256 | 23.6711 | -23.4246 | -25.2699 |
| 2000/04 | 20 | 149 | 0.9798 | 17.7759 | -17.7129 | -17.0298 |
| 2001/04 | 20 | 171 | 0.9631 | 12.1294 | -12.0992 | -11.1415 |
| 2001/10 | 20 | 164 | 0.9636 | 10.6477 | -10.6138 | -10.3075 |
| 2002/04 | 17.9 | 164 | 0.9582 | 10.9299 | -10.8924 | -10.4722 |
| 2002/10 | 18 | 197 | 0.9720 | 7.9589 | -7.9615 | -7.3338 |
| 2003/04 | 20 | 87 | 0.9554 | 8.6503 | -8.6073 | -8.8525 |
| 2003/10 | 20 | 140 | 0.9883 | 6.8776 | -6.8851 | -6.2405 |
| 2004/04 | 13 | 112 | 0.9836 | 4.2614 | -4.2352 | -4.2271 |

Note: 1. $\tau$ is the speed of decay.

2. $\underset{\beta_0\beta_1\beta_2\tau}{Min}\sum_{i=1}^{n}(P_i - B_i)^2 = \sum_{i=1}^{n}\left\{P_i - \sum_{m=t}^{T}\frac{CF_{im}}{[1+R(m)]^{t_{im}}}\right\}^2$, where

$$R(m) = \hat{\beta}_0 + \hat{\beta}_1(\tau/m)\left[1-\exp(-\tau/m)\right] + \hat{\beta}_2(\tau/m)\left[1-\exp(-m/\tau)(m/\tau+1)\right].$$

Where $P_i$ denotes the current market price of i^th bond; $B_i$ denotes the theoretical value of i^th bond; i means i^th bond; n denotes the number of bonds; $CF_{im}$ is the cash flow of i^th bond at m point in time; R(m) denotes the spot rate m time from now on; $t_{im}$ is the time interval for i^th bond at t point in time to next coupon-paying date.

$\hat{\beta}_2$ are the key factors in determining the shape of the term structure. They can be divided into four categories based on the positive or negative value of $\hat{\beta}_1$ and $\hat{\beta}_2$ as follows

I. $\hat{\beta}_1 < 0$ $\hat{\beta}_2 < 0$: The term structure is positive slope shape.

II. $\hat{\beta}_1 < 0$ $\hat{\beta}_2 > 0$: The term structure is positive slope with humped shape.

III. $\hat{\beta}_1 > 0$ $\hat{\beta}_2 < 0$: The term structure is negative slope with valley shape.

IV. $\hat{\beta}_1 > 0$ $\hat{\beta}_2 > 0$: The term structure is negative slope.

It's clear that regression coefficients, $\hat{\beta}_1$ and $\hat{\beta}_2$ in table 4.2, are all less than zero, thus, the shape of term structure is positive slope shape.

Table 4.3 shows the comparison of performance among seven immunization strategies over historical data, and M-Vector(M3, M4, and M5) strategies rank the first three. Moreover, the performance of traditional duration strategies (bullet and barbell strategies) which fall behind the other strategies could result from the change of interest rate by a large scale or slope and curvature in the term structure. Most of the immunization strategies can not prevail over the target return of zero-coupon bond, which may result from the frequency of modifying is too few so that the bond portfolio encounters enormous interest risk or too few bonds to be priced precisely. Eventually, as Table 4.3 shown, M3, M4, and M5 strategies are among all immunization strategies which track the target return closely.

Table 4.4 shows the comparison of performance among all immunization strategies over simulated data while the variation of term structure goes down; on the contrary, Table 4.5 shows the comparison of performance among all immunization strategies over simulated data while the variation of term structure goes up.

It's worth noting that the performance of bullet and M-Square strategies are complete the same over two sorts of simulated data, which match Bierwag, Fooladi, and Roberts[1] (1993) study. Moreover, M3, M4, and M5 strategies mostly rank the first three whether the simulated variation of term structure goes up or down, representing its great capability against interest rate risk. Such kind of results meet the Kuo(1998) study and theoretical nature of M-Vector strategies. Recall back to Section 2.2--the four categories according to its characteristics of immunization[2], M-Absolute strategy should perform well in the scenario of variation of $\hat{\beta}_1$ and $\hat{\beta}_2$, though, the result in this study don't show a consistent performance that M-Absolute strategy beat other strategies.

M-Square strategy, in theory, ought to prevail over traditional duration strategies (bullet and barbell) and M-Absolute strategies, however, performs quite badly among all strategies, which matches Yang(2002) result. The reason may be due to imperfect bond market in Taiwan-too few bonds that can't satisfy the restrictions of M-Square result in its inefficiency. Eventually, the performance for traditional duration strategies on these two simulated data is identical to that on historical data, which almost falls

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[1] Let $D^k = \dfrac{\sum_{j=1}^{m} c_j p(s_j) S_j^k}{I_0}$, if the following conditions are hold, then, the portfolio selected by bullet and M-Square strategies are the same.

(1) $D_1^1 < D_2^1 < \cdots < D_n^1$ (subscript denotes i[th] bond)

(2) $D_1^2 < D_2^2 < \cdots < D_n^2$

(3) $D_j^1$ should be non-convexity. (i.e. $(D_j^1 < D_{j-1}^1) \le (D_{j-1}^1 - D_{j-2}^1)$ )

(4) $D_j^2$ should be strictly convex. (i.e. $(D_j^2 < D_{j-1}^2) > (D_{j-1}^2 - D_{j-2}^2)$ )

[2] I. Variation of term structure on parallel shift: M1 and traditional duration strategies. (bullet and barbell strategies)

II. Variation of term structure on slope: M2 strategy.

III. Variation of term structure on curvature: M2 and M3 strategies.

IV. Variation of term structure on slope and curvature: M-Absolute strategy.

behind as last three. Besides, the performance of traditional duration strategies over the historical data and two sorts of simulated data matches Wu(1997) and Yang(2002) empirical results.

Table 4.6 & 4.7 show the value of estimated coefficients and variation of $\hat{\beta}_0$ and $\hat{\beta}_1$; Figure 4.1 & 4.2 portray the corresponding curve based on the value in Table 4.6 & 4.7. Table 4.8 & 4.9 show the value of estimated coefficients and variation of $\hat{\beta}_0$ and $\hat{\beta}_2$; Figure 4.3 & 4.4 portray the corresponding curve based on the value in Table 4.8 & 4.9. Table 4.10 & 4.11 show the value of estimated coefficients and variation of $\hat{\beta}_1$ and $\hat{\beta}_2$; Figure 4.5 & 4.6 portray the corresponding curve based on the value in Table 4.10 & 4.11.

TABLE 4. 3

The Comparison of Performance among Immunization Strategies over Historical Data

| Returns of Immunization Strategies | Historical Data | DIF (Target Return: 1.26567) |
|---|---|---|
| Bullet | 1.172(8) | -0.093587 |
| Barbell | 1.2005(6) | -0.065190 |
| M-Absolute | 1.2144(4) | -0.051356 |
| M-Square | 1.1897(7) | -0.075837 |
| M1 | 1.1514(9) | -0.114221 |
| M2 | 1.2122(5) | -0.053470 |
| M3 | 1.2515(3) | -0.014132 |
| M4 | 1.2769(2) | 0.011233 |
| M5 | 1.2884(1) | 0.022770 |

Note: 1. The number listed behind the return in the parenthesis is the rank among all immunization strategies.

2. DIF= $(R_r - R_t)$ , where $R_r$ denotes the realized return at the end of investment horizon, and $R_t$ denotes the target return determined at the beginning of investment horizon.

3. Target return is the return of zero-coupon bond (default-free) five years from now derived from Nelson-Siegel model.

TABLE 4. 4 The Comparison of Performance among Immunization Strategies over Simulated Data with down Variation of Term Structure

| Returns of Immunization Strategies | Simulated Data-Change of $\hat{\beta}_0$ & $\hat{\beta}_1$ | Simulated Data-Change of $\hat{\beta}_0$ & $\hat{\beta}_2$ | Simulated Data-Change of $\hat{\beta}_1$ & $\hat{\beta}_2$ |
|---|---|---|---|
| Bullet | 1.12136(7) | 1.05608(7) | 1.06498(7) |
| Barbell | 1.09348(9) | 1.03868(9) | 1.04837(9) |
| M-Absolute | 1.14927(1) | 1.06739(4) | 1.07658(1) |
| M-Square | 1.12136(7) | 1.05608(7) | 1.06498(7) |
| M1 | 1.12938(2) | 1.05868(6) | 1.06659(6) |
| M2 | 1.12761(3) | 1.06542(5) | 1.07013(5) |
| M3 | 1.12525(4) | 1.06781(3) | 1.0711(4) |
| M4 | 1.12446(5) | 1.06862(2) | 1.07192(3) |
| M5 | 1.12319(6) | 1.06874(1) | 1.07194(2) |

Note: 1.The number listed behind the return in the parenthesis is the rank among all immunization strategies.

2. $\beta_0$ is the long-term component, which causes the yield curve to shift by the same amount; $\beta_1$ denotes the short-term component, which measures the change of yield curve's slope; $\beta_2$ denotes the medium-term component, which describes the curvature variation of yield.

TABLE 4. 5 The Comparison of Performance among Immunization Strategies over Simulated Data with Up Variation of Term Structure

| Returns of Immunization Strategies | Simulated Data-Change of $\hat{\beta}_0$ & $\hat{\beta}_1$ | Simulated Data-Change of $\hat{\beta}_0$ & $\hat{\beta}_2$ | Simulated Data-Change of $\hat{\beta}_1$ & $\hat{\beta}_2$ |
|---|---|---|---|
| Bullet | 0.89351(8) | 0.93927(7) | 0.93932(7) |
| Barbell | 0.91105(5) | 0.92224(9) | 0.92222(9) |
| M-Absolute | 0.9039(6) | 0.95495(5) | 0.955(5) |
| M-Square | 0.89351(8) | 0.93927(7) | 0.93932(7) |
| M1 | 0.90133(7) | 0.94417(6) | 0.94564(6) |
| M2 | 0.91763(4) | 0.95677(4) | 0.9596(4) |
| M3 | 0.9265(3) | 0.96295(3) | 0.96657(3) |
| M4 | 0.93084(2) | 0.96594(2) | 0.96996(2) |
| M5 | 0.93364(1) | 0.96785(1) | 0.97196(1) |

Note: 1. The number listed behind the return in the parenthesis is the rank among all immunization strategies.

2. $\beta_0$ is the long-term component, causing the yield curve to shift by the same amount; $\beta_1$ denotes the short-term component, measuring the change of yield curve's slope; $\beta_2$ denotes the medium-term component, describing the curvature variation of yield.

TABLE 4. 6

The Value of Estimated Coefficients and Variation of Term Structure

| Date | $\hat{\beta}_0$ | $\hat{\beta}_1$ | $\hat{\beta}_2$ |
|---|---|---|---|
| 2001/04 | 12.1294 | 12.0992 | 11.1415 |
| 2001/10 | 12.1194 | 12.0892 | 11.1415 |
| 2002/04 | 12.1094 | 12.0792 | 11.1415 |
| 2002/10 | 12.0994 | 12.0692 | 11.1415 |
| 2003/04 | 12.0894 | 12.0592 | 11.1415 |
| 2003/10 | 12.0794 | 12.0492 | 11.1415 |

Note: 1. $\hat{\beta}_0$ and $\hat{\beta}_1$ are subtracted by 0.01 at each period as time goes by.

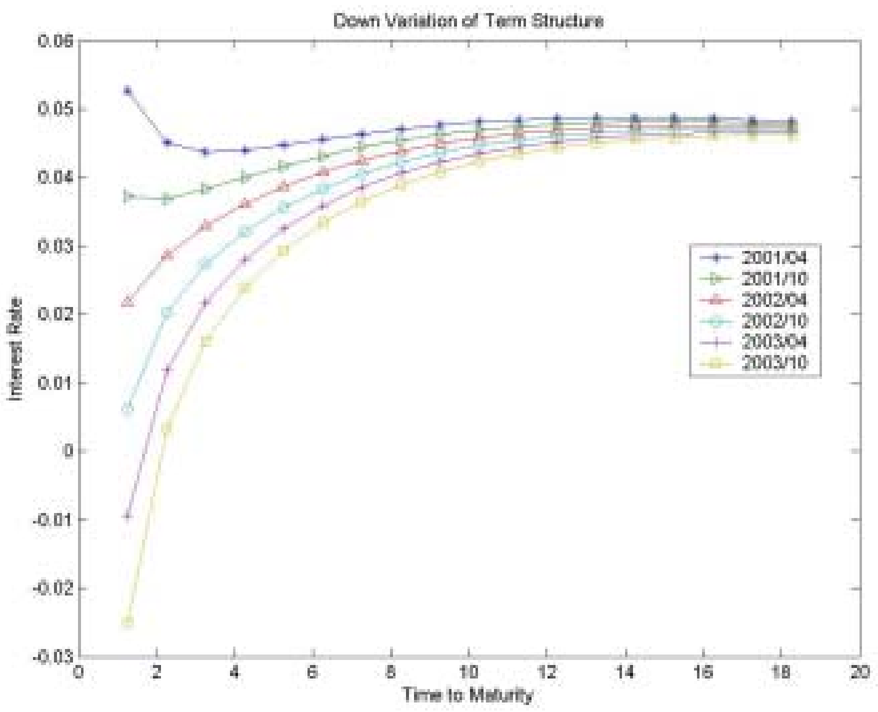


FIGURE 4. 1

The Variation of Term Structure

TABLE 4. 7

The Value of Estimated Coefficients and Variation of Term Structure

| Date | $\hat{\beta}_0$ | $\hat{\beta}_1$ | $\hat{\beta}_2$ |
|---|---|---|---|
| 2001/04 | 12.1294 | 12.0992 | 11.1415 |
| 2001/10 | 12.1394 | 12.1092 | 11.1415 |
| 2002/04 | 12.1494 | 12.1192 | 11.1415 |
| 2002/10 | 12.1594 | 12.1292 | 11.1415 |
| 2003/04 | 12.1694 | 12.1392 | 11.1415 |
| 2003/10 | 12.1794 | 12.1492 | 11.1415 |

Note: 1. $\hat{\beta}_0$ and $\hat{\beta}_1$ are added by 0.01 at each period as time goes by.

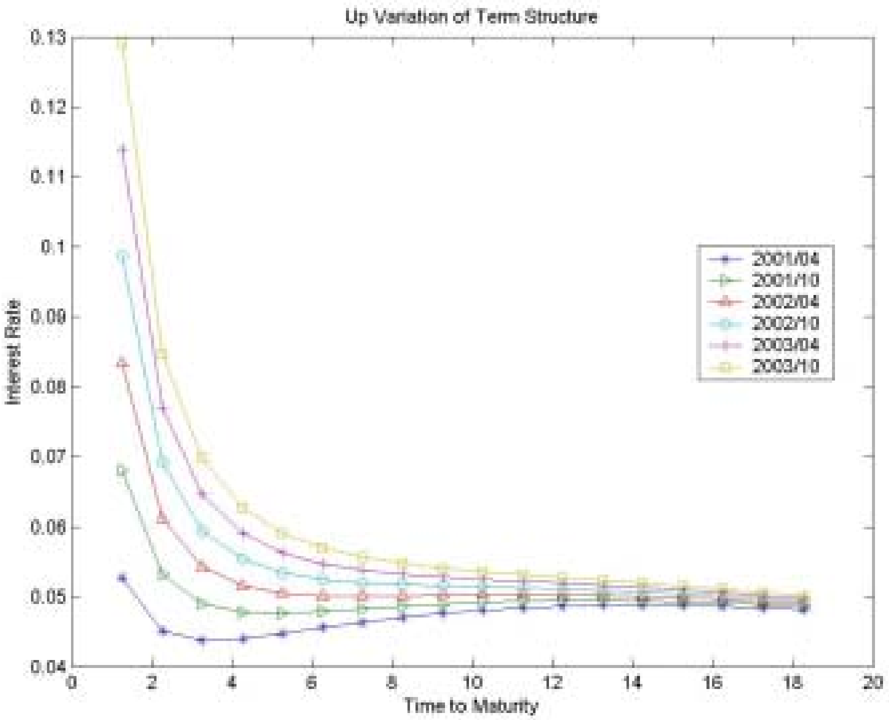


FIGURE 4. 2

The Variation of Term Structure

TABLE 4. 8

The Value of Estimated Coefficients and Variation of Term Structure

| Date | $\hat{\beta}_0$ | $\hat{\beta}_1$ | $\hat{\beta}_2$ |
|---|---|---|---|
| 2001/04 | 12.1294 | 12.0992 | 11.1415 |
| 2001/10 | 12.1194 | 12.0992 | 11.1315 |
| 2002/04 | 12.1094 | 12.0992 | 11.1215 |
| 2002/10 | 12.0994 | 12.0992 | 11.1115 |
| 2003/04 | 12.0894 | 12.0992 | 11.1015 |
| 2003/10 | 12.0794 | 12.0992 | 11.0915 |

Note: 1. $\hat{\beta}_0$ and $\hat{\beta}_2$ are subtracted by 0.01 at each period as time goes by.

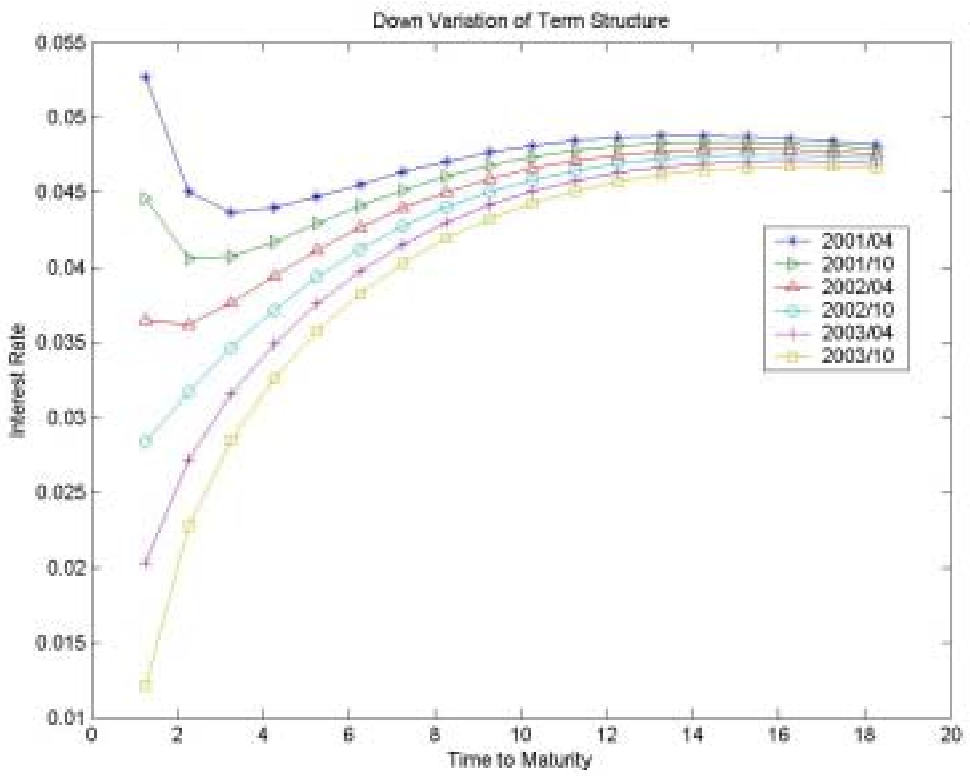


FIGURE 4. 3

The Variation of Term Structure

TABLE 4. 9

The Value of Estimated Coefficients and Variation of Term Structure

| Date | $\hat{\beta}_0$ | $\hat{\beta}_1$ | $\hat{\beta}_2$ |
|---|---|---|---|
| 2001/04 | 12.1294 | 12.0992 | 11.1415 |
| 2001/10 | 12.1394 | 12.0992 | 11.1515 |
| 2002/04 | 12.1494 | 12.0992 | 11.1615 |
| 2002/10 | 12.1594 | 12.0992 | 11.1715 |
| 2003/04 | 12.1694 | 12.0992 | 11.1815 |
| 2003/10 | 12.1794 | 12.0992 | 11.1915 |

Note: 1. $\hat{\beta}_0$ and $\hat{\beta}_2$ are added by 0.01 at each period as time goes by.

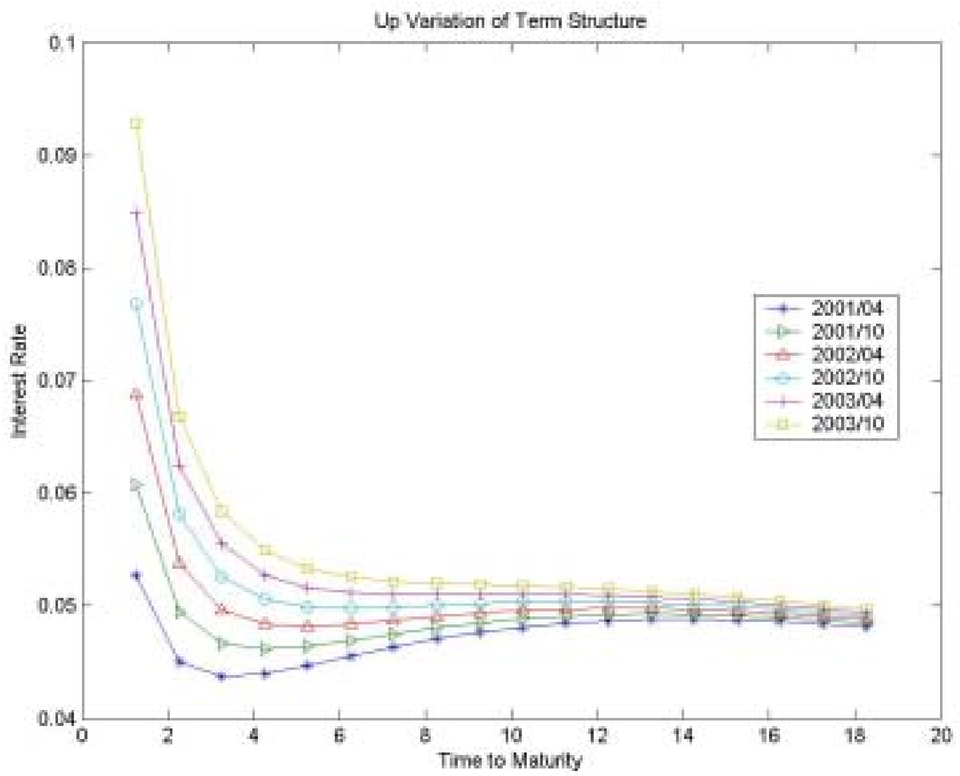


FIGURE 4. 4

The Variation of Term Structure

TABLE 4. 10

The Value of Estimated Coefficients and Variation of Term Structure

| Date | $\hat{\beta}_0$ | $\hat{\beta}_1$ | $\hat{\beta}_2$ |
|---|---|---|---|
| 2001/04 | 12.1294 | 12.0992 | 11.1415 |
| 2001/10 | 12.1294 | 12.0892 | 11.1315 |
| 2002/04 | 12.1294 | 12.0792 | 11.1215 |
| 2002/10 | 12.1294 | 12.0692 | 11.1115 |
| 2003/04 | 12.1294 | 12.0592 | 11.1015 |
| 2003/10 | 12.1294 | 12.0492 | 11.0915 |

Note: 1. $\hat{\beta}_1$ and $\hat{\beta}_2$ are subtracted by 0.01 at each period as time goes by

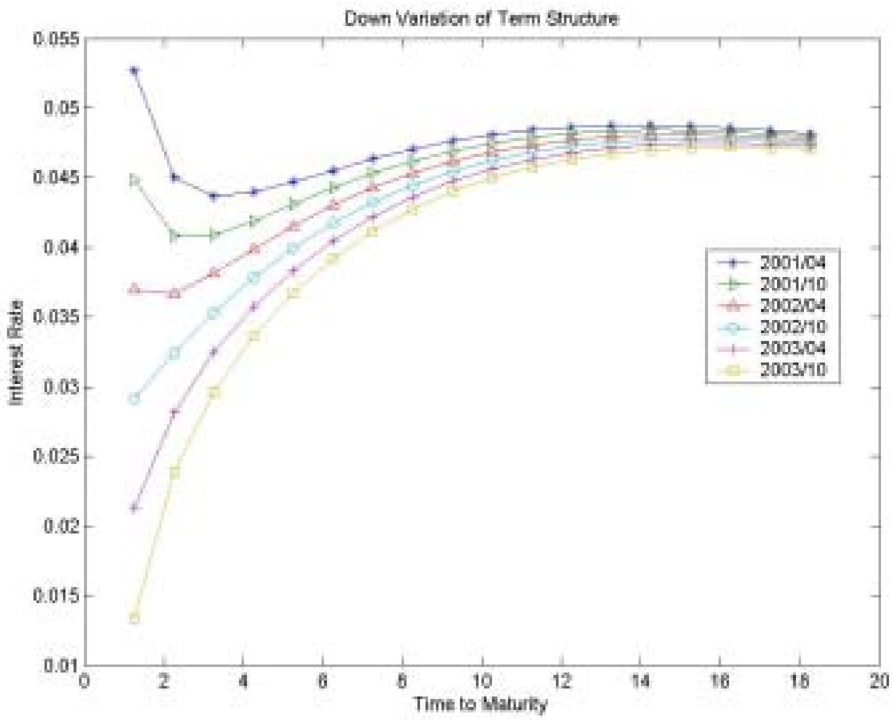


FIGURE 4. 5
The Variation of Term Structure

TABLE 4. 11

The Value of Estimated Coefficients and Variation of Term Structure

| Date | $\hat{\beta}_0$ | $\hat{\beta}_1$ | $\hat{\beta}_2$ |
|---|---|---|---|
| 2001/04 | 12.1294 | 12.0992 | 11.1415 |
| 2001/10 | 12.1294 | 12.1092 | 11.1515 |
| 2002/04 | 12.1294 | 12.1192 | 11.1615 |
| 2002/10 | 12.1294 | 12.1292 | 11.1715 |
| 2003/04 | 12.1294 | 12.1392 | 11.1815 |
| 2003/10 | 12.1294 | 12.1492 | 11.1915 |

Note: 1. $\hat{\beta}_1$ and $\hat{\beta}_2$ are added by 0.01 at each period as time goes by.

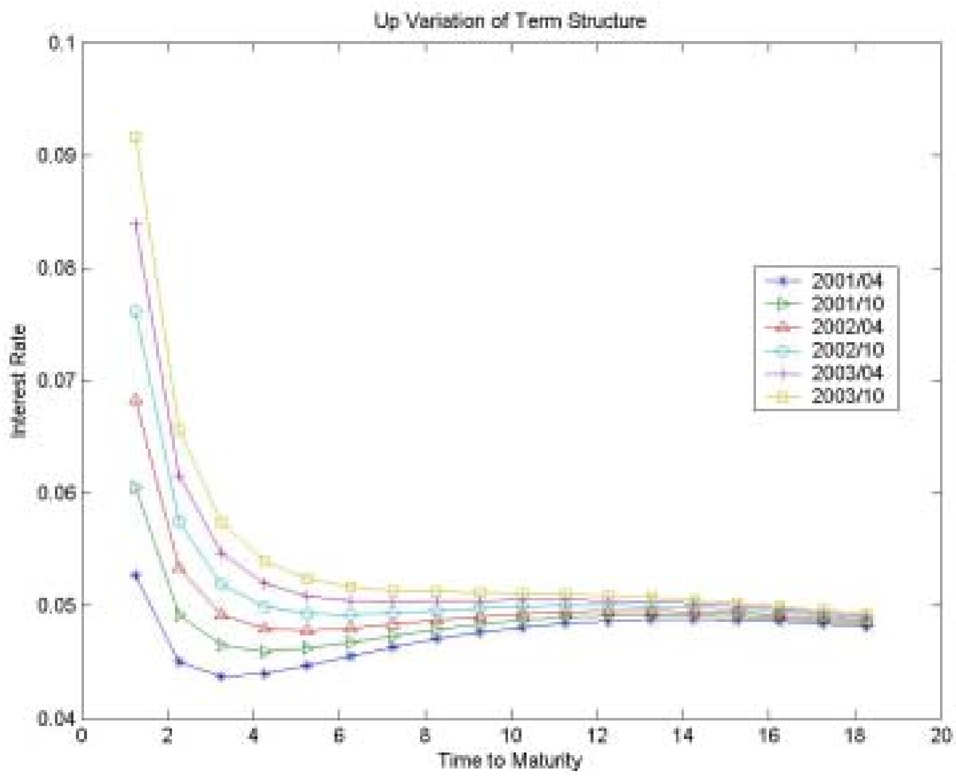


FIGURE 4. 6

The Variation of Term Structure

# 5. Conclusion

Taiwan administration started to take aggressive actions on finance reform over past few years. In a way, this reform did activate capital market—merger and acquisition among financial institutions were nothing new and the trading amounts in bond market have been more than those in stock market since 2001. In the light of premature bond market compared to stock market, it's worth being devoted to probe into this field.

There are two issues discussed in this study as follows:

1. Determine the optimal immunization strategies under historical and simulated variation of term structure of interest rate.
2. Derive the precise term structure by means of Nelson & Siegel (1987) model.

The investment horizon of historical data is ranging from 1999 to 2004 and the investment horizon of virtual data is ranging from 2001 to 2004. The bond samples sums up at most 75 samples in 2004, at least 19 samples in 1999. There are three conclusions can be drawn, they are:

1. The M-Vector (M3, M4, and M5) strategies are ranked as top three no matter which sorts of data are used. However, it is difficult to make a decision as which is the best for the sake of inconsistent rank among three strategies.
2. Traditional duration strategies and M-Square strategies can't protect against the change in the risk of interest rate, especially when interest rate changes by a large scale.
3. Nelson & Siegel (1987) model do a good job in fitting the term structure which is demonstrated from high R-Square.

The premature bond market, however, does hinder the progress of this research, such as the illiquid off the run bonds and unreasonable regulations on the division of bonds so that the market can't respond the true value instantly and precisely. Since government toot actions in 2001, bond market in Taiwan kept pace with developed countries gradually.

At last, some suggestions are raised for the improvement of future research:

1. Besides Nelson-Siegel model, a few sublime term structure models haven't been empirically tested in Taiwan, such as the method, Svensson model, which is used widely among many Central Bank around the world.
2. Use other immunization strategies which hasn't yet been tested in Taiwan to see its performance, for example, duration vector by Donald R. Chambers, Willard T. Carleton, and Richard W. McEnally(1988).
3. In nonlinear regression problems, steepest descent and the Marquardt algorithm methods are frequently used in addition to Gauss-Newton method, so, future research can try to use these methods.
4. This study only concentrates on the variation of term structure by small range; hence, future research can try to variate the term structure by larger range or variate it by irregular range(for example, add 0.01 on the estimated coefficients at this period but subtract 0.01 on them at next period.) to see if the performance of all immunization strategies in accord with past empirical results.

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# Appendix

To find initial starting values for $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$, I follow Chang'(2001)s approach. At first, let

$$M_{im} = (\tau / m)[1 - \exp(-m/\tau)].$$

$$N_{im} = (\tau / m)[1 - (m/\tau + 1)\exp(-m/\tau)].$$

Thus, we can simplify (3.4) as follows

$$P_i = \sum_{m=t}^{T} CF_{im}[1 - (\beta_0 + \beta_1 M_{im} + \beta_2 N_{im})]^{-t_{im}} + \varepsilon_i, 1 \le i \le n.$$

Then, to approximate $[1 - (\beta_0 + \beta_1 M_{im} + \beta_2 N_{im})]^{-t_{im}}$ by first order Taylor expansion, the following result is obtained

$$P_i = \sum_{m=t}^{T} CF_{im}[1 - t_{im}(\beta_0 + \beta_1 M_{im} + \beta_2 N_{im})] + \delta_i, 1 \le i \le n$$

where $\delta_i = \varepsilon_i + O((\beta_0 + \beta_1 M_{im} + \beta_0 N_{im})^2)$, at last, the above equation will become the following linear regression

$$\frac{\sum_{m=t}^{T} CF_{im} - P_i}{\sum_{m=t}^{T} CF_{im} \times t_{o,}} = \beta_0 + \beta_1 \left[ \frac{\sum_{m=t}^{T} CF_{im} \times M_{im} \times t_{im}}{\sum_{m=t}^{T} CF_{im} \times t_{im}} \right] + \beta_2 \left[ \frac{\sum_{m=t}^{T} CF_{im} \times N_{im} \times t_{im}}{\sum_{m=t}^{T} CF_{im} \times t_{im}} \right] + \in_i$$

where $\in_i = \dfrac{\delta_i}{\sum_{m=t}^{T} CF_{im} \times t_{im}}$. The estimated coefficients are determined after the above procedure, that is, the initial starting values are determined.

TABLE A. 1

The Number of Bonds for Each Strategy

| Bond Number | Bullet | Barbell | M-Absolute | M-Square | M1 | M2 | M3 | M4 | M5 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 583 | 905 | 0 | 600 | 105 | 82 | 69 | 65 | 63 |
| 2 | 376 | 0 | 0 | 0 | 102 | 80 | 67 | 63 | 61 |
| 3 | 0 | 0 | 0 | 0 | 99 | 80 | 68 | 64 | 62 |
| 4 | 0 | 0 | 963 | 367 | 94 | 83 | 68 | 65 | 63 |
| 5 | 0 | 0 | 0 | 0 | 78 | 83 | 68 | 65 | 62 |
| 6 | 0 | 0 | 0 | 0 | 77 | 80 | 67 | 63 | 61 |
| 7 | 0 | 0 | 0 | 0 | 67 | 76 | 64 | 61 | 59 |
| 8 | 0 | 0 | 0 | 0 | 62 | 73 | 64 | 61 | 59 |
| 9 | 0 | 0 | 0 | 0 | 61 | 74 | 65 | 62 | 60 |
| 10 | 0 | 0 | 0 | 0 | 58 | 72 | 64 | 61 | 59 |
| 11 | 0 | 0 | 0 | 0 | 55 | 72 | 66 | 64 | 62 |
| 12 | 0 | 0 | 0 | 0 | 50 | 72 | 70 | 69 | 67 |
| 13 | 0 | 0 | 0 | 0 | 23 | 37 | 49 | 53 | 55 |
| 14 | 0 | 0 | 0 | 0 | 19 | 34 | 47 | 52 | 54 |
| 15 | 0 | 0 | 0 | 0 | 14 | 26 | 43 | 49 | 53 |
| 16 | 0 | 0 | 0 | 0 | 12 | 24 | 41 | 47 | 51 |
| 17 | 0 | 0 | 0 | 0 | -12 | -39 | -14 | -8 | -2 |
| 18 | 0 | 0 | 0 | 0 | -12 | -40 | -16 | -11 | -5 |
| 19 | 0 | 0 | 0 | 0 | -16 | -44 | -20 | -16 | -11 |

Note: 1. The portfolio for each strategy is created in April, 1999.

2. The dash represents that sale-short of this bond in this strategy.