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# Multichannel nucleon–alpha and alpha–alpha interactions†

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**Abstract.** The nucleon–alpha and alpha–alpha interactions are obtained for the multichannel case in which the internal structure of the alpha particle is approximately represented by a two-state system. In this model the nucleon–alpha and alpha–alpha interactions are represented by $l$-dependent $2 \times 2$ and $3 \times 3$ matrices respectively. The corresponding Schrödinger equations are diagonalized by a unitary transformation to obtain exact expressions for the total cross section and the phase shift in terms of the potential parameters which are determined from the low-energy two-body scattering data.

## 1. Introduction

The subject of nucleon–alpha and alpha–alpha interactions occupies so important a role in nuclear structure problems that an entire understanding of these interactions is necessary. The tight structure of the $\alpha$ particle helps it to interact with many systems while retaining its identity, especially when the energies involved are not too high. In particular, there is a limited range of about 20 MeV for which an N$\alpha$ or $\alpha\alpha$ interaction can be usefully regarded as a two-body problem. This concept has received ample experimental confirmation. Representing the N$\alpha$ or $\alpha\alpha$ interaction by an effective two-body potential has some distinct advantages. In principle, it gives a concrete realization to the scheme of the resonating group. In practice, effective N$\alpha$ or $\alpha\alpha$ potentials are extremely useful for calculation of the energy levels in nuclear systems in which the $\alpha$ particles can be regarded as distinct entities, such as in the systems $^{6}$Li, $^{9}$Be and $^{12}$C.

The N$\alpha$ and $\alpha\alpha$ interactions have been studied by a large number of workers. Sack *et al* (1954) fitted the s and p wave phase shifts using a phenomenological, local, energy-independent N$\alpha$ interaction, having a central term and a spin–orbit term—the spin–orbit term being required to be the derivative of the central term. Three different shapes were used in fitting the phase shifts, but Sack *et al* concluded that the Gauss well gave the best overall fit to the phase shifts. In spite of the accuracy of this potential for use in the bound-state calculation, the mathematical complexities involved in using the Gauss well limited its use. One of the most comprehensive phenomenological attempts to represent an N$\alpha$ force by a two-body potential is that due to Gammel and Thaler (1958), who were able to give almost precise fits to N$\alpha$ scattering data up to about 40 MeV. Mitra *et al* (1962) took a similar view, but with a

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closer eye on the practical possibility of subjecting the Nα potential to some simple three-body systems. This necessarily implied a simple choice of the potential so as to make the subsequent three-body problem mathematically tractable. However, they considered an energy range of 40 MeV too large for neglecting the structure effects of the α particle.

The earliest phenomenological αα interaction was proposed by Haefner (1951), who assumed a repulsive potential for small $r$, attractive for intermediate $r$, and Coulombic for larger $r$. Van der Spuy and Pienaar (1958) made an αα scattering analysis up to a bombarding energy of about 6 MeV. They considered a square-well potential to investigate the velocity dependence of this interaction. Igo (1960) made an optical-model analysis of the elastic αα scattering from 23·1 to 47·1 MeV by using a complex potential. Among all the existing works, the most realistic αα interaction is perhaps the one obtained by Darriulat *et al* (1965). This interaction is $l$-dependent, consists of a Saxon–Woods repulsive core and a complex Saxon–Woods attractive well of larger radius, and can reproduce the behaviour of the phase shifts over the entire energy range from 0·15 to 120 MeV. However, all the above-mentioned works assume that the α particle is an elementary particle and neglect its internal structure. While this assumption simplifies the calculation greatly, its justification is not really convincing.

Recently, considerable interest has been shown in the three-body cluster model of light nuclei (Chuu *et al* 1973, 1974) in which the internal structure of the α particle is approximately represented by a two-state system. The results of such calculations show that the internal structure of the α particle plays an important role in calculating the binding energies of the nuclear systems. Thus there is evidence that calls for an extension of ordinary single-channel Nα and αα potentials to inelastic channels. Up to now, the multichannel Nα and αα interactions have been studied only through a resonance approximation method (Chuu *et al* 1973, 1974). Although such a method gives a very good fit to the two-body scattering data and the results in the three-body nuclear calculations are also in good agreement with experiment, the validity of the method is still in question.

Calculations on multichannel scattering problems have been investigated by many authors. Weidenmüller (1964) generalized Levinson's theorem to the case of many-channel scattering and gave a detailed analysis of the properties of an exactly soluble many-channel scattering problem. Kermode (1967) investigated a two-channel αα potential by considering that one of the α particles could be in its first excited state in the αα scattering. The effective range formulae for the multichannel scattering problem were derived from the assumed interaction for each channel. In a later calculation, Kamal and Kruzer (1970) presented three soluble two-channel problems in potential scattering with square-well, $\delta$-function and separable potentials respectively. The structure of the cross sections and the presence of resonances and bound states were discussed in some numerical detail.

In this paper we present an exact solution for the multichannel Nα and αα interactions with a form of square-well type. In our model the α particle is still assumed to be a two-state system. This picture has effectively taken into account all the inelastic channel contributions except for the break-up of the α particle. The Nα interaction is given by a two-channel potential, and the αα interaction is represented by a $3 \times 3$ matrix because of symmetry considerations. Both potentials are $l$-dependent with each matrix element represented by a square well. By the use of a unitary transformation, the set of coupled equations can be transformed to a new system in which the Schrödinger equation becomes uncoupled and thus can be solved exactly.

The potential parameters can then be obtained by fitting the experimental data. In §2 we describe the Nα interaction. In §3 we solve the problem for the αα interaction. The results and discussion are given in §4.

## 2. Multichannel nucleon–alpha interaction

We assume that the α particle has only two non-degenerate internal states, the excited state $|\uparrow\rangle$ with energy $\epsilon_\uparrow$ and the ground state $|\downarrow\rangle$ with energy $\epsilon_\downarrow$. Furthermore, we also assume that both these states have spin and parity $0^+$ for simplicity. The Schrödinger equation for the Nα system, after partial wave decomposition, can be written as (Newton 1968)

$$\begin{aligned}\frac{d^2u_{1l}}{dr^2}+\left(k^2-\frac{l(l+1)}{r^2}-\frac{2\mu}{\hbar^2}(\epsilon_\downarrow+V_{10})\right)u_{1l}&=\frac{2\mu}{\hbar^2}V_{30}u_{2l}\\ \frac{d^2u_{2l}}{dr^2}+\left(k^2-\frac{l(l+1)}{r^2}-\frac{2\mu}{\hbar^2}(\epsilon_\uparrow+V_{20})\right)u_{2l}&=\frac{2\mu}{\hbar^2}V_{30}u_{1l}.\end{aligned}\tag{1}$$

Define $U_l$ as the column matrix

$$U_l=\begin{pmatrix}u_{1l}\\u_{2l}\end{pmatrix}\tag{2}$$

and assume that $V_{10}$, $V_{20}$ and $V_{30}$ have a common range $r_0$ but with different widths:

$$V_{i0}=\begin{cases}-V_i & r<r_0\\ 0 & r>r_0\end{cases}\qquad i=1,2,3.$$

Equation (1) becomes

$$U_l''+A_lU_l=0\tag{3}$$

where

$$A_l=\begin{cases}[k^2-l(l+1)/r^2]\,\mathbb{1}-\epsilon+V & r<r_0\\ [k^2-l(l+1)/r^2]\,\mathbb{1}-\epsilon & r>r_0\end{cases}\tag{4a}$$

$$V=\frac{2\mu}{\hbar^2}\begin{pmatrix}V_1 & V_3\\ V_3 & V_2\end{pmatrix}\tag{4b}$$

$$\epsilon=\frac{2\mu}{\hbar^2}\begin{pmatrix}\epsilon_\downarrow & 0\\ 0 & \epsilon_\uparrow\end{pmatrix}.\tag{4c}$$

$k^2$ is related to the kinetic energy of the nucleon, $\mu$ is the reduced mass of the Nα system and $\mathbb{1}$ is the unit matrix. Let us define

$$K^2=\begin{pmatrix}k^2-\epsilon_\downarrow(2\mu/\hbar^2) & 0\\ 0 & k^2-\epsilon_\uparrow(2\mu/\hbar^2)\end{pmatrix}\equiv\begin{pmatrix}k_1^2 & 0\\ 0 & k_2^2\end{pmatrix}.\tag{5}$$

Thus $A_l$ can be rewritten as

$$A_l=\begin{cases}K^2+V-[l(l+1)/r^2]\,\mathbb{1} & r<r_0\\ K^2-[l(l+1)/r^2]\,\mathbb{1} & r>r_0.\end{cases}\tag{6}$$

One can observe that equation (3) is still a coupled equation inside the width of the potential. In order to make the problem soluble, it is more convenient to make a transformation within the width $r_0$ as follows. Let

$$T = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \tag{7}$$

be a unitary matrix which diagonalizes $A_l$ by a unitary transformation

$$a^2 = T^+ A_l T = \begin{pmatrix} a_{11}^2 & 0 \\ 0 & a_{22}^2 \end{pmatrix}. \tag{8}$$

If we define

$$k_1'^2 = k_1^2 + \frac{2\mu}{\hbar^2} V_1 - \frac{l(l+1)}{r^2} \qquad k_2'^2 = k_2^2 + \frac{2\mu}{\hbar^2} V_2 - \frac{l(l+1)}{r^2} \tag{9}$$

and substitute equations (5)–(7) into equation (8), we obtain

$$a_{11}^2 = a_1^2 - [l(l+1)/r^2] \tag{10a}$$

$$a_{22}^2 = a_2^2 - [l(l+1)/r^2] \tag{10b}$$

where

$$a_1^2 = \left(k_1^2 + \frac{2\mu}{\hbar^2} V_1\right)\cos^2\alpha + \left(k_2^2 + \frac{2\mu}{\hbar^2} V_2\right)\sin^2\alpha - \frac{4\mu}{\hbar^2} V_3 \sin\alpha\cos\alpha \tag{11a}$$

$$a_2^2 = \left(k_1^2 + \frac{2\mu}{\hbar^2} V_1\right)\sin^2\alpha + \left(k_2^2 + \frac{2\mu}{\hbar^2} V_2\right)\cos^2\alpha + \frac{4\mu}{\hbar^2} V_3 \sin\alpha\cos\alpha \tag{11b}$$

$$\tan 2\alpha = \frac{V_3}{k_2'^2 - k_1'^2}\frac{4\mu}{\hbar^2} = \frac{4\mu}{\hbar^2} V_3 \frac{1}{k_2^2 - k_1^2 + (2\mu/\hbar^2)(V_2 - V_1)}. \tag{11c}$$

Equation (3) thus becomes

$$T^{-1}U_l'' + T^{-1}A_l T T^{-1} U_l = 0. \tag{12}$$

Define $T^{-1}U_l = F_l$ and

$$d^2 = \begin{pmatrix} a_1^2 & 0 \\ 0 & a_2^2 \end{pmatrix}$$

then equation (12) becomes

$$F_l'' + \{d^2 - [l(l+1)/r^2]\,\mathbb{1}\}F_l = 0. \tag{13}$$

This equation can be solved exactly with the solution

$$F_l = \begin{pmatrix} a_1 r j_l(a_1 r) B_{11} \\ a_2 r j_l(a_2 r) B_{22} \end{pmatrix}. \tag{14}$$

Thus

$$U_l = TF_l = \begin{cases} \begin{pmatrix} \cos\alpha(a_1 r) j_l(a_1 r) B_{11} + \sin\alpha(a_2 r) j_l(a_2 r) B_{22} \\ -\sin\alpha(a_1 r) j_l(a_1 r) B_{11} + \cos\alpha(a_2 r) j_l(a_2 r) B_{22} \end{pmatrix} & r < r_0 \\ \begin{pmatrix} C_{11}[\cos\delta_l(k_1 r) j_l(k_1 r) - \sin\delta_l(k_1 r) n_l(k_1 r)] \\ C_{22} k_2 r h_l(k_2 r) \end{pmatrix} & r > r_0 \end{cases} \tag{15}$$

where $n_l$ and $h_l$ are the modified spherical Neumann and Hankel functions respectively. Equation (15) comes from the fact that the only incoming waves are in the first channel, i.e. we assume that the α particle originally stays in its ground state. Therefore the excited-state channel exists only in the outgoing waves. The asymptotic behaviour will be in the form

$$U_l \rightarrow \begin{pmatrix} C_{11} \sin(k_1 r - \frac{1}{2} l\pi + \delta_l) \\ C_{22} \exp\{-\mathrm{i}[k_2 r - \frac{1}{2}(l+1)\pi]\} \end{pmatrix}. \tag{16}$$

By matching the wavefunctions and their first derivatives at $r = r_0$, the phase shift $\delta_l$ can be expressed in terms of the square-well potential parameters as

$$\cot \delta_l = \frac{n_l'(k_1 r_0) - \Phi n_l(k_1 r_0)}{j_l'(k_1 r_0) - \Phi j_l(k_1 r_0)} \tag{17}$$

where

$$\begin{aligned} \Phi &= (r_0 e_2 - 1)/r_0 e_1 \\ e_1 &= \cos\alpha\, p_1 (B_{11}/B_{22}) + p_2 \sin\alpha \\ e_2 &= \cos\alpha(p_1 + q_2)(B_{11}/B_{22}) + \sin\alpha(p_2 + q_2) \\ p_i &= a_i r_0 j_l(a_i r_0) \qquad q_i = a_i r_0 j_l'(a_i r_0) \qquad i = 1, 2 \\ \frac{B_{11}}{B_{22}} &= \frac{r_0 h_l(k_2 r_0)\cos\alpha(p_2 + q_2) - [h_l(k_2 r_0) - r_0 h_l'(k_2 r_0)] p_2 \cos\alpha}{r_0 h_l(k_2 r_0)\sin\alpha(p_1 + q_1) - [h_l(k_2 r_0) - r_0 h_l'(k_2 r_0)] p_1 \sin\alpha}. \end{aligned} \tag{18}$$

The differential cross section for elastic scattering is

$$\mathrm{d}\sigma/\mathrm{d}\Omega = |\sum_l [(2l+1)/k] \exp(\mathrm{i}\delta_l)\sin\delta_l P_l(\cos\theta)|^2. \tag{19}$$

## 3. Multichannel alpha–alpha interaction

To describe the internal structure of a system of two α particles, each having two internal states, we can treat such a system in analogy with a system of spin-$\frac{1}{2}$ particles. Thus we introduce the operator $R = R(1) + R(2)$ corresponding to the total spin operator. The $z$ component of $R$ is called $R_z$ such that

$$R_z(i)|\uparrow\rangle = \tfrac{1}{2}|\uparrow\rangle, \qquad R_z(i)|\downarrow\rangle = -\tfrac{1}{2}|\downarrow\rangle. \tag{20}$$

The internal states $|RM\rangle$ of the two-α system are given by

$$\begin{aligned} |11\rangle &= |\uparrow\uparrow\rangle \\ |10\rangle &= (1/\sqrt{2})(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\ |1-1\rangle &= |\downarrow\downarrow\rangle \\ |00\rangle &= (1/\sqrt{2})(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle). \end{aligned} \tag{21}$$

The Schrödinger equation in the centre-of-mass frame is

$$[-(\hbar^2/2\mu)\nabla_r^2 + \mathscr{H} + V]\Phi = E\Phi \tag{22}$$

where $\mathscr{H}$ represents the internal structure energy operator and $V$ is the multichannel

potential energy operator. In $|RM\rangle$ representation they can be written as

$$\mathscr{H} = \begin{pmatrix} 2\epsilon_\downarrow & 0 & 0 & 0 \\ 0 & \epsilon_\uparrow + \epsilon_\downarrow & 0 & 0 \\ 0 & 0 & 2\epsilon_\uparrow & 0 \\ 0 & 0 & 0 & \epsilon_\uparrow + \epsilon_\downarrow \end{pmatrix} \qquad V = \begin{pmatrix} V_1 & V_5 & V_6 & 0 \\ V_5 & V_2 & V_7 & 0 \\ V_6 & V_7 & V_3 & 0 \\ 0 & 0 & 0 & V_4 \end{pmatrix}. \tag{23}$$

Since the $\alpha$ particles are assumed to have zero spin in any state, the matrix element of the spin–orbit force and the tensor force vanish identically. Therefore we only have to consider the central force. Equation (23) implies that if the two $\alpha$ particles are both in their ground states initially, the two-$\alpha$ system can never be found in the anti-symmetric state. Consequently the Schrödinger equation (22) reduces to a set of three coupled equations, and we can write the total wavefunction as

$$\Phi(r, RM) = \sum_l \frac{2l+1}{k_0 r} \mathrm{i}^l P_l(\cos\theta) \begin{pmatrix} u_{1l} \\ u_{2l} \\ u_{3l} \end{pmatrix}. \tag{24}$$

The set of three coupled equations are

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2} \begin{pmatrix} u_{1l} \\ u_{2l} \\ u_{3l} \end{pmatrix} + \left( k_0^2 \mathbb{1} - \frac{l(l+1)}{r^2} \mathbb{1} - U(r) \right) \begin{pmatrix} u_{1l} \\ u_{2l} \\ u_{3l} \end{pmatrix} = 0 \tag{25}$$

where $U(r) = (2\mu/\hbar^2)\, V(r) + \epsilon.\epsilon$ is a diagonal matrix coming from the internal structure Hamiltonian $\mathscr{H}$ and has the form

$$\epsilon = \frac{2\mu}{\hbar^2} \begin{pmatrix} 2\epsilon_\downarrow & 0 & 0 \\ 0 & \epsilon_\uparrow + \epsilon_\downarrow & 0 \\ 0 & 0 & 2\epsilon_\uparrow \end{pmatrix}. \tag{26}$$

To solve this problem, we write equation (25) as

$$\frac{\mathrm{d}^2 u}{\mathrm{d}r^2} + \left( k_0^2 \mathbb{1} - \frac{l(l+1)}{r^2} \mathbb{1} - U(r) \right) u = 0 \tag{27}$$

where

$$u = \begin{pmatrix} u_{1l} \\ u_{2l} \\ u_{3l} \end{pmatrix}. \tag{28}$$

We transform $u$ to a new system $u'$ by a unitary matrix $A$ as

$$u' = A^{-1} u = \begin{pmatrix} u'_{1l} \\ u'_{2l} \\ u'_{3l} \end{pmatrix}. \tag{29}$$

Equation (27) becomes

$$[\mathrm{d}^2/\mathrm{d}r^2 + k_0^2 - l(l+1)/r^2]\, u' - U' u' = 0 \tag{30}$$

where

$$U' = A^{+}UA. \tag{31}$$

In order to decouple the equations, we have to find the unitary transformation to diagonalize $U$. This transformation matrix $A = (a_{ij})$ and the eigenvalues $\lambda_i (i = 1, 2, 3)$ of $U$ can be obtained by the usual diagonalization method. Thus, in the new system, we have

$$\frac{\mathrm{d}^2u'}{\mathrm{d}r^2} + \left(k_0^2 - \frac{l(l+1)}{r^2} - \hat{\lambda}\right)u' = 0 \tag{32}$$

where $\hat{\lambda}$ is the diagonal matrix with the eigenvalues $\lambda_i$ as the diagonal elements. The solutions of equation (32) satisfying the boundary conditions at the origin are

$$u' = \begin{pmatrix} c_1 r j_l(k_1 r) \\ c_2 r j_l(k_2 r) \\ c_3 r j_l(k_3 r) \end{pmatrix} \tag{33}$$

where

$$k_i^2 = k_0^2 - \lambda_i. \tag{34}$$

Thus we obtain the wavefunctions for $r < r_0$:

$$u_{il} = \sum_{j=1}^{3} c_j a_{ij} r j_l(k_j r) \qquad i = 1, 2, 3. \tag{35}$$

For $r > r_0$ we have

$$\begin{aligned} u_{1l} &= D_1(\cos\delta_l F_l(K_1 r) + \sin\delta_l G_l(K_1 r)) \\ u_{2l} &= D_2 H_l(K_2 r) \\ u_{3l} &= D_3 H_l(K_3 r) \end{aligned} \tag{36}$$

where $F_l$ and $G_l$ are the regular and irregular Coulomb wavefunctions, $H_l = G_l + \mathrm{i}F_l$, $\delta_l$ is the nuclear phase shift and

$$\begin{aligned} K_1^2 &= k_0^2 - (2\mu/\hbar^2)2\epsilon_{\downarrow} \\ K_2^2 &= k_0^2 - (2\mu/\hbar^2)(\epsilon_{\uparrow} + \epsilon_{\downarrow}) \\ K_3^2 &= k_0^2 - (2\mu/\hbar^2)2\epsilon_{\uparrow}. \end{aligned}$$

Matching the wavefunctions and their first derivatives at $r = r_0$, we get

$$\tan\delta_l = \frac{F_l'(K_1 r_0) - \Phi' F_l(K_1 r_0)}{G_l'(K_1 r_0) - \Phi' G_l(K_1 r_0)} \tag{38}$$

where

$$\begin{aligned} \Phi' &= \mathrm{d}(\log\psi)/\mathrm{d}r \\ \psi &= a_{11} j_l(k_1 r_0) + \beta a_{12} j_l(k_2 r_0) + \gamma a_{13} j_l(k_3 r_0) \\ \beta &= (a_{21}a_{33}d_{12}d_{33} - a_{23}a_{31}d_{13}d_{23})/(a_{23}a_{32}d_{23} - a_{22}a_{33}d_{22}d_{33}) \\ \gamma &= (a_{22}a_{31}d_{22}d_{13} - a_{21}a_{32}d_{12}d_{32})/(a_{23}a_{32}d_{32}d_{23} - a_{22}a_{32}d_{22}d_{33}) \\ d_{ij} &= j_l(k_j r_0)H_l'(\mathrm{K}_i r_0) - j_l'(k_j r_0)H_l(K_i r_0). \end{aligned}$$

Equation (38) is an exact expression for the nuclear phase shift as a function of the energy for the coupled channels. From this equation we can obtain the $\alpha\alpha$ interaction parameters by fitting the experimental scattering data.

## 4. Results and discussion

For the N$\alpha$ interaction we have fitted the total scattering cross section from 1 to 20 MeV by an $l$-dependent potential. With a set of initially chosen potential parameters, the numerical program calculates the square deviation of the total cross section at 20 different energies. An iteration procedure then starts to minimize the square sum deviation by optimizing the parameters until the RMS error for the total cross sections is about 0·038 b. The calculated and observed total cross sections are shown in figure 1. The s and p wave parameters obtained in this way are listed in table 1. For the $\alpha\alpha$ interaction we have fitted the s and D wave phase shifts from 0·5 to 23 MeV also by an $l$-dependent potential. The RMS error for the s wave phase shifts from 0·5 to 23 MeV with 26 different angles is 1·81° and that of the D wave phase shifts with 22 different angles is 1·92°. The calculated and experimental phase shifts are shown in figure 2, and the parameters obtained are given in table 2. Here, the $\alpha$ excitation energy is taken to be 20 MeV.

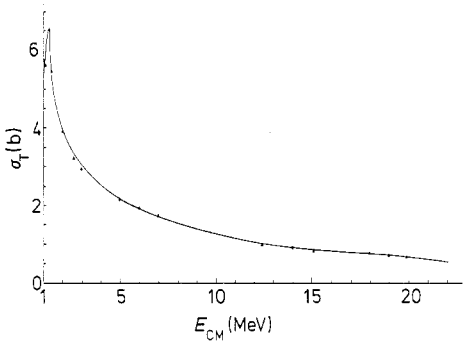


**Figure 1.** N$\alpha$ total cross section as a function of neutron bombardment energy: ——, calculated; ▲, experimental (Los Alamos Physics and Cryogenics Groups 1959).

**Table 1.** Parameters of the N$\alpha$ potential.

| | $l$ | $V_1$ (MeV) | $V_2$ (MeV) | $V_3$ (MeV) | $r_0$ (fm) |
|---|---|---|---|---|---|
| Resonance method | 0 | 46·815 | 31·14 | 0·205 | 3·581 |
| | 1 | 21·507 | 46·86 | 0·325 | 3·493 |
| Exact solution | 0 | 47·207 | 30·84 | 0·276 | 3·66 |
| | 1 | 22·085 | 45·94 | 0·369 | 3·587 |

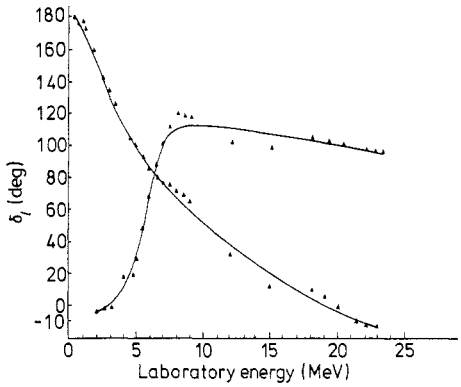


**Figure 2.** αα scattering phase shifts as a function of laboratory energy: ——, calculated from equation (38); ▲, experimental (Heydenberg and Temmer 1956, Russell *et al* 1956, Nilson *et al* 1958, Jones *et al* 1960).

**Table 2.** Parameters for the αα potential.

| | $l$ | $V_1$ (MeV) | $V_2$ (MeV) | $V_3$ (MeV) | $V_5$ (MeV) | $V_6$ (MeV) | $V_7$ (MeV) | $r_0$ (fm) |
|---|---|---|---|---|---|---|---|---|
| Resonance method | 0 | 23·3 | 23·32 | 24·50 | 8·19 | 0·5 | 2·52 | 4·806 |
| | 2 | 24·9 | 29·50 | 29·40 | 4·98 | 10·2 | 11·56 | 4·64 |
| Exact solution | 0 | 22·4 | 23·41 | 23·70 | 6·90 | 1·03 | 2·50 | 4·81 |
| | 2 | 24·6 | 29·50 | 26·80 | 4·81 | 8·53 | 13·52 | 4·64 |

Comparing the results of the exact calculations and of the resonance approximation method (as listed in tables 1 and 2), we see that the diagonal matrix elements are almost the same, but the off-diagonal elements differ by about 15%. From the calculations of the three-body cluster models of $^6$Li, $^9$Be and $^{12}$C, we know that the contribution to the binding energies of these nuclei is mainly due to the diagonal elements. The small deviations in the off-diagonal elements can be expected to have little effect on the binding energies of the three-body systems. Thus our present work shows two important facts: (i) the resonance approximation method is indeed a very good approximation; and (ii) with the consideration of the internal structure of the α particle, the scattering data can be fitted without a spin–orbit term in the Nα interaction or a repulsive core in the αα interaction. This is in contrast to the single-channel potential in which an *LS* coupling term or a repulsive core is required. Therefore, although the internal structure of the α particle gives only a weak coupling, it is significant in the Nα and αα interactions and in nuclear structure problems such as the α particle model of light nuclei.

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