CHAPTER 6

ROBOTIC MANIPULATOR CONTROL BASED ON UNIFORMLY DISTRIBUTED SIMPLEX SLIDING-MODE CONTROL

In this chapter, the UDSSMC algorithm is developed to deal with the position tracking control of the robotic manipulators suffering from the system uncertainty and external disturbance. Section 6.1 gives the problem formulation and Section 6.2 proposes the UDSSMC algorithm for the tracking control of robotic manipulators. Finally, a two-link robotic manipulator as an example is simulated to demonstrate the success of the proposed UDSSMC algorithm in Section 6.3.

6.1 Problem Formulation

In general, the dynamic equation of an m-link robotic manipulator is expressed as [53]

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{u} + \boldsymbol{d}(t) \tag{6.1}$$

where $\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}} \in \Re^{\boldsymbol{m}}$ respectively represent the joint position, velocity, and acceleration vectors. Besides, $\boldsymbol{M}(\boldsymbol{q}) \in \Re^{\boldsymbol{m}\times\boldsymbol{m}}$ is a positive-definite and symmetric inertia matrix, $\boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}}) \in \Re^{\boldsymbol{m}\times\boldsymbol{m}}$ is a matrix containing the Coriolis force and centrifugal terms, $\boldsymbol{G}(\boldsymbol{q}) \in \Re^{\boldsymbol{m}}$ is a vector of gravitational terms, and $\boldsymbol{u} \in \Re^{m}$ is composed of the joint torque or forces. Note that $\boldsymbol{M}(\boldsymbol{q})$ and $\boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})$ must satisfy the following condition:

$$\boldsymbol{z}^{\boldsymbol{T}}\left(\dot{\boldsymbol{M}}(\boldsymbol{q}) - 2\boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\boldsymbol{z} = 0 \tag{6.2}$$

where $\boldsymbol{z} \in \Re^{\boldsymbol{m}}$. In other words, $\dot{\boldsymbol{M}}(\boldsymbol{q}) - 2\boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})$ belongs to a skew-symmetric matrix, which the elements in the diagonal are all equal to zero. Furthermore, since the uncertainties exist in the system (6.1), the matrices $\boldsymbol{M}(\boldsymbol{q}), \boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})$, and $\boldsymbol{G}(\boldsymbol{q})$ are decomposed into

$$\begin{aligned} \boldsymbol{M}(\boldsymbol{q}) &= \boldsymbol{M}_0(\boldsymbol{q}) + \Delta\boldsymbol{M}(\boldsymbol{q}) \\ \boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}}) &= \boldsymbol{B}_0(\boldsymbol{q},\dot{\boldsymbol{q}}) + \Delta\boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}}) \\ \boldsymbol{G}(\boldsymbol{q}) &= \boldsymbol{G}_0(\boldsymbol{q}) + \Delta\boldsymbol{G}(\boldsymbol{q}) \end{aligned} \tag{6.3}$$

where $\Delta\boldsymbol{M}(\boldsymbol{q}), \Delta\boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})$, and $\Delta\boldsymbol{G}(\boldsymbol{q})$ are respectively the uncertainties deviated from the nominal parts $\boldsymbol{M}_0(\boldsymbol{q}), \boldsymbol{B}_0(\boldsymbol{q},\dot{\boldsymbol{q}})$, and $\boldsymbol{G}_0(\boldsymbol{q})$, which is available. As to $\boldsymbol{d}(\boldsymbol{x},t) \in \Re^m$, it represents the external disturbance. For these uncertainties and disturbance, they are constrained by

$$\begin{aligned} &\|\Delta\boldsymbol{M}(\boldsymbol{q})\| \le \delta_{\boldsymbol{M}}, \quad \|\Delta\boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})\| \le \delta_{\boldsymbol{B}}(\dot{\boldsymbol{q}}), \\ &\|\Delta\boldsymbol{G}(\boldsymbol{q})\| \le \delta_{\boldsymbol{G}}, \quad \|\boldsymbol{d}(\boldsymbol{x},t)\| \le \delta_{\boldsymbol{d}} \end{aligned} \tag{6.4}$$

where these upper bounds $\delta_{\boldsymbol{M}}, \delta_{\boldsymbol{B}}(\dot{\boldsymbol{q}}), \delta_{\boldsymbol{G}}$, and $\delta_{\boldsymbol{d}}$ are available.

Since the considered problem here is the position tracking control, let's first define the tracking error as

$$\boldsymbol{e} = \boldsymbol{q} - \boldsymbol{q}_d(t) \tag{6.5}$$

where $\boldsymbol{q}_d(t)$ means the desired trajectory. As a result, the sliding vector is selected as

$$\boldsymbol{\sigma} = \dot{\boldsymbol{e}} + \boldsymbol{C}\boldsymbol{e} \tag{6.6}$$

where $\boldsymbol{\sigma} = [\sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_{\boldsymbol{m}}]^T \in \Re^{\boldsymbol{m}}$, $\boldsymbol{M}(\boldsymbol{q})$ is the inertia matrix in (6.1), and $\boldsymbol{C} = diag\{c_1, c_2, \cdots, c_{\boldsymbol{m}}\}$ is a diagonal matrix with diagonal terms $c_i > 0$ for

$i = 1,\cdots,m$. Clearly, when the system is successfully controlled to stay on the sliding surface σ=0, its trajectory must satisfy

$$\dot{\boldsymbol{e}} + \boldsymbol{C}\boldsymbol{e} = 0 \tag{6.7}$$

or

$$\dot{e}_i + c_i e_i = 0, \qquad i = 1,2,\cdots,m \tag{6.8}$$

With the fact of $c_i > 0$, it can be obtained from (6.8) that $e_i \to 0$ as $t \to \infty$. For this reason, $\boldsymbol{e} = \boldsymbol{q} - \boldsymbol{q}_d(t) \to 0$ as $t \to \infty$. Therefore, the tracking $\boldsymbol{q} \to \boldsymbol{q}_d$ is successfully achieved when the system is completely restricted in the sliding mode σ=0. Next, it will focus on the development of UDSSMC algorithm to drive (6.1) to reach the sliding mode σ=0 in a finite time and then stay thereafter.

6.2 UDSSMC algorithm

With the tracking error (6.5), the dynamic equation of the robot manipulator (6.1) can be rearranged as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{e}} + \boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{e}} = \boldsymbol{u} + \boldsymbol{d}(t) - \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}_d - \boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_d - \boldsymbol{G}(\boldsymbol{q}) \tag{6.9}$$

Since $\boldsymbol{M}(\boldsymbol{q})$ is positive-definite, the candidate of Lyapunov function can be selected as

$$\boldsymbol{V} = \sigma^T \boldsymbol{M}(\boldsymbol{q})\sigma/2 \tag{6.10}$$

From (6.2) and (6.6), taking the first derivative of $\boldsymbol{V}$ leads to

$$\begin{aligned}\dot{V} &= \boldsymbol{\sigma}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{\sigma}} + \boldsymbol{\sigma}^T \dot{\boldsymbol{M}}(\boldsymbol{q})\boldsymbol{\sigma}/2 \\ &= \boldsymbol{\sigma}^T \boldsymbol{M}(\boldsymbol{q})(\ddot{\boldsymbol{e}} + \boldsymbol{C}\dot{\boldsymbol{e}}) + \boldsymbol{\sigma}^T \boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})(\dot{\boldsymbol{e}} + \boldsymbol{C}\boldsymbol{e}) \\ &= \boldsymbol{\sigma}^T [\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{e}} + \boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{e}} + \boldsymbol{M}(\boldsymbol{q})\boldsymbol{C}\dot{\boldsymbol{e}} + \boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{C}\boldsymbol{e}]\end{aligned} \tag{6.11}$$

With the use of (6.3) and (6.9), (6.11) becomes

$$\begin{aligned}\dot{V} &= \boldsymbol{\sigma}^T [\boldsymbol{u} + \boldsymbol{d}(t) - \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{M}(\boldsymbol{q})(\boldsymbol{C}\dot{\boldsymbol{e}} - \ddot{\boldsymbol{q}}_{\boldsymbol{d}}) + \boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})(\boldsymbol{C}\boldsymbol{e} - \dot{\boldsymbol{q}}_{\boldsymbol{d}})] \\ &= \boldsymbol{\sigma}^T [\boldsymbol{u} + \boldsymbol{d}(t) - (\boldsymbol{G}_0(\boldsymbol{q}) + \Delta\boldsymbol{G}(\boldsymbol{q})) + (\boldsymbol{M}_0(\boldsymbol{q}) + \Delta\boldsymbol{M}(\boldsymbol{q}))(\boldsymbol{C}\dot{\boldsymbol{e}} - \ddot{\boldsymbol{q}}_{\boldsymbol{d}}) \\ &\qquad + (\boldsymbol{B}_0(\boldsymbol{q},\dot{\boldsymbol{q}}) + \Delta\boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}}))(\boldsymbol{C}\boldsymbol{e} - \dot{\boldsymbol{q}}_{\boldsymbol{d}})]\end{aligned} \tag{6.12}$$

For this reason, the UDSSMC algorithm is designed as

$$\boldsymbol{u} = \boldsymbol{G}_0(\boldsymbol{q}) - \boldsymbol{M}_0(\boldsymbol{q})(\boldsymbol{C}\dot{\boldsymbol{e}} - \ddot{\boldsymbol{q}}_{\boldsymbol{d}}) - \boldsymbol{B}_0(\boldsymbol{q},\dot{\boldsymbol{q}})(\boldsymbol{C}\boldsymbol{e} - \dot{\boldsymbol{q}}_{\boldsymbol{d}}) + \bar{\boldsymbol{u}} \tag{6.13}$$

$$\bar{\boldsymbol{u}} = \left(m \cdot \left(\delta_{\boldsymbol{d}} + \delta_{\boldsymbol{G}} + \delta_{\boldsymbol{M}}\|\boldsymbol{C}\dot{\boldsymbol{e}} - \ddot{\boldsymbol{q}}_{\boldsymbol{d}}\| + \delta_{\boldsymbol{B}}\|\boldsymbol{C}\boldsymbol{e} - \dot{\boldsymbol{q}}_{\boldsymbol{d}}\|\right) + \varepsilon\right)\boldsymbol{u}^{\boldsymbol{i}},\ \varepsilon > 0, \quad \text{for } \boldsymbol{\sigma} \text{ in } \boldsymbol{\Sigma}_i \tag{6.14}$$

where $\delta_{\boldsymbol{d}}, \delta_{\mathrm{G}}, \delta_{\boldsymbol{M}}$, and $\delta_{\boldsymbol{B}}$ are respectively the upper bounds of the uncertainties and disturbances given in (6.4). Note that $\boldsymbol{u}^i$, i=1,2,…,m+1, represent the uniformly distributed simplex set described in Chapter 3 and $\boldsymbol{\Sigma}_i$ are the disjointed open sub-regions given in (2.4). With use of the uniformly distributed simplex set, the sliding vector $\boldsymbol{\sigma}$ could be represented as

$$\boldsymbol{\sigma} = \sum_{j=1, j\neq \boldsymbol{i}}^{m+1} \gamma_j \boldsymbol{u}^j = \boldsymbol{U}_{\boldsymbol{i}}\gamma_{\boldsymbol{i}}, \qquad \gamma_j > 0 \tag{6.15}$$

where $\boldsymbol{U}_{\boldsymbol{i}} = [\boldsymbol{u}^1 \ \cdots \ \boldsymbol{u}^{i-1} \ \ \boldsymbol{u}^{i+1} \ \cdots \ \boldsymbol{u}^{m+1}]$ and $\gamma_{\boldsymbol{i}} = [\gamma_1 \ \cdots \ \gamma_{\boldsymbol{i}-1} \ \ \gamma_{\boldsymbol{i}+1} \ \cdots \ \gamma_{m+1}]^T$.

Hence, it leads to

$$\boldsymbol{\sigma}^{\boldsymbol{T}}\boldsymbol{u}^{\boldsymbol{i}} = \gamma_{\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{U}_{\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{u}^{\boldsymbol{i}} \tag{6.16}$$

To employ the truth of $(\boldsymbol{u}^i)^T \boldsymbol{u}^j = -\frac{1}{m}$, $i, j = 1,2,\cdots,m+1$, $i \neq j$, (6.16) results in

$$\begin{aligned}\boldsymbol{\sigma}^T \boldsymbol{u}^i &= [\gamma_1 \quad \cdots \quad \gamma_{i-1} \quad \gamma_{i+1} \quad \cdots \quad \gamma_{m+1}]\underbrace{[-1/m \quad \cdots \quad -1/m]}_{m}^T \\ &= -\frac{1}{m}(\gamma_1 + \cdots + \gamma_{i-1} + \gamma_{i+1} + \cdots + \gamma_{m+1})\end{aligned} \tag{6.17}$$

Since the 1-norm of any vector possesses the maximum value in all norm definitions, (6.17) leads to

$$\boldsymbol{\sigma}^T \boldsymbol{u}^i = -\frac{1}{m}(\gamma_1 + \cdots + \gamma_{i-1} + \gamma_{i+1} + \cdots + \gamma_{m+1}) \le -\frac{1}{m}\|\sigma\| \tag{6.18}$$

Now, substituting (6.13) and (6.14) into (6.12) yields

$$\begin{aligned}\dot{\boldsymbol{V}} &= \boldsymbol{\sigma}^T \big[\big(m\cdot\big(\delta_d + \delta_G + \delta_M\|\boldsymbol{C}\dot{\boldsymbol{e}} + \ddot{\boldsymbol{q}}_d\| + \delta_B\|\boldsymbol{C}\boldsymbol{e} - \dot{\boldsymbol{q}}_d\|\big) + \varepsilon\big)\boldsymbol{u}^i \\ &\qquad + \boldsymbol{d}(t) - \Delta\boldsymbol{G}(\boldsymbol{q}) + \Delta\boldsymbol{M}(\boldsymbol{q})(\boldsymbol{C}\dot{\boldsymbol{e}} - \ddot{\boldsymbol{q}}_d) + \Delta\boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})(\boldsymbol{C}\boldsymbol{e} - \dot{\boldsymbol{q}}_d)\big] \\ &= \big(m\cdot\big(\delta_d + \delta_G + \delta_M\|\boldsymbol{C}\dot{\boldsymbol{e}} + \ddot{\boldsymbol{q}}_d\| + \delta_B\|\boldsymbol{C}\boldsymbol{e} - \dot{\boldsymbol{q}}_d\|\big) + \varepsilon\big)\boldsymbol{\sigma}^T \boldsymbol{u}^i \\ &\quad + \boldsymbol{\sigma}^T\big(\boldsymbol{d}(t) - \Delta\boldsymbol{G}(\boldsymbol{q}) + \Delta\boldsymbol{M}(\boldsymbol{q})(\boldsymbol{C}\dot{\boldsymbol{e}} - \ddot{\boldsymbol{q}}_d) + \Delta\boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})(\boldsymbol{C}\boldsymbol{e} - \dot{\boldsymbol{q}}_d)\big)\end{aligned} \tag{6.19}$$

By means of (6.18), (6.19) becomes

$$\begin{aligned}\dot{\boldsymbol{V}} &\le -\frac{1}{m}\big(m\cdot\big(\delta_d + \delta_G + \delta_M\|\boldsymbol{C}\dot{\boldsymbol{e}} + \ddot{\boldsymbol{q}}_d\| + \delta_B\|\boldsymbol{C}\boldsymbol{e} - \dot{\boldsymbol{q}}_d\|\big) + \varepsilon\big)\|\sigma\| \\ &\quad + \|\sigma\|\big(\|\boldsymbol{d}(t)\| + \|\Delta\boldsymbol{G}(\boldsymbol{q})\| + \|\Delta\boldsymbol{M}(\boldsymbol{q})\|\|(\boldsymbol{C}\dot{\boldsymbol{e}} - \ddot{\boldsymbol{q}}_d)\| + \|\Delta\boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})\|\|(\boldsymbol{C}\boldsymbol{e} - \dot{\boldsymbol{q}}_d)\|\big)\end{aligned} \tag{6.20}$$

With the use of (6.4), it leads to

$$\dot{\boldsymbol{V}} \le -\frac{\varepsilon}{m}\|\sigma\| \tag{6.21}$$

where $\varepsilon > 0$. It means $\dot{\boldsymbol{V}}$ is negative until $\boldsymbol{\sigma} = 0$. Therefore, $\boldsymbol{V}$ is really a Lyapunov function and decreases all the time and reaches zero in a finite time. In other words, the UDSSMC algorithm (6.13) and (6.14) could drive the system (6.1) to reach the sliding mode σ=0 in a finite time.

Similarly, the control algorithm (6.14) still inevitably confronts with the chattering problem, which happens not only in the sliding mode but also in the approach mode. Applied the smoothing strategy described in Section 4.4, it leads to

$$
\begin{aligned}
&\text{In the approaching mode:}\\
&\bar{\boldsymbol{u}} = \begin{cases} \left(m\cdot\left(\delta_{\boldsymbol{d}} + \delta_{\boldsymbol{G}} + \delta_{\boldsymbol{M}}\left\|\boldsymbol{C}\dot{\boldsymbol{e}} - \ddot{\boldsymbol{q}}_{\boldsymbol{d}}\right\| + \delta_{\boldsymbol{B}}\left\|\boldsymbol{C}\boldsymbol{e} - \dot{\boldsymbol{q}}_{\boldsymbol{d}}\right\|\right) + \varepsilon\right)\boldsymbol{u}^{\boldsymbol{i}}, & \text{if } dist_{\min} \geq \xi_1 \\ \text{Unchanged} & \text{if } dist_{\min} < \xi_1 \end{cases}\\
&\text{In the sliding mode:}\\
&\bar{\boldsymbol{u}} = \begin{cases} \left(m\cdot\left(\delta_{\boldsymbol{d}} + \delta_{\boldsymbol{G}} + \delta_{\boldsymbol{M}}\left\|\boldsymbol{C}\dot{\boldsymbol{e}} - \ddot{\boldsymbol{q}}_{\boldsymbol{d}}\right\| + \delta_{\boldsymbol{B}}\left\|\boldsymbol{C}\boldsymbol{e} - \dot{\boldsymbol{q}}_{\boldsymbol{d}}\right\|\right) + \varepsilon\right)\boldsymbol{u}^{\boldsymbol{i}}, & \text{if } \left\|\boldsymbol{\sigma}\right\| \geq \xi_2 \\ \text{Unchanged} & \text{if } \left\|\boldsymbol{\sigma}\right\| < \xi_2 \end{cases}
\end{aligned}
\tag{6.22}
$$

where $\boldsymbol{\sigma} \in \boldsymbol{\Sigma}_{\boldsymbol{i}}$, $\xi_1, \xi_2 > 0$ and $dist_{min}$ is defined as (4.39). Next, a numeric example will demonstrate the success of UDSSMC in suppressing the uncertainties and matched disturbance.

6.3 Numeric Example and Simulation Results

For the demonstration of UDSSMC algorithm for robotic manipulators, we consider the position tracking control of a two-link robotic manipulator model shown in Figure 4.1 as an example.

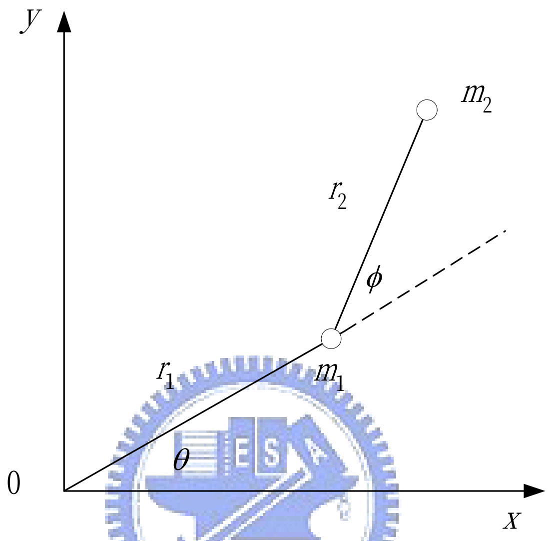

Figure 6.1 Two-link robotic manipulator model

The dynamic equation of a two-link robotic manipulator is given as [53]

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{u} + \boldsymbol{d}(t) \tag{6.23}$$

where

$$\boldsymbol{q} = [\theta \quad \phi]^T$$

$$\boldsymbol{M} = \begin{bmatrix} (m_1 + m_2)r_1^2 + m_2 r_2^2 + 2 m_2 r_1 r_2 \cos\phi + J_1 & m_2 r_2^2 + m_2 r_1 r_2 \cos\phi \\ m_2 r_2^2 + m_2 r_1 r_2 \cos\phi & m_2 r_2^2 + J_2 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} -m_2 r_1 r_2 \dot{\phi} \sin\phi & -m_2 r_1 r_2 (\dot{\theta} + \dot{\phi}) \sin\phi \\ m_2 r_1 r_2 \dot{\theta} \sin\phi & 0 \end{bmatrix}$$

$$\boldsymbol{G} = \begin{bmatrix} [(m_1 + m_2) r_1 \cos\theta + m_2 r_2 \cos(\theta + \phi)] g \\ [m_2 r_2 \cos(\theta + \phi)] g \end{bmatrix}$$

The corresponding parameters in (6.23) are selected as follows:

$$r_1 = 1m, \quad r_2 = 0.8m, \quad m_1 = 1kg, \quad m_2 = 0.6 \sim 0.8kg,$$
$$J_1 = J_2 = 5kg \cdot m^2, \quad \boldsymbol{d}(t) = \begin{bmatrix} sin(2t) \\ cos(3t) \end{bmatrix} nt \cdot m$$

For the robotic manipulator (6.23), the nominal parts represent the condition that the tip of robotic manipulator is without payload, i.e. $m_2 = 0.6kg$. Hence, the upper bound of the uncertainties and disturbance could be obtained as

$$\begin{aligned} &\|\Delta \boldsymbol{M}(\boldsymbol{q})\| \le \delta_{\boldsymbol{M}} = \Delta m_2 r_1^2 + 4\Delta m_2 r_2^2 + 4\Delta m_2 r_1 r_2 = 1.352, \\ &\|\Delta \boldsymbol{B}(\boldsymbol{q},\dot{\boldsymbol{q}})\| \le \delta_{\boldsymbol{B}}(\dot{\boldsymbol{q}}) = 2\Delta m_2 r_1 r_2 \times (\dot{\theta} + \dot{\phi}) = 0.32 \times (\dot{\theta} + \dot{\phi}), \\ &\|\Delta \boldsymbol{G}(\boldsymbol{q})\| \le \delta_{\boldsymbol{G}} = \Delta m_2 (r_1 + 2r_2) g = 5.096, \\ &\|\boldsymbol{d}(\boldsymbol{x},t)\| \le \delta_{\boldsymbol{d}} = \sqrt{2} \end{aligned} \qquad (6.24)$$

In addition, the desired position vector is chosen as

$$\boldsymbol{q_d}(t) = \begin{bmatrix} \theta_d \\ \phi_d \end{bmatrix} = \begin{bmatrix} sin(3t) \\ cos(t) \end{bmatrix}$$

With the use of the tracking error, the parameter of the sliding vector given in (6.6) is selected as

$$\boldsymbol{C} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

Hence, the state variables are chosen as

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \theta - \theta_d \\ \dot{\theta} - \dot{\theta}_d \\ \phi - \phi_d \\ \dot{\phi} - \dot{\phi}_d \end{bmatrix}$$

Further following the systematic procedure described in Section 6.2, the uniformly distributed simplex vectors can be found as

$$\boldsymbol{u}^1 = [1 \quad 0]^T$$
$$\boldsymbol{u}^2 = \left[-1/2 \quad \sqrt{3}/4\right]^T$$
$$\boldsymbol{u}^3 = \left[-1/2 \quad -\sqrt{3}/4\right]^T$$

To improve the chattering phenomenon, the control algorithm (6.22) with the proposed smoothing strategy is adopted in this simulation. For this simulation, the thickness values in (6.22) are chosen as $\xi_1 = 0.1, \xi_2 = 0.2$. Figure 6.2 to Figure 6.6 are simulation results with the initial condition $\boldsymbol{x}(0) = [\pi/2 \quad 0 \quad \pi/2 \quad 0]^T$. Figure 6.2 shows the motion of the norm of the sliding vector and Figure 6.3 gives the trajectory of the sliding vector in the sliding plane. It is clear that the two-link robotic manipulator is successfully driven to the layer $\|\sigma\| < \xi_2$. In Figure 6.4, it illustrates the tracking error $\boldsymbol{e} = \boldsymbol{q} - \boldsymbol{q}_d(t)$ all converge near zero. Figure 6.5 shows the trajectories of the practical and desired joint positions $\boldsymbol{q} = [\theta \quad \phi]^T$. From simulation results, it reveals that the developed UDSSMC algorithm has been successfully tracked the desired joint position for the two-link robotic manipulator model. From Figure 6.6, it is also evident that the proposed smoothing strategy (6.22) can efficiently improve the high frequency chattering problem.

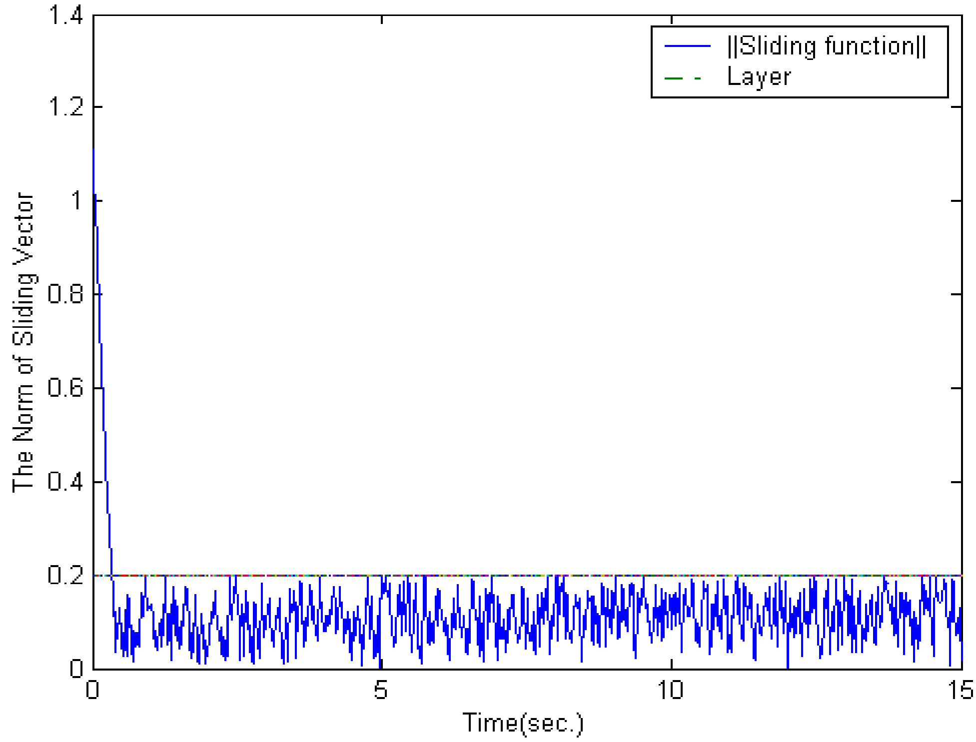

Figure 6.2 The norm value of the sliding vector

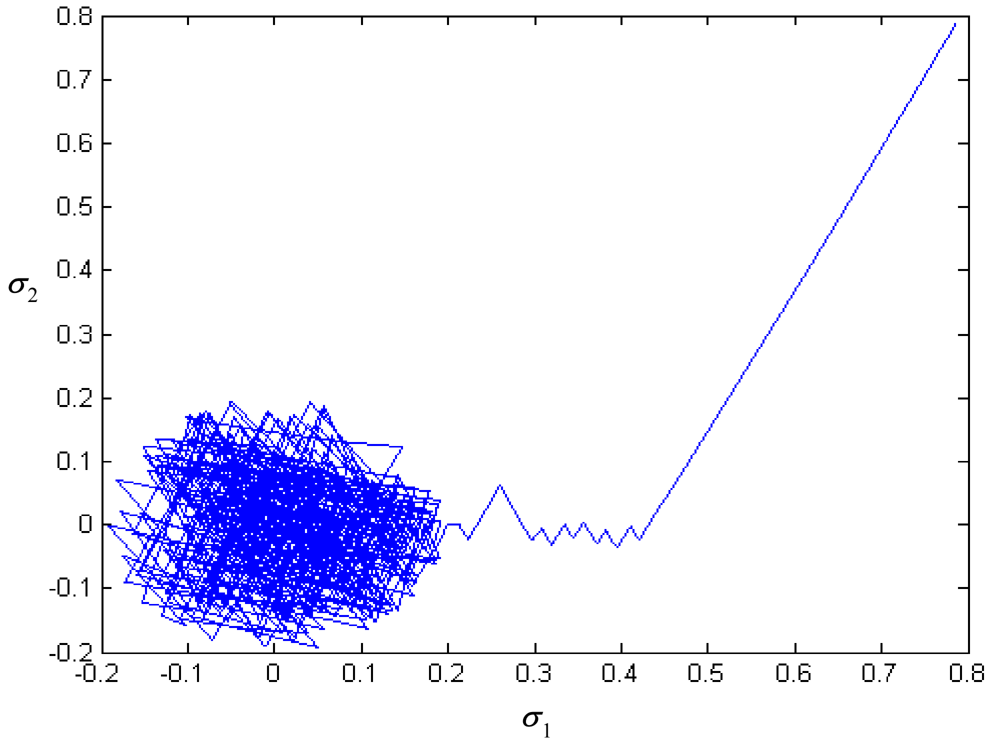

Figure 6.3 The trajectory of the sliding vector in the σ space

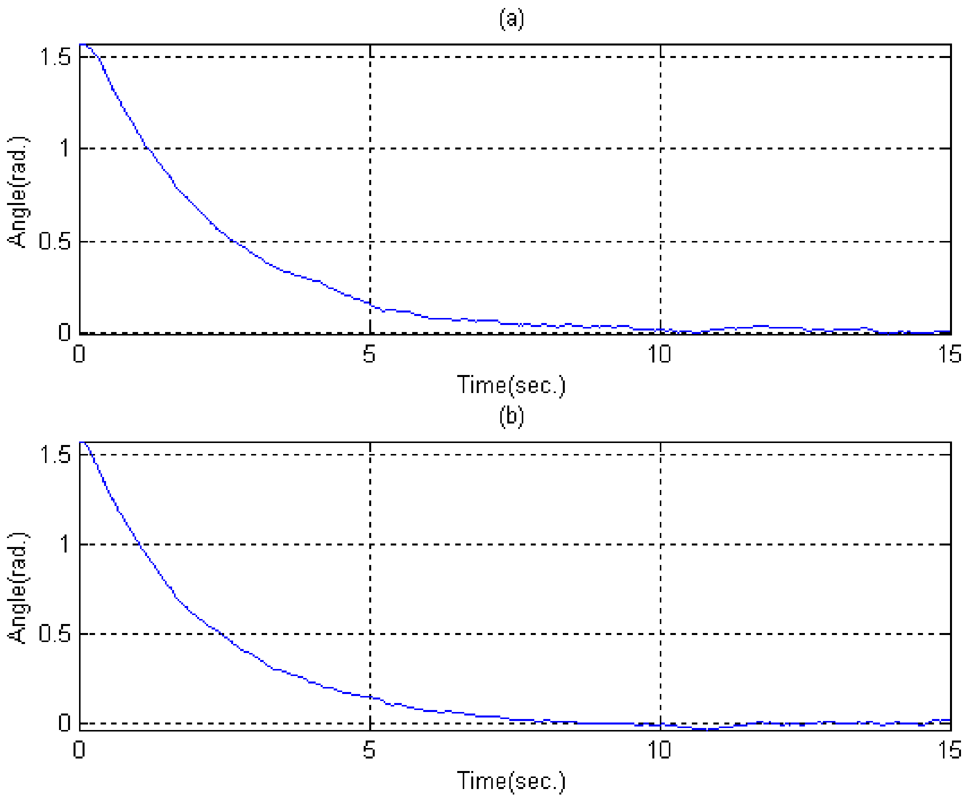

Figure 6.4 Tracking error (a) $x_1 = \theta - \theta_d$ (b) $x_3 = \phi - \phi_d$

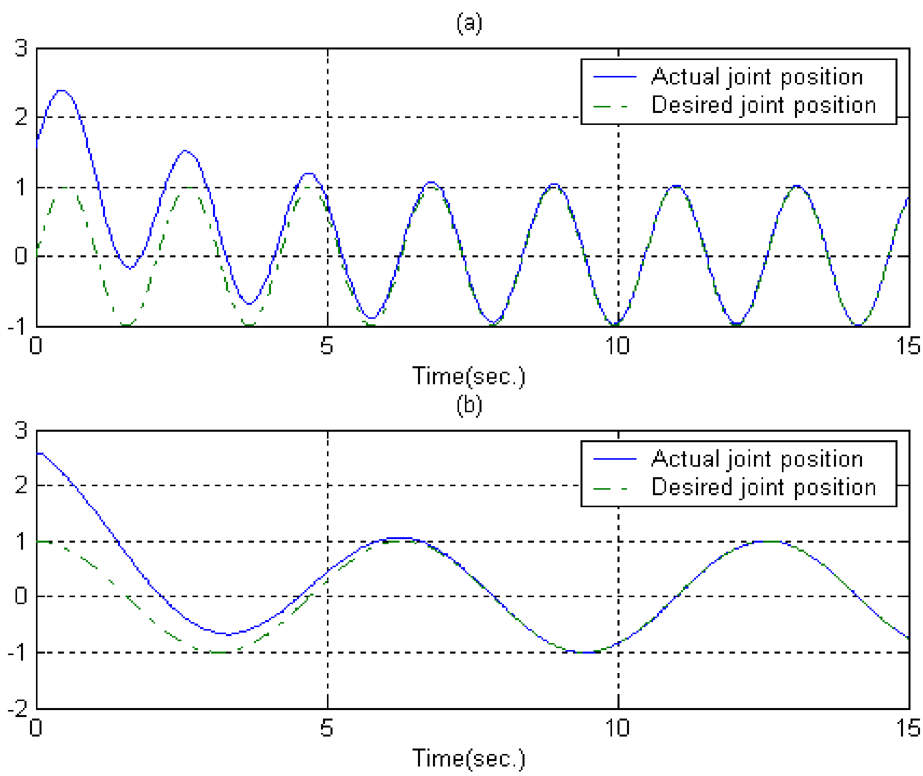

Figure 6.5 The trajectories of the desired and actual joint positions (a) θ, θ_d (b) ϕ, ϕ_d

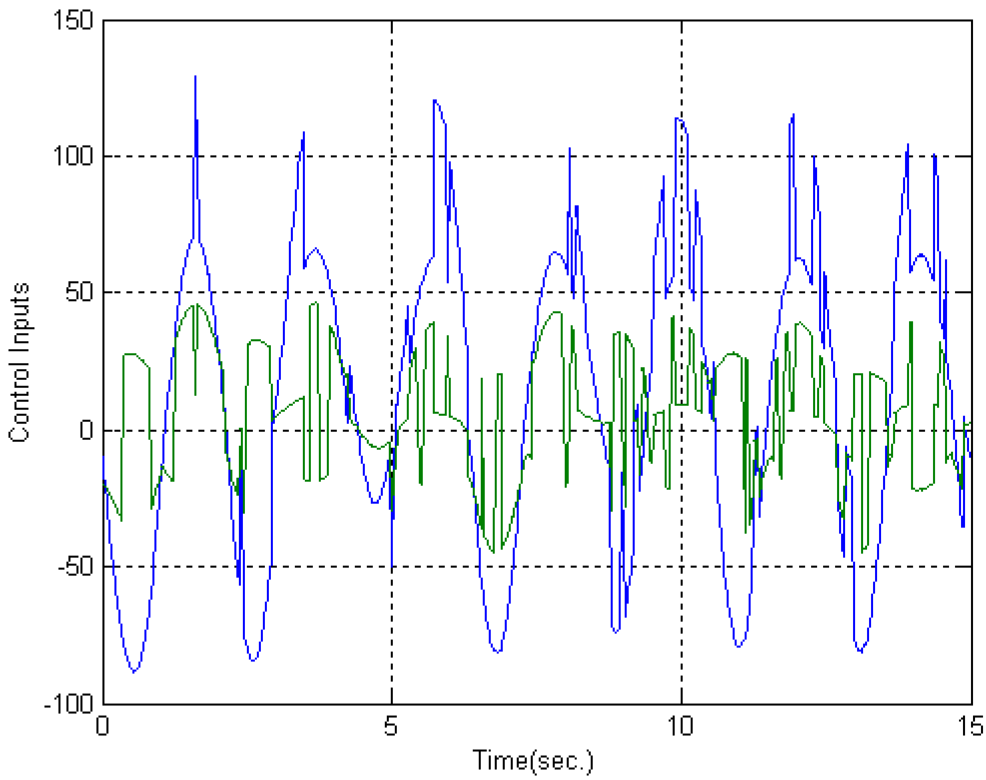

Figure 6.6 Control Inputs