CHAPTER 4

UNIFORMLY DISTRIBUTED SIMPLEX SLIDING-MODE CONTROL FOR MATCHED DISTURBANCES

Based on the uniformly distributed simplex set presented in Chapter 3, a novel simplex sliding-mode control, uniformly distributed simplex sliding-mode control (UDSSMC), is introduced in this chapter. The system description is indicted in Section 4.1. Section 4.2 shows the sliding vector design. In Section 4.3, the UDSSMC algorithm is developed to guarantee system trajectory could reach the sliding mode in a finite time. Besides, a new smoothing strategy is employed to solve the chattering caused by the UDSSMC in Section 4.4. Finally, a numeric example is simulated to demonstrate the usefulness of the developed UDSSMC in Section 4.5.

4.1 System Description

Consider a linear time-invariant system encountering matched disturbance, expressed as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{B}\boldsymbol{d}(\boldsymbol{x},t) \tag{4.1}$$

where $\boldsymbol{x} \in \Re^n$ is the state, $\boldsymbol{u} \in \Re^m$ is the control input, and $\boldsymbol{d}(\boldsymbol{x},t) \in \Re^m$ represents the matched disturbance. Without loss of generality, the pair ($\boldsymbol{A}$,$\boldsymbol{B}$) is assumed to be controllable and $\boldsymbol{B}$ is of full rank. Besides, the matched disturbance is constrained by

$$\|\boldsymbol{d}(\boldsymbol{x},t)\| \le \delta_{max}(\boldsymbol{x},t) \tag{4.2}$$

where the upper bound $\delta_{max}(\boldsymbol{x},t)$ is available. Similar to the conventional simplex sliding-mode control (SSMC), the design procedure of the uniformly distributed simplex sliding-mode Control (UDSSMC) is mainly divided into two steps. In the first step, an appropriate sliding vector is selected such that the system is stabilized in the sliding mode. In the second step, the UDSSMC algorithm is derived such that the system trajectory could reach the sliding mode in a finite time and then stay thereafter. Next, these two steps will be respectively discussed in detail.

4.2 Sliding Vector Design

To efficiently eliminate the matched disturbance, the method proposed by Chang and Chen [49] will be employed in the UDSSMC to choose the sliding vector. In this section, it will be briefly introduced as below.

Since (4.1) is controllable and $\boldsymbol{B}$ is full rank, a state-feedback gain $\boldsymbol{K}$ could be obtained from the pole-placement method by assigning n eigenvalues $\{\lambda_1,\cdots,\lambda_{n-m},\omega_1,\cdots,\omega_m\}$ to $\boldsymbol{A}-\boldsymbol{BK}$ [50]. To design the sliding vector, $\{\omega_1,\cdots,\omega_m\}$ are purposely set to be the same and negative, i.e., $\omega_j=\omega<0$ for j=1,2,…,m, and $\{\lambda_1,\cdots,\lambda_{n-m}\}$ are selected to be $\lambda_i<0$ for i=1,2,…,n-m and $\lambda_i\neq\lambda_j$ for $i\neq j$. Besides, let $\lambda_i\neq\omega$, i.e., ω is not in the spectrum of $\boldsymbol{A}$. Chang and Chen then presented the sliding vector as

$$\sigma=\boldsymbol{C}\boldsymbol{x} \tag{4.3}$$

where $\boldsymbol{C}$ consists of m independent left eigenvectors of $\boldsymbol{A}-\boldsymbol{BK}$ corresponding to ω, i.e.,

$$\boldsymbol{C}\left(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{K}\right)=\omega\boldsymbol{C} \tag{4.4}$$

To rearrange (4.4), it becomes

$$\boldsymbol{C}\boldsymbol{A}-\omega\boldsymbol{C}=\boldsymbol{C}\boldsymbol{B}\boldsymbol{K} \tag{4.5}$$

Since the matrix $\boldsymbol{C}$ includes m independent eigenvectors, i.e. Rank($\boldsymbol{C}$)=m. Based on the fact which the chosen eigenvalue ω isn't in the spectrum of $\boldsymbol{A}$, it is derived

$$\text{Rank}(\boldsymbol{C})=\text{Rank}(\boldsymbol{C}\boldsymbol{A}-\omega\boldsymbol{C})=\text{Rank}(\boldsymbol{C}\boldsymbol{B}\boldsymbol{K})=m \tag{4.6}$$

By utilizing the matrix theory [51], it leads to

$$\text{Rank}\left(\boldsymbol{C}\boldsymbol{B}\right)\geq m \tag{4.7}$$

In view of the fact $\boldsymbol{C}\boldsymbol{B}\in\Re^{m\times m}$, it must coincide with

$$\text{Rank}\left(\boldsymbol{C}\boldsymbol{B}\right)\leq m \tag{4.8}$$

From (4.7) and (4.8), it results in

$$\text{Rank}\left(\boldsymbol{C}\boldsymbol{B}\right)=m \tag{4.9}$$

In other words, it shows the fact that the $m\times m$ square matrix $\boldsymbol{C}\boldsymbol{B}$ is invertible, i.e., $\left(\boldsymbol{C}\boldsymbol{B}\right)^{-1}$ exists. Because of the fact that $\omega_j=\omega<0$ for $j=1,2,\ldots,m$ and $\lambda_i<0$ for $i=1,2,\ldots,n-m$, the chosen sliding vector (4.3) could guarantee that the system trajectory will approach the destination along the sliding surface when the system is in the sliding surface.

Based on (4.3) and the truth that $\left(\boldsymbol{C}\boldsymbol{B}\right)^{-1}$ exists, a modified sliding vector will be employed in the UDSSMC, expressed as

$$\boldsymbol{\sigma} = (\boldsymbol{CB})^{-1}\boldsymbol{Cx} \tag{4.10}$$

where $\boldsymbol{\sigma} = [\sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_{\boldsymbol{m}}]^{\boldsymbol{T}} \in \Re^m$. Apparently, the system trajectory will be also reach the destination in the sliding mode by employing the modified sliding vector (4.10).

4.3 UDSSMC Algorithm

This section will develop the UDSSMC algorithm using (4.10) as the sliding vector. Let the control law be

$$\boldsymbol{u} = -\boldsymbol{Kx} + \overline{\boldsymbol{u}} \tag{4.11}$$

with

$$\overline{\boldsymbol{u}} = \left(\sqrt{\frac{2m}{m+1}} \cdot \delta_{max}(\boldsymbol{x},t) + \varepsilon\right)\boldsymbol{u}^i, \quad \varepsilon > 0, \quad \text{for } \boldsymbol{\sigma} = (\boldsymbol{CB})^{-1}\boldsymbol{Cx} \text{ is in } \boldsymbol{\Sigma}_i \tag{4.12}$$

where $\boldsymbol{u}^i$ for i=1,2,…,m+1 represent the uniformly distributed simplex set S_U described in Chapter 3. By means of the uniformly distributed simplex set, the sliding vector $\boldsymbol{\sigma} \in \Re^{\boldsymbol{m}}$ in $\boldsymbol{\Sigma}_i$ can be uniquely expressed as

$$\boldsymbol{\sigma} = \sum_{j \neq \boldsymbol{i}, j=1}^{m+1} \gamma_j \boldsymbol{u}^j, \gamma_j > 0 \tag{4.13}$$

Note that the use of $\sqrt{\frac{2m}{m+1}} \cdot \delta_{max}(\boldsymbol{x},t) + \varepsilon$ will be explained later.

The most important thing for the use of (4.12) is to determine which sub-region $\boldsymbol{\Sigma}_i$ the sliding vector $\boldsymbol{\sigma}$ belongs to. Actually, this is not an easy job, especially when the number of control inputs is increased higher than 3. To deal with such problem, an efficient scheme is proposed as below.

First, reviewing an important equation (3.3) derived from the uniformly distributed simplex set, it has $\left[\frac{m}{m+1}\left(\boldsymbol{U}_{m+1}-\boldsymbol{V}_{m+1}\right)^T\right]\boldsymbol{U}_{m+1}=\boldsymbol{I}_{m\times m}$ where $\boldsymbol{U}_{m+1}=\left[\boldsymbol{u}^1,\boldsymbol{u}^2,\cdots,\boldsymbol{u}^m\right]$ and $\boldsymbol{V}_{m+1}=\left[\boldsymbol{u}^{m+1},\boldsymbol{u}^{m+1},\cdots,\boldsymbol{u}^{m+1}\right]$. It also implies

$$\boldsymbol{U}_{m+1}\left[\frac{m}{m+1}\left(\boldsymbol{U}_{m+1}-\boldsymbol{V}_{m+1}\right)^T\right]=\boldsymbol{I}_{m\times m} \tag{4.14}$$

Then, define a checking vector $\boldsymbol{\gamma}'=\left[\gamma_1',\gamma_2',\cdots,\gamma_m'\right]^T$ as

$$\boldsymbol{\gamma}'=\left(\boldsymbol{U}_{m+1}-\boldsymbol{V}_{m+1}\right)^T\boldsymbol{\sigma} \tag{4.15}$$

With this checking vector, the sub-region $\boldsymbol{\Sigma}_i$ can be easily determined according to the following lemma:

Lemma 1: Let $\gamma_{\boldsymbol{p}}'$ be the smallest element of $\boldsymbol{\gamma}'=\left[\gamma_1',\gamma_2',\cdots,\gamma_m'\right]^T$, i.e., $\gamma_j'\geq\gamma_{\boldsymbol{p}}'$ for $1\leq j\leq m$. If $\gamma_{\boldsymbol{p}}'>0$, the sliding vector $\boldsymbol{\sigma}$ belongs to $\boldsymbol{\Sigma}_{\boldsymbol{m}+1}$. If $\gamma_{\boldsymbol{p}}'<0$ and $\gamma_j'>\gamma_{\boldsymbol{p}}'$ for $j\neq p$, then the sliding vector $\boldsymbol{\sigma}$ belongs to $\boldsymbol{\Sigma}_{\backslash\boldsymbol{p}}$. Otherwise, the sliding vector $\boldsymbol{\sigma}$ doesn't belong to any open sub-regions $\boldsymbol{\Sigma}_i$, i=1,2,…,m+1. It is on one of the boundaries of these m+1 open sub-regions.

Proof:

From (4.14), pre-multiplying $\frac{m}{m+1}\boldsymbol{U}_{\boldsymbol{m}+1}$ into (4.15) becomes

$$\boldsymbol{\sigma}=\frac{m}{m+1}\boldsymbol{U}_{m+1}\boldsymbol{\gamma}'=\frac{m}{m+1}\left(\gamma_1'\boldsymbol{u}^1+\gamma_2'\boldsymbol{u}^2+\cdots+\gamma_m'\boldsymbol{u}^m\right)=\frac{m}{m+1}\sum_{j=1}^{m}\gamma_j'\boldsymbol{u}^j \tag{4.16}$$

Clearly, if $\gamma_{\boldsymbol{p}}'>0$ then $\gamma_j'\geq\gamma_{\boldsymbol{p}}'>0$ for j=1,2,…,m since $\gamma_{\boldsymbol{p}}'$ is the smallest element. This implies that the sliding vector $\boldsymbol{\sigma}$ can be expressed by a linear combination of $\boldsymbol{u}^1$,

$\boldsymbol{u}^2$, …, $\boldsymbol{u}^m$ with positive coefficients. Due to (4.13), it is easy to find that the sliding vector $\boldsymbol{\sigma}$ belongs to $\boldsymbol{\Sigma}_{m+1}$ by directly setting $\gamma_j = \frac{m}{m+1}\gamma'_j$.

If $\gamma'_p < 0$ and $\gamma'_j > \gamma'_p$ for $j \neq p$, then it has $\gamma'_j + |\gamma'_p| > \gamma'_p + |\gamma'_p| = 0$ for $j \neq p$. Now, by letting $\gamma'_{m+1} = 0$ and $\gamma_i = \frac{m}{m+1}\left(\gamma'_i + |\gamma'_p|\right)$ for i=1,2,…,m+1, (4.16) can be rearranged as

$$\boldsymbol{\sigma} + \frac{m}{m+1}|\gamma'_p|\sum_{j=1}^{m+1}\boldsymbol{u}^j = \gamma_1\boldsymbol{u}^1 + \gamma_2\boldsymbol{u}^2 + \cdots + \gamma_{m+1}\boldsymbol{u}^{m+1} = \sum_{j\neq p, j=1}^{m+1}\gamma_j\boldsymbol{u}^j \qquad (4.17)$$

where $\gamma_p = \frac{m}{m+1}\left(\gamma'_p + |\gamma'_p|\right) = 0$, $\gamma_{m+1} = \frac{m}{m+1}|\gamma'_p|$ and $\gamma_j = \frac{m}{m+1}\left(\gamma'_j + |\gamma'_p|\right) > 0$ for $j \neq p$. From (3.11) and (3.12), it can be obtained that $\sum_{i=1}^{m+1}\boldsymbol{u}^i = 0$. Hence, (4.17) becomes

$$\boldsymbol{\sigma} = \sum_{j\neq p, j=1}^{m+1}\gamma_j\boldsymbol{u}^j, \gamma_j > 0 \qquad (4.18)$$

According to (4.13), the sliding vector $\boldsymbol{\sigma}$ belongs to $\boldsymbol{\Sigma}_p$.

For the other cases, i.e., when $\gamma'_p = 0$ or when $\gamma'_j = \gamma'_p < 0$ for some $j \neq p$, it is not difficult to find that the sliding vector $\boldsymbol{\sigma}$ doesn’t belong to any sub-regions $\boldsymbol{\Sigma}_i$ because in addition to $\gamma_p = 0$, at least one of the coefficients γ_j, $j \neq p$, is zero.

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According to the proposed checking vector (4.15) and Lemme1, it can be easily to determine which sub-region $\boldsymbol{\Sigma}_i$ the sliding vector $\boldsymbol{\sigma}$ belongs to. Next, let’s show that the system (4.1) will be driven to the sliding mode $\boldsymbol{\sigma} = 0$ in a finite time by using the UDSSMC algorithm (4.11) and (4.12).

For the system (4.1) encountering matched disturbance $\boldsymbol{d}$, the use of control input (4.11) leads to

$$\dot{\boldsymbol{x}} = (\boldsymbol{A} - \boldsymbol{BK})\boldsymbol{x} + \boldsymbol{B}\overline{\boldsymbol{u}} + \boldsymbol{Bd}(\boldsymbol{x},t) \tag{4.19}$$

with $\overline{\boldsymbol{u}}$ given in (4.12). Let the sliding vector $\boldsymbol{\sigma}$ be chosen as (4.10), then its first derivative becomes

$$\begin{aligned}\dot{\boldsymbol{\sigma}} &= (\boldsymbol{CB})^{-1}\boldsymbol{C}\dot{\boldsymbol{x}} \\ &= (\boldsymbol{CB})^{-1}\boldsymbol{C}(\boldsymbol{A} - \boldsymbol{BK})\boldsymbol{x} + \overline{\boldsymbol{u}} + \boldsymbol{d} \\ &= \omega(\boldsymbol{CB})^{-1}\boldsymbol{Cx} + \overline{\boldsymbol{u}} + \boldsymbol{d} \\ &= \omega\boldsymbol{\sigma} + \overline{\boldsymbol{u}} + \boldsymbol{d}\end{aligned} \tag{4.20}$$

where (4.4) has been adopted. Without loss of generality, let's consider the following case that the sliding vector $\boldsymbol{\sigma}$ is currently in Σ_p, $1 \le p \le m+1$, i.e.,

$$\boldsymbol{\sigma} = \sum_{j \ne p, j=1}^{m+1} \gamma_j \boldsymbol{u}^j = \boldsymbol{U}_p \gamma_p, \qquad \gamma_j > 0 \tag{4.21}$$

where $\boldsymbol{U}_p$ is defined in (3.1) and $\gamma_p = \begin{bmatrix}\gamma_1 & \cdots & \gamma_{p-1} & \gamma_{p+1} & \cdots & \gamma_{m+1}\end{bmatrix}^T$. Hence, it leads to

$$\overline{\boldsymbol{u}} = \left(\sqrt{\frac{2m}{m+1}} \cdot \delta_{max}(\boldsymbol{x},t) + \varepsilon\right)\boldsymbol{u}^p \tag{4.22}$$

where $\delta_{max}(\boldsymbol{x},t)$ means the upper bound of the matched disturbance given in (4.2). Furthermore, the truth of $det\,\boldsymbol{U}_p \ne 0$ in (C1) implies that all the columns of $\boldsymbol{U}_p$ form a basis of $\Re^m$, and then the matched disturbance $\boldsymbol{d}$ can be uniquely expressed as

$$\boldsymbol{d} = \sum_{j \ne p, j=1}^{m+1} \delta_j \boldsymbol{u}^j = \boldsymbol{U}_p \delta_p \tag{4.23}$$

where $\delta_p = \begin{bmatrix}\delta_1 & \cdots & \delta_{p-1} & \delta_{p+1} & \cdots & \delta_{m+1}\end{bmatrix}^T$. Note that all the elements of δ_p are unknown and may be negative. Now, by pre-multiplying $(\boldsymbol{u}^k - \boldsymbol{u}^p)^T$ into (4.23) for $k \neq p$, it leads to

$$\left(\boldsymbol{u}^k - \boldsymbol{u}^p\right)^T \boldsymbol{d} = \left(\boldsymbol{u}^k - \boldsymbol{u}^p\right)^T \sum_{j \neq p, j=1}^{m+1} \delta_j \boldsymbol{u}^j = \delta_k \left(\boldsymbol{u}^k - \boldsymbol{u}^p\right)^T \boldsymbol{u}^k = \delta_k \left(\frac{m+1}{m}\right) \tag{4.24}$$

where $\left(\boldsymbol{u}^k - \boldsymbol{u}^p\right)^T \boldsymbol{u}^j = 0$ for $k \neq j$. Clearly,

$$\left|\delta_k\right| = \frac{m}{m+1}\left|\left(\boldsymbol{u}^k - \boldsymbol{u}^p\right)^T \boldsymbol{d}\right| \leq \frac{m}{m+1}\left\|\boldsymbol{u}^k - \boldsymbol{u}^p\right\| \cdot \delta_{max}(\boldsymbol{x}, t) \tag{4.25}$$

where $\|\boldsymbol{d}\| < \delta_{max}(\boldsymbol{x}, t)$. With the use of (C3) and (C4), it causes that $\left\|\boldsymbol{u}^k - \boldsymbol{u}^p\right\|^2 = \left(\boldsymbol{u}^k - \boldsymbol{u}^p\right)^T \left(\boldsymbol{u}^k - \boldsymbol{u}^p\right) = 2\left(\frac{m+1}{m}\right)$. That means $\left\|\boldsymbol{u}^k - \boldsymbol{u}^p\right\| = \sqrt{2\left(\frac{m+1}{m}\right)}$ and then (4.25) is rewritten as

$$\left|\delta_k\right| \leq \frac{m}{m+1}\sqrt{2\left(\frac{m+1}{m}\right)} \cdot \delta_{max}(\boldsymbol{x}, t) = \sqrt{\frac{2m}{m+1}} \cdot \delta_{max}(\boldsymbol{x}, t) \tag{4.26}$$

Now, substituting (4.21), (4.22), and (4.23) into (4.20) yields

$$\boldsymbol{U}_p \dot{\gamma}_p = \omega \boldsymbol{U}_p \gamma_p + \left(\sqrt{\frac{2m}{m+1}} \cdot \delta_{max}(\boldsymbol{x}, t) + \varepsilon\right) \boldsymbol{u}^p + \boldsymbol{U}_p \boldsymbol{\delta}_p \tag{4.27}$$

Similar to (4.24), by pre-multiplying (4.27) with $(\boldsymbol{u}^k - \boldsymbol{u}^p)^T$ for $k \neq p$, it becomes

$$\dot{\gamma}_k = \omega \gamma_k - \left(\sqrt{\frac{2m}{m+1}} \cdot \delta_{max}(\boldsymbol{x}, t) + \varepsilon\right) + \delta_k \leq \omega \gamma_k - \varepsilon \tag{4.28}$$

where the truth of (4.26) is adopted. Since $\gamma_{\boldsymbol{k}} > 0$ for $k \neq p$ in $\boldsymbol{\Sigma}_p$, the candidate of Lyapunov function can be selected as

$$V = \sum_{\substack{k=1 \\ k \neq p}}^{m+1} \gamma_k \tag{4.29}$$

From (4.28), taking the first derivative of V leads to

$$\dot{V} = \sum_{\substack{k=1 \\ k\neq p}}^{m+1} \dot{\gamma}_k \le \omega \sum_{\substack{k=1 \\ k\neq p}}^{m+1} \gamma_k - m\varepsilon = \omega V - m\varepsilon \tag{4.30}$$

Since ω<0, it leads to

$$\dot{V} < -m\varepsilon \tag{4.31}$$

Obviously, V is really a Lyapunov function. Besides, it has been shown that V is continuous, i.e., V doesn't change discontinuously when $\boldsymbol{\sigma}$ switches from Σ_p to the other sub-regions Σ_j, $j \neq p$ [18]. Therefore, V decreases all the time and will become zero in a finite time, i.e., $V(t) \equiv 0$ for $t - t_0 > t'$, where t_0 is the initial time and $t' = V(t_0)/m\varepsilon$. Note that $V(t) \equiv 0$ implies $\gamma_k = 0$ for k=1,2,…,m+1 or $\boldsymbol{\sigma} = \sum_{j\neq p, j=1}^{m+1} \gamma_j \boldsymbol{u}^j = 0$. In other words, the system trajectory will reach the sliding mode σ=0 in a finite time.

4.4 Smoothing Strategy for UDSSMC

By virtue of the above derivation, it demonstrates that the developed UDSSMC algorithms (4.11) and (4.12) could efficiently suppress the matched disturbance. However, it still inevitably confronts with the chattering problem, which happens not only in the sliding mode but also in the approach mode. If the concept of sliding layer is directly adopted, which modifies (4.12) as

$$\bar{\boldsymbol{u}} = \begin{cases} \left(\sqrt{\dfrac{2m}{m+1}} \cdot \delta_{max}(\boldsymbol{x}, t) + \varepsilon \right) \boldsymbol{u}^i, & \text{for } \boldsymbol{\sigma} \in \Sigma_i \quad \text{if } \|\boldsymbol{\sigma}\| \ge \xi \\ \text{Unchanged} & \text{if } \|\boldsymbol{\sigma}\| < \xi \end{cases} \tag{4.32}$$

then (4.32) could only smooth away the chattering in the sliding layer $\|\boldsymbol{\sigma}\| \le \xi$. As for the chattering in the approaching mode, (4.32) is still unable to suppress such unwanted high frequency behavior. In fact, such chattering before the sliding mode exists due to the switching function in the UDSSMC, which is excited when the system trajectory moves around two connected open sub-regions given in (2.4).

In order to improve the above weakness, a novel scheme with two different strategies is proposed here. First, let the system trajectory be currently in $\boldsymbol{\Sigma}_i$, $1 \le i \le m+1$, i.e., the sliding vector can be expressed as

$$\boldsymbol{\sigma} = \sum_{j \ne \boldsymbol{i}, j=1}^{m+1} \gamma_j \boldsymbol{u}^j = \boldsymbol{U}_i \boldsymbol{\gamma}_i, \qquad \gamma_j > 0 \tag{4.33}$$

where $\boldsymbol{U}_i$ is defined in (3.1) and $\boldsymbol{\gamma}_i = [\gamma_1 \ \cdots \ \gamma_{i-1} \ \ \gamma_{i+1} \ \cdots \ \gamma_{m+1}]^T$. Further rearrange (4.33) as

$$\boldsymbol{\sigma} = \gamma_q \boldsymbol{u}^q + \sum_{\substack{j=1, \\ j \ne \boldsymbol{i}, j \ne \boldsymbol{q}}}^{m+1} \gamma_j \boldsymbol{u}^j, \qquad \sum_{\substack{j=1, \\ j \ne \boldsymbol{i}, j \ne \boldsymbol{q}}}^{m+1} \gamma_j \boldsymbol{u}^j \in \boldsymbol{\Sigma}_{iq} \tag{4.34}$$

where $q \ne i$ and $\boldsymbol{\Sigma}_{iq} = \left\{ \boldsymbol{\sigma}_{iq} \middle| \boldsymbol{\sigma}_{iq} = \sum_{\substack{j=1, \\ j \ne \boldsymbol{i}, j \ne \boldsymbol{q}}}^{\boldsymbol{m}+1} \gamma_j \boldsymbol{u}^j, \gamma_j > 0 \right\}$ represents the sub-region formed by the simplex vectors $\boldsymbol{u}^j$, $1 \le j \le m+1$, $j \ne i,q$. Actually, $\boldsymbol{\Sigma}_{iq}$ is the boundary separating $\boldsymbol{\Sigma}_i$ and $\boldsymbol{\Sigma}_q$, which is depicted in Figure 4.1 as an example.

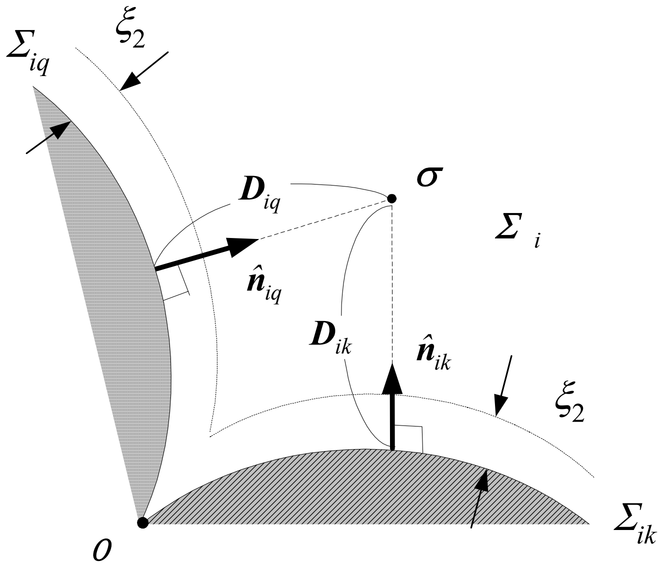

Figure 4.1. The sub-region formed by the uniformly distributed simplex set

Now, let's define the distance between the sliding vectors $\boldsymbol{\sigma}$ and the boundary $\boldsymbol{\Sigma}_{iq}$ by introducing the following unit vector

$$\hat{\boldsymbol{n}}_{iq} = \frac{\boldsymbol{u}^i - \boldsymbol{u}^q}{\left\| \boldsymbol{u}^i - \boldsymbol{u}^q \right\|} \tag{4.35}$$

Note that any vector in $\boldsymbol{\Sigma}_{iq}$ can be expressed as $\boldsymbol{\sigma}_{iq} = \sum_{\substack{j=1, \\ j \neq i, j \neq q}}^{m+1} \gamma_j \boldsymbol{u}^j$ and the inner product $\boldsymbol{\sigma}_{iq} \cdot \hat{\boldsymbol{n}}_{iq}$ is then attained as

$$\boldsymbol{\sigma}_{iq} \cdot \hat{\boldsymbol{n}}_{iq} = \left(\sum_{\substack{j=1, \\ j \neq i, j \neq q}}^{m+1} \gamma_j \boldsymbol{u}^j \right) \cdot \left(\frac{\boldsymbol{u}^i - \boldsymbol{u}^q}{\left\| \boldsymbol{u}^i - \boldsymbol{u}^q \right\|} \right) = \sum_{\substack{j=1, \\ j \neq i, j \neq q}}^{m+1} \frac{\gamma_j \left(\boldsymbol{u}^j \cdot \boldsymbol{u}^i - \boldsymbol{u}^j \cdot \boldsymbol{u}^q \right)}{\left\| \boldsymbol{u}^i - \boldsymbol{u}^q \right\|} = 0 \tag{4.36}$$

where the truth of $\boldsymbol{u}^j \cdot \boldsymbol{u}^i - \boldsymbol{u}^j \cdot \boldsymbol{u}^q = 0$ can be seen from (C4). It is clear that $\hat{\boldsymbol{n}}_{iq}$ is perpendicular to $\boldsymbol{\Sigma}_{iq}$, i.e., $\hat{\boldsymbol{n}}_{iq} \perp \boldsymbol{\Sigma}_{iq}$. Hence, the distance between the sliding vectors $\boldsymbol{\sigma}$ and the boundary $\boldsymbol{\Sigma}_{iq}$ can be implemented as

$$
\begin{aligned}
dist(\boldsymbol{\sigma},\boldsymbol{\Sigma}_{iq}) &= \left|\boldsymbol{\sigma}\cdot\hat{\boldsymbol{n}}_{iq}\right| = \left|\left(\gamma_q\boldsymbol{u}^q + \sum_{\substack{j=1\\ j\neq \boldsymbol{i},\, j\neq \boldsymbol{q}}}^{m+1}\gamma_j\boldsymbol{u}^j\right)\cdot\hat{\boldsymbol{n}}_{iq}\right| = \left|\left(\gamma_q\boldsymbol{u}^q+\boldsymbol{\sigma}_{iq}\right)\cdot\hat{\boldsymbol{n}}_{iq}\right| \\
&= \gamma_q\boldsymbol{u}^q\cdot\hat{\boldsymbol{n}}_{iq}
\end{aligned}
\tag{4.37}
$$

which can be further calculated from (C3) and (C4) as

$$
\begin{aligned}
dist(\boldsymbol{\sigma},\boldsymbol{\Sigma}_{iq}) &= \gamma_q\boldsymbol{u}^q\cdot\hat{\boldsymbol{n}}_{iq} = \left|\left(\gamma_q\boldsymbol{u}^q\right)\cdot\left(\frac{\boldsymbol{u}^i-\boldsymbol{u}^q}{\left\|\boldsymbol{u}^i-\boldsymbol{u}^q\right\|}\right)\right| \\
&= \left|\gamma_q\left(-\frac{1}{m}-1\right)\Big/\sqrt{2+\frac{2}{m}}\right| \\
&= \gamma_q\left(1+\frac{1}{m}\right)\Big/\sqrt{2+\frac{2}{m}} = \frac{\gamma_q}{\sqrt{2}}\sqrt{1+\frac{1}{m}}
\end{aligned}
\tag{4.38}
$$

Furthermore, the minimum distance between the sliding vector $\boldsymbol{\sigma}$ and the boundaries for the region $\boldsymbol{\Sigma}_i$ could be defined as below:

$$
dist_{min} = \frac{\gamma_{min}}{\sqrt{2}}\sqrt{1+\frac{1}{m}}
\tag{4.39}
$$

where γ_{min} is the minimum value among $\boldsymbol{\gamma}_i = \begin{bmatrix}\gamma_1 & \cdots & \gamma_{\boldsymbol{i}-1} & \gamma_{\boldsymbol{i}+1} & \cdots & \gamma_{m+1}\end{bmatrix}^T$ for $\boldsymbol{\sigma} = \sum_{j\neq \boldsymbol{i},\, j=1}^{m+1}\gamma_j\boldsymbol{u}^j = \boldsymbol{U}_i\boldsymbol{\gamma}_i \in \boldsymbol{\Sigma}_i$. In other words, if $\gamma_{min} = \gamma_j$, then the minimum distance $dist_{min} = \frac{\gamma_j}{\sqrt{2}}\sqrt{1+\frac{1}{m}}$ represents the distance between the sliding vector $\boldsymbol{\sigma}$ and $\boldsymbol{\Sigma}_{ij}$, which is the boundaries of $\boldsymbol{\Sigma}_i$ and $\boldsymbol{\Sigma}_j$.

To totally get rid of the chattering problem for the simplex sliding mode control, the novel scheme, which includes two different strategies, is designed as

In the approaching mode:

$$\overline{\boldsymbol{u}} = \begin{cases} \left(\sqrt{\dfrac{2m}{m+1}} \cdot \delta_{max}(\boldsymbol{x},t) + \varepsilon \right) \boldsymbol{u}^{\boldsymbol{i}}, \quad \text{for } \boldsymbol{\sigma} \in \boldsymbol{\Sigma}_i & \text{if } dist_{\min} \geq \xi_1 \\ \text{Unchanged} & \text{if } dist_{\min} < \xi_1 \end{cases}$$

In the sliding mode:

$$\overline{\boldsymbol{u}} = \begin{cases} \left(\sqrt{\dfrac{2m}{m+1}} \cdot \delta_{max}(\boldsymbol{x},t) + \varepsilon \right) \boldsymbol{u}^{\boldsymbol{i}}, \quad \text{for } \boldsymbol{\sigma} \in \boldsymbol{\Sigma}_i & \text{if } \|\boldsymbol{\sigma}\| \geq \xi_2 \\ \text{Unchanged} & \text{if } \|\boldsymbol{\sigma}\| < \xi_2 \end{cases} \tag{4.40}$$

where $dist_{min}$ is defined as (4.39). In addition, $\xi_1, \xi_2 > 0$ are treated as the thickness of the layer illustrated in Figure 4.2 for the case of *m*=2.

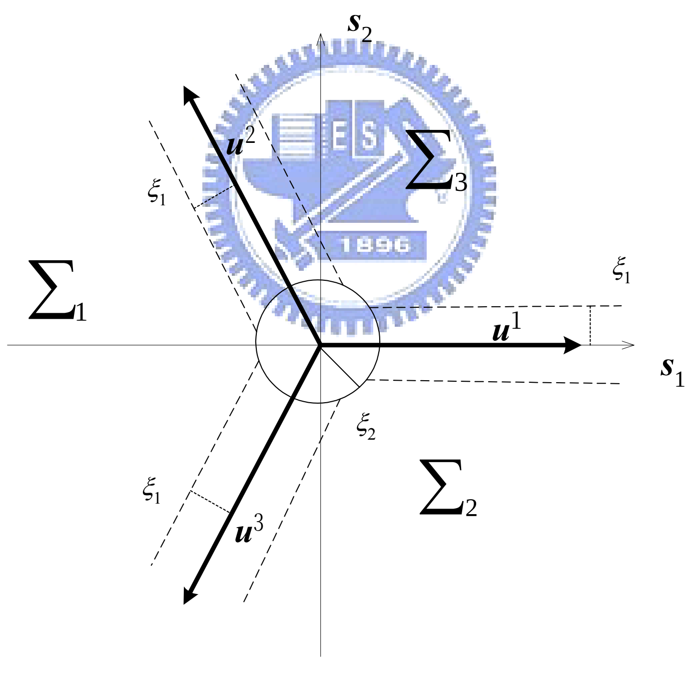

Figure 4.2 The novel smoothing strategy

4.5 Numeric Example and Simulation Results

In this section, two examples with different dimensions of control inputs are simulated to demonstrate the usefulness of UDSSMC in suppressing the matched disturbance. Next, these examples and simulation results will be explicitly shown in the following.

Example 4.1

Consider a linear time-invariant system (4.1) suffering from the matched disturbance, with the following numeric data:

$$\boldsymbol{A}=\begin{bmatrix} -0.0506 & 0 & -1 & 0.2380 \\ -0.7374 & -1.3345 & 0.3696 & 0 \\ 0.01 & 0.1074 & -0.3320 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\text{and} \quad \boldsymbol{B}=\begin{bmatrix} 0.0409 & 0 \\ 1.2714 & -20.3106 \\ -2.0625 & 1.3350 \\ 0 & 0 \end{bmatrix}$$

The control input and system state are respectively represented by $\boldsymbol{u}=[u_1 \quad u_2]^T$ and $\boldsymbol{x}=[x_1 \quad x_2 \quad x_3 \quad x_4]^T$. The matched disturbance $\boldsymbol{d}(\boldsymbol{x},t)=[d_1(\boldsymbol{x},t) \quad d_2(\boldsymbol{x},t)]^T$ is assumed as

$$\boldsymbol{d}(\boldsymbol{x},t)=\begin{bmatrix} sin(2t)+0.2cos(0.5t)\times x_1 \\ cos(3t)+0.2sin(t)\times x_3 \end{bmatrix} \tag{4.41}$$

Apparently, the upper bound of the matched disturbance are obtained as

$$\|\boldsymbol{d}(\boldsymbol{x},t)\| \le \delta_{max}(t)=2+0.2|x_1|+0.2|x_3| \tag{4.42}$$

The first step of the UDSSMC design is to choose a sliding vector $\boldsymbol{\sigma} = (\boldsymbol{CB})^{-1}\boldsymbol{Cx}$ as given in (4.10). Following the procedure described in Section 4.2, the eigenvalues for $\boldsymbol{A}-\boldsymbol{BK}$ are assigned to be

$$\lambda_1 = -1,\ \lambda_2 = -2,\ \ \omega_1 = \omega_2 = -5 \tag{4.43}$$

where ω_1 and ω_2 are purposely set to be the same and negative. By the aid of MATLAB, the state-feedback gain $\boldsymbol{K}$ and the left eigenvectors of $\boldsymbol{A}-\boldsymbol{BK}$ corresponding to ω_1 and ω_2 could be calculated as

$$\boldsymbol{K} = \begin{bmatrix} 2.3642 & -0.1194 & -2.7990 & 0.3789 \\ 0.1622 & -0.2864 & -0.1894 & -0.4697 \end{bmatrix}$$

and

$$\boldsymbol{C} = \begin{bmatrix} 0.2581 & -0.4385 & -0.3104 & -0.8030 \\ -0.3152 & -0.3758 & 0.2914 & -0.8213 \end{bmatrix}$$

Hence, it obtains

$$(\boldsymbol{CB})^{-1}\boldsymbol{C} = \begin{bmatrix} 0.4738 & -0.0326 & -0.4955 & 0.0532 \\ 0.0252 & -0.0513 & -0.0311 & -0.0951 \end{bmatrix}$$

According to the sliding vector design described in Section 4.2, it is evident that the system stability in the sliding mode is guaranteed since all the eigenvalues are allocated in the left half plane. Based on the new construction proposed in Section 3.3, the uniformly distributed simplex set for $m = 2$ can be selected as

$$\begin{aligned} \boldsymbol{u}^1 &= \begin{bmatrix} 1 & 0 \end{bmatrix}^T \\ \boldsymbol{u}^2 &= \begin{bmatrix} -1/2 & \sqrt{3}/2 \end{bmatrix}^T \\ \boldsymbol{u}^3 &= \begin{bmatrix} -1/2 & -\sqrt{3}/2 \end{bmatrix}^T \end{aligned} \tag{4.44}$$

For $m = 2$ and (4.42), the corresponding coefficient in (4.12) is chosen as

$$\sqrt{\frac{2m}{m+1}}\cdot\delta_{max}(\boldsymbol{x},t)+\varepsilon=2.5+0.25|\boldsymbol{x}_1|+0.25|\boldsymbol{x}_3| \tag{4.45}$$

To demonstrate the effectiveness of the UDSSMC and the new smoothing strategy described in Section 4.4, there are three cases to simulate in the following:

Case 1: In this case, the UDSSMC control algorithm without any smooth strategy is adopted. Due to (4.45), the UDSSMC control algorithm is

$$\begin{aligned}&\boldsymbol{u}=-\boldsymbol{Kx}+\overline{\boldsymbol{u}}\\&\overline{\boldsymbol{u}}=\left(2.5+0.25|\boldsymbol{x}_1|+0.25|\boldsymbol{x}_3|\right)\boldsymbol{u}^{\boldsymbol{i}},\quad \varepsilon>0,\quad \text{for } \boldsymbol{\sigma}=(\boldsymbol{CB})^{-1}\boldsymbol{Cx} \text{ is in } \boldsymbol{\Sigma}_i\end{aligned} \tag{4.46}$$

Figure 4.3 to Figure 4.6 are simulation results with initial condition $\boldsymbol{x}(0)=[5 \quad 5 \quad -7 \quad 5]^{\boldsymbol{T}}$ for Case 1. Figure 4.3 shows the time response of the sliding vector $\boldsymbol{\sigma}=[\sigma_1 \quad \sigma_2]^{\boldsymbol{T}}$ and Figure 4.4 gives the trajectory of sliding vector in the $\boldsymbol{\sigma}$ space. It is clear that the system is successfully driven to the destination, i.e. $\boldsymbol{\sigma}=\boldsymbol{0}$. Figure 4.5 and Figure 4.6 are respectively the time response of the state variables and the control input. In Figure 4.5, it illustrates the system state variables all converge to $\boldsymbol{x}=\boldsymbol{0}$. However, Figure 4.6 shows the control input with serious chattering problem. To overcome the chattering problem, the following cases are simulated to test the feasibility of the smoothing strategies, the conventional sliding layer (4.32) and the new smoothing strategy (4.40).

Case 2: To improve the chattering phenomenon, the conventional sliding layer is adopted in this case. For this reason, the control algorithm in Case 2 becomes

$$\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{x} + \overline{\boldsymbol{u}}$$
$$\overline{\boldsymbol{u}} = \begin{cases} \left(2.5 + 0.25|\boldsymbol{x}_1| + 0.25|\boldsymbol{x}_3|\right)\boldsymbol{u}^i, \quad \text{for } \boldsymbol{\sigma} \in \boldsymbol{\Sigma}_i & \text{if } \|\boldsymbol{\sigma}\| \geq \xi \\ \text{Unchanged} & \text{if } \|\boldsymbol{\sigma}\| < \xi \end{cases} \tag{4.47}$$

With the use of initial condition $\boldsymbol{x}(0) = [5 \quad 5 \quad -7 \quad 5]^T$ and $\xi = 0.02$, Figure 4.7 to Figure 4.10 show simulation results for Case 2. Figure 4.7 and Figure 4.8 respectively illustrate the time response of the sliding vector $\boldsymbol{\sigma}$ and the trajectory of sliding vector in the $\boldsymbol{\sigma}$ space. Figure 4.9 gives the trajectory of the state variables and Figure 4.10 is the time response of the control input. From simulation results, it is obvious that the conventional sliding layer could only smooth away the chattering in the sliding layer $\|\boldsymbol{\sigma}\| \leq \xi$. However, it still exists the chattering phenomenon before the sliding mode. In other words, it implies that the conventional sliding layer is unable to totally suppress the chattering caused by the SSMC.

Case 3: To verify the usefulness of the new smoothing strategy, the UDSSMC control algorithm (4.40) is applied in Case 3. With the use of (4.45), the control algorithm in this case is

$$\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{x} + \overline{\boldsymbol{u}}$$

and

In the approaching mode:
$$\overline{\boldsymbol{u}} = \begin{cases} \left(2.5 + 0.25|\boldsymbol{x}_1| + 0.25|\boldsymbol{x}_3|\right)\boldsymbol{u}^i, \quad \text{for } \boldsymbol{\sigma} \in \boldsymbol{\Sigma}_i & \text{if } dist_{\min} \geq \xi_1 \\ \text{Unchanged} & \text{if } dist_{\min} < \xi_1 \end{cases} \tag{4.48}$$
In the sliding mode:
$$\overline{\boldsymbol{u}} = \begin{cases} \left(2.5 + 0.25|\boldsymbol{x}_1| + 0.25|\boldsymbol{x}_3|\right)\boldsymbol{u}^i, \quad \text{for } \boldsymbol{\sigma} \in \boldsymbol{\Sigma}_i & \text{if } \|\boldsymbol{\sigma}\| \geq \xi_2 \\ \text{Unchanged} & \text{if } \|\boldsymbol{\sigma}\| < \xi_2 \end{cases}$$

Figure 4.11 to Figure 4.14 are simulation results for Case 3 under the same conditions of $\boldsymbol{x}(0)=[5 \quad 5 \quad -7 \quad 5]^T$, $\xi_1=0.01$, and $\xi_2=0.02$. In Figure 4.11 and Figure 4.12, it respectively shows the time response of the sliding vector $\boldsymbol{\sigma}$ and the trajectory of sliding vector in the $\boldsymbol{\sigma}$ space. Figure 4.13 is the trajectory of the state variables and Figure 4.14 gives the time response of the control inputs. From these simulation results, it illustrates that the new smoothing strategy could really get rid of the chattering problem caused by the switching function in the UDSSMC not only in the sliding mode but also in the approach mode.

Example 4.2

To demonstrate the usefulness of the UDSSMC for the high-dimension system, the considered system in this example is the linear time-invariant system (4.1) with the following numeric data:

$$\boldsymbol{A}=\begin{bmatrix} -0.7872 & 2.1790 & -6.8820 & 2.6148 & 15.2879 & -16.3675 & 6.9081 \\ -0.0254 & -0.3073 & 1.0126 & -1.7008 & -2.9374 & 3.8520 & -1.4969 \\ -0.0402 & -0.3638 & -1.4163 & 2.5400 & 3.5547 & -4.7931 & 2.2363 \\ 0.2033 & -0.5108 & -0.3787 & 1.3152 & 0.5825 & -1.1385 & 0.7788 \\ -0.1071 & -0.7761 & -2.1877 & 4.1729 & 5.5411 & -7.5789 & 3.5906 \\ -0.1372 & -0.5795 & -0.2683 & 1.0650 & 0.8279 & -1.3620 & 0.7549 \\ -0.3000 & 2.2109 & -1.5576 & -0.6625 & 4.5384 & -4.4192 & 1.0167 \end{bmatrix}$$

$$\text{and} \quad \boldsymbol{B}=\begin{bmatrix} 2.6512 & 0.1492 & 1.9140 & 1.3748 \\ 1.4108 & 0.9988 & -1.9156 & -2.5232 \\ 1.2948 & -1.2088 & 1.4372 & 1.7348 \\ 0.7752 & 1.6712 & -1.0328 & -0.3252 \\ 1.4236 & 1.2760 & 1.1072 & 1.6308 \\ 1.1012 & 2.3764 & -1.2796 & -0.5728 \\ 1.2104 & 1.8576 & -1.6344 & -1.2808 \end{bmatrix} \tag{4.49}$$

Clearly, the system state $\boldsymbol{x} = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7]^T \in \Re^7$ and there are four control variables, denoted as $\boldsymbol{u} = [u_1 \quad u_2 \quad u_3 \quad u_4]^T$. Besides, the pair $(\boldsymbol{A},\boldsymbol{B})$ is controllable. Then, the matched disturbance will be chosen as

$$\boldsymbol{d} = \begin{bmatrix} 0.5\,sin(7t) \\ 0.5cos(3t)+0.5(\boldsymbol{x}(1)+\boldsymbol{x}(2))sin(0.5t) \\ 0.5\,sin(7t) \\ 0.5cos(4t)+0.5(\boldsymbol{x}(3)+\boldsymbol{x}(4))sin(t) \end{bmatrix} \tag{4.50}$$

Obviously, the upper bound of the matched disturbance could be obtained as

$$\|\boldsymbol{d}(\boldsymbol{x},t)\| \le \delta_{max}(t) = 2 + 0.5|\boldsymbol{x}(1)+\boldsymbol{x}(2)| + 0.5|\boldsymbol{x}(3)+\boldsymbol{x}(4)| \tag{4.51}$$

Now, choose a sliding vector $\boldsymbol{\sigma} = (\boldsymbol{CB})^{-1}\boldsymbol{Cx}$ as given in (4.10). Based on the procedure described in Section 4.2, the eigenvalues for $\boldsymbol{A}-\boldsymbol{BK}$ are assigned to be

$$\lambda_1 = -1,\ \lambda_2 = -4,\ \lambda_3 = -5,\ \omega_1 = \omega_2 = \omega_3 = \omega_4 = -6 \tag{4.52}$$

where ω_i, i=1,2,3,4, are the same. By the aid of MATLAB, it gets

$$\boldsymbol{K} = \begin{bmatrix} -0.7115 & 3.9350 & -1.1581 & 14.4688 & 8.0927 & -18.4880 & 4.0333 \\ 2.3067 & 0.6071 & -8.2486 & 11.0521 & 8.3284 & -15.9111 & 4.2407 \\ 3.9504 & 1.3034 & -10.1507 & -6.5267 & 9.7626 & -9.9789 & 5.6934 \\ -2.1713 & -2.4858 & 5.8846 & 9.7104 & -3.8529 & 2.2005 & -3.2839 \end{bmatrix}$$

and

$$(\boldsymbol{CB})^{-1}\boldsymbol{C} = \begin{bmatrix} -0.2343 & 0.6595 & -0.0550 & 2.0352 & 1.1045 & -2.6746 & 0.4604 \\ 0.3511 & -0.1555 & -0.8990 & 1.6897 & 0.4117 & -1.6285 & 0.2890 \\ 0.8008 & -0.3884 & -0.9273 & -0.8262 & -0.1436 & -0.0152 & 0.4025 \\ -0.4835 & 0.0489 & 0.6094 & 1.1667 & 0.2529 & -0.3239 & -0.3999 \end{bmatrix}$$

Note that the system stability in the sliding mode is guaranteed since all the eigenvalues are allocated in the left half plane. Further, the uniformly distributed simplex set can be found as

$$
\begin{aligned}
\boldsymbol{u}^1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T \\
\boldsymbol{u}^2 &= \begin{bmatrix} -1/4 & \sqrt{15}/4 & 0 & 0 \end{bmatrix}^T \\
\boldsymbol{u}^3 &= \begin{bmatrix} -1/4 & -\sqrt{15}/12 & \sqrt{30}/6 & 0 \end{bmatrix}^T \\
\boldsymbol{u}^4 &= \begin{bmatrix} -1/4 & -\sqrt{15}/12 & -\sqrt{30}/12 & \sqrt{90}/12 \end{bmatrix}^T \\
\boldsymbol{u}^5 &= \begin{bmatrix} -1/4 & -\sqrt{15}/12 & -\sqrt{30}/12 & -\sqrt{90}/12 \end{bmatrix}^T
\end{aligned} \tag{4.53}
$$

from the systematic procedure described in Section 4.3. Due to the fact of m=4 and (4.51), the corresponding coefficient in (4.12) is chosen as

$$
\sqrt{\frac{2m}{m+1}} \cdot \delta_{max}(\boldsymbol{x},t) + \varepsilon = 2.6 + 0.7|\boldsymbol{x}(1) + \boldsymbol{x}(2)| + 0.7|\boldsymbol{x}(3) + \boldsymbol{x}(4)| \tag{4.54}
$$

Hence, the UDSSMC algorithm in (4.11) and (4.40) becomes

$$
\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{x} + \bar{\boldsymbol{u}}
$$

and

$$
\begin{aligned}
&\text{In the approaching mode:} \\
&\bar{\boldsymbol{u}} = \begin{cases} (2.6 + 0.7|\boldsymbol{x}_1 + \boldsymbol{x}_2| + 0.7|\boldsymbol{x}_3 + \boldsymbol{x}_4|)\boldsymbol{u}^i, \quad \text{for } \boldsymbol{\sigma} \in \boldsymbol{\Sigma}_i & \text{if } dist_{\min} \geq \xi_1 \\ \text{Unchanged} & \text{if } dist_{\min} < \xi_1 \end{cases} \\
&\text{In the sliding mode:} \\
&\bar{\boldsymbol{u}} = \begin{cases} (2.6 + 0.7|\boldsymbol{x}_1 + \boldsymbol{x}_2| + 0.7|\boldsymbol{x}_3 + \boldsymbol{x}_4|)\boldsymbol{u}^i, \quad \text{for } \boldsymbol{\sigma} \in \boldsymbol{\Sigma}_i & \text{if } \|\boldsymbol{\sigma}\| \geq \xi_2 \\ \text{Unchanged} & \text{if } \|\boldsymbol{\sigma}\| < \xi_2 \end{cases}
\end{aligned} \tag{4.55}
$$

Figure 4.15 to Figure 4.18 are simulation results with the condition of $\boldsymbol{x}(0) = [0.5 \quad -0.5 \quad 1 \quad 0 \quad -0.5 \quad 1 \quad 1]^T$, $\xi_1 = 0.05$, and $\xi_2 = 0.1$. Figure 4.15 shows the sliding vector $\sigma_i, i = 1,\cdots,4$ and Figure 4.16 gives the trajectory of the norm of the sliding vector, $\|\boldsymbol{\sigma}\|$. From simulation results, it is demonstrably that the system is successfully driven to the sliding layer $\|\boldsymbol{\sigma}\| < \xi$. Figure 4.17 and Figure 4.18 are respectively the trajectories of the state variables and control inputs. In Figure 4.17, it illustrates the system state variables all converge to $\boldsymbol{x}=\boldsymbol{0}$. From simulation results, it demonstrates the practicability of the developed UDSSMC for high-dimension systems even the number of the control input is increased higher than three. In addition, it also reveals that the chattering problem could be effectively improved by the UDSSMC algorithm (4.55).

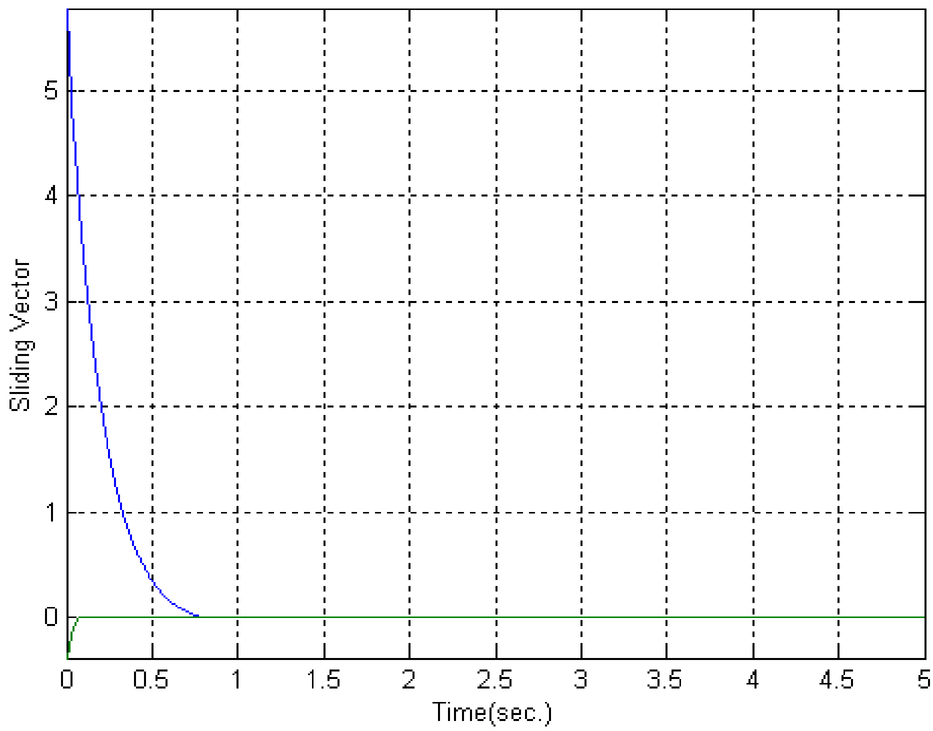

Figure 4.3 Time response of the sliding vector for Case 1 of Example 1

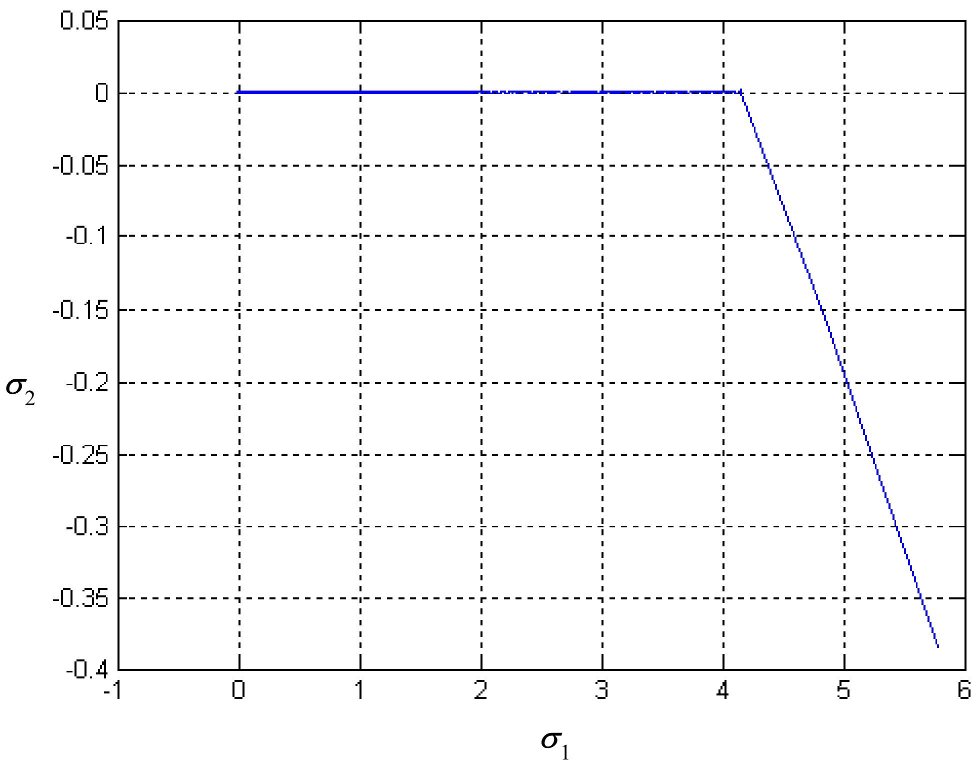

Figure 4.4 The trajectory of the sliding vector in the σ space for Case 1 of Example 1

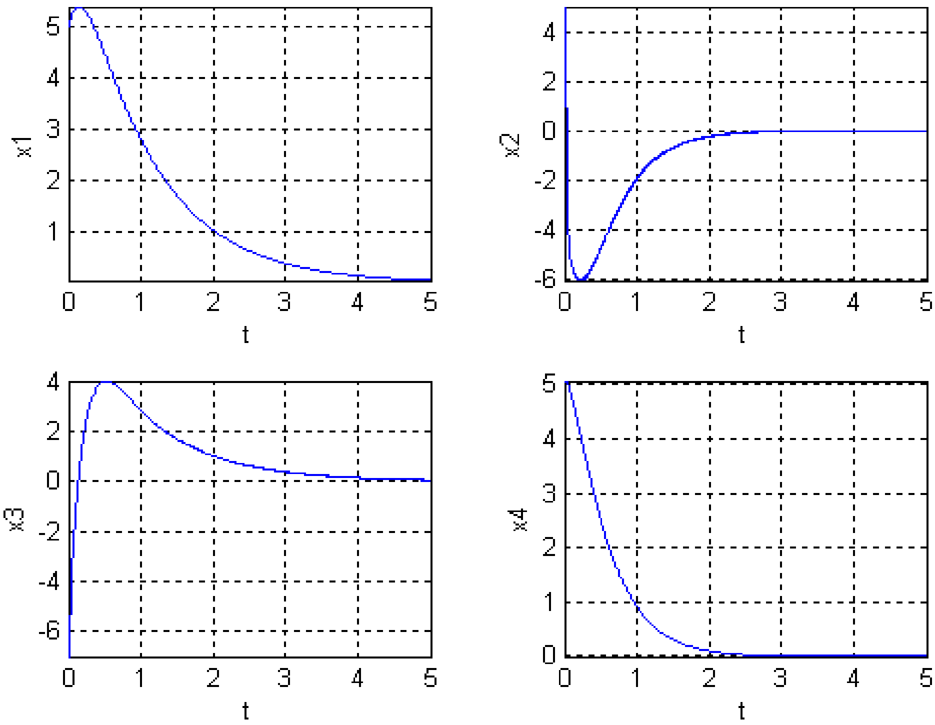

Figure 4.5 State variables x_1-x_4 for Case 1 of Example 1

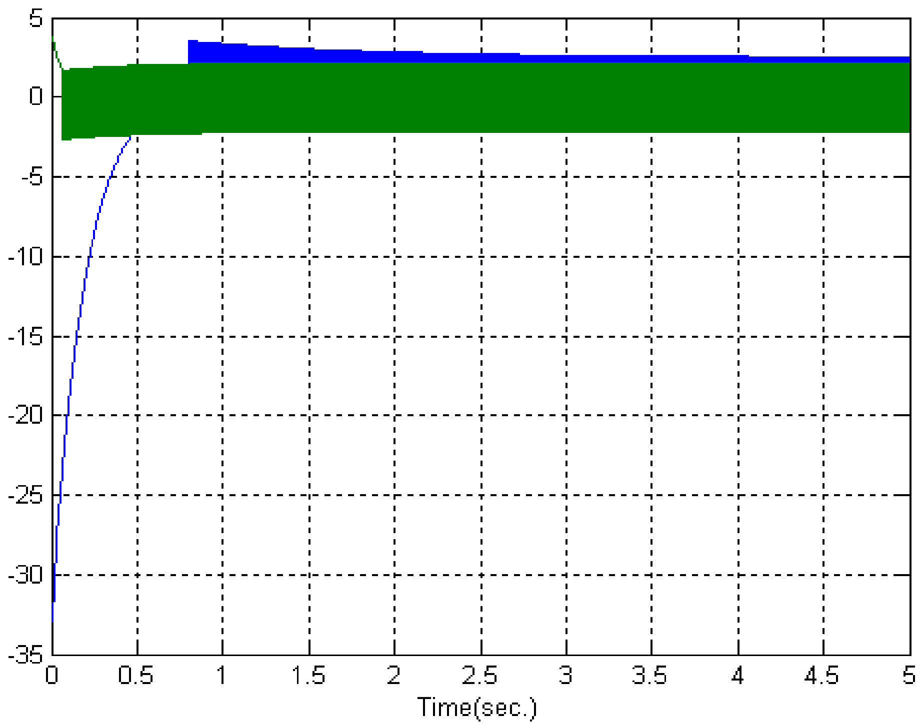

Figure 4.6 Control Inputs for Case 1 of Example 1

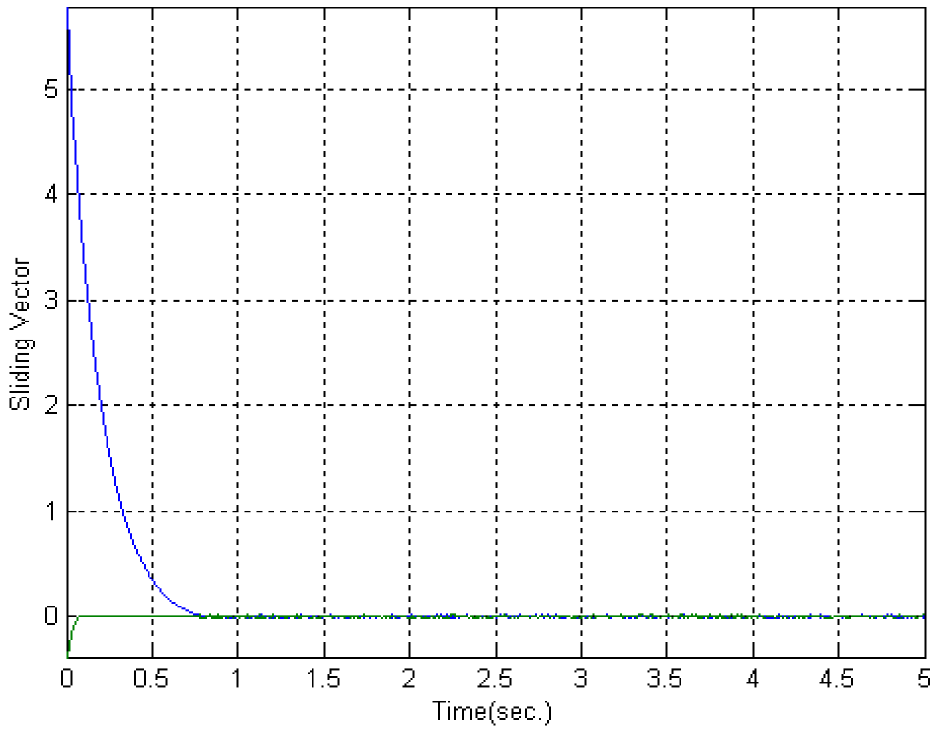

Figure 4.7 Time response of the sliding vector for Case 2 of Example 1

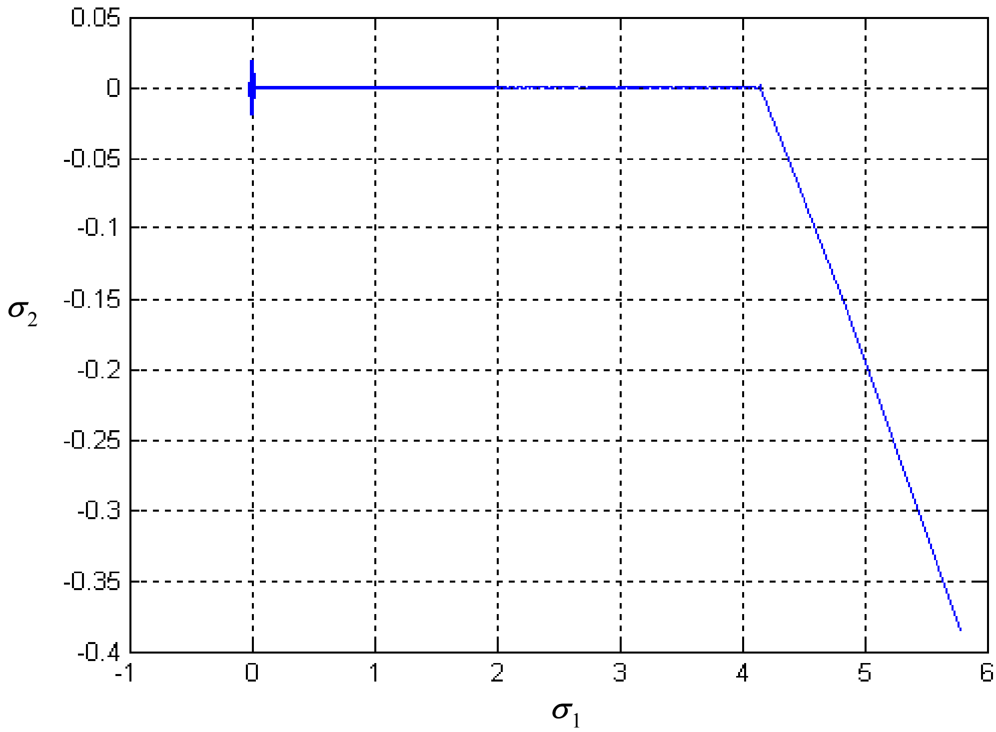

Figure 4.8 The trajectory of the sliding vector in the σ space for Case 2 of Example 1

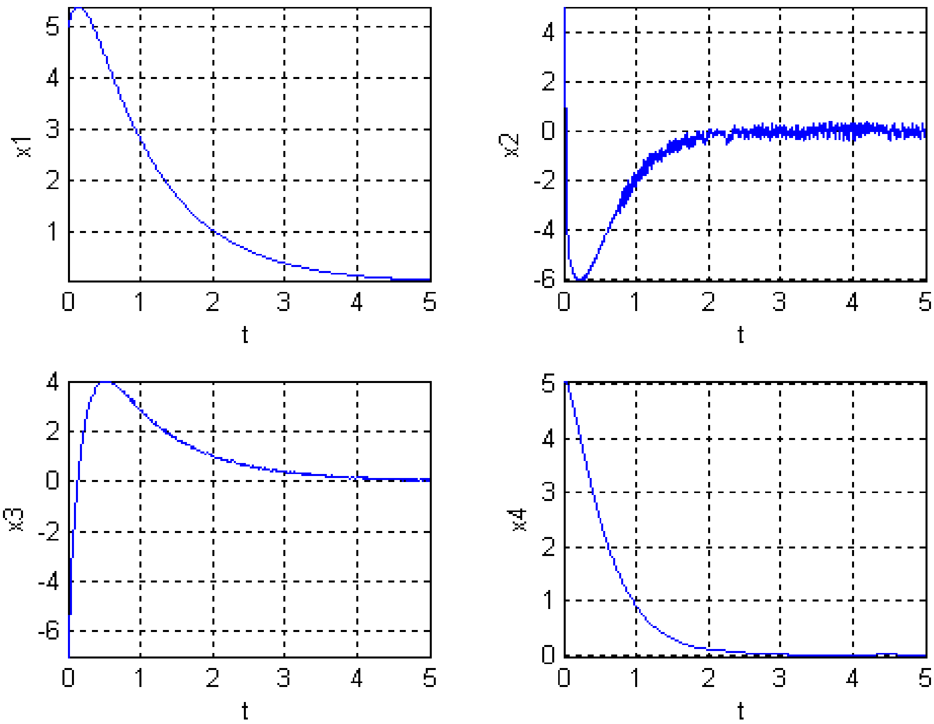

Figure 4.9 State variables x_1-x_4 for Case 2 of Example 1

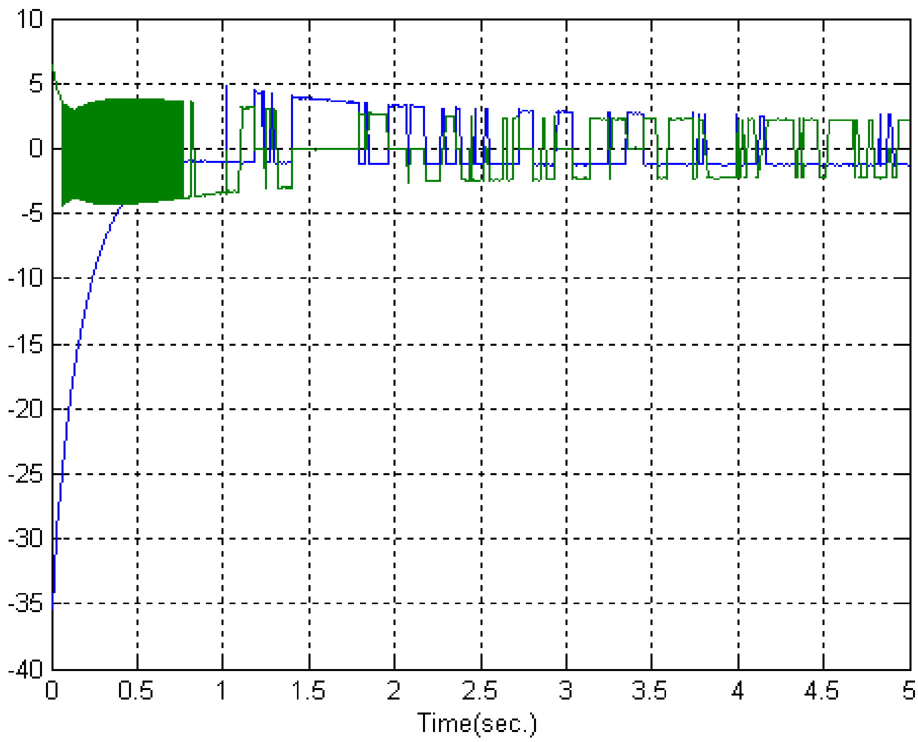

Figure 4.10 Control Inputs for Case 2 of Example 1

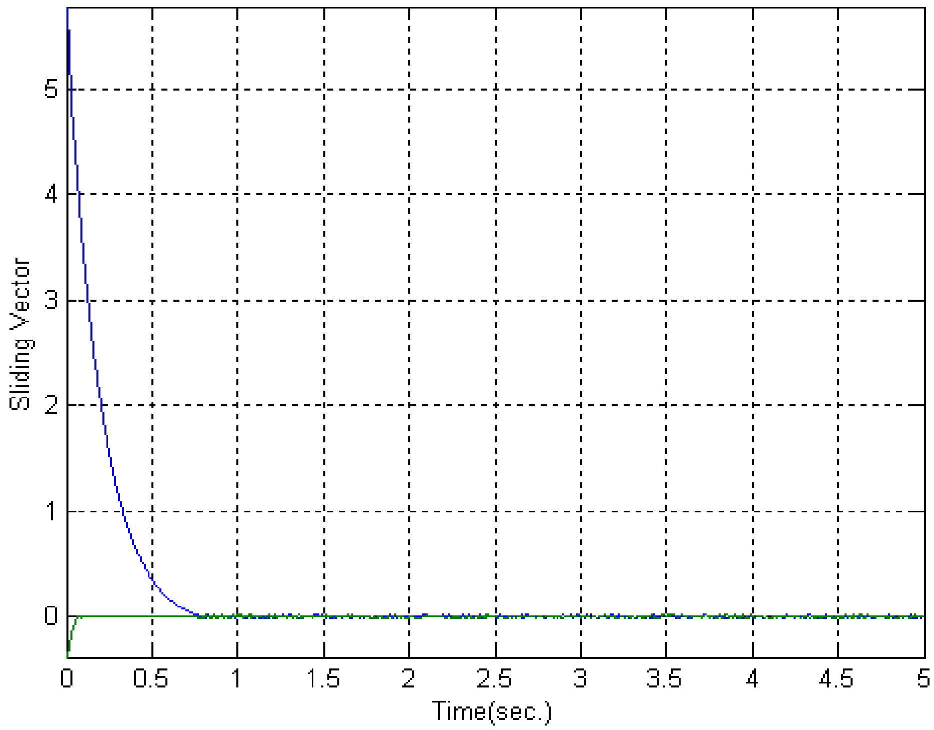

Figure 4.11 Time response of the sliding vector for Case 3 of Example 1

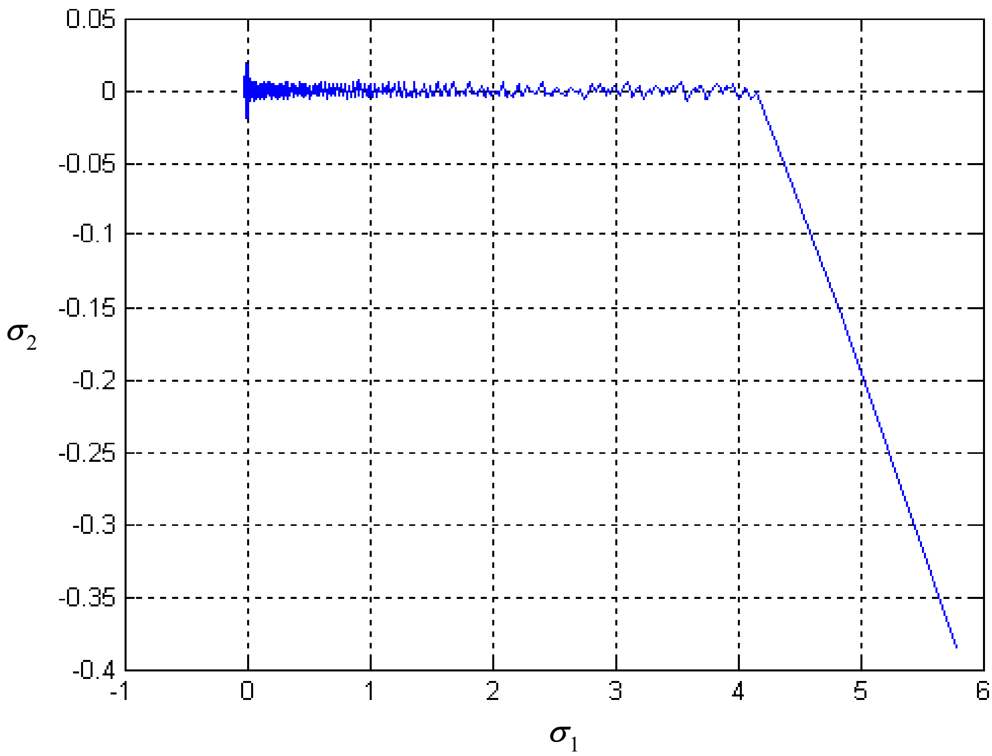

Figure 4.12 The trajectory of the sliding vector in the σ space for Case 3 of Example 1

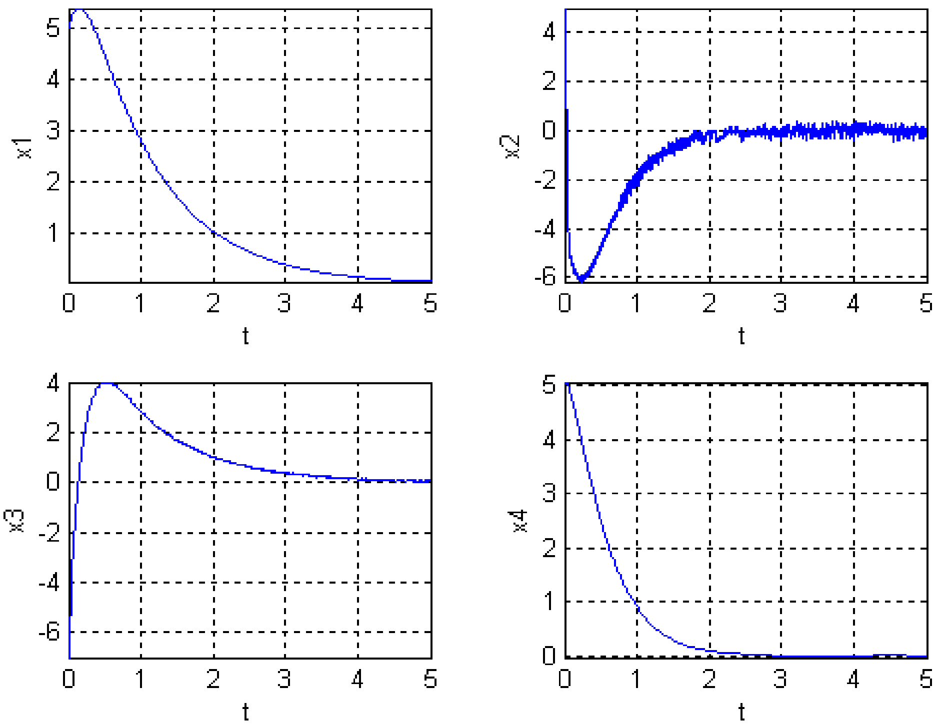

Figure 4.13 State variables x_1-x_4 for Case 3 of Example 1

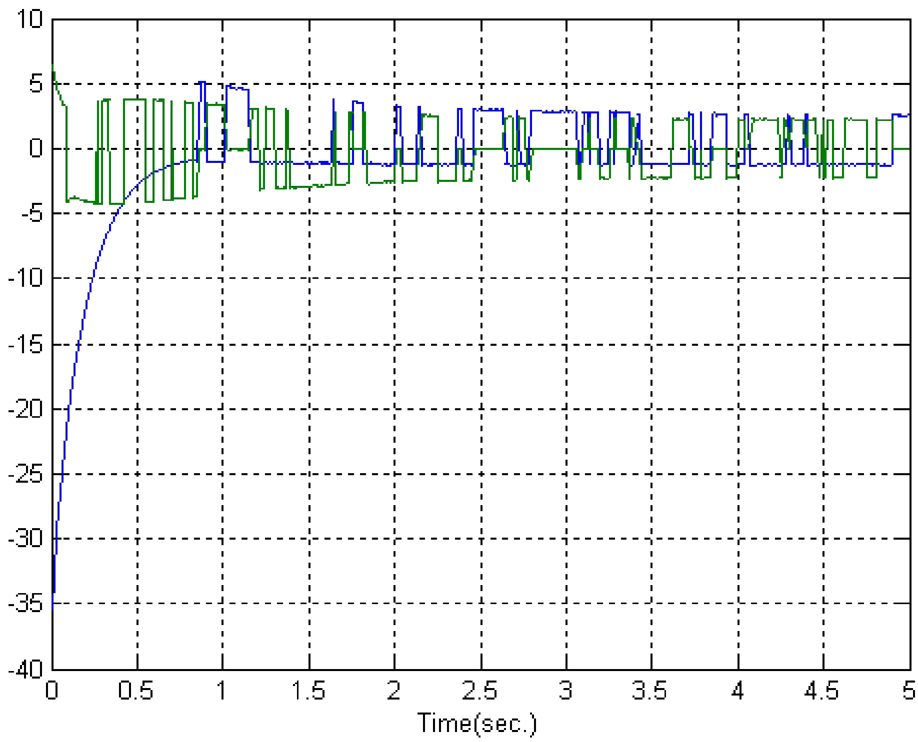

Figure 4.14 Control Inputs for Case 3 of Example 1

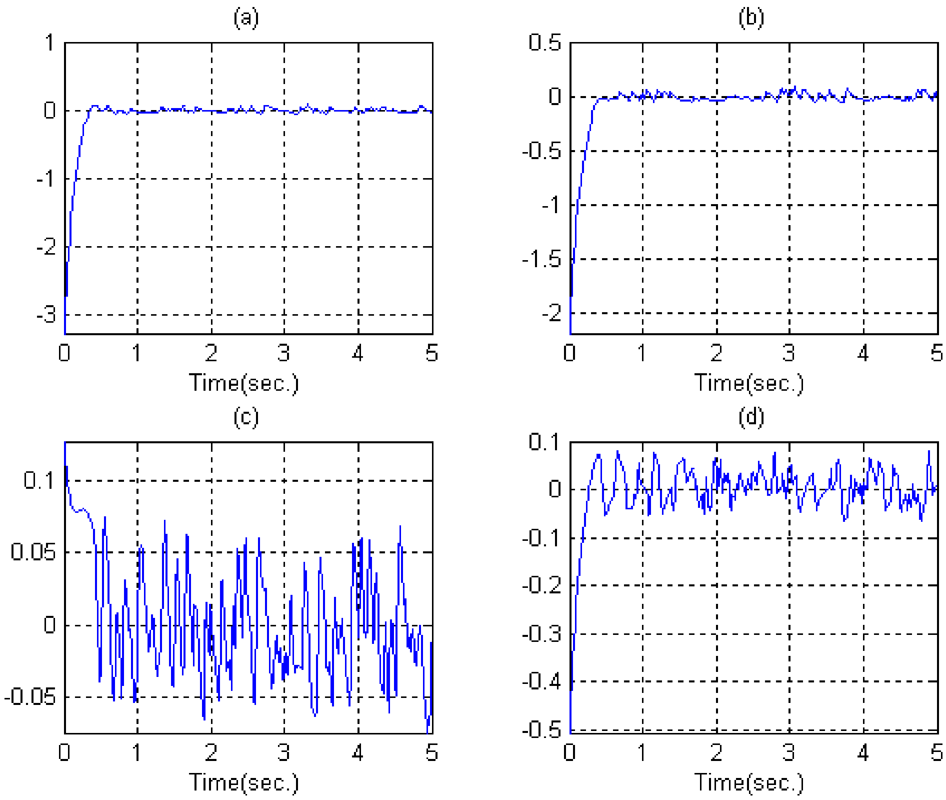

Figure 4.15 Time response of the sliding vector σ_1-σ_4 for Example 2

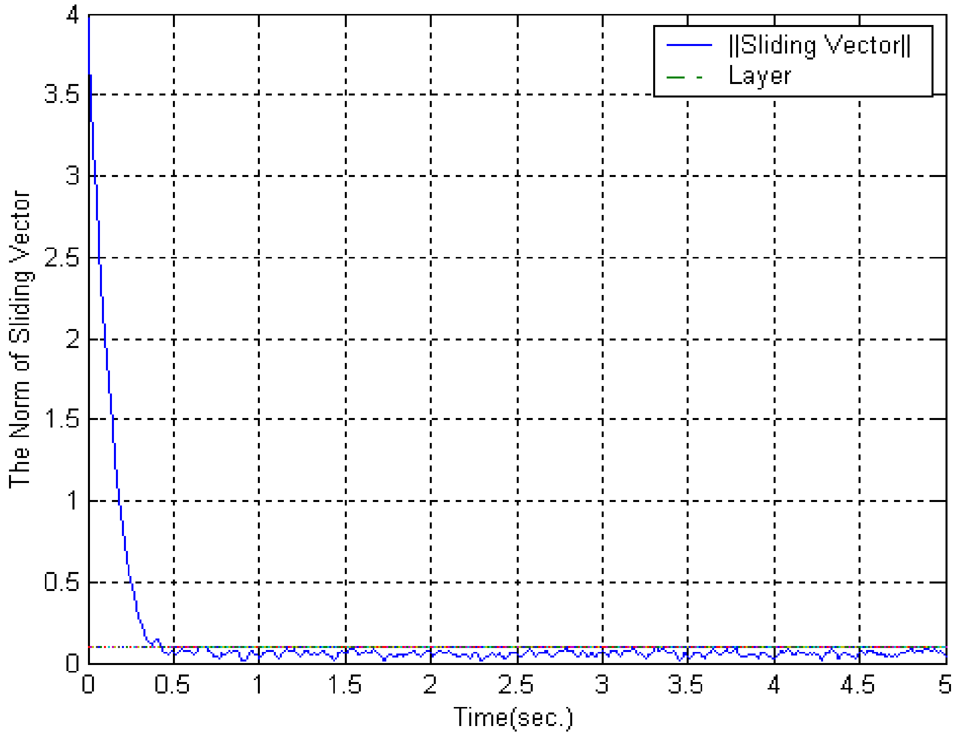

Figure 4.16 The norm of the sliding vector for Example 2

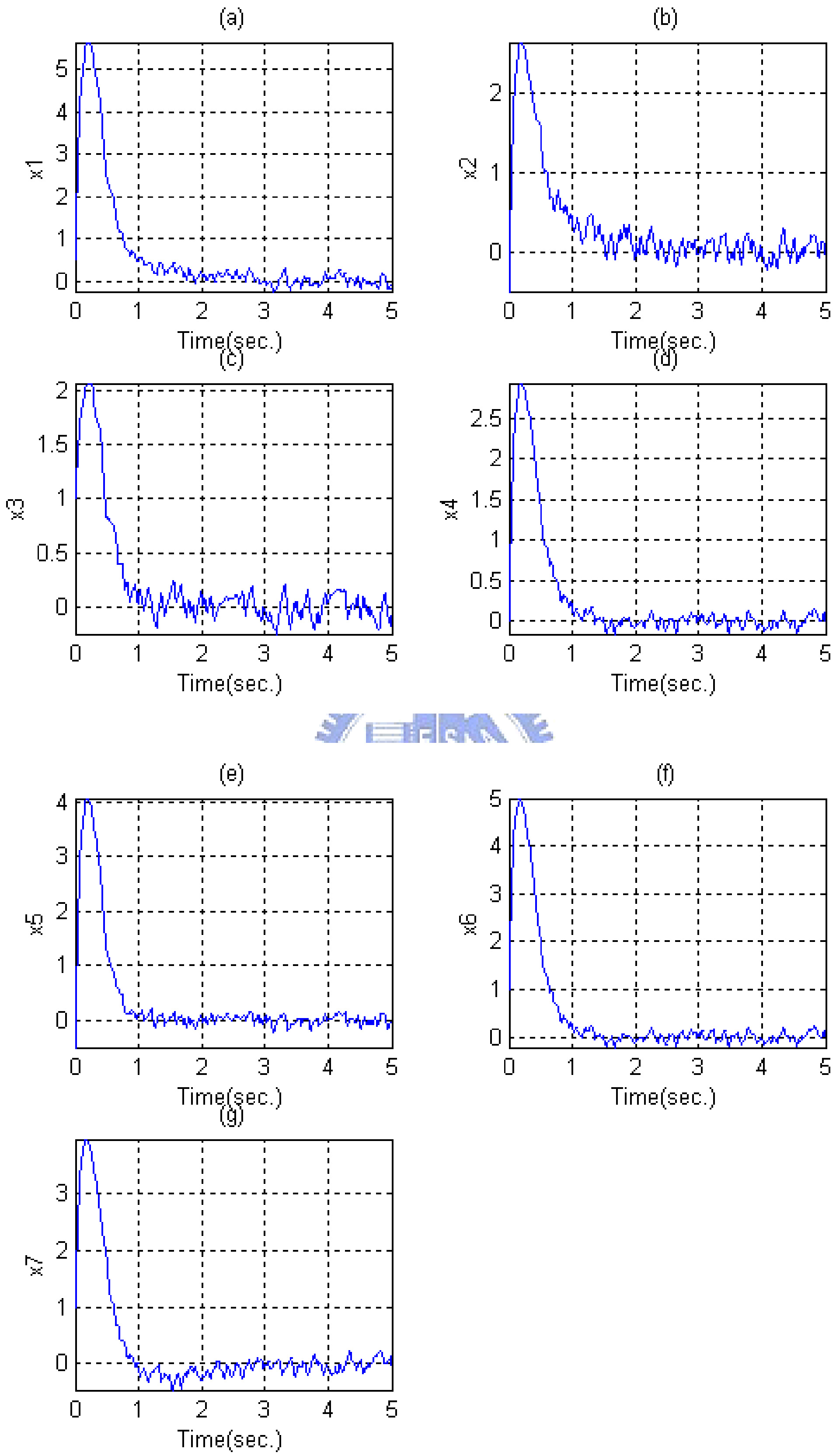

Figure 4.17 State variables x_1-x_7 for Example 2

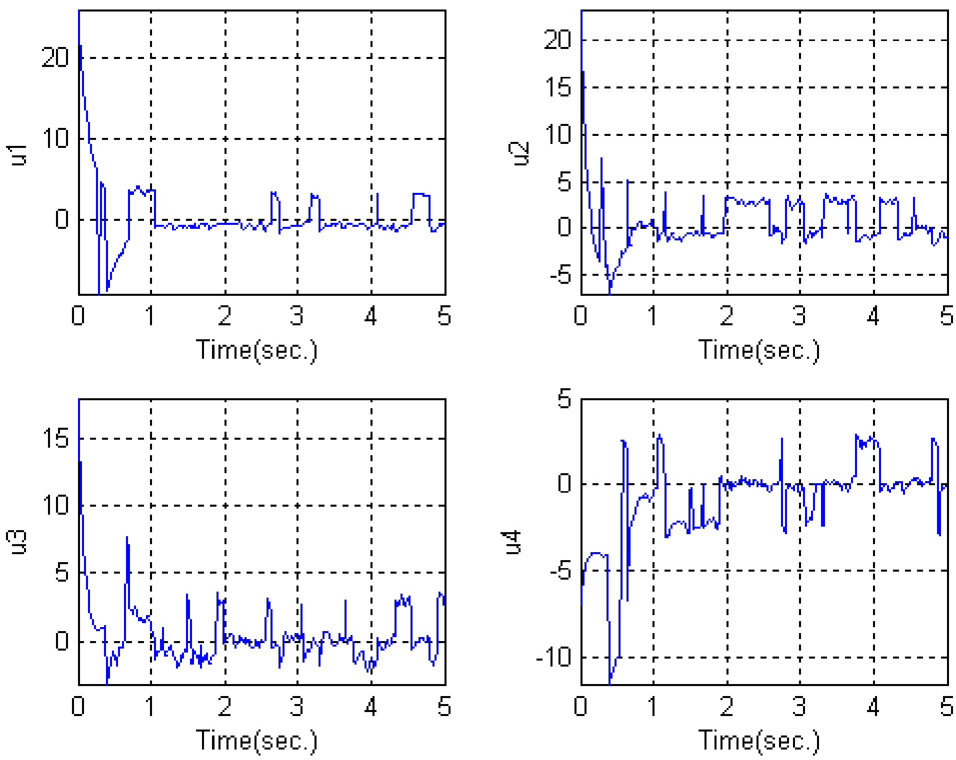

Figure 4.18 Control Inputs for Example 2