國 立 交 通 大 學

工業工程與管理學系碩士班

碩士論文

製程能力指標於供應商決策之應用

The Application for Supplier Selection Problem Based on Process Capability Indices

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製程能力指標於供應商決策之應用

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摘要

製程能力指標（Probability Capability Indices）是藉由一個指標的數值來衡量製程的能力與績效。在本篇論文中共應用了單邊指標 C_{pu} ，C_{pl} 以及雙邊指標 C_{pm} 分別來分辨兩家供應商製程能力，分別利用了兩個決策法則（1）Chou 在 1994 利用所提出，建立了三個單邊檢定來比較兩個相互競爭的供應商的製程能力（2）Huang and Lee 在 1995 年基於指標 C_{pm} 所提出的數學逼近的法則，主要功能在於由一群候選的供應商中選出一組包含有最佳供應商的集合，本研究並應用了上述的決策方法分別建立了一個實用的決策程序供使用者能用作於供應商決策時使用，由於我們無法直接地對兩個供應商作比較，我們必須分別從兩個供應商的產品進行抽樣，並使用統計分析來了解何者具有較佳的製程能力，即可決定是否要更換現有的供應商。為了證明本研究的可靠性，我們利用了模擬工具做了準確度分析，了解在欲達到的目標檢定力之下，所必須抽取的樣本數目為何。而兩階層的決策程序首先能選出較佳的供應商，再進一步的分別求出兩供應商製程能力的差距。最後，本研究應用了實際的例子，分別是：STN-LCD，TFT-LCD 及汽車玻璃三種產品製程的樣本，套用本研究的決策程序來做供應商的選擇。

The Application for Supplier Selection Problem Based on Process Capability Indices

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Abstract

Process capability indices (PCIs) have been used in the manufacturing industry to provide quantitative measures on process potential and performance. In this paper, we obtain the unilateral index C_{pu}, C_{pl} and bilateral index C_{pm} to distinguish which supplier has better process capability, so we apply the selection method proposed by Chou (1994) developed three one-sided tests for comparing two process capability indices to choose between competing processes. And based on C_{pm} index a mathematically complicated approximation method is developed by Huang and Lee (1995) for selecting a subset of processes containing the best supplier from a given set of processes. We implement this method, and develop a practical step-by-step procedure for practitioners to use in making supplier selection decisions. Since we can't compare these two suppliers directly, we have to sample some products made by these two suppliers, then use the statistical analysis to realize which one has better process capability. Then we decide whether switch the present supplier or not. To make our research realizable, we make an accuracy analysis by building tables to make users convenient to know the required sample size under an objective selection power. Accuracy of the selection method is investigated using simulation technique. The accuracy study provides useful information about the sample size required for designated selection power. A two- phase selection procedure is developed to select better supplier and further examine the magnitude of the difference between the two suppliers. Finally, we also investigate a real-world case on the STN-LCD (Super Twisted Nematic Liquid Crystal Display) ,TFT-LCD (Thin Film Transistor Liquid Crystal Display) and automobile window manufacturing process, and apply the selection procedure using actual data collected from the factories, to reach a decision in supplier selections.

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Notations

T ：target value

LSL ：the lower specification limits preset by the process engineers

USL ：the lower specification limits preset by the process engineers

π_i ： supplier i, for $i = 1, 2$

X_{ij} ：the measurements of samples independently drawn from supplier i, for $i = 1, 2$

n ：the number of the sample size drawn from supplier 1(for case C_{pu},C_{pl})

m ：the number of the sample size drawn from supplier 2(for case C_{pu},C_{pl})

n_i ：the number of the sample size drawn from supplier $i = 1, 2$ (for case C_{pm})

μ_i ：the population mean of supplier i, for $i = 1, 2$

σ_i^2 ：the population variation of supplier i, for $i = 1, 2$

$\bar{x}_i$ ：the sample mean calculated from data of supplier i, for $i = 1, 2$

S_i ：the sample variation calculated from data of supplier i, for $i = 1, 2$

Y_i^2 ：the MLE of σ_i^2

γ^2 ：the average loss of group

$\hat{\gamma}_i^2$ ：the unbiased estimator of the average loss of a group of supplier i, for $i = 1, 2$

$\gamma_{[i]}^2$ ：the ordered γ^2

$\hat{\gamma}_{[i]}^2$ ：the ordered $\hat{\gamma}_i^2$

$\pi_{(i)}$ ：the population associated with $\gamma_{[i]}^2$

$\tilde{C}_{pu}$ ：the UMVUE of C_{pu}

$\tilde{C}_{pl}$ ：the UMVUE of C_{pl}

$\hat{C}_{pm(CCS)}$ ：an estimator of C_{pm}

$\hat{C}_{pm(B)}$ ：the MLE of C_{pm}

C_{pu0} ：the minimal requirement of C_{pu} values for two candidate processes

C_{pl0} ：the minimal requirement of C_{pl} values for two candidate processes

C_{pm0} ：the minimal requirement of C_{pm} values for two candidate processes

δ ：the minimal difference of PCIs between these two suppliers

A ：the likelihood ratio test statistics

c ：the critical value

w ：the weight number used to decide the range of a subset including the best supplier

$p*$ ：the least probability of a correct selection, $0.5 < p* < 1$

q ：the notable magnitude of the difference between these two suppliers

1. Introduction

Process capability indices (PCIs), the purpose of which is to provide numerical measures of whether the ability of a manufacturing process meets a predetermined level of production tolerance or not, have received considerable research attention and increased usage in process assessments and purchasing decisions in the automotive industry during last decade.

In this paper, we obtain the unilateral index C_{pu}, C_{pl} and bilateral index C_{pm} to distinguish which supplier has better process capability. For this purpose, we apply the selection method proposed by Chou (1994) developing three one-sided tests to select between competing processes that which is more capable. Using the hypothesis test to find the larger C_{pu}, C_{pl}. And based on C_{pm} index, a mathematically complicated approximation method is developed by Huang and Lee (1995) for selecting a subset of processes containing the best supplier from a given set of processes. Under the circumstance, to search the larger C_{pm} which are used to provide unitless measure of the process performance is equivalent to look for the smaller γ^2.

2. Literature Review

2.1 Literature Review

Process capability indices have been used in the manufacturing industry to provide quantitative measures on process potential and performance. Including C_p, C_{pu}, C_{pl}, C_{pk}, C_{pm}, C_{pmk}, (see Kane (1986), Chan et al. (1988), Boyles (1991) and Pearn et al. (1992)). While C_p, C_{pk}, C_{pm} and C_{pmk} are appropriate measures for processes with two-sided specifications (which require both LSL and USL), C_{pu} and C_{pl} have been designed specifically for processes with one-sided specification limit (which require only LSL or USL). Those indices are effective tools for process capability analysis and quality assurance, and the formula for those indices are easy to understand and straightforward to apply. The C_p index was developed by Kane (1986), which considers the overall process variability relative to the manufacturing tolerance to measure process precision (product consistency). Due to simplicity of the design, C_p cannot reflect the tendency of process centering (targeting).

$$C_p = \frac{USL - LSL}{6\sigma}.$$

The index C_{pu} measures the capability of a smaller-the-better process with an upper specification limit USL, whereas the index C_{pl} measures the capability of a larger-the-better process with a lower specification limit LSL. Pearn and Chen (2001) develop a similar procedure using these one-sided capability indices C_{pu} and C_{pl} to test whether practitioners' processes meet the capability requirement. And set a convenient table display the critical value for various α-risk, sample sizes n and the desired quality condition. Further, Pearn and Lin provide the information of *p*-value required for making decisions.

When the process mean is off center of the specification, the index C_{pk} results in that one specification limit (the one closer to the process mean). And two calculations, C_{pu} and C_{pl}, have to be computed. In other words, the C_{pk} index is the minimum of C_{pu} and C_{pl}. The index C_{pk} defined as:

$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right\}$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean and σ is the process standard deviation, and T is the target value. The index C_{pk} was developed because C_p does not adequately deal with cases where process mean μ is not centered. However, C_{pk} alone still cannot provide adequate measure of process centering. That is, a large value of C_{pk} does not really tell us anything about the location of the mean in the tolerance interval. When a process is centered, C_p and C_{pk} will be the same number, therefore, C_{pk} is preferred because it's not dependent on the process being centered. The index C_{pk} takes the process mean into consideration but it can fail to distinguish between on-target processes from off-target processes (Pearn et al.(1992)). "Although the process capability indices C_p and C_{pk} are widely used to provide useful measures of process potential and performance. These indices don't adequately address the issue of process centering" (Boyles(1991)). In other words, they are not related to the cost of failing to meet customer desires.

To overcome this deficiency, several indices have been proposed that include the deviations from the target value when assessing the capability of a particular process. Hsiang and Taguchi (1985) considered am extension of C_p to address the issue directly. And it also named C_{pm} by Chan et al. (1988). Process capability index C_{pm} incorporates with the variation of production items with respect to the target value and the specification limits preset in the factory. The index C_{pm} is defined as:

$$C_{pm} = \frac{USL-LSL}{6\sqrt{\sigma^2+(\mu-T)^2}} = \frac{USL-LSL}{6\gamma}$$

we note $\gamma^2 = \sigma^2 + (\mu-T)^2 = E\left[(X-T)^2\right]$ to be the major part of the denominator of C_{pm}, which incorporates two variation components: (i) variation to the process mean and (ii) deviation of the process mean from the target. Since $E\left[(X-T)^2\right]$ was the expected loss where we have noted that the loss of a characteristic X missing the target is often assumed to be well approximated by the symmetric squared error loss function, $loss(X)=(X-T)^2$. Hence, the capability index C_{pm} is a loss-based index.

For on-target processes, the value of C_{pm} index reaches its maximum, implying that the process capability runs under the desired condition. On the other hand, the smaller value of C_{pm} means the higher expected loss and the poorer process capability. Therefore, the index C_{pm} is considered to be more sensitive than C_p and C_{pk} in reflecting the process loss.

Boyles (1991) has provided a definitive analysis of C_{pm} and its usefulness in measuring process centering. He notes that both C_{pk} and C_{pm} coincide with C_p when $\mu=T$ and decrease as μ moves away from T. However, $C_{pk}< 0$ for $\mu>USL$ or $\mu<LSL$, whereas C_{pm} of process with $|\mu-T|=\Delta>0$ is strictly bounded above by the C_p value of a process with $\sigma=\Delta$.

In the initial stage of production setting, the decision maker usually faces the problem of selecting the best manufacturing supplier from several available manufacturing suppliers. There are many factors, such as quality, cost, service and so on, which need to be considered in selecting the best suppliers. Several selection rules have been proposed for selecting the means or variances in analysis of variance (see Gibbons, Olkin and Sobel (1977), Gupta and Panchapakesan (1979), Gupta and Huang (1981) for more details). PCIs are useful management tools, particularly in the manufacturing industry, which provide common quantitative measures on manufacturing capability and production quality. In the situation of the manufacturing process being control, we assume that the quality characteristic X is normally distributed, USL and LSL are usually fixed and determined in advance, the larger C_p is equivalent to looking for the smallest σ^2. Tseng and Wu (1991) considered the problem of selecting the best manufacturing process from k available manufacturing processes based on the precision index C_p and a modified likelihood ratio (MLR) selection rule is proposed. Chou (1994) developed three one-sided tests (C_p, C_{PU}, C_{PL}) for comparing two process capability indices to choose between competing processes when the sample sizes are equal. Based on C_{pm} index, a mathematically complicated approximation method is developed by Huang and Lee (1995) for selecting a subset of processes containing the best supplier from a given set of processes.

Since we couldn't compare these two suppliers directly, we have to sample some products made by these two suppliers, then use the statistical analysis to realize which one has better process capability. Then we decide whether to switch the present supplier or not. To make our research realizable, we make an accuracy analysis by building tables to make users convenient to know the required sample size under an objective selection power. Accuracy of the selection method is investigated using simulation technique. The accuracy study provides useful information about the sample size required for designated selection power. A two- phase selection procedure is developed to select better supplier and further examine the magnitude of the difference between the two suppliers. Finally, we also investigate a real-world case on the STN-LCD (Super Twisted Nematic Liquid Crystal Display), TFT-LCD (Thin Film Transistor Liquid Crystal Display) and automobile window manufacturing process, and apply the selection procedure using actual data collected from the factories, to reach a decision in supplier selections.

2.2 Distribution of the PCIs

In this paper, we obtain the unilateral index C_{pu}, C_{pl} and bilateral index C_{pm} to distinguish which supplier has better process capability. The formula for these indices are easy to understand and straightforward to apply. In practice, sample data must be collected to calculate these indices. Therefore, a great degree of uncertainty may most practitioners simply look at the value of the estimators calculated from the sample data, then make a conclusion on whether their processes meet the preset capability requirement. This approach is highly unreliable since sampling errors are ignored. Thus, we then introduce the distributional properties of the estimated index C_{pu}, C_{pl}, C_{pm} and the unbiased estimator of loss function, $\hat{\gamma}^2$, is considered.

2.2.1 Distribution of Estimated C_{pu} and C_{pl}

C_{pu} and C_{pl} have been designed particularly for processes with one-sided specifications (which require only the upper or the lower specification limit). Chou and Owen (1989) considered the natural estimators of C_{pu} and C_{pl}, $\hat{C}_{pu}$ and $\hat{C}_{pl}$, which are defined as the following:

$$\hat{C}_{pu} = \frac{USL - \overline{X}}{3S}, \quad \hat{C}_{pl} = \frac{\overline{X} - LSL}{3S},$$

where $\overline{X} = \sum_{i=1}^{n} x_i / n$, $S^2 = \sum_{i=1}^{n} (x_i - \overline{x})^2 / (n-1)$, USL and LSL are the upper and the lower specification limits preset by the process engineers or product designers. Under normality assumption, Chou and Owen (1989) show that the estimator $\hat{C}_{pu}$ is distributed as $c_n t_{n-1}(\delta)$, where $c_n = (3\sqrt{n})^{-1}$, and $t_{n-1}(\delta)$ is a non-central t distribution with $(n-1)$ degrees of freedom and non-centrality parameter $\delta = 3\sqrt{n} C_{pu}$, the same distribution of $\hat{C}_{pl}$ (with $\delta = 3\sqrt{n} C_{pl}$). But both $\hat{C}_{pu}$ and $\hat{C}_{pl}$ are unbiased. Pearn and Chen (2001) added the correction factor $b_{n-1} = [2/(n-1)]^{1/2} \Gamma[(n-1)/2] / \Gamma[(n-2)/2]$ to correct the natural estimators of C_{pu} and C_{pl}, and obtain these unbiased estimators, $\tilde{C}_{pu}$ and $\tilde{C}_{pl}$, which are defined as the following:

$$\tilde{C}_{pu} = b_{n-1}\hat{C}_{pu} = \frac{b_{n-1}(USL - \bar{X})}{3S},$$

$$\tilde{C}_{pl} = b_{n-1}\hat{C}_{pl} = \frac{b_{n-1}(\bar{X} - LSL)}{3S},$$

then we have $E(\tilde{C}_{pu}) = C_{pu}$, and $E(\tilde{C}_{pl}) = C_{pl}$, since $b_{n-1} < 1$, then $Var(\tilde{C}_{pu}) < Var(\hat{C}_{pu})$ and $Var(\tilde{C}_{pl}) < Var(\hat{C}_{pl})$. Since both estimators depend only on the sufficient and complete statistics $(\bar{X}, S^2)$ of (μ, σ^2) and $\tilde{C}_{pu}$ and $\tilde{C}_{pl}$ are UMVUEs of C_{pu} and C_{pl}. The *r*-th moment and the variance of C_{pu} are as the following:

$$E\left[\tilde{C}_{pu}\right]^r = \frac{(\Gamma[(n-1)/2])^{r-1}\Gamma[(n-1-r)/2]}{(3\sqrt{n})^r(\Gamma[(n-2)/2])^r}E(Z)^r,$$

$$Var\left[\tilde{C}_{pu}\right] = \left\{\frac{\Gamma[(n-1)/2]\Gamma[(n-3)/2]}{(\Gamma[(n-2)/2])^2} - 1\right\}\left[C_{pu}\right]^2 + \frac{1}{9n}\frac{\Gamma[(n-1)/2]\Gamma[(n-3)/2]}{(\Gamma[(n-2)/2])^2}$$

where $Z = \sqrt{n}(USL - \bar{X})/\sigma$, it's easy to verify that $E(\tilde{C}_{pu}) = C_{pu}$. The results of the *r*-th moment, the expected value and the variance of estimator $\tilde{C}_{pl}$ are the same. And Pearn and Lin use the UMVUEs of C_{pu} and C_{pl} to calculate the critical values and the *p*-value for making decisions.

Further the PDF (probability density function) of $\tilde{C}_{pu}$ and $\tilde{C}_{pl}$ was be obtained as:

$$f(x) = \frac{3\sqrt{n/(n-1)}\cdot 2^{-n/2}}{b_{n-1}\sqrt{\pi}\Gamma[(n-1)/2]} \times \int_0^\infty y^{(n-2)/2}\exp\left\{-\frac{1}{2}\left[y + \left(\frac{3x\sqrt{ny}}{b_{n-1}\sqrt{n-1}} - \delta\right)^2\right]\right\}dy$$

where $b_{n-1} = [2/(n-1)]^{1/2}\Gamma[(n-1)/2]/\Gamma[(n-2)/2]$ and $\delta = 3\sqrt{n}C_{pu}$ (or $\delta = 3\sqrt{n}C_{pl}$).

2.2.2 Distribution of the Estimated C_{pm}

Since the process mean μ and the process variance σ^2 must be estimated from the sample. Thus, the estimated index $\hat{C}_{pm}$ is obtained by replacing μ and σ^2 by their estimators. Chan, Cheng, and Spring (1988) and Boyles (1991) proposed two different estimators of C_{pm} respectively defined as the following:

$$\hat{C}_{pm(CCS)} = \frac{d}{3\sqrt{s^2 + (\bar{x}-T)^2}} \quad \text{and} \quad \hat{C}_{pm(B)} = \frac{d}{3\sqrt{s_n^2 + (\bar{x}-T)^2}},$$

where $d = (USL - LSL)/2$ is the half width of the specification interval, $\bar{x} = \sum_{i=1}^{n} x_i / n$, $s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2/(n-1)$ and $s_n^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2 / n$. In fact, the two estimators, $\hat{C}_{pm(CCS)}$ and $\hat{C}_{pm(B)}$, are asymptotical equivalent. Assuming that the process data are normally distributed and $T = M$, Chan, Cheng, and Spring (1988) derived the probability density function of $\hat{C}_{pm(CCS)} = Y$ as

$$f_Y(y) = \frac{a}{2^{n/2-1} y^3} \exp\left[-\frac{1}{2}\left(\frac{a}{y^2} + \lambda\right)\right] \sum_{j=1}^{\infty} \frac{\lambda^j (a/y^2)^{n/2+j-1}}{j!\Gamma(n/2+j)2^{2j}}, \quad y > 0.$$

where $a = C_{pm}^2(1+\lambda/n)(n-1)$ and $\lambda = n(\mu - T)^2/\sigma^2$. Experts in statistical distributions will easily recognize that $\hat{C}_{pm(CCS)}$ can be shown to be functions of the inverse moments of a non-central chi-square distribution. An alternative equivalent formula was provided by Pearn, Kotz and Johnson (1992).

The distributional properties of $\hat{C}_{pm(CCS)}$ are intractable for asymmetrical specifications ($(USL + LSL)/2 \neq T$). When the case of $(USL + LSL)/2 = T$, $\hat{C}_{pm(CCS)}$ is a biased estimator of C_{pm}, but is asymptotically unbiased. Detailed descriptions and proofs of the properties of $\hat{C}_{pm(CCS)}$ are given in Chan, Cheng, and Spring (1988). On the other hand, Boyles (1991) considered that it would be more appropriate to replace the factor $n-1$ by n in the denominator since the term $\hat{\gamma}_{(B)} = s_n^2 + (\bar{x}-T)^2$ in the denominator of $\hat{C}_{pm(B)}$ is the uniformly minimum variance unbiased estimator (UMVUE) of the term $\sigma^2 + (\mu - T)^2$. We note that $\bar{x}$ and s_n^2 are the maximum likelihood estimators (MLEs) of μ and σ^2, respectively. Hence, the estimated $\hat{C}_{pm(B)}$ is also the MLE of C_{pm}.

The approach by simply looking at the calculated values of the estimated indices and then make a conclusion on whether the given process is capable, is highly unreliable as the sampling errors have been ignored. As the use of the capability indices grows more widespread, users are becoming educated and sensitive to the impact of the estimators and their sampling distributions on constructing confidence intervals and performing hypothesis testing. Under the assumption of normality, Kotz and Johnson (1993) obtained the r-th moment, and calculated the first two moments, the mean, and the variance of $\hat{C}_{pm}$. Cheng (1994) has developed a hypothesis testing procedure where tables of the approximate p-values were provided for some commonly used capability requirements, using the natural estimator of C_{pm}. The practitioners can use the obtained results to determining if their process satisfies the targeted quality condition. But Cheng's approach requires further estimation of the distribution characteristic $(\mu - T)/\sigma$ when calculating the p-values, which introduces additional sampling errors thus making the decisions made less reliable. Zimmer and Hubele (1997) provided tables of exact percentiles for the sampling distribution of the estimator $\hat{C}_{pm}$. Zimmer, Hubele and Zimmer (2001) proposed a graphical procedure to obtain exact confidence intervals for C_{pm}, where the parameter $(\mu - T)/\sigma$ is assumed to be a known constant. On the other hand, using the method similar to that presented in Vännman (1997), Pearn and Lin (2002) obtained an exact form of the cumulative distribution function of $\hat{C}_{pm}$. Under the assumption of normality, the cumulative distribution function of $\hat{C}_{pm}$ can be expressed in terms of a mixture of the chi-square distribution and the normal distribution:

$$F_{\hat{C}_{pm}}(x) = 1 - \int_0^{b\sqrt{n}/(3x)} G\left(\frac{b^2 n}{9x^2} - t^2\right)\left[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})\right]dt, \qquad (1)$$

C_{pl} $x > 0$, where $b = d/\sigma$, $\xi = (\mu - T)/\sigma$, $G(\cdot)$ is the cumulative distribution function of the chi-square distribution with degree of freedom $n-1$, χ^2_{n-1}, and $\phi(\cdot)$ is the probability density function of the standard normal distribution N(0, 1). It is noted that we would obtain an identical equation if we substitute ξ by $-\xi$ into equation (1) for fixed values of x and n.

3. Selection Method

3.1 Selection Method

Based on the distribution properties of the estimated PCIs C_{pu} and C_{pl}, Chou (1994) developed one-sided tests to select between competing processes that which is more capable. And Huang and Lee (1995) developed based on C_{pm} index a mathematically complicated approximation method for selecting a subset of processes containing the best supplier from a given set of processes.

3.1.1 Selection Method of C_{pu} and C_{pl}

Chou (1994) developed three one-sided tests for comparing two process capability indices (C_p, C_{pu}, C_{pl}) to choose between competing processes when the sample sizes are equal.

Based on the hypothesis testing comparing the two C_{pu} values, $H_0: C_{pu1} \geq C_{pu2}$ versus $H_1: C_{pu1} < C_{pu2}$. If the test rejects the null hypothesis $H_0: C_{pu1} \geq C_{pu2}$, then we have sufficient information to conclude that the new supplier II is better than the present supplier I, and we may switch to the new supplier II.

Let $X_{11}, X_{12}, \ldots, X_{1n}$ and $X_{21}, X_{22}, \ldots, X_{2m}$ be the measurements of two samples independently drawn from two suppliers π_i following the normal distributions $N(\mu_i,\sigma_i^2)$, for $i=1,2$. In practice, the number of the sample size n, m ($n=m$) should be decided first based on C_{pu0}, δ, and the preset power. Using those Tables7-14, the practitioners may perform the capability testing without having to run the computer programs. The sample mean and the sample standard deviation, $\bar{x}_i$ and S_i, are calculated from supplier i, for $i=1,2$. The estimator $\hat{C}_{pui}$ can be calculated.

Nevertheless, the estimator $\hat{C}_{pui}$ has distributions which are proportional to non-central t distribution. It is complicated to find the critical value of the test statistics to make a decision. Therefore Chou (1994) made a variable transformation that $O_{ij}=U-X_{ij}$ following the normal distributions $N(U-\mu_i,\sigma_i^2)$, for $i=1,2$. The sample mean and the sample standard deviation, $\overline{O}_i$ and S_i, are calculated from supplier i, for $i=1,2$. Then the test could be transferred to test $H_0: o_1/\sigma_1 \ge o_2/\sigma_2$ versus $H_1: o_1/\sigma_1 < o_2/\sigma_2$. And the equality test of two coefficients of variation (publish by Miller & Karson (1977)) could be used. By using the likelihood ratio test, the reject region was defined follows:

$\overline{O}_1/S_1 < \overline{O}_2/S_2$ and $A<c$

it is equivalent to

$\hat{C}_{pu1} < \hat{C}_{pu2}$ and $A<c$

Using the likelihood test, the test statistic A given as:

$$A=\left[\frac{2Y_1Y_2}{\left[(\overline{O}_1^2+2Y_1^2)^{1/2}(\overline{O}_2^2+2Y_2^2)^{1/2}-\overline{W}_1\overline{W}_2\right]}\right]^n$$

which is equivalent to

$$A=\left[\frac{2}{\left[(a\hat{C}_{pu1}^2+2)^{1/2}(a\hat{C}_{pu2}^2+2)^{1/2}-a\hat{C}_{pu1}\hat{C}_{pu2}\right]}\right]^n$$

where $Y_i^2=(n-1)S_i^2/n$, $a=9n/(n-1)$

Under the process measurements follow a normal distribution. $\hat{C}_{pu}$ has a pdf. proportional to a non-central t distribution. Since A is a function of $\hat{C}_{pu1}$ and $\hat{C}_{pu2}$, it's difficult to determine the distribution of A. Hence it's impossible to find c such that $\Pr\{A<c\,|\,H_0\}$ equal an appropriate value of α. Using this fact, we can show that $-2\ln A$ has an approximate chi-square distribution with one degree under H_0 is true. Then we can find the critical value of the test, c, as follows:

$$c \approx \exp\left[-\frac{\chi_1^2(1-2\alpha)}{2}\right], 0 < c < 1.$$

3.1.2 Selection Method of C_{pm}

Huang and Lee (1995) considered the supplier selection problem based on the index C_{pm}, and developed a rather complicated method for supplier selection applications. The method essentially compares the average loss of a group of candidate processes, and select a subset of these processes with small process loss γ^2, which with certain level of confidence containing the best process.

Due to the specification limits are usually fixed and determined in advance, searching the largest C_{pm} is equivalent to looking for the smallest γ^2. The selection rule of Huang and Lee (1995) is that retain the population i in the selected subset if and only if $\hat{\gamma}_i^2 \le w \times \min_{i \ne j}^{1 \le j \le k} \hat{\gamma}_j^2$, where the value of w is determined by a function of parameters, which can be determined by calculating from collected samples. And we note that choose the value of w is larger than 1 and choose the value as small as possible.

The method, however, provides no indication on how one could further proceed with selecting the best population among those chosen subset of populations. We investigate this method for cases with two candidate processes. Let π_i be the population with mean μ_i and variance σ_i^2, $i = 1,2$, and $X_{i1}, X_{i2}, \ldots, X_{in_i}$ are the independent random samples from π_i, $i = 1,2$. When the populations are ranked in terms of $\hat{\gamma}_i^2$, our interest is to select the better process with smaller value γ^2. We denote a correct selection as CS, and assume that the ordered γ^2 as $\gamma_{[1]}^2 < \gamma_{[2]}^2$.

Let us denote $\pi_{(i)}$ as the population associated with $\gamma_{[i]}^2$, $i = 1,2$. Then, the better population is $\pi_{(i)}$. We wish to define a procedure with selection rule R such that the probability of a correct selection is no less than a pre-assigned number $p*$ and $0.5 < p* < 1$. That is, $\Pr(\mathrm{CS} \mid R) \ge p*$. We refer to this requirement as the $p*$-condition. The selection rule R based on the unbiased and consistent estimators $\hat{\gamma}_i^2$ of γ_i^2, $i = 1,2$, and $\hat{\gamma}_i^2$ is defined as follows:

$$\hat{\gamma}_i^2 = \frac{\sum_{j=1}^{n_i} (x_{ij} - T)^2}{n_i} = \frac{(n_i - 1)S_i^2 + n_i(\bar{x}_i - T)^2}{n_i},$$

$$S_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2, \quad \bar{x}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} x_{ij},$$

For cases with two candidate processes, comparing $\hat{C}_{pm1}$ and $\hat{C}_{pm2}$ is equivalent to compare $\hat{\gamma}_1^2$ and $\hat{\gamma}_2^2$. Hence, by the result of Pearn, Kotz and Johnson (1992),

$$\hat{\gamma}_i^2 \sim \frac{\sigma_1^2}{n_i} \chi_{n_i}^2(\lambda_i), \quad \lambda_i = n_i \left(\frac{\mu_1 - T}{\sigma_1} \right)^2.$$

where $\chi^2_{n_i}(\lambda_i)$ is the non-central chi-squared distribution with degrees of freedom and non-centrality parameter λ_i.

Selection rule R: Consider the problem of selecting two populations with the smaller $\hat{\gamma}^2$. The selection rule R is that: Consider π_i as the better supplier if and only if $\hat{\gamma}_i^2 \le w \times \hat{\gamma}_j^2$ and $\hat{\gamma}_j^2 > w \times \hat{\gamma}_i^2$, $i = 1,2$ and $i \ne j$. For satisfying the p^*-condition, then

$$w_1 = \exp\left\{-2L_1\sqrt{\frac{1}{\hat{v}_{[1]}} + \left(\frac{1}{\hat{v}_{[1]}} - \frac{1}{\hat{v}_{[k]}}\right)\sqrt{\frac{\hat{v}_{[k]}}{\hat{v}_{[1]}}}}\right\}$$

and ,

$$w_2 = \exp\left\{-2L_2\sqrt{\frac{1}{\hat{v}_{[1]}} + \left(\frac{1}{\hat{v}_{[1]}} - \frac{1}{\hat{v}_{[k]}}\right)\sqrt{\frac{\hat{v}_{[k]}}{\hat{v}_{[1]}}}}\right\}$$

.

Choose the value of w which is larger than 1 and choose the value as small as possible, so

$w = \min\{w_1, w_2\}$, if $w_1 > 1$ and $w_2 > 1$

$w = w_1$, if $w_1 > 1$ and $w_2 \le 1$, $w = w_2$, if $w_2 > 1$ and $w_1 \le 1$;

where

$$L_1 = \frac{-d_2 + \sqrt{d_2^2 - 4d_1d_3}}{2d_1}, L_2 = \frac{-d_2 - \sqrt{d_2^2 - 4d_1d_3}}{2d_1},$$

$$d_1 = a\sum_{i=1}^{k-1}\left(1 + \frac{a_k}{a_1}\right) + \frac{a^2}{a^*}\left(\sum_{i=1}^{k-1}\frac{\sqrt{a_i + a_k}\sqrt{a_k}}{a_i}\right)^2,$$

$$d_2 = b\sum_{i=1}^{k-1}\sqrt{1 + \frac{a_k}{a_1}} + \frac{ab}{a^*}\left(\sum_{i=1}^{k-1}\frac{\sqrt{a_i + a_k}\sqrt{a_k}}{a_i}\right)\left(\sum_{i=1}^{k-1}\sqrt{\frac{a_k}{a_1}}\right),$$

,

$$d_3 = \frac{b^2}{4a^*}\left(\sum_{i=1}^{k-1}\sqrt{\frac{a_k}{a_i}}\right)^2 - \ln(p^* 2^{k-1}\sqrt{2a^*}),$$

$$a^* = 0.5 - a\sum_{i=1}^{k-1}\frac{a_k}{a_i},\ a_j = \frac{1}{\hat{v}_{[j]}},\ b = -0.513277\ ,\ a = -0.085514$$

$$\hat{v}_i = \frac{(n_i + \hat{\lambda}_i)^2}{(n_i + 2\hat{\lambda}_i)},\ \hat{\lambda}_i = n_i\left(\frac{\overline{x}_i - T}{S_i}\right)^2,$$

where $\hat{v}_i$ is used to estimate v_i, $i = 1,2$, and ordered $\hat{v}_i$ are denoted by $\hat{v}_{[1]} \le \hat{v}_{[2]}$.

3.2 Selection Procedure

Chou (1994) developed one-sided tests for comparing two process capability indices to select between competing processes. And based on C_{pm} index a mathematically complicated approximation method is developed by Huang and Lee (1995) for selecting a subset of processes containing the best supplier from a given set of processes. After probing into these selection method, we develop the practical step-by-step procedure for practitioners to use in making supplier selection decisions. The main steps in tests are developed as:

3.2.1 Selection Procedure of C_{pu} and C_{pl}

To make users do this selection work conveniently, we summarized a selection procedure based the selection method proposed by Chou (1994) using the process capability index C_{pu} and C_{pl}.

Step1. Determine the specification limits USL. Check the appropriate Table1-4 to find the corresponding n based on C_{pu0}, δ, and the preset power, where $n = m$. Then input the sample data of size n, m.

Step2. Calculate the sample mean $\bar{x}_i$, and sample standard deviation S_i, the test statistic $\hat{C}_{pui}$, $i=1,2$ and the value of a

$$\bar{x}_i = \frac{1}{n}\sum_{j}^{n} x_{ij}\ ,\quad S_i = \left[\frac{1}{n-1}\sum_{j}^{n}(x_{ij}-\bar{x}_i)^2\right]^{1/2}$$

$$\hat{C}_{pui} = \frac{USL-\bar{x}_i}{3S_i}\ ,\quad a = \frac{9n}{n-1}$$

Step3. Calculate the value of A and c.

$$A = \left[\frac{2}{(a\hat{C}_{pu1}^2+2)^{1/2}(a\hat{C}_{pu2}^2+2)^{1/2} - a\hat{C}_{pu1}^2\hat{C}_{pu2}^2}\right]^n,$$

$$c = \exp\left\{-\chi_1^2(1-2\alpha)/2\right\}$$

Step4. Use the decision rule to conclude which supplier is better:

If $\hat{C}_{pu1} < \hat{C}_{pu2}$ and $A < c$ then we conclude that that π_1 is better supplier. Otherwise, we conclude π_2 is better supplier.

3.2.2 Selection Procedure of C_{pm}

Huang and Lee (1995) developed the mathematically complicated approximation method for dealing the selected problem. To make this method practical for in-plant applications, the selection procedure may be summarized and expand in our form as follows:

Step 1: Input the original sample data of size n_i, $i=1,2$, set the specification limits USL, LSL, target value T, the probability p^*, and the constants $a=-0.085514$, $b=-0.513277$.

Step 2: Calculate the sample mean $\bar{x}_i$, sample standard deviation S_i, the value of $\hat{\gamma}_i^2$, $i=1,2$.

$$\hat{\gamma}_i^2 = \frac{\sum_{j=1}^{n_i}(x_{ij}-T)^2}{n_i} = \frac{(n_i-1)S_i^2 + n_i(\bar{x}_i - T)^2}{n_i},$$

$$S_i^2 = \frac{1}{n_i-1}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_i)^2, \quad \bar{x}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} x_{ij},$$

Step 3: Calculate the value of $\hat{\lambda}_i, \hat{v}_i$, $a_j (a_j = 1/\hat{v}_{[i]})$, and $a^*, i=1,2$

$$a^* = 0.5 - a\sum_{i=1}^{k-1}\frac{a_k}{a_i}, a_j = \frac{1}{\hat{v}_{[j]}}, b = -0.513277, a = -0.085514$$

$$\hat{v}_i = \frac{(n_i+\hat{\lambda}_i)^2}{(n_i+2\hat{\lambda}_i)}, \hat{\lambda}_i = n_i\left(\frac{\bar{x}_i - T}{S_i}\right)^2,$$

Step 4: Calculate d_1, d_2, d_3, and the value of L_1, L_2.

$$L_1 = \frac{-d_2 + \sqrt{d_2^2 - 4d_1d_3}}{2d_1}, L_2 = \frac{-d_2 - \sqrt{d_2^2 - 4d_1d_3}}{2d_1},$$

$$d_1 = a\sum_{i=1}^{k-1}\left(1+\frac{a_k}{a_1}\right) + \frac{a^2}{a^*}\left(\sum_{i=1}^{k-1}\frac{\sqrt{a_i + a_k}\sqrt{a_k}}{a_i}\right)^2,$$

$$d_2 = b\sum_{i=1}^{k-1}\sqrt{1+\frac{a_k}{a_1}} + \frac{ab}{a^*}\left(\sum_{i=1}^{k-1}\frac{\sqrt{a_i + a_k}\sqrt{a_k}}{a_i}\right)\left(\sum_{i=1}^{k-1}\sqrt{\frac{a_k}{a_1}}\right),$$

$$d_3 = \frac{b^2}{4a^*}\left(\sum_{i=1}^{k-1}\sqrt{\frac{a_k}{a_i}}\right)^2 - \ln(p^* 2^{k-1}\sqrt{2a^*}),$$

Step 5: Calculate the value of w

$w = \min\{w_1, w_2\}$, if $w_1 > 1$ and $w_2 > 1$

$w = w_1$, if $w_1 > 1$ and $w_2 \le 1$, $w = w_2$, if $w_2 > 1$ and $w_1 \le 1$;

$$w_1 = \exp\left\{-2L_1\sqrt{\frac{1}{\hat{v}_{[1]}}} + \left(\frac{1}{\hat{v}_{[1]}} - \frac{1}{\hat{v}_{[k]}}\right)\sqrt{\frac{\hat{v}_{[k]}}{\hat{v}_{[1]}}}\right\}$$

and .

$$w_2 = \exp\left\{-2L_2\sqrt{\frac{1}{\hat{v}_{[1]}}} + \left(\frac{1}{\hat{v}_{[1]}} - \frac{1}{\hat{v}_{[k]}}\right)\sqrt{\frac{\hat{v}_{[k]}}{\hat{v}_{[1]}}}\right\}$$

Step 6: Conclude which supplier is better using the following rule R:

If $\hat{\gamma}_1^2 \le w \times \hat{\gamma}_2^2$ and $\hat{\gamma}_2^2 > w \times \hat{\gamma}_2^2$ then we conclude that the process of π_1 is more capable.

If $\hat{\gamma}_2^2 \le w \times \hat{\gamma}_1^2$ and $\hat{\gamma}_1^2 > w \times \hat{\gamma}_2^2$ then we conclude that the process of π_2 is more capable.

If $\hat{\gamma}_1^2 \le w \times \hat{\gamma}_2^2$ and $\hat{\gamma}_2^2 \le w \times \hat{\gamma}_1^2$, we doesn't have enough information to make supplier selection.

4. Accuracy Analysis

In this case, we want to distinguish which supplier has better process capability by the index C_{pu} and C_{pl}, so we apply the selection method proposed by Chou (1994).

The use of loss functions in quality assurance settings has grown with the introduction of Taguchi's philosophy. The index C_{pm} incorporates with the variation of production items with respect to the target value and the specification limits preset in the factory. Huang and Lee (1995) proposed a mathematically complicated approximation method for selecting a subset of processes containing the best supplier from a given set of processes based on the index C_{pm}. The method essentially compares the average loss of a group of candidate processes, and select a subset of these processes with small process loss γ^2, which with certain level of confidence containing the best process.

Since we can not compare these two suppliers directly, we have to sample some products made by these two suppliers, then use the statistical analysis to realize which one has better process capability. Then we decide whether switch the present supplier or not. Before sampling, we have to decide how many sample sizes we should sample to achieve our objective power. And we use statistical simulation program, S-plus, to investigate the accuracy of the selection method. Then build up Table7-19 to make users convenient to know the required sample size under an objective selection power.

4.1 Selection Power Analysis for C_{pu} and C_{pl}

4.1.1 Sample size required for designated selection power

Replacing the supplier will cause huge affection (no matter it's visible or invisible). So the new supplier has to make sufficient information to prove that it is more capable. Otherwise we will not run risks of the disadvantage caused by wrong decision. We have to sample a required number of products made by these two suppliers to make a believable comparison under the designated selection power.

In order to satisfy the user's need to distinguish which supplier has better process capability, we have to set two factors first, (1) the minimum of C_{pu}, C_{pu0}. In a purchasing contract, a minimum value of the PCI is usually specified. Montgomery (2001) recommended the minimum quality requirements of C_{pu} and C_{pl} for processes runs under some designated capable conditions. In particular, 1.25 for existing processes, and 1.45 for new processes; 1.45 also for existing processes on safety, strength, or critical parameter, and 1.60 for new processes on safety, strength, or critical parameter. (2) the minimal difference of C_{pu} between these two suppliers, $\delta = C_{pu2} - C_{pu1}$, then we can know how many sample sizes we should sample with determined power by the selection method. If $\hat{C}_{pu1} < \hat{C}_{pu2}$ and $A < c$ then we conclude that the process capability of the new supplier better than that of the present supplier. By the way, it means that we have sufficient evidence to reject the null hypothesis $H_0 : C_{pu1} \geq C_{pu2}$, otherwise we can not believe that the new supplier has better process capability to replace the present supplier. For the accuracy of this selection method, we use simulation program, S-plus, with 20,000 numbers to establish Tables1-4 which present the required sample to distinguish which supplier has better process capability under power condition = 0.95, and minimum of C_{pu} = 1.00, 1.25, 1.45, 1.60, the minimal difference of C_{pu} between these two suppliers δ = 0.05(0.05)1.00 with power = 0.90, 0.95, 0.975, 0.99, the power here means the probability of rejecting the null hypothesis $H_0 : C_{pu1} \geq C_{pu2}$ when $C_{pu1} < C_{pu2}$ is true. And we make an example about the response time of LCD, the minimum of C_{pu} = 1.00 and the minimal difference of C_{pu} between these two suppliers, $\delta = 0.25$, the determined selection power = 0.95, then we can know we have to take 257 samples.

According to Table 7-14, which present the required sample to distinguish which supplier has better process capability under power condition = 0.95. And we find two phenomenon (1) Within fixed selection power, the larger the difference δ between two suppliers, the larger the required sample size. (2) With fixed δ and minimum of C_{pu}, the selection power increases, the required sample size increases. It's only because when we want this selection analysis more realizable, we most draw more products to avoid the variation of statistic estimation and risks of by wrong decision.

4.1.2 Phase I—Supplier Selection

Based on the hypothesis testing comparing the two C_{pu} values, $H_0: C_{pu1} \geq C_{pu2}$ versus $H_1: C_{pu1} < C_{pu2}$ If the test rejects the null hypothesis $H_0: C_{pu1} \geq C_{pu2}$, then we have sufficient information to conclude that the new supplier II is better than the present supplier I, and we may switch to a new supplier II (We want to avoid type I error happened. Since switching the supplier will cause a huge cost, over a span then we find it has been a great loss).

For the Phase I of Supplier Selection problem, the user should input the preset minimum requirement of C_{pu} values, and the minimal difference that must be differentiated between suppliers with designated selection power. The user may alternatively check Tables 7-14 for required sample size for selection power = 0.95, with designated selection power = 0.90, 0.95, 0.975, 0.99. In this case, we only need to compare the test statistic $\hat{C}_{pu1}$ and $\hat{C}_{pu2}$, and the selection value $A \& c$ based on the test statistic and the required sample sizes.

4.1.3 Phase II—Magnitude Outperformed Detection

Because replacing the supplier will cause a huge cost, we have to compare process capability indices C_{pu} of these two suppliers. Although the process capability of the new supplier is better than that of the present supplier, the difference between these two suppliers may be too small to be noticed. At this situation, we may not decide to replace the present supplier, unless we can prove that there is a notable magnitude of the difference between these two suppliers. This action of changing the supplier will be meaningful. So we further investigate the magnitude of the difference between these two suppliers in this stage.

Based on the selection method using the hypothesis test, we set a specified constant q, the notable magnitude of the difference between these two suppliers, and $q > 0$, to realize the value of q, we will test $H_0: C_{pu1} + q \geq C_{pu2}$ (the new supplier is not as capable as the present supplier with a magnitude, q) versus $H_1: C_{pu1} + q < C_{pu2}$ (the new supplier is more capable than the present supplier with a magnitude, q). By comparing these test statistics $\hat{C}_{pu1}$, $\hat{C}_{pu2}$, and the selection value $A \& c$ based on the test statistic and the required sample sizes. If the test apply to reject H_0 ($\hat{C}_{pu1} + q < \hat{C}_{pu2}$ and $A < c$), we can conclude that the new supplier is more capable than the present supplier at least a magnitude, q. In other words, We note that C_{pu2} must be greater than the preset capability requirement, and $C_{pu2} = C_{pu1} + q$, where q = max{ q' | test rejects

$C_{pu1} + q \geq C_{pu2}$ }. Then we decide to switch the present supplier to avoid waste such a huge exchanging cost.

4.2 Selection Power Analysis for C_{pm}

4.2.1 Sample size required for designated selection power

In practice, if a new supplier II wants to join competing the orders by claiming its capability better than the existing supplier I, then the new supplier II must furnish convincing information justifying the claim with prescribed level of confidence. Thus, the sample size required for designated selection power must be determined to collect actual data from the factories. The method, however, applies some approximating results and provides no indication on how one could further proceed with selecting the best population among those chosen subset of populations. We investigate this method for cases with two candidate processes.

If the minimum requirement of C_{pm} values for two candidate processes, C_{pm0}, and the minimal difference $\delta = C_{pm2} - C_{pm1}$ are determined then the sample size required need to sample such that the suppliers must be differentiated with designated selection power. Thus, based on the proposed selection procedures, if If $\hat{\gamma}_2^2 \leq w \times \hat{\gamma}_1^2$ and $\hat{\gamma}_1^2 > w \times \hat{\gamma}_2^2$ then we conclude that π_2 is better supplier. Otherwise, we would believe that the existing supplier I is better than the new supplier II since we don't have sufficient information to reject the null hypothesis. We investigate the selection method and accuracy analysis using simulation technique with simulated 10,000 numbers. For users' convenience in applying our procedure in practice, we tabulate the sample size required for various designated selection power = 0.90, 0.95, 0.975, 0.99. The selection power is calculating the probability of rejecting the null hypothesis $H_0 : C_{pm1} \geq C_{pm2}$, while actually $C_{pm1} \geq C_{pm2}$ is true, using simulation technique. Tables 1-4 summarize the sample size required for various capability requirements C_{pm} = 1.00, 1.33, 1.50, 1.67 and the difference δ = 0.05(0.05)1.00 under the $p*$-condition = 0.95, respectively. For example, if the capability requirement of suppliers C_{pm} is set to 1.00 and δ = 0.30, we would suggest to collect 151 samples to satisfy the designated selection power = 0.95.

We note that the sample size required is a function of C_{pm}, the difference δ between two suppliers and the designated selection power. From these tables, it can be seen that the larger the value of the difference δ between two suppliers, the smaller the sample size required for fixed selection power. For fixed δ and C_{pm}, the sample size required increases as designated selection power increases. This phenomenon can be explained easily, since the smaller of the difference and the larger designated selection power, the more collected sample is required to account for the smaller uncertainty in the estimation.

4.2.2 Phase I—Supplier Selection

In most applications, the supplier selection decisions would be solely based on the hypothesis testing comparing the two C_{pm} values, $H_0 : C_{pm1} \geq C_{pm2}$ versus $H_1 : C_{pm1} < C_{pm2}$. If the test rejects the null hypothesis $H_0 : C_{pm1} \geq C_{pm2}$, then one has sufficient information to conclude that the new supplier II is superior to the original supplier I, and the decision of the replacement would be suggested.

For the Phase I of Supplier Selection problem, the practitioner should input the preset minimum requirement of C_{pm} values, and the minimal difference that must be differentiated between suppliers with designated selection power. The practitioner may alternatively check Tables 1-4 for sample size required for $p*$-condition = 0.95, with designated selection power = 0.90, 0.95, 0.975, 0.99. In this case one only need to compare the test statistic $\hat{\gamma}_i^2$, $i = 1.2$, with the selection value w based on the selection procedure corresponding to the preset capability requirement and the required sample sizes.

4.2.3 Phase II—Magnitude Outperformed Detection

In Phase I of supplier selection problem, the supplier selection decisions would be solely based on the hypothesis testing comparing the two C_{pm} values without further investigating the magnitude of the difference between the two suppliers.

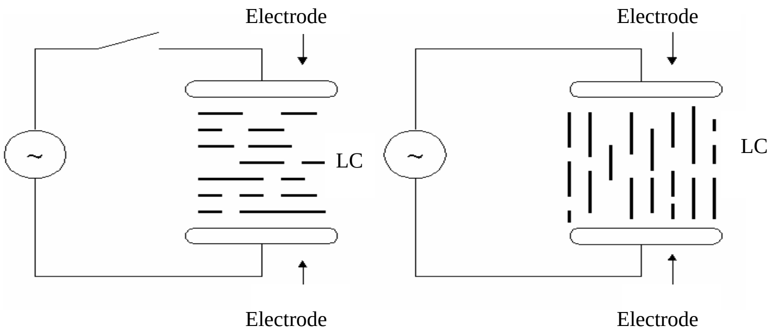
In other applications, the supplier selection decisions would be based on the hypothesis testing comparing the two C_{pm} values, $H_0 : C_{pm1} + q \geq C_{pm2}$, versus $H_2 : C_{pm1} + q < C_{pm2}$, where $q > 0$ is a specified constant. If the test rejects the null hypothesis $H_0 : C_{pm1} + q \geq C_{pm2}$ then one has sufficient information to conclude that supplier II is significantly better than supplier I by a magnitude of q, and the replacement would then be made due to expensive cost for the supplier replacement. In this case one would have to compare the test statistic $\hat{\gamma}_i^2$, $i = 1,2$, with the selection value w corresponding to the preset capability requirement for given sample and designated selection power, to ensure that the magnitude of the difference between the two suppliers exceeds q. We note that C_{pm1} must be greater than the preset capability requirement, and $C_{pm2} = C_{pm1} + q$, where q = $\max\{q' \mid \text{test rejects } C_{pm1} + q' \geq C_{pm2}\}$. The basic problem is checking whether or not the two suppliers meeting the preset capability requirement could be done by finding the lower confidence bounds on their process capabilities.

5. Example

5.1 Application Example TFT-LCD

LCD (liquid crystal display) is the technology used for displays in notebook and other smaller computers. Like light-emitting diode (LED) and gas-plasma technologies, LCDs allow displays to be thinner than cathode ray tube (CRT) technology. LCDs consume much less power than LED and gas-display displays because they work on the principle of blocking light rather than emitting it.

To achieve the color on a pixel in an LCD panel, a current is applied to the crystals at that pixel to change the state of the crystals. Response times refer to the amount of time it takes for the crystals in the panel to move from an on to off state. A rising response time refers to the amount of time it takes to turn on the crystals and the falling time is the amount of time it takes for the crystals to move from an on to off state. Rising times tend to be very fast on LCDs, but the falling time tends to be much slower. This tends to cause a slight ghosting effect on bright moving images on black backgrounds. Simply to say, it's the time takes for pixels to come up (become lit) and come down (become dark). The lower the response time, the less of a ghosting effect there will be on the screen. The electronic field effect of the liquid crystal is displayed in Figure 1, when the electronic field which between the electrode started to driving, it will attract to the electronic field works to make the liquid crystal turn its direction. And the optics effects will be produced. The picture tube theorem is showed in the Figure 2, An LCD is made with either a passive matrix or an active matrix display grid. The active matrix LCD is also known as a thin film transistor (TFT) display. The passive matrix LCD has a grid of conductors with pixels located at each intersection in the grid. A current is sent across two conductors on the grid to control the light for any pixel. An active matrix has a transistor located at each pixel intersection, requiring less current to control the luminance of a pixel.The current in an active matrix display can be switched on and off more frequently, improving the screen refresh time.



Turn off the electronic field Turn on the electronic field

Figure 1. The electronic field effect of the liquid crystal

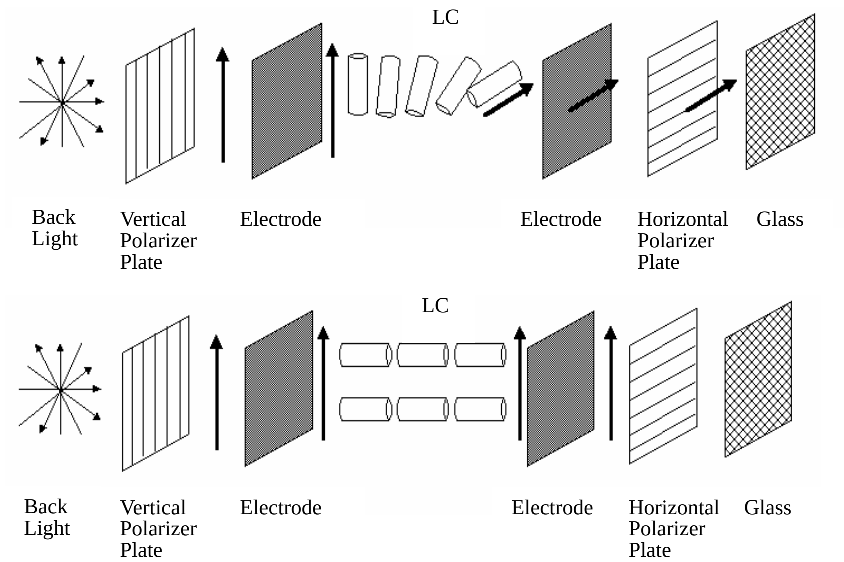


Figure 2. The picture tube theorem

To illustrate which has better process capability between the two suppliers, we presented a case study on TFT-LCD manufacturing processes, which located on the Science-Based Industrial Park in Taiwan. These factories manufacture carious types of the LCD. For a particular model of the TFT-LCD investigated, the upper specification limit, USL of the response time is set to 20ms (ms , milliseconds) . If the characteristic data does not fall under the USL , the performance of the TFT-LCD will be discounted. We will use the software "LaCie calibration probe" to do the variable set of the LCD, then calculating the time takes for pixels to come up and come down.

5.1.1 Data Analysis and Supplier Selection

Before doing the data analysis, we set two factors first, (1) the minimum of C_{pu} (2) the minimal difference of C_{pu} between these two suppliers, $\delta = C_{pu2} - C_{pu1}$, then we can know how many sample sizes we should sample with determined power by the selection method. In this example, we set the minimum of C_{pu} =1.00 and the minimal difference of C_{pu} between these two suppliers, $\delta = 0.25$, the determined selection power = 0.95, then we can know we have to take 257 samples by checking Table 1. Then we present the data drew from these two suppliers in Table. In order to affirm these data as normal distributed, we show the distribution of these data in Figure 3-4. And we set these data to be a histogram in Figure 5-6.

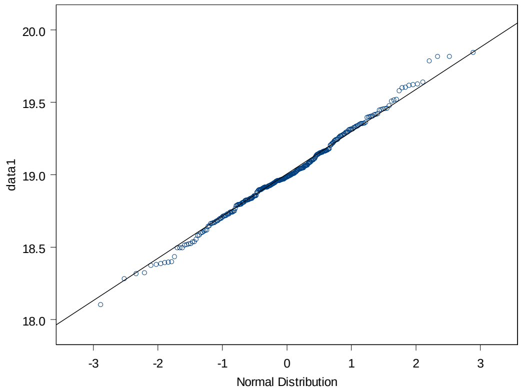


Figure 3. Normal probability plot for response time data of Supplier I.

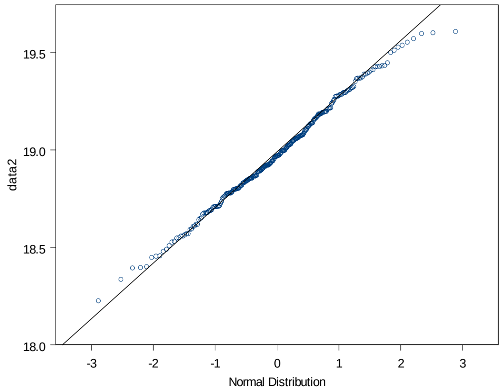


Figure 4. Normal probability plot for response time data of Supplier II.

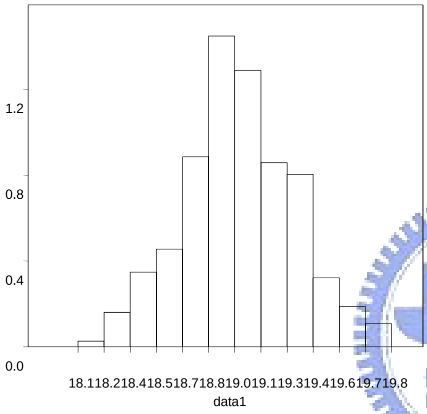


Figure 5. Histogram for supplier I.

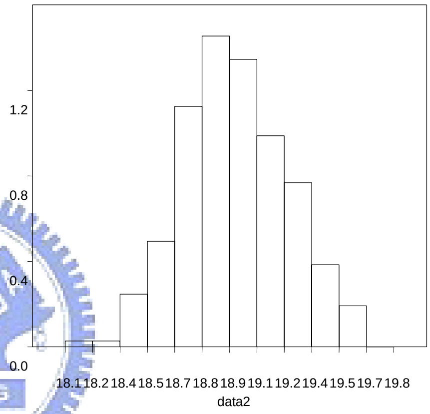


Figure 6. Histogram for supplier II.

5.1.2 Phase I—Supplier Selection

We will test $H_0 : C_{pu1} \geq C_{pu2}$ versus $H_1 : C_{pu1} < C_{pu2}$ by comparing these test statistics $\hat{C}_{pu1}$, $\hat{C}_{pu2}$, and the selection value A & c based on the test statistic and the required sample sizes. If $\hat{C}_{pu1} < \hat{C}_{pu2}$ and $A < c$ then we conclude that the process capability of the new supplier better than that of the present supplier. The calculated sample statistics for two suppliers are summarized in Table1.

Table 1. The calculated sample statistics for two suppliers.(C_{pu})

Population	$\bar{X}$	S	$\hat{C}_{pu}$
I	19.00094	0.3072499	1.083872
II	18.97955	0.2724119	1.248655

Based on the selection method, the values $\hat{C}_{pu1} = 1.083872$ and $\hat{C}_{pu2} = 1.248655$. In this case one only need to compare the test statistic $\hat{C}_{pu1}$ and $\hat{C}_{pu2}$, by $A = 0.1102599$ and $c = 0.2585227$, the outcome presents $\hat{C}_{pu1} < \hat{C}_{pu2}$ and $A < c$, then we conclude that the process of this new supplier is capable.

5.1.3 Phase II—Magnitude Outperformed Detection

To realize the lower bound value of the magnitude, *h*, we will test $H_0 : C_{pu1} + h \geq C_{pu2}$ versus $H_1 : C_{pu1} + h < C_{pu2}$. By comparing these test statistics $\hat{C}_{pu1}$, $\hat{C}_{pu2}$, and the selection value $A \& c$ based on the test statistic and the required sample sizes. From the estimation of Phase I, we list the obtained selection values A and c and the decision based on the selection procedure for h = 0.01, 0.03(0.001)0.035 in Table 2.

Therefore, from the analysis of magnitude outperformed detection based on sample statistics, the magnitude of the difference between the two suppliers is *h* = 0.034. By the way, we can conclude that the new supplier is more capable than the present supplier at least a magnitude, *h*=0.034.

Table 2. Magnitude outperformed detection of selection procedure. (C_{pu})

$\hat{C}_{pu1}$	1.093872	1.113872	1.116872	1.117872	1.118872
$\hat{C}_{pu2}$	1.248655	1.248655	1.248655	1.248655	1.248655
h	0.01	0.03	0.033	0.034	0.035
A	0.1449597	0.2361393	0.2523842	0.2579458	0.2635801
c	0.2585227	0.2585227	0.2585227	0.2585227	0.2585227
Decision	Reject Ho	Reject Ho	Reject Ho	Reject Ho	Don't Reject Ho

5.2 Application Example Automobile Windows

Up to now, the number of registered vehicle (including the intercity bus, truck, car and wagon) has tended to 18,215,069. Thus, with the growing number of vehicle, there is a need for automobile windows. For the safety, the automobile window always be the sandwich glass. The sandwich glass inserted with the special membrane (PVB film) between two pieces of tempered glass was dealt with by the high pressure of high temperature. The structure of the sandwich was displayed in Figure 7. After the glass is broken, chip can still be glued together, it is a kind of safe type glass. The sandwich glass can absorb the ultraviolet ray in the sunlight effectively; protect the personal safety in maximum.

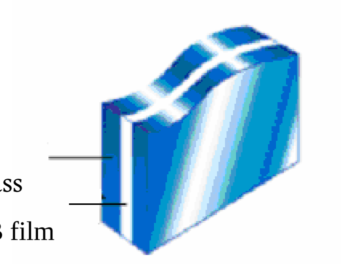


Figure 7. The structure of the sandwich glass.

Tempered glass is commonly used with various applications in our real life. Especially, it is used in automobile's side windows, front windows (displayed in Figure 8, 9). There are some characteristic of the tempered glass: (1) the strength against still-mode impact resistance is three to five times over that of regular glass. (2) Resilient to sudden temperature drop with its heat endurance much superior than common glass. (3) When broken, its fragments differ from usual pointy shards but rather in curd configuration, which greatly reduces the impact of cuts. Based on these characteristics of the tempered glass, we have to temper the glass to avoid the dangers coming with the broken glass in some special occasions, like the automobile window, the microwave oven and so on. It is a high-impact glass with its broken fragments in curds featuring an optimal performance in safety.

Figure 8. Automobile's front window

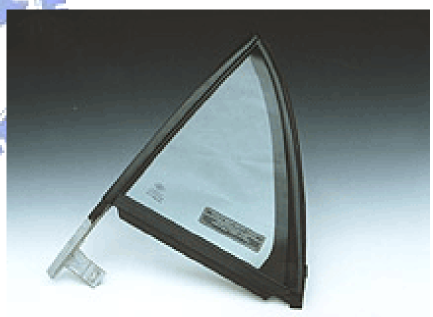


Figure 9. Automobile's side window

Tempered glass is derived by heating the raw glass sheeting to a temperature of near-melting point, with an evenly distributed cool air for rapid cooling to form a surface hardening process in order to overcome physical expandability found in glass. The outer surface is quickly cooled for a reinforced characteristic, which is known as tempered glass. In order to keep the high optical quality, we have to ask the thickness of the tempered glass at least 0.5mm. , no any distortion, wave and other defects on the surface due to it's treatment temperature lower then the thermo tempered glass, so easy to laminating fabrication. Too thin tempered glass will result in danger when it broken (more break pattern) and increasing the difficult when it be processed and can't suffer the outside force impact

To illustrate which has better capability between the two suppliers, we present a case study on the automobile window manufacturing process, which located on the Tafa industrial region in Taiwan. These factories manufacture various types of the tempered glass. For the particular model of the automobile window investigated, the lower specification limit, LSL of an automobile side window's thickness is set to be 0.5mm. And we use thickness gauge to inspect the inspection for thickness. If the characteristic data does not fall over the tolerance LSL, the safety of the automobile window will be discounted.

5.2.1 Data Analysis and Supplier Selection

Before doing the data analysis, we set two factors first, (1) the minimum of C_{pl} (2) the minimal difference of C_{pl} between these two suppliers, $\delta = C_{pl2} - C_{pl1}$, then we can know how many sample sizes we should sample with determined power by the selection method. In this example, we set the minimum of C_{pl} =1.00 and the minimal difference of C_{pl} between these two suppliers, $\delta = 0.25$, the determined selection power = 0.95, then we can know we have to take 257 samples by checking Table 1. Then we present the data drew from these two suppliers in Table. In order to affirm these data as normal distributed, we show the distribution of these data in Figure 10-11. And we set these data to be a histogram in Figure 12-13

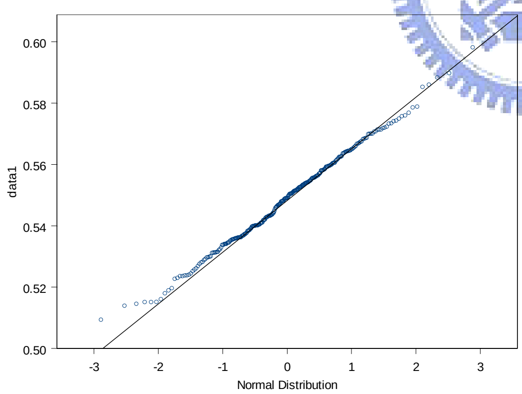


Figure 10. Normal probability plot for thickness data of Supplier I.

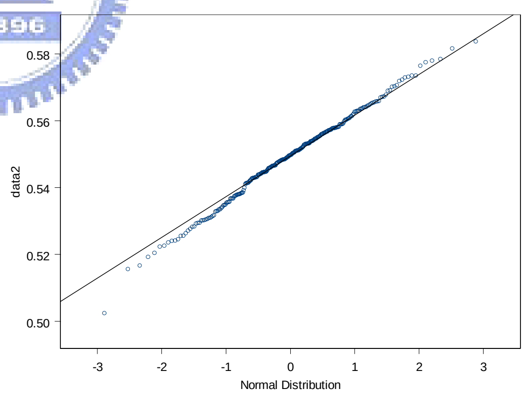
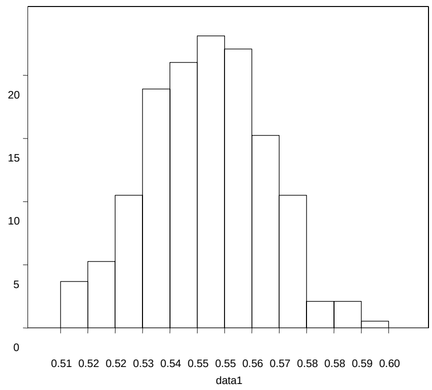


Figure 11. Normal probability plot for thickness data of Supplier II.



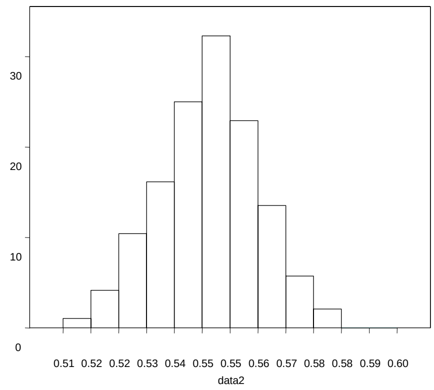


Figure 12. Histogram for supplier I.

Figure 13. Histogram for supplier II.

5.2.2 Phase I－Supplier Selection

We will test $H_0: C_{pl1} \geq C_{pl2}$ versus $H_1: C_{pl1} < C_{pl2}$ by comparing these test statistics $\hat{C}_{pl1}$, $\hat{C}_{pl2}$, and the selection value $A \& c$ based on the test statistic and the required sample sizes. If $\hat{C}_{pl1} < \hat{C}_{pl2}$ and $A < c$ then we conclude that the process capability of the new supplier better than that of the present supplier. The calculated sample statistics for two suppliers are summarized in Table 3.

Table 3. The calculated sample statistics for two suppliers.(C_{pl})

Population	$\bar{X}$	S	$\hat{C}_{pl}$
I	0.5487296	0.01592503	1.019979
II	0.548967	0.01335757	1.221954

Based on the selection method, the values $\hat{C}_{pl1} = 1.019979$ and $\hat{C}_{pl2} = 1.221954$. In this case one only need to compare the test statistic $\hat{C}_{pl1}$ and $\hat{C}_{pl2}$, by $A = 0.02891871$ and $c = 0.2585227$, the outcome presents $\hat{C}_{pl1} < \hat{C}_{pl2}$ and $A < c$, then we conclude that the process of this new supplier is capable.

5.2.3 Phase II－Magnitude Outperformed Detection

To realize the lower bound value of the magnitude, q, we will test $H_0: C_{pl1} + q \geq C_{pl2}$ versus $H_1: C_{pl1} + q < C_{pl2}$. By comparing these test statistics $\hat{C}_{pl1}$, $\hat{C}_{pl2}$, and the selection value $A \& c$ based on the test statistic and the required sample sizes. From the estimation of Phase I, we list the obtained selection values A and c and the decision based on the selection procedure for h = 0.01, 0.05, 0.07(0.001)0.074 in Table 4.

Therefore, from the analysis of magnitude outperformed detection based on sample statistics, the magnitude of the difference between the two suppliers is q = 0.034. By the way, we can conclude that the new supplier is more capable than the present supplier at least a magnitude, q =0.074.

Table 4. Magnitude outperformed detection of selection procedure. (C_{pl})

$\hat{C}_{pl1}$	1.029979	1.069979	1.089979	1.090979	1.091979	1.092979	1.093979
$\hat{C}_{pl2}$	1.221954	1.221954	1.221954	1.221954	1.221954	1.221954	1.221954
q	0.01	0.05	0.07	0.071	0.072	0.073	0.074
A	0.04169824	0.1447203	0.2377967	0.2432623	0.2488044	0.2544226	0.2601165
c	0.2585227	0.2585227	0.2585227	0.2585227	0.2585227	0.2585227	0.2585227
Decision	Reject Ho	Reject Ho	Reject Ho	Reject Ho	Reject Ho	Reject Ho	Don't Reject Ho

5.3 Application Example STN–LCD

Liquid crystals have been employed for display applications with various configurations. Most of the displays produced recently involve the use of either Twisted Nematic (TN) or Super Twisted Nematic (STN) liquid crystals, the technology of the STN display was introduced recently to improve the performance of LCD without using the TFT. A larger twist angle results in a significantly larger electro-optical distortion. This leads to a substantial improvement in the contract and viewing angles over TN displays. The STN-LCD products are popularly used in making the PDAs, notebook personal computers, word processors, and other peripherals. A typical assembly drawing for the STN-LCD product is depicted in Figure 14 and the custom glass and modules of the STN-LCD product is displayed in Figure 15.

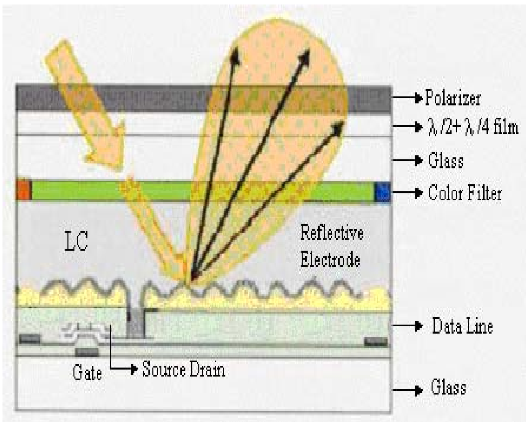


Figure 12. An assembly drawing for the STN-LCD product.

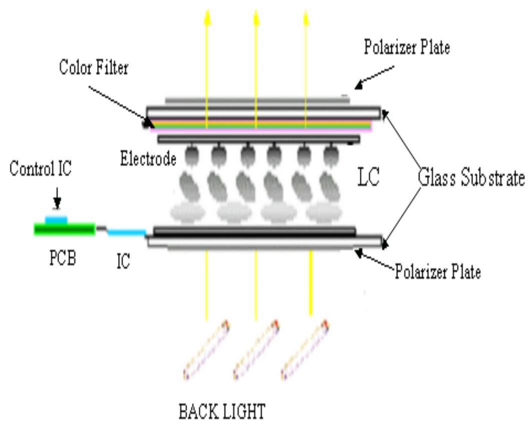


Figure 2. The custom glass and modules of the STN-LCD product.

With an increasing number of personal computers are now network-ready and multimedia capable, equipped with CD-ROM drives. Due to advances in telecommunications technology, simple monochromatic displays are no longer in popular demand. The next generation of telecommunication products will require displays with rich, graphic quality images and personal interfaces. So future displays must be become clearer, sharper to meet these demands. Until this point, STN-LCD have been used mainly to display still images, and because of the slow response time needed to process still images, STN-LCD have not been able to reproduce animated images with an adequate contrast level. Thus, with the growing popularity of multimedia applications, there is a need for PCs equipped with color STN-LCD that are capable of processing animated pictures instead of only still images. The space between the glass substrate is filled with liquid crystal material, the thickness of the LC is kept uniform by using glass fibers or plastic balls as spacer, So the STN-LCD is sensitive in the thickness of the glass substrates.

To illustrate which has better process capability between the two suppliers, we present a case study on STN-LCD (Super Twisted Nematic Liquid Crystal Displays) manufacturing processes, which located on the Science-Based Industrial Park in Taiwan. These factories manufacture various types of the LCD. For a particular model of the STN-LCD investigated, the upper specification limit, USL of a glass substrate's thickness is set to 0.77 mm, the lower specification limit, LSL of a glass substrate's thickness is set to 0.63 mm, and the target value is set to T = 0.70 mm. If the characteristic data does not fall within the tolerance (LSL, USL) , the lifetime or reliability of the STN-LCD will be discounted.

5.3.1 Data Analysis and Supplier Selection

For the Phase I of Supplier Selection problem, the practitioner should input the preset minimum requirement of C_{pm} values, and the minimal difference that must be differentiated between suppliers with designated selection power. If minimum requirement of STN-LCD product is C_{pm}=1.00, and $\delta = 0.25$ with selection power = 0.95. By checking Table 1 the sample size required for estimation is 204. Thus, the glass substrate's thickness data taken from two LCD suppliers are displayed in Table 6. To confirm if the data of both suppliers normally distributed, we do the Shapiro-Wilk test for normality as shown in Figures 3-4. Because the p-values are larger than 0.05, we don't reject the null hypothesis that the data are normally distributed. Histograms of both data for the two suppliers are displayed in Figures 5-6.

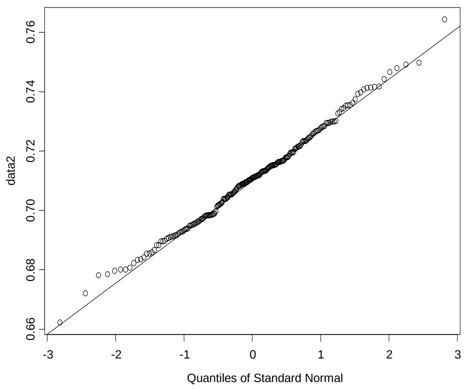


Figure 14. Normal probability plot for thickness data of Supplier I.

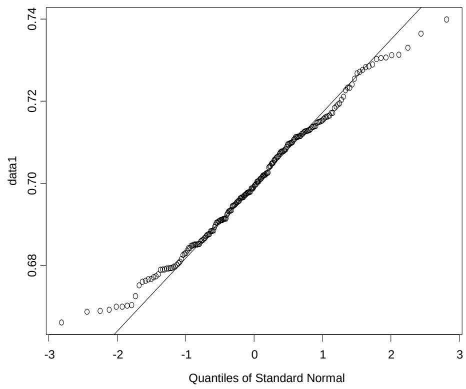


Figure 15. Normal probability plot for thickness data of Supplier II.

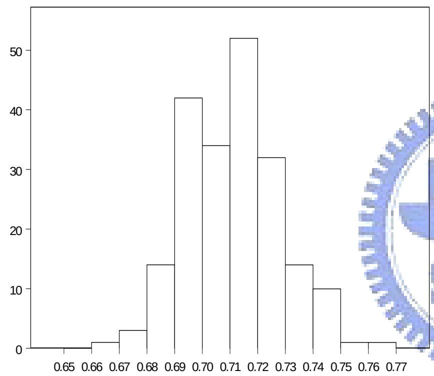


Figure 16. Histogram for supplier I.

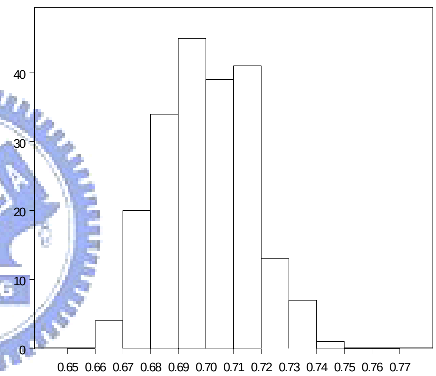


Figure 17. Histogram for supplier II.

5.3.2 Phase I—Supplier Selection

To determine whether supplier II has better process capability than supplier I, that is, do the hypothesis testing comparing the two C_{pm} values, $H_0: C_{pm1} \geq C_{pm2}$ versus $H_1: C_{pm1} < C_{pm2}$. First, we calculate the sample means, sample standard deviations, the sample estimators of $\hat{C}_{pm}$, $\hat{\gamma}^2$, and $\hat{v}$ for supplier I and supplier II, which summarized in Table 7.

Table 5. The calculated sample statistics for two suppliers.(C_{pm})

Population	$\bar{X}$	S	$\hat{C}_{pm}$	$\hat{\gamma}^2$	Rank $\hat{\gamma}^2$
I	0.7106	0.01695	1.1705	3.974×10^{-3}	2
II	0.6998	0.01593	1.4687	2.524×10^{-3}	1

Based on the selection procedure, the values $w_1 = 1.241426$ and

$w_2 = 1.478218$. Choose the value of w which is larger than 1 and choose the value as small as possible, so $w = \min\{w_1, w_2\} = 1.241426$. In this case one only need to compare the test statistic $\hat{\gamma}_i^2$, $i = 1,2$, with the selection value w. Since $\hat{\gamma}_2^2 \le w \times \hat{\gamma}_1^2$ and $\hat{\gamma}_1^2 > w \times \hat{\gamma}_2^2$ then we conclude that the new supplier is better supplier with larger process capability C_{pm}.

5.3.3 Phase II—Magnitude Outperformed Detection

To further investigate the magnitude of the capability difference between the two suppliers, the supplier selection decisions would find a magnitude of q such that $C_{pm2} = C_{pm1} + q$, where $q = \max\{ q' \mid \text{ test rejects } C_{pm1} + q' \ge C_{pm2} \}$. From the estimation of Phase I, we list the obtained selection values w and the decision based on the selection procedure for q = 0.01, 0.05, 0.10, 0.12(0.01)0.15 in Table 8.

Therefore, from the analysis of magnitude outperformed detection based on sample statistics, the magnitude of the difference between the two suppliers is q = 0.14. That is, we conclude that $C_{pm2} > C_{pm1} + 0.14$.

Table 6. Magnitude outperformed detection of selection procedure. (C_{pm})

$\hat{C}_{pm1}$	1.1805	1.2205	1.2705	1.2905	1.3005	1.3105	1.3205
$\hat{C}_{pm2}$	1.4687	1.4687	1.4687	1.4687	1.4687	1.4687	1.4687
q	0.01	0.05	0.10	0.12	0.13	0.14	0.15
w	1.241459	1.241602	1.241821	1.241922	1.241976	1.242032	1.242091
Decision	Reject Ho	Reject Ho	Reject Ho	Reject Ho	Reject Ho	Reject Ho	Don’t reject Ho

6. Conclusion

Replacing the supplier will cause huge affection (no matter visible or invisible). So the new supplier has to make sufficient information to prove that it is more capable. Otherwise we will not run risks of the disadvantage caused by wrong decision. We have to sample a required number of products made by these two suppliers to make a believable compare under designated selection power. In the initial stage of production setting, the decision maker usually faces the problem of selecting better manufacturing supplier from two available manufacturing suppliers. Chou (1994) developed one-sided tests to select between competing processes that which is more capable. According to today's modern quality improvement theory, reduction of the process loss is as important as increasing the process yield. The use of loss functions in quality assurance settings has grown with the introduction of Taguchi’s philosophy. The index C_{pm}

incorporates with the variation of production items with respect to the target value and the specification limits preset in the factory. Based on C_{pm} index a mathematically complicated approximation method is developed by Huang and Lee (1995) for selecting a subset of processes containing the best supplier from a given set of processes. But the required sample size for doing the selection method isn't notified definitely.

In this paper, we implement these two methods, provide a effective sample size information before doing the selection and develop a practical step-by-step procedure for practitioners to use in making supplier selection decisions. Accuracy of the selection method is investigated by using simulation technique. The accuracy analysis provides useful information about the sample size required for designated selection power. A two- phase selection procedure is developed to select better supplier and further examine the magnitude of the difference between the two suppliers. Finally, we also investigate a real-world case on the STN-LCD (Super Twisted Nematic Liquid Crystal Display) ,TFT-LCD (Thin Film Transistor Liquid Crystal Display) and automobile window manufacturing process, and apply the selection procedure using actual data collected from the factories, to reach a decision in supplier selections.

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Appendix A. The sample sizes information

Table 7. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under $p^* = 0.95$, with $C_{pu1} = 1.00$, $C_{pu2} = 1.05(0.05)2.00$.

Cpu1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Cpu2	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
90%	4376	1140	524	308	205	147	111	89	74	61
95%	5499	1427	662	391	257	191	144	111	93	78
97.5%	6650	1710	794	467	308	224	171	134	108	93
99%	8009	2050	955	556	376	270	207	164	134	112

Cpu1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Cpu2	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
90%	52	45	40	36	32	29	27	24	23	21
95%	66	58	51	46	40	37	34	31	29	27
97.5%	79	68	60	54	49	44	39	37	34	32
99%	95	84	75	65	58	54	49	44	39	38

Table 8. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under $p^* = 0.95$, with $C_{pu1} = 1.25$, $C_{pu2} = 1.30(0.05)2.25$.

Cpu1	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25
Cpu2	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.75
90%	6326	1628	749	437	291	207	160	125	102	85
95%	8020	2084	961	549	362	262	198	157	128	105
97.5%	9651	2500	1146	665	436	310	239	191	154	128
99%	11577	3001	1405	803	534	380	289	228	189	156

Cpu1	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25
Cpu2	1.80	1.85	1.90	1.95	2.00	2.05	2.10	2.15	2.20	2.25
90%	71	62	55	49	43	39	36	33	31	29
95%	91	78	69	61	55	50	46	41	38	36
97.5%	108	94	84	74	66	58	54	50	46	43
99%	132	115	102	88	80	71	66	60	55	51

Table 9. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under p^* = 0.95, with C_{pu1} = 1.45, C_{pu2} = 1.50(0.05)2.45.

Cpu1	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45
Cpu2	1.50	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95
90%	8193	2118	960	563	369	267	200	157	128	117
95%	10400	2676	1221	707	467	333	251	200	162	137
97.5%	12521	3223	1463	849	559	400	305	236	195	162
99%	15201	3870	1985	1032	675	490	365	285	233	196

Cpu1	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45
Cpu2	2.00	2.05	2.10	2.15	2.20	2.25	2.30	2.35	2.40	2.45
90%	90	78	69	61	54	49	44	40	37	35
95%	113	98	86	76	68	61	55	51	46	43
97.5%	137	119	103	91	82	75	67	62	56	52
99%	165	144	127	110	98	88	80	74	68	64

Table 10. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under p^* = 0.95, with C_{pu1} = 1.60, C_{pu2} = 1.65(0.05)2.60.

Cpu1	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60
Cpu2	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00	2.05	2.10
90%	9901	2512	1149	666	439	310	235	187	152	125
95%	12496	3177	1454	840	552	393	299	235	191	157
97.5%	14906	3869	1747	1002	659	469	361	281	225	191
99%	18007	4652	2140	1220	806	572	434	340	277	228

Cpu1	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60
Cpu2	2.15	2.20	2.25	2.30	2.35	2.40	2.45	2.50	2.55	2.60
90%	106	91	79	71	63	57	51	47	42	39
95%	134	115	98	89	79	71	65	59	54	50
97.5%	161	137	120	106	95	85	78	71	65	60
99%	195	166	147	128	115	102	95	85	78	74

Table 11. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under p^* = 0.95, with C_{pl1} = 1.00, C_{pl2} = 1.05(0.05)2.00.

Cpl1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Cpl2	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
90%	4377	1141	524	309	205	147	111	89	74	61
95%	5500	1428	663	391	257	191	144	111	93	78
97.5%	6649	1710	795	467	308	224	171	134	108	93
99%	8009	2051	955	557	376	270	207	164	134	112

Cpl1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Cpl2	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
90%	52	45	40	36	32	29	27	24	23	21
95%	66	58	51	46	40	37	34	31	29	27
97.5%	79	68	60	54	49	44	39	37	34	32
99%	95	84	75	65	58	54	49	44	40	38

Table 12. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under p^* = 0.95, with C_{pl1} = 1.25, C_{pl2} = 1.30(0.05)2.25.

Cpl1	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25
Cpl2	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.75
90%	6326	1629	751	438	292	209	159	124	102	86
95%	8020	2085	961	550	363	263	198	156	129	107
97.5%	9651	2500	1146	666	436	313	239	191	155	128
99%	11577	3002	1405	805	535	381	289	228	190	157

Cpl1	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25
Cpl2	1.80	1.85	1.90	1.95	2.00	2.05	2.10	2.15	2.20	2.25
90%	71	63	55	49	44	39	36	33	31	29
95%	91	79	69	61	55	50	46	41	38	36
97.5%	109	95	83	74	66	59	54	50	46	43
99%	132	115	102	88	80	71	66	60	56	51

Table 13. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under p^* = 0.95, with C_{pl1} = 1.45, C_{pl2} = 1.50(0.05)2.45.

Cpl1	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45
Cpl2	1.50	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95
90%	8193	2118	960	563	370	267	200	157	129	107
95%	10400	2676	1221	707	471	334	251	200	162	137
97.5%	12521	3223	1463	850	560	400	305	237	195	162
99%	15201	3871	1981	1032	677	490	365	285	233	197

Cpl1	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45	1.45
Cpl2	2.00	2.05	2.10	2.15	2.20	2.25	2.30	2.35	2.40	2.45
90%	91	79	69	61	54	49	45	41	37	35
95%	113	98	86	76	69	61	56	51	47	43
97.5%	137	119	103	91	82	75	67	62	57	52
99%	165	144	127	111	98	88	80	74	68	64

Table 14. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under p^* = 0.95, with C_{pl1} = 1.60, C_{pl2} = 1.65(0.05)2.60.

Cpl1	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60
Cpl2	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00	2.05	2.10
90%	9901	2513	1149	666	439	311	235	187	152	125
95%	12496	3178	1455	840	553	394	299	235	191	157
97.5%	14907	3870	1747	1002	660	469	361	281	225	191
99%	18007	4653	2141	1221	807	573	434	340	277	228

Cpl1	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60
Cpl2	2.15	2.20	2.25	2.30	2.35	2.40	2.45	2.50	2.55	2.60
90%	106	92	79	71	63	57	51	47	42	39
95%	134	115	98	90	79	71	65	59	54	50
97.5%	162	137	120	107	95	85	78	71	65	60
99%	196	166	147	129	115	102	95	85	78	74

Table 15. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under p^* = 0.95, with C_{pm1} = 1.00, C_{pm2} = 1.05(0.05)2.00.

C_{pm1}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
C_{pm2}	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
0.90	3408	898	414	240	165	118	90	71	59	50
0.95	4351	1120	520	307	204	151	115	91	73	63
0.975	5130	1356	640	371	250	180	137	109	91	76
0.99	6131	1631	785	451	303	220	171	135	110	93

C_{pm1}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
C_{pm2}	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
0.90	43	37	32	29	26	24	22	22	19	18
0.95	53	48	41	37	33	31	29	28	27	26
0.975	65	57	50	45	40	37	34	30	28	27
0.99	80	70	61	56	49	45	40	38	35	33

Table 16. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under p^* = 0.95, with C_{pm1} = 1.33, C_{pm2} = 1.38(0.05)2.33.

C_{pm1}	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33
C_{pm2}	1.38	1.43	1.48	1.53	1.58	1.63	1.68	1.73	1.78	1.83
0.90	5900	1520	694	400	269	194	147	115	94	79
0.95	7493	1297	896	530	343	246	191	149	119	102
0.975	9014	2350	1060	622	401	301	231	178	147	120
0.99	10999	2859	1315	765	499	368	272	222	175	149

C_{pm1}	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33
C_{pm2}	1.88	1.93	1.98	2.03	2.08	2.13	2.18	2.23	2.28	2.33
0.90	67	59	52	45	41	36	33	32	29	26
0.95	85	73	65	59	52	46	43	39	35	33
0.975	103	90	78	69	64	56	51	48	43	39
0.99	127	109	95	85	76	70	64	56	52	49

Table 17. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under p^* = 0.95, with C_{pm1} = 1.50, C_{pm2} = 1.55(0.05)2.50.

C_{pm1}	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50
C_{pm2}	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
0.90	7394	1941	891	513	338	245	184	145	118	96
0.95	9506	2460	1120	657	430	308	232	180	151	125
0.975	11503	3001	1338	801	515	376	283	220	180	151
0.99	13502	3540	1634	974	627	457	340	268	221	177

C_{pm1}	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50
C_{pm2}	2.05	2.10	2.15	2.20	2.25	2.30	2.35	2.40	2.45	2.50
0.90	83	71	62	55	49	45	39	38	35	32
0.95	106	91	79	71	63	56	51	48	44	40
0.975	125	109	95	85	75	69	63	57	53	50
0.99	155	134	115	103	92	83	75	71	65	60

Table 18. Sample size required for power = 0.90, 0.95, 0.975, 0.99 under p* = 0.95, with $C_{pm}1$ = 1.67, $C_{pm}2$ = 1.72(0.05)2.67.

C_{pm1}	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67
C_{pm2}	1.72	1.77	1.82	1.87	1.92	1.97	2.02	2.07	2.12	2.17
0.90	9291	2360	1091	630	408	292	223	173	141	115
0.95	12000	3034	1387	807	531	371	282	220	177	151
0.975	14297	3700	1650	970	629	448	338	260	218	180
0.99	17990	4400	2000	1163	765	544	400	325	255	220

C_{pm1}	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67	1.67
C_{pm2}	2.22	2.27	2.32	2.37	2.42	2.47	2.52	2.57	2.62	2.67
0.90	100	85	75	66	60	52	49	43	39	38
0.95	125	108	95	85	75	67	63	55	51	48
0.975	154	130	115	102	91	82	75	66	63	56
0.99	185	159	140	120	112	99	91	83	74	69

Appendix B. The sample data for application

Table 19. Sample data collected from suppliers (using C_{pm})

Supplier I						Supplier II					
0.688	0.719	0.666	0.698	0.707	0.709	0.709	0.698	0.695	0.693	0.692	0.683
0.725	0.706	0.679	0.731	0.697	0.715	0.697	0.716	0.690	0.697	0.708	0.695
0.711	0.701	0.706	0.696	0.699	0.685	0.690	0.698	0.719	0.750	0.693	0.695
0.712	0.702	0.697	0.679	0.698	0.700	0.708	0.717	0.729	0.693	0.749	0.728
0.698	0.717	0.683	0.688	0.691	0.706	0.730	0.718	0.706	0.717	0.712	0.741
0.687	0.699	0.730	0.709	0.708	0.710	0.741	0.727	0.713	0.698	0.724	0.698
0.712	0.702	0.695	0.716	0.679	0.677	0.715	0.717	0.699	0.713	0.710	0.718
0.679	0.694	0.700	0.695	0.700	0.708	0.730	0.697	0.678	0.719	0.733	0.710
0.707	0.723	0.711	0.693	0.670	0.723	0.694	0.728	0.709	0.708	0.705	0.721
0.691	0.713	0.680	0.719	0.691	0.680	0.696	0.747	0.707	0.739	0.721	0.688
0.686	0.684	0.727	0.705	0.685	0.670	0.711	0.730	0.715	0.696	0.715	0.709
0.714	0.695	0.685	0.696	0.733	0.710	0.702	0.735	0.728	0.728	0.735	0.688
0.679	0.673	0.715	0.680	0.691	0.706	0.726	0.709	0.727	0.678	0.737	0.707
0.684	0.691	0.708	0.716	0.679	0.718	0.723	0.690	0.705	0.710	0.710	0.721
0.705	0.704	0.729	0.698	0.716	0.689	0.726	0.711	0.729	0.722	0.704	0.730
0.709	0.711	0.719	0.678	0.669	0.711	0.729	0.727	0.685	0.684	0.692	0.704
0.684	0.713	0.691	0.731	0.691	0.710	0.713	0.710	0.710	0.734	0.691	0.723
0.688	0.708	0.670	0.693	0.696	0.703	0.715	0.711	0.713	0.726	0.704	0.714
0.676	0.685	0.728	0.713	0.685	0.697	0.709	0.690	0.694	0.694	0.698	0.718
0.693	0.699	0.710	0.699	0.711	0.681	0.715	0.682	0.703	0.713	0.701	0.748
0.696	0.698	0.691	0.693	0.700	0.720	0.742	0.697	0.702	0.735	0.662	0.711
0.677	0.669	0.690	0.724	0.690	0.685	0.699	0.698	0.712	0.705	0.691	0.764
0.704	0.712	0.690	0.716	0.693	0.714	0.717	0.721	0.706	0.700	0.723	0.725
0.736	0.721	0.679	0.713	0.728	0.730	0.720	0.736	0.699	0.722	0.686	0.698
0.707	0.683	0.700	0.683	0.715	0.723	0.722	0.705	0.740	0.691	0.709	0.716
0.676	0.711	0.702	0.714	0.701	0.702	0.693	0.720	0.704	0.716	0.696	0.704
0.675	0.697	0.685	0.695	0.740	0.697	0.712	0.715	0.684	0.714	0.692	0.733
0.705	0.691	0.699	0.716	0.701	0.681	0.691	0.705	0.724	0.704	0.744	0.716
0.687	0.714	0.688	0.706	0.702	0.695	0.695	0.717	0.711	0.680	0.696	0.685
0.682	0.685	0.727	0.686	0.712	0.717	0.702	0.680	0.680	0.711	0.725	0.734
0.688	0.728	0.694	0.701	0.715	0.687	0.712	0.712	0.741	0.696	0.687	0.742
0.702	0.713	0.677	0.731	0.708	0.677	0.723	0.724	0.714	0.703	0.708	0.718
0.692	0.669	0.710	0.708	0.704	0.686	0.702	0.681	0.713	0.720	0.713	0.672
0.688	0.713	0.687	0.715	0.670	0.697	0.715	0.710	0.699	0.706	0.716	0.715

Table 20. Sample data collected from suppliers for (using C_{pu})

Supplier I

19.50887	18.49534	19.01191	19.58026	19.03893	19.41706	19.00949	18.93824
18.55457	19.27253	19.81652	18.89563	19.24296	19.29311	19.42069	19.03118
18.82897	19.06534	18.71327	18.76645	19.04814	18.51734	18.74375	19.15274
18.37392	19.08271	19.23258	19.15462	19.14358	18.61067	18.82561	18.95075
18.83704	18.94513	18.93034	18.99778	19.21342	18.97118	18.53654	18.91620
19.35235	18.99165	19.29487	18.96002	18.83323	19.31604	19.15266	19.44910
19.04188	18.96122	19.14016	19.06402	19.00688	19.04279	18.90722	18.91623
19.26781	19.14902	19.33575	18.89807	19.05814	18.93584	19.14677	18.38613
19.40426	18.96595	18.98250	18.95521	18.82830	19.10985	19.35341	19.61786
19.12376	18.39881	18.91442	19.17434	19.60094	18.98125	18.82489	19.01936
19.04415	19.03912	18.43395	18.94474	19.07810	19.18113	19.10499	18.64332
18.79539	19.40191	19.52051	19.25382	19.11344	19.02177	18.79392	18.66889
19.23620	19.09896	19.28412	19.01550	18.71444	18.84828	18.72923	18.93971
18.69903	19.60415	19.16601	19.45659	18.61767	18.82487	19.35499	19.10017
19.36216	18.78598	18.89032	18.67590	18.80744	19.27476	19.29530	19.44674
19.81644	18.79661	18.61695	18.74314	18.81000	19.10232	19.03217	18.69078
18.98861	18.98333	19.39603	18.92354	18.68095	18.51426	19.06710	19.24419
19.16216	18.71872	19.28059	18.64794	18.28189	18.66298	19.62193	19.20634
18.83678	19.07669	18.10243	18.72877	18.85579	18.53593	19.21832	18.85377
18.96684	18.78936	19.34159	19.16857	18.39330	19.84420	18.72959	19.15650
18.79488	18.66942	19.33462	19.16296	19.30823	19.26709	19.05669	19.45820
18.99997	19.17676	18.85470	18.68293	19.35543	19.34733	19.39894	18.52424
18.89965	18.39666	18.31666	18.95589	19.31446	18.71866	19.00800	18.92302
18.49540	19.24302	19.32774	19.47645	19.32612	18.59760	18.84302	18.70581
18.60216	18.96178	18.91078	19.41534	18.91465	19.16897	18.92578	18.96048
18.90525	18.69745	18.97376	18.96916	18.58000	18.89592	18.98732	18.91505
18.66344	19.45461	18.99426	19.25963	19.03363	18.38067	18.89573	18.95902
18.79612	18.96935	19.40599	19.78532	19.08909	18.74275	18.32309	18.75040
18.91448	19.31341	18.75103	19.04676	19.20317	19.06308	19.22384	18.99315
18.83664	18.92697	18.96523	19.04757	19.62643	19.08792	19.31578	19.00386
18.85597	18.58410	19.51615	18.88083	19.63993	18.97679	18.52285	19.13188
19.13807	18.81165	19.16163	18.93206	18.80539	18.49568	18.82101	18.87773
18.73990							

Supplier II

19.18338	19.05785	19.06293	19.32024	19.42831	18.92743	18.97491	18.89443
18.54733	19.07784	18.96980	19.10482	18.97141	18.93092	19.43333	19.07702
18.61789	18.39431	19.32511	18.96637	19.21269	19.21590	19.07455	19.41158
18.85440	18.56997	19.42615	19.51079	18.80263	18.98351	18.86589	18.99828
18.71082	18.45416	18.55770	19.21430	18.70881	19.36842	18.49003	18.79611
19.29471	19.38879	18.72604	19.44648	19.16593	19.28478	18.85318	18.68894
19.23815	19.13883	18.83626	19.50036	18.68572	19.31185	19.07355	18.70961
19.06961	19.10228	18.86276	19.27669	18.78214	18.99557	19.00537	18.71028
18.87018	18.86720	18.64760	18.90438	18.67536	19.29426	18.94428	19.36563
18.75626	19.13649	18.64156	19.19490	19.28434	18.84490	18.92320	18.83933
18.83337	19.30753	18.68874	19.17186	19.02514	18.79810	19.10085	19.52701
18.79478	19.18746	18.96797	18.81091	18.86431	18.70334	18.69295	18.86545
18.67778	19.43123	18.95796	19.15569	18.84638	18.44806	19.05808	18.93428
19.20781	18.97086	18.88805	18.95540	18.91824	18.77698	18.61861	18.99556
18.52633	19.02732	19.53650	19.42770	19.18412	19.35152	19.04429	18.81704
18.83928	19.05518	19.19239	19.39402	18.80130	19.55276	18.67561	18.65084
18.96573	19.04594	19.11811	18.99807	19.15976	18.80283	19.38945	18.89159
18.45645	19.59680	19.60726	18.94409	19.05710	18.85655	18.89143	19.01728
19.03834	18.91664	19.31819	18.60662	19.26010	18.90850	18.76065	19.24267
18.77883	18.80609	18.90769	19.13785	19.19606	19.57086	19.27991	19.02291
18.88595	19.25139	19.27507	18.93002	18.54880	19.19778	18.94639	18.98606
19.30562	18.99572	18.67080	19.11132	18.77853	19.07156	19.08467	19.02913
18.91499	18.92307	18.73536	18.89761	19.19294	19.60152	19.12179	18.86852
18.70872	18.56414	18.81653	19.29368	18.33544	19.12481	19.00913	18.81978
19.27540	18.82925	18.85348	19.15198	19.40912	19.16288	19.27299	18.79681
18.76342	19.30063	18.80186	18.89850	18.84458	19.18776	18.47895	18.40057
18.50817	18.97977	18.70939	18.59426	19.02008	18.85019	19.09217	19.06307
18.92304	19.19785	19.21540	18.60994	19.36940	18.67803	19.17542	19.07209
18.59139	19.04923	18.77615	18.39602	18.75090	18.91602	18.83088	19.37463
19.13396	18.22555	19.18360	18.77076	18.53105	18.99660	19.28701	18.93843
18.71434	19.39902	19.01172	18.88908	19.36774	19.04825	18.97310	18.78602
18.56919	18.77832	19.03165	19.06472	18.85475	19.12924	18.92176	18.77584
18.55797							

Table 21. Sample data collected from suppliers (using C_{pl})

Supplier I

0.5274734	0.5700074	0.5784949	0.5328325	0.5138712	0.5635044	0.5644245
0.5344980	0.5473243	0.5550764	0.5361334	0.5358392	0.5398198	0.5400457
0.5400450	0.5626833	0.5385397	0.5741802	0.5787484	0.5563115	0.5440541
0.5313819	0.5537756	0.5360569	0.5635766	0.5767652	0.5592020	0.5349897
0.5532349	0.5409493	0.5707085	0.5382040	0.5344477	0.5409347	0.5533137
0.5188521	0.5311647	0.5717385	0.5684969	0.5503480	0.5698865	0.5858994
0.5524737	0.5144656	0.5643579	0.5371923	0.5298309	0.5314141	0.5488809
0.5512894	0.5733451	0.5641299	0.5604639	0.5673236	0.5296863	0.5674069
0.5319827	0.5748534	0.5237188	0.5466400	0.5401797	0.5719410	0.5579063
0.5432531	0.5616061	0.5406766	0.5712774	0.5561603	0.5539683	0.5446132
0.5431837	0.5425337	0.5669723	0.5591799	0.5490190	0.5624640	0.5568565
0.5290802	0.5464258	0.5419978	0.5287505	0.5598825	0.5602706	0.5279462
0.5397406	0.5478922	0.5488729	0.5639776	0.5150664	0.5150857	0.5712911
0.5546193	0.5461799	0.5384562	0.5243750	0.5542116	0.5299714	0.5336589
0.5416983	0.5571611	0.5159757	0.5441056	0.5516461	0.5356530	0.5503606
0.5706059	0.5351348	0.5458234	0.5758798	0.5507319	0.5645299	0.5579892
0.5419777	0.5366381	0.5598177	0.5739995	0.5519784	0.5579771	0.5178984
0.5489711	0.5234956	0.5339526	0.5226060	0.5712792	0.5555796	0.5980706
0.5310751	0.5195507	0.5503758	0.5312199	0.5563953	0.5504876	0.5436689
0.5366522	0.5650374	0.5260805	0.5336566	0.5470464	0.5356031	0.5509019
0.5583560	0.5612652	0.5699557	0.5513068	0.5358854	0.5368253	0.5587643
0.5150442	0.5389153	0.5511619	0.5374255	0.5557851	0.5234580	0.5756039
0.5311680	0.5623737	0.5256325	0.5659715	0.5657026	0.5295880	0.5666076
0.5721986	0.5427547	0.5239634	0.5530300	0.5605421	0.5398827	0.5539843
0.5498441	0.5537779	0.5594738	0.5251152	0.5604109	0.5546862	0.5342689
0.5686305	0.5698142	0.5340236	0.5544884	0.5560820	0.5896714	0.5851981
0.5478539	0.5430561	0.5487576	0.5416362	0.5480857	0.5653327	0.5882674
0.5591843	0.5482130	0.5479563	0.5429086	0.5568631	0.5237858	0.5398917
0.5536332	0.5525371	0.5400630	0.5642429	0.5581467	0.5428101	0.5361280
0.5523178	0.5382098	0.5229395	0.5558505	0.5361735	0.5468325	0.5732125
0.5514554	0.5459234	0.5340347	0.5468102	0.5434653	0.5527672	0.5268228
0.5409873	0.5452164	0.5515293	0.5400705	0.5092596	0.5434005	0.5396555
0.5486324	0.5494740	0.5322607	0.5638068	0.5359084	0.5611129	0.5281036
0.5648720	0.5619083	0.5403331	0.5354617	0.5555665	0.5667387	0.5541134
0.5354209	0.5582502	0.5519141	0.5533746	0.5362108	0.5376194	0.5595529
0.5594596	0.5525673	0.5682778	0.5681431	0.5391295	0.5431753	0.5501607
0.5403710	0.5553848	0.5555365	0.5624079	0.5372093		

Supplier II

0.5464737	0.5494429	0.5281629	0.5611731	0.5511057	0.5470282	0.5559287
0.5304957	0.5291391	0.5670360	0.5540639	0.5450239	0.5308764	0.5262580
0.5328757	0.5529731	0.5513210	0.5420235	0.5578576	0.5526925	0.5458560
0.5416706	0.5382090	0.5484786	0.5509645	0.5409197	0.5721556	0.5340741
0.5520913	0.5378711	0.5430062	0.5480865	0.5778785	0.5427121	0.5254567
0.5244643	0.5499248	0.5481714	0.5531491	0.5374584	0.5203668	0.5497475
0.5372811	0.5384473	0.5633606	0.5643172	0.5456437	0.5458232	0.5639853
0.5676825	0.5690094	0.5576675	0.5730622	0.5648107	0.5486031	0.5413949
0.5552136	0.5440219	0.5538077	0.5589139	0.5603631	0.5428620	0.5376773
0.5495234	0.5444606	0.5427447	0.5308379	0.5420385	0.5463971	0.5353378
0.5516222	0.5413076	0.5191446	0.5316395	0.5580431	0.5763535	0.5303686
0.5652460	0.5333419	0.5378999	0.5471756	0.5446361	0.5479695	0.5504003
0.5553642	0.5596418	0.5511026	0.5412252	0.5465541	0.5607439	0.5430754
0.5612297	0.5366586	0.5504782	0.5222420	0.5653856	0.5529271	0.5588247
0.5509787	0.5155494	0.5529732	0.5668445	0.5476572	0.5355431	0.5437269
0.5527147	0.5576667	0.5541438	0.5591222	0.5773793	0.5574758	0.5275427
0.5521596	0.5442366	0.5282561	0.5717580	0.5550697	0.5432270	0.5293177
0.5537051	0.5518465	0.5588115	0.5650840	0.5165956	0.5688504	0.5628087
0.5549124	0.5337865	0.5483538	0.5366638	0.5638934	0.5601523	0.5465105
0.5545933	0.5488800	0.5572349	0.5531386	0.5509955	0.5235046	0.5444652
0.5601507	0.5483673	0.5391154	0.5561946	0.5570951	0.5563029	0.5438024
0.5023482	0.5572015	0.5580945	0.5412386	0.5656902	0.5452622	0.5376225
0.5656328	0.5516094	0.5347015	0.5549957	0.5379528	0.5533349	0.5815071
0.5463701	0.5270096	0.5474197	0.5224999	0.5558872	0.5538273	0.5836291
0.5733894	0.5494646	0.5490610	0.5487046	0.5625949	0.5482964	0.5398580
0.5239627	0.5354702	0.5444169	0.5240820	0.5640050	0.5577703	0.5356923
0.5440018	0.5334012	0.5311627	0.5545585	0.5502345	0.5497013	0.5671413
0.5702173	0.5579676	0.5635291	0.5301101	0.5461340	0.5465717	0.5558840
0.5566609	0.5783592	0.5531069	0.5461542	0.5429828	0.5734166	0.5513483
0.5366636	0.5446041	0.5543715	0.5366370	0.5314226	0.5565067	0.5626622
0.5628171	0.5619661	0.5327255	0.5644786	0.5428007	0.5293852	0.5383554
0.5301921	0.5329116	0.5537567	0.5457277	0.5728265	0.5501609	0.5530735
0.5345607	0.5491278	0.5546326	0.5474037	0.5568713	0.5254830	0.5628985
0.5478464	0.5576496	0.5424531	0.5700641	0.5605010	0.5507329	0.5380821
0.5437597	0.5477360	0.5348374	0.5482121	0.5588086	0.5570494	0.5636199
0.5579025	0.5562411	0.5446164	0.5555567	0.5705444	0.5300522	0.5616418
0.5566462	0.5657742	0.5508917	0.5606110	0.5447110		