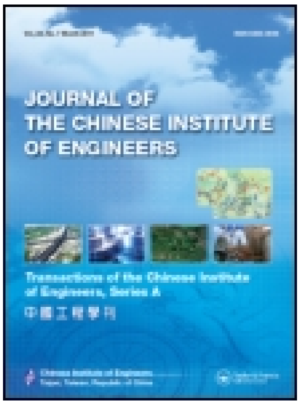

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A new model and heuristic algorithms for the multiple-depot vehicle scheduling problem

Jin-Yuan Wang [a] & Chih-Kang Lin [b]

[a] Department of Transportation Technology and Management , National Chiao Tung University , Hsinchu, 300, Taiwan, R.O.C. Phone: 886-3-5131336 Fax: 886-3-5131336 E-mail:

[b] Department of Transportation Technology and Management , National Chiao Tung University , Hsinchu, 300, Taiwan, R.O.C.

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A NEW MODEL AND HEURISTIC ALGORITHMS FOR THE MULTIPLE-DEPOT VEHICLE SCHEDULING PROBLEM

Jin-Yuan Wang and Chih-Kang Lin*

ABSTRACT

The multiple-depot vehicle scheduling problem (MDVSP) addresses the work of assigning vehicles to serve a given set of time trips with the consideration of certain requirements representing the market rules. Extensive studies in the literature address the MDVSP, but because of the complexity of the problem, the findings of those researchers are still not enough to represent real world situations in Taiwan. Formulation for the MDVSP typically contains the following assumptions: (1) the size of fleet or maximum number of available vehicles at each depot is known already, (2) all trip serving costs are usually simplified as a single term in the objective function, which fails to reflect public transit operator concerns, (3) the applied deadheading strategy is static less flexibility, (4) there is no discussion of differences of route change frequency in the problem. This paper presents a new MDVSP model to address the above issues. A greedy heuristic algorithm based on the divide-and-conquer technique is also proposed to solve the MDVSP effectively. Computational tests are performed on the Kinmen Bus Administration (KBA) and results demonstrate that the proposed new model and the greedy heuristic algorithm for the MDVSP are effective in solving real world problems.

Key Words: transportation, multiple-depot vehicle scheduling problem (MDVSP), heuristic, greedy algorithm.

I. INTRODUCTION

Vehicle scheduling is one of the most important transportation industry jobs, especially for public transit services. This problem addresses the work of assigning vehicles to serve a given set of time trips, considering certain market rule requirements. An optimal schedule generally satisfies minimum fleet size of vehicles and minimum operational costs including costs for vehicle idle time at depots and the number of deadhead trips. Extensive studies in the literature address the multiple-depot vehicle scheduling problem (MDVSP). Bodin and Golden (1983) (1981), and Ball *et al*. (1995) point out some critical requirements for this problem formulation, such as vehicle idle time, number of depots, fleet size, and deadhead trips. Minimizing the number of required vehicles and operational costs which combine vehicle idle time and deadhead trip costs are the main components in the MDVSP objective function. Previous studies propose some approximate exact solution algorithms to deal with the MDVSP. Those efforts help efficiently solve the MDVSP and obtain a reasonable answer. Studies by Carraresi and Gallo (1983), Carpaneto *et al*. (1989), Mesquita and Paixão (1990), Lamatsch (1992), Forbes *et al*. (1994), Ribeiro and Soumis (1994), Beasley and Cao (1996), Kokott and Löbel (1996), Löbel (1997), Haghni *et al*. (2002), Haghani and Banihashemi (2003), Gintner *et al*. (2005), Pepin *et al*. (2006), Kliewer *et al*. (2006) and Hadjar *et al*. (2006) also show these efforts.

The assumptions mentioned above however, are still not enough to represent real world situations in Taiwan (Wang and Lin, 2003). Formulation for the MDVSP typically contains the following assumptions: (1) the size of fleet or maximum number of available vehicles at each depot is known already, (2) all trip serving costs are usually simplified as a single term

*Corresponding author. (Tel: 886-3-5131336; Fax: 886-3-5725804; Email: john101@faculty.nctu.edu.tw)

The authors are with the Department of Transportation Technology and Management, National Chiao Tung University, Hsinchu 300, Taiwan, R.O.C.

in the objective function, which fails to reflect public transit operator concerns, (3) sufficient flexibility for adopting different deadheading strategies in formulation is not provided, (4) there is no discussion of different route change frequencies in the problem. These assumptions leave significant limitations while applying the MDVSP to solve real world problems, especially for public transit.

Operators are usually troubled in deciding the number of available vehicles at each depot in practice. In particular, assigning too many vehicles to one depot can cause vehicle shortages at other depots. Moreover, finding a good solution is crucial for the Taiwan public transit industry, since prices are generally kept low for the public welfare. As a result, most Taiwan public transit operators focus on reducing vehicle fleet size as much as possible even in the schedule planning stage. Some studies (Bodin and Golden, 1981; Carraresi and Gallo, 1983; Ball *et al*., 1995) argue that it is not necessary to minimize the number of required vehicles since it is not a short term planning concern. Extra vehicles, if they do exist, can be used for other profit-making activities such as chartering, if required vehicles are kept to a minimum.

Different types of costs need to be considered separately in order to address different decision makers' preferences. Considering public transit operation needs in terms of cost, one should represent more than one perspective. Most researchers use only a single deterministic term to present the concept of cost accommodated in the objective function, which is not a problem from a mathematical point of view. However, it could be a problem for the decision maker of public transit operation by this approach, since it is not easy to integrate different types of cost into a single one.

Deadhead trip arrangement is a common trade off influencing operating efficiency. Most researchers base model formulation on a pre-determined network. Associated deadhead arcs and costs are added to the network if deadhead trips are allowed. Again, this is perfectly suitable from the theoretical point of view. However, public transit operators in the real world might want to try adopting different deadhead strategies before making decisions. It is a flexible way to incorporate the deadhead trip consideration into the model formulation, as constraint functions.

Changing service routes is not considered a good public transit practice. Real world drivers are usually assigned to a specific vehicle route they are familiar with. A vehicle assigned to serve more routes requires a driver familiar with all those routes. Most operators agree that keeping route change frequency as low as possible is important (Wang and Lin, 2006). Most studies, unfortunately, seem to ignore this issue.

Literature findings show that the computation capability of existing optimization technologies for solving the MDVSP are limited according to the problem size (i.e. number of trips) since it is characterized as NP-hard (Bertossi *et al*., 1987). Finding the optimal solution within an acceptable time is a problem when exceeding 850 trips for the MDVSP (Hadjar *et al*., 2006). For handling large scale MDVSPs, there are some applied techniques such as the column generation algorithm (Ribeiro and Soumis, 1994; Löbel, 1997), the Lagrangian relaxation algorithm (Mesquita and Paixão, 1990; Lamatsch, 1992; Kokott and Löbel, 1996), metaheuristics algorithms: the Tabu search algorithm (Cordeau *et al*., 2001; Pepin *et al*., 2006), Genetic Algorithms (Su and Yu, 2006), and many other proposed heuristic algorithms: an auction algorithm for the quasi- assignment formulation (Freling, 1997), a schedule first-cluster second approach (Daduna and Paixão, 1995) and a cluster first-schedule second approach (Carraresi and Gallo, 1983) can be found in related studies. Most of these algorithms solve the MDVSP efficiently and effectively with the prerequisite of having a particular network structure to ease the solution process (Pepin *et al*., 2006). However, time-consuming network constructions are not acceptable in real world public transit operations. Therefore, solving large scale MDVSPs efficiently and effectively remains a challenge. It also determines whether or not the identified solutions can be applied in real world operations. This is particularly true for transit operations that require quick responses.

This paper proposes a new MDVSP model with the objectives of minimizing vehicle idle time, minimizing fleet size, minimizing service route change frequency, and deadhead trip strategy concerns. This study also presents a heuristic solution procedure by dividing the MDVSP into relatively easy problems to solve this model efficiently and effectively. The rest of the paper is organized as follows: This work first introduces a new MDVSP model formulation, and then illustrates a greedy algorithm for solving the MDVSP formulation. Finally, a real case in Taiwan is adopted to show the efficiency and effectiveness of the theoretical approach in this study.

II. A NEW MDVSP MODEL FORMULATION

1. The Basic Assumptions Made for the MDVSP

Figure 1 graphically illustrates the MDVSP. The horizontal and vertical axes represent time span and depot locations respectively. N trips have to be performed by a minimal number of vehicles daily. Each box represents a single trip, which includes service duration, departure depot, arrival depot, and service vehicle index. The arc, connecting two boxes, indicates the sequence of trips served by a vehicle. Each block includes trips that satisfy constraints and can

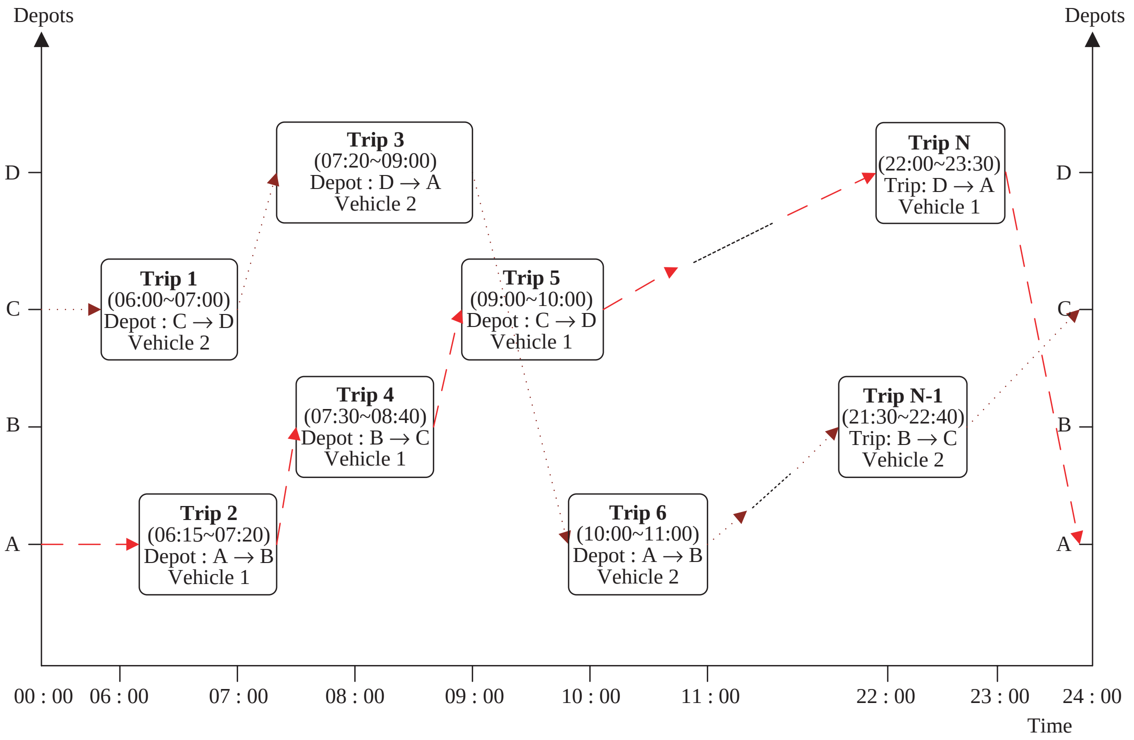


Fig. 1 Daily operation diagram for multiple depot vehicle scheduling

be served by one vehicle, and each vehicle must return to the same depot from where it departs. Blocks in the MDVSP are acyclic, i.e. there are no cycles in the block, because of the time dimension. A number of basic assumptions for this MDVSP made by this study are listed below.

(1) Each trip must be served by one and only one vehicle;
(2) All trips need to be served by a minimal number of vehicles;
(3) The following attributes of each trip are known. That is, each trip i is associated with the information of departure depot, arrival depot, departure time, length of service time (or duration), specific route number, and cost of serving the trip;
(4) Different types of deadhead trip (to be discussed later) are allowed;
(5) For any feasible solution, the costs of performing all trips ($\sum_{i=1}^{N} oc_i$) are the same, so this cost can be ignored.

The MDVSP is defined formally as: where $N = \{1, 2, \cdots, n\}$ represents the known trip set, and I denotes the feasible set of pairs of trips, each pair is connected by an arc. With each depot $p \in P$, this formulation associates the graph $G^p = (V^p, A^p)$, where $n + p$ denotes the pth depot, $V^p = N \cup \{n + p\}$, and $A^p = I \cup \{n + p\} \times N) \cup (N \times \{n + p\})$. The arc cost c_{ij}, $(i, j) \in A^p$, is independent of p if $(i, j) \in I$, while $c_{n+p,j}$ for $j \in N$, and $c_{i,n+p}$ for $i \in N$, which depends on p. x_{ij}^p is the flow for type p through arc $(i, j) \in A^p$.

2. Model Formulation

According to the notations of graph theory mentioned above, the proposed mathematical formulation for the MDVSP used in this study is introduced as follows.

Minimize

$$\alpha(\sum_{p \in P}\sum_{i,j \in N} c_{ij}x_{ij}^p)$$

$$+\beta\sum_{p \in P}(\sum_{j \in N} x_{n+p,j}^p + \sum_{j \in N} x_{j,n+p}^p)$$

$$+\lambda(\sum_{p \in P}\sum_{i,j \in N}(\left|\frac{|\gamma_j - \gamma_i|}{(\gamma_j + \gamma_i)}\right| x_{ij}^p) + \omega\sum_{p \in P}(\sum_{i,j \in N}\xi_{ij}x_{ij}^p$$

$$+\sum_{j \in N}\xi_{j,n+p}x_{j,n+p}^p + \sum_{j \in N}\xi_{n+p,j}x_{n+p,j}^p) \qquad (1)$$

s.t.

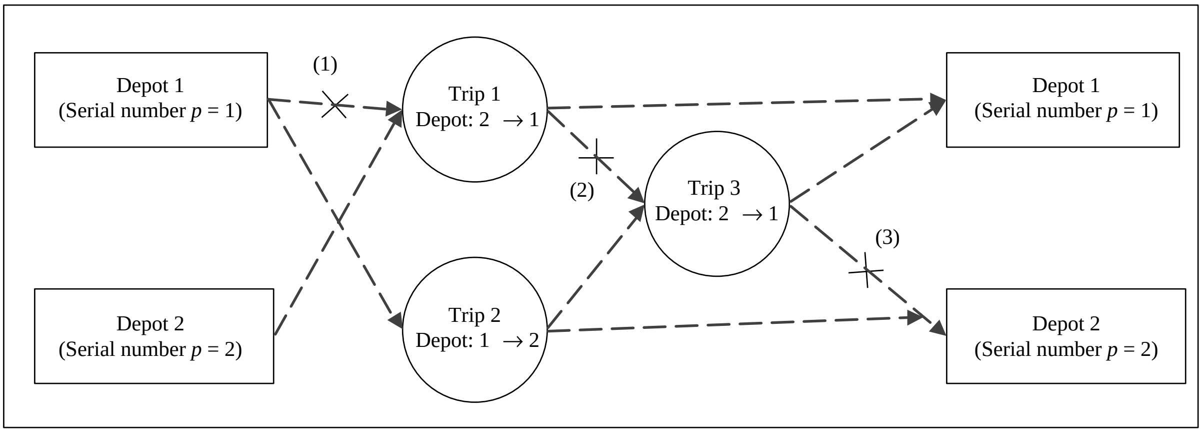
$$\sum_{i \in N} x_{ij}^p + x_{n+p,j}^p = \sum_{i \in N} x_{ji}^p + x_{j,n+p}^p \quad \forall p \in P \;\; \forall j \in N \qquad (2)$$

$$\sum_{p \in P}(\sum_{j \in N} x_{ij}^p + x_{i,n+p}^p) = 1 \qquad \forall i \in N \qquad (3)$$

$$\sum_{i \in N}(p - s_i)x_{n+p,i}^p \le 0 \qquad \forall p \in P \qquad (4)$$

$$\sum_{j \in N}(p - e_j)x_{j,n+p}^p \le 0 \qquad \forall p \in P \qquad (5)$$

$$\sum_{p \in P}(e_i - s_j)x_{ij}^p \le 0 \qquad \forall (i, j) \in I \qquad (6)$$



(1) It makes sure that deadhead occurring from depot 1 to departure terminal of trip 1 will not happen when Eq. (4) exists in the model formulation.

(2) It makes sure that no deadhead occurrence between trip 1 and trip 3 when Eq. (6) exists in the model formulation.

(3) It makes sure that deadhead occurring from arrival terminal of trip 3 to depot 2 will nothappenwhen Eq. (5) exists in the model formulation.

Fig. 2 Example of explanation for Eqs. (4), (5), and (6)

$$x_{ij}^p \in \{0, 1\} \qquad \forall p \in P \qquad (7)$$

$$x_{ij}^p = \begin{cases} 1 & \text{if trips } i \text{ and } j \text{ are served} \\ & \text{consecutively by avehicle} \\ & \text{from depot } p \\ 0 & \text{otherwise;} \end{cases}$$

The objective function (Eq. (1)) can be decomposed into four parts, seeking to minimize total costs. Its logical meaning can be described as follows.

The first part $(\alpha(\sum_{p \in P} \sum_{i, j \in N} c_{ij} x_{ij}^p))$ of Eq. (1) represents the summation of total vehicle idle time cost, since extra idle time at depots usually increases personnel cost or makes the crew scheduling problem more difficult (Baita *et al.*, 2000). The cost of vehicle idle time can be measured based on the pay rate (dollars/min) for a hired driver, represented in the model as parameter α. The second part $(\beta \sum_{p \in P} (\sum_{j \in N} x_{n+p,j}^p + \sum_{j \in N} x_{j,n+p}^p))$ of Eq. (1) represents the total cost of vehicle depreciation, measured based on its purchase cost and its reasonable lifetime. The third part $(\lambda(\sum_{p \in P} \sum_{i, j \in N} (\left\lceil \frac{|\gamma_j - \gamma_i|}{(\gamma_j + \gamma_i)} \right\rceil x_{ij}^p))$ of Eq. (1) represents the total routes changing time with the best arrangement. Parameter λ represents the penalty for every route change. For any two trips i, j, γ_i and γ_j representing the specific route numbers respectively, they are compatible when serving trip j immediately after trip i using the same vehicle. If $\gamma_i \neq \gamma_j$, the value of the the ceiling function $(\left\lceil \frac{|\gamma_j - \gamma_i|}{(\gamma_j + \gamma_i)} \right\rceil)$ is one, which represents one route-change per block, and the cost value of λ increases in the objective function. On the contrary, if $\gamma_i = \gamma_j$, the value of the ceiling function $(\left\lceil \frac{|\gamma_j - \gamma_i|}{(\gamma_j + \gamma_i)} \right\rceil)$ is zero, which means there is no penalty to the objective function value. The last part $(\omega \sum_{p \in P} \cdot (\sum_{i, j \in N} \xi_{ij} x_{ij}^p + \sum_{j \in N} \xi_{j,n+p} x_{j,n+p}^p + \sum_{j \in N} \xi_{n+p,j} x_{n+p,j}^p))$ of Eq. (1) represents the total cost of performing deadhead trips. This formulation allows deadhead trips between different depots if the arrival terminal of trip i and the departure terminal of trip j are not the same when these two trips i, j are compatible. Based on this definition, this part of the objective function consists of three items: the weighted cost for deadhead trips occurring from depots to departure terminal of trips, between the departure and arrival terminals of any trips, and from departure terminal of trips to all depots. While deadhead trips take place in all blocks in the situation described above, the cost of $\omega \times \xi_{ij}$ increases in the objective function. These four parts of Eq. (1) are commonly used in solving vehicle scheduling problems.

Equation (2) is a flow conservation constraint which makes sure the number of arcs leaving a trip is equal to the arcs entering the same trip. Eq. (3) is a constraint to ensure that each trip is served by one and only one vehicle. Eqs. (4), (5), and (6) can be regarded as Either-Or constraints which limit the chance of a deadhead trip occurring. Eq. (4) prohibits deadhead occurrence from depots to any trips when the serial number of a depot is not the same as departure terminal of a trip, as Fig. 2 shows. Based on this constraint, if p (number of depots) is not the same as s_j (departure terminal of trip j), then the variable $x_{n+p,j}^p$ must be equal to zero. This ensures that a deadhead trip from a depot p to departure terminal of trip j will not happen in any block. Eq. (5) is similar to Eq. (4), which prohibits a deadhead trip from any trips to depots when arrival terminal of that trip is not the same as the serial number of a depot, as Fig. 2 shows. In this constraint, if p is not the same as e_i (arrival terminal of trip i), then the variable $x^p_{i,\, n+p}$ must be equal to zero. This ensures that a deadhead trip from arrival terminal of trip i to a depot p will not happen in any block. Likewise, Eq. (6) ensures no deadhead occurrence between any two trips when arrival terminal of a trip is not the same as departure terminal of the other trip, as Fig. 2 shows. According to this constraint, if s_j is not the same as e_i, then the variable x^p_{ij} must be equal to zero. Eq. (7) indicates all variables are binary. The solution for this MDVSP problem formulation finds a set of blocks to minimize the objective value so an ideal vehicle allocation plan with a minimal fleet size can be identified.

3. Deadhead Trips Adopting Strategies

Deadhead trips can be classified into two categories, the "*depot deadhead trip*" occurring between depot and trip, and the "*street deadhead trip*" occurring between two trips. The deadhead trip is a common status in vehicle scheduling problems. The current study adopts four strategies representing deadhead trip tolerance of these two categories. Constraint Eqs (4), (5) and (6), defined below, decide the different strategies.

(1) *All depot deadhead trips and street deadhead trips are allowed*: Eqs. (4), (5) and (6) should be included in the model.

(2) *Depot deadhead trips are allowed and street deadhead trips are not allowed*: Eqs. (4) and (5) should be included in the model.

(3) *Depot deadhead trips are not allowed and street deadhead trips are allowed*: only Eq. (6) should be included in the model.

(4) *All depot deadhead trips and street deadhead trips are not allowed*: none of Eqs. (4), (5) or (6) should be included in the model.

If the fourth strategy is adopted, the total number of trips departing from each depot must be equal to the total number of trips returning to the same depot, i.e. Eq. (8) should be included in the model. Otherwise, the solution is not feasible.

$$\sum_{i \in N} x^p_{n+p,\, i} = \sum_{j \in N} x^p_{j,\, n+p} \quad \forall p \in P \tag{8}$$

III. A GREEDY HEURISTIC ALGORITHM FOR SOLVING THE MDVSP MODEL

It is already known that the MDVSP is characterized as NP-hard when depot number exceeds two, and the single depot problem can be solved in the polynomial time (Hadjar *et al*., 2006). The vehicle scheduling problem with single depot can be modeled as a minimum cost flow problem and solved with efficient algorithms to optimality (Gintner *et al*., 2005). This work develops a heuristics algorithm applying the divide-and-conquer technique to solve the MDVSP model, according to the above-mentioned discussion.

1. Problem Solving Procedure

According to the graph theory concept, the proposed MDVSP formulation includes P networks, where P is the number of depots (i.e. there are P depots-as-nodes). There are N trips-as-nodes in each network, where N is the total number of trips. Besides, all networks correspond to P depots-as-nodes. Under the proposed heuristic algorithm, the MDVSP can be divided into several simplified sets of networks based on P depots. Each network corresponds to a single depot vehicle scheduling problem (SDVSP), while each problem takes into account all depots. Select a SDVSP associated with the chosen depot from the MDVSP formulation as a starting point, and ignore other models for a moment. Mathematically, the simplified SDVSP can be solved efficiently. Follow this with a search for feasible blocks in which *depot deadhead trips* are not allowed for the chosen depot. The variables for these feasible blocks are fixed in the following searches. This process narrows the search scope and increases search efficiency in the remaining SDVSP models. The problem solving procedure repeats until every depot is selected and the MDVSP solution is obtained by aggregating all feasible SDVSP solutions.

This heuristic can be regarded as a greedy algorithm, and the process of solving each SDVSP model considers all the interactions between the total number of trips and all depots. A greedy algorithm can generally achieve a solution by making sequential choices, each of which simply looks at the best choice at the moment (Neapolitan and Naimipour, 2004). The traditional greedy rule performs following a defined criterion which satisfies some local optimal considerations at a time. According to the requirement of MDVSP, it is regarded as a reasonable rule for finding the maximum number of trips served by each depot in the proposed greedy heuristic algorithm. It is known that, although a greedy algorithm is no guarantee of an optimal solution, one often leads to an efficient solution fast (Neapolitan and Naimipour, 2004).

This solving procedure can be defined as below:

Step 1. Assign depot number in ascendant order following the number of trips. And divide the given MDVSP formulation into a set of

SDVSP models.

Step 2. Set $p := 1$.

Step 3. If $p := |P|$, go to Step 7, otherwise proceed to Step 4.

Step 4. Solve the SDVSP with an efficient algorithm while only considering depot p.

Step 5. Add every feasible block in which *depot deadhead trips* are not allowed into set S and fix the variables in these feasible blocks.

Step 6. Define, $p := p + 1$, and proceed to Step 3.

Step 7. Output set S as a final solution to the given MDVSP.

2. Selection Rule for A Starting Point in the Greedy Heuristic Algorithm

Selecting a good starting point rule is generally an important step for a greedy algorithm. Each depot is first assigned a serial number continuously following the number of trips in our solving procedure. This study adopts three different rules to precede problem computation: Following increasing order, decreasing order, and random order of depots in order to compare the results of selecting different rules. The selection rules are described as follows.

(a) *Select depots following increasing serial number order*: This heuristic algorithm first divides the given MDVSP into a serial of SDVSP's following an increasing depot number order. The solution procedure is introduced in section III.

(b) *Select depot following decreasing serial number order*: The steps of this heuristic algorithm are similar to (a), except for following the decreasing order of depot serial number. The solution procedure is the same as the one mentioned in section III except steps 2, 3 and 6 are replaced by:

Step 2. Set $p := |P|$, and ignore the other depots for a moment.

Step 3. If $p := 1$ go to Step 7, otherwise proceed to Step 4.

Step 6. Define $p := p – 1$, and returns to Step 3.

(c) *Select depot serial number arbitrarily*: The steps of this heuristic algorithm are also similar to both heuristics mentioned above, except instead of following the increasing or decreasing order of depot serial numbers, it selects the depot serial number arbitrarily. The solution procedure is also similar to the one mentioned in section III except that Step 2, 3 and 6 are replaced by the following:

Step 2. Set $U = P$. Select an initial depot arbitrarily $p_i \in U$, and $U = U – p_i$.

Step 3. If $U = \varnothing$, go to Step 7, otherwise proceed to Step 4.

Step 6. Select another depot arbitrarily $p_j \in U$, and define $U = U – p_j$, then return to Step 3.

IV. COMPUTATIONAL TESTS

1. Results of A Case Study

The adopted greedy heuristic solving procedures solve a real case to demonstrate the efficiency and effectiveness of the proposed formulation and the heuristic algorithm. A MDVSP model is set up to replicate the situation occurring in the Kinmen Bus Administration (KBA). The theoretical optimal solution and existing arrangement are used to compare with solutions obtained by the proposed problem solving algorithms. The computations are executed on a personal computer with a Pentium-IV CPU 3.2GHz and 3.25GB main memory. All models in the numerical experiments are solved with ILOG CPLEX 7.0.

The KBA case includes three depots, A, B, C, currently serving six routes (A-A, A-B, A-C, B-B, B-C, C-C). Twenty-nine vehicles are operated daily, offering a total of 395 trips. The three starting point selection rules have all show possible computation result differences. The parameters' values provided by the manager in the KBA case are set as follows: α = 3.33(NT dollar/min), β = 1,644(NT dollar/per vehicle per day), λ = 10 (NT dollar/per round), and ω = 4.67(NT dollar/min).

The experiments adopt only the strategy of *all depot deadhead trips and street deadhead trips are allowed*, for computation convenience. Table 1 shows the computation results. Compared to the existing arrangement represented as KBA, the solutions obtained by the three different starting point selection rules reduce the total cost by 19.99%, 19.94%, and 20.05% respectively. Similarly, compared to KBA, the theoretical optimal solution reduces the total cost by 21.77%, showing that the solutions obtained by the proposed heuristic algorithms are close to the optimal solution. Cost saving mainly comes from improving vehicle idle time and preventing unnecessary route changing. Improved vehicle idle time percentages are from 44.00% to 44.40%, the theoretical optimal solution improves 48.82%. Improved percentages on cost of changing route are from 41.79% to 43.28%, the theoretical optimal solution improves 41.79%. Three vehicles are saved by all three selection rules applying in the heuristic algorithm and the percentage of vehicle capital cost saving are all 10.34%. The savings of deadhead trips cost in the three results obtained by the heuristic algorithm are all worse than KBA and the theoretical optimal solution. This result shows that applying the heuristic algorithm is not

Table 1 Test results for the KBA problem

Note: The exchange rate was NT $1 = US $0.03051

	Vehicle idle time (NT dollars)	Vehicle depreciation cost (NT dollars)	Changing route cost (NT dollars)	Deadhead trip cost (NT dollars)	Total cost (NT dollars)	Gap with exact (%)	# of vehicles used
Heuristic 1[a]	13,905	42,744	760	1,987	59,396	2.23	26
KBA	25,008	47,676	1,340	215	74,239	–	29
Heuristic 2[b]	14,004	42,744	760	1,926	59,434	2.29	26
KBA	25,008	47,676	1,340	215	74,239	–	29
Heuristic 3[c]	13,904	42,744	780	1,926	59,354	2.16	26
KBA	25,008	47,676	1,340	215	74,239	–	29
Exact	12,797	44,388	780	107	58,072		27

	Saving over the solution of KBA (%)					Computation time (Sec)	Saving time (%)
	Vehicle idle time	Vehicle depreciation cost	Changing route cost	Deadhead trip	Total cost		
Heuristic 1[a]	44.39	10.34	43.28	-824.18	19.99	2.9	99.43
KBA	–	–	–	–	–	–	–
Heuristic 2[b]	44.00	10.34	43.28	-795.81	19.94	3.5	99.32
KBA	–	–	–	–	–	–	–
Heuristic 3[c]	44.40	10.34	41.79	-795.81	20.05	4.3	99.16
KBA	–	–	–	–	–	–	–
Exact	48.82	6.89	41.79	50.23	21.77	511.0	

[a]: Using the selection rule with increasing order of depot serial number
[b]: Using the selection rule with decreasing order of depot serial number
[c]: Using the selection rule with depot serial number at random

necessarily a guarantee of saving improvement in all cost items. Compared to the theoretical optimal solution, the total cost reduction obtained by the heuristic algorithm using the three different starting point selection rules ranges from 2.16% to 2.29%, close enough in this case. The computation times of the heuristic algorithm by three different starting point selection rules are all significantly less than that for finding the theoretical optimal solution. The time saving rates are 99.43%, 99.32% and 98.16% by three different selection rules respectively. This result indicates that the heuristic approach solves the MDVSP much more efficiently, a very important result in practical application.

This research modifies the KBA case by adding the number of trips arbitrarily to evaluate the performance of the greedy algorithm applied to a larger scale problem such as a global airline or maritime operation. Testing is conducted for cases composing 593, 790, 988, 1,185, 1,383, and 1,580 trips. Table 2 summarizes computation results. Findings show that the number of variables in the problem increases significantly following the increasing problem scale (i.e numbers of trips). The result shows that under the same computer processing capability, the optimal solution cannot be found if the number of trips exceeds 1,383. The result also shows that compared to objective values in solutions obtained by the heuristic algorithm compared to theoretical optimal solutions, differences remain within the range of 1.41% to 2.99%. Computation times on the other hand, save more than 97.99%.

2. Sensitivity Analysis of the MDVSP Parameter Values

Sensitivity analysis shows the difference in results by changing MDVSP parameter values. The two important parameters adopted in the proposed MDVSP model are vehicle depreciation cost and cost of deadhead trips per minute because the necessary fleet size in the KBA case is a crucial concern for decision makers and the savings of deadhead trips cost in the three results obtained by the heuristic algorithm are all worse than KBA and the theoretical optimal solution. For testing purposes, ten different scenarios for each parameter value scenario (vehicle depreciation cost, deadhead trip cost) are increased 10% arbitrarily every time (seen Table 3).

Table 2 Test results for six scenarios

No. of trips	Objective value (dollars)				Computation time (second)				No. of variables				No. of vehicles used (vehicles)			
	H1	H2	H3	E.O.[a]	H1	H2	H3	E.O.[a]	H1[b]	H2[b]	H3[b]	E.O.[a]	H1	H2	H3	E.O.[a]
395	59,396 (2.23%)	59,434 (2.29%)	59,354 (2.16%)	58,072	2.9 [99.43%]	3.5 [99.32%]	4.3 [98.16%]	511.0	69,473	69,473	69,473	208,419	26	26	26	27
593	100,468 (2.27%)	100,123 (1.94%)	99,583 (1.41%)	98,180	18.8 [98.61%]	20.6 [98.48%]	27.2 [97.99%]	1,353.3	155,847	155,847	155,847	467,541	42	41	41	42
790	136,645 (2.76%)	136,970 (2.99%)	136,015 (2.31%)	132,867	49.7 [98.76%]	52.1 [98.71%]	71.0 [98.23%]	4,023.8	275,705	275,705	275,705	827,115	59	59	60	61
988	172,631 (2.42%)	173,002 (2.62%)	173,389 (2.84%)	168,464	82.4 [99.02%]	90.5 [98.92%]	117.9 [98.60%]	8,415.8	430,306	430,306	430,306	1,290,918	75	74	72	76
1185	201,534 (2.15%)	201,496 (2.13%)	203,286 (2.99%)	197,200	137.4 [99.15%]	173.6 [98.92%]	228.3 [98.59%]	16,187.0	619,240	619,240	619,240	1,857,720	88	88	87	92
1383	228,668 (–)	227,864 (–)	229,055 (–)	c	228.0 [–]	302.5 [–]	360.8 [–]	c	842,998	842,998	842,998	2,528,994	99	98	99	c
1580	265,192 (–)	264,106 (–)	265,847 (–)	c	368.1 [–]	502.1 [–]	593.4 [–]	c	1,100,416	1,100,416	1,100,416	3,301,248	115	115	114	c

[a] : Exact optimization;

[b] : This value is the most variables in three submodels;

[c] :The 1,383 and 1,580 trips can not be solved by the exact optimization on the test machine;

( ) : The percentage in parentheses is the gap with optimal solution;

[ ] : The percentage in braces is the time-saving rate.

Table 3 Sensitivity analysis of depreciation cost for the KBA problem

The exchange rate was NT $1 = US $0.03051

Depreciation cost of one vehicle (dollars/ vehicle)	Total cost				Vehicle idle time				Vehicle depreciation cost				Changing routes cost				Deadhead trips cost			
	H1	H2	H3	E.O.[a]	H1	H2	H3	E.O.[a]	H1	H2	H3	E.O.[a]	H1	H2	H3	E.O.[a]	H1	H2	H3	E.O.[a]
1644.0	59,396 (2.23%)	59,434 (2.29%)	59,354 (2.16%)	58,072	13,905	14,004	13,904	12,797	42,744 [26]	42,744 [26]	42,744 [26]	44,388 [27]	760	760	780	780	1,987	1,926	1,926	107
1808.4	63,326 (1.47%)	63,326 (1.47%)	63,588 (1.87%)	62,398	15,732	15,732	15,732	14,453	45,210 [26]	45,210 [25]	45,210 [25]	47,018 [25]	720	720	720	750	1,664	1,664	1,926	177
1972.8	67,436 (1.17%)	67,497 (1.25%)	67,698 (1.55%)	66,650	15,732	15,732	15,732	16,256	49,320 [25]	49,320 [25]	49,320 [25]	49,320 [25]	720	720	720	720	1,664	1,725	1,926	354
2137.2	72,070 (1.82%)	71,284 (0.74%)	71,808 (1.46%)	70,760	15,732	15,733	15,733	16,256	53,430 [25]	53,430 [25]	53,430 [25]	53,430 [25]	720	720	720	720	2,188	1,402	1,926	354
2301.6	75,717 (1.12%)	75,394 (0.70%)	75,656 (1.04%)	74,870	15,732	15,733	15,733	16,256	57,540 [25]	57,540 [25]	57,540 [25]	57,540 [25]	720	720	720	720	1,725	1,402	1,664	354
2466.0	80,089 (1.38%)	79,504 (0.66%)	79,766 (0.99%)	78,980	15,732	15,732	15,732	16,256	61,650 [25]	61,650 [25]	61,650 [25]	61,650 [25]	720	720	720	720	1,987	1,402	1,664	354
2630.4	83,937 (1.01%)	83,937 (1.01%)	83,876 (0.94%)	83,090	15,732	15,732	15,732	16,256	65,760 [25]	65,760 [25]	65,760 [25]	65,760 [25]	720	717	723	720	1,725	1,725	1,664	354
2794.8	88,047 (0.96%)	88,047 (0.96%)	87,986 (0.89%)	87,200	15,732	15,732	15,732	16,256	69,870 [25]	69,870 [25]	69,870 [25]	69,870 [25]	720	720	720	720	1,725	1,725	1,664	354
2959.2	92,157 (0.92%)	92,157 (0.92%)	92,096 (0.85%)	91,310	15,732	15,732	15,732	16,256	73,980 [25]	73,980 [25]	73,980 [25]	73,980 [25]	720	720	720	720	1,725	1,725	1,664	354
3123.6	96,267 (0.88%)	96,267 (0.88%)	96,206 (0.82%)	95,420	15,732	15,733	15,733	16,256	78,090 [25]	78,090 [25]	78,090 [25]	78,090 [25]	720	720	720	720	1,725	1,725	1,664	354

[a] : Exact Optimization;

( ) : The percentage in parentheses is the gap with optimal solution;

[ ] : The value in braces is the number of vehicles used.

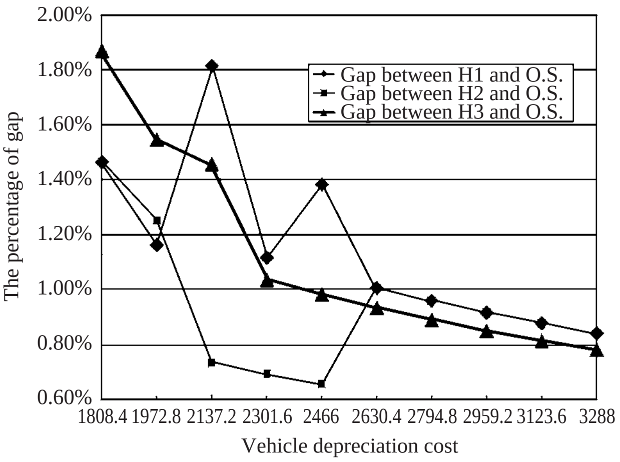


Fig. 3 The percentage of gap between the heuristic algorithms and the theoretical optimal solutions (o.s.) vs. vehicle depreciation cost

The first sensitivity analysis shows the difference in resulting from changing vehicle depreciation cost. Table 3 shows that total cost increases if vehicle depreciation cost increases. But it is quite obvious that the other three cost items, such as vehicle idle time, changing routes and deadhead trips, all remain the same no matter which starting point selection rule is adopted while vehicle depreciation cost exceeds 2,630.4. The total cost increases steadily as vehicle depreciation cost increases, as shown. Results show that vehicle depreciation cost is insensitive to the other three cost items and total cost while the value is larger than 2,630.4. When vehicle depreciation cost reaches 1972.8, the number of required vehicles remains twenty-five, which seems to be the minimum number needed in the KBA case. Results show that the difference between the results of total cost obtained by the heuristic algorithm and the theoretical optimal solution reduce as vehicle depreciation cost increases. Fig. 3 shows the percentage of gap between the heuristic algorithm using the three different starting point selection rules and the theoretical optimal solutions versus vehicle depreciation cost for the KBA.

The second sensitivity analysis shows the difference in results from changing deadhead trip cost. Table 4 again shows that total cost increases if deadhead trip cost increases. Results in the ten scenarios show that vehicle idle time costs remain nearly the same for each selection rule, but compared to the theoretical optimal solution, the vehicle idle time costs increase as deadhead trip costs increase. Deadhead trip cost on the other hand, seems to have no effect on vehicle depreciation costs and changing routes cost because vehicle depreciation cost and changing routes cost remain nearly the same. Results show that deadhead trip cost has little impact on solutions obtained by the heuristic algorithm, but has most effect on the theoretical optimal solution. A large gap in deadhead trip cost between the heuristic algorithm using the three different starting point selection rules and the theoretical optimal solution exists, as shown. This gap indicates that the difference between total cost obtained by the heuristic algorithm and the theoretical optimal solution is mostly reflected in deadhead trip cost. Results also show that the differences between the results of total cost obtained by the heuristic algorithm and the theoretical optimal solution tend to increase as deadhead trip cost increases. Fig. 4 shows the percentage of gap between the heuristic algorithm using the three different starting point selection rules and the theoretical optimal solutions versus deadhead trip cost for the KBA.

V. CONCLUSIONS

This paper presents a new model formulation for the MDVSP for public transit services. This study also proposes a greedy heuristic algorithm based on divide-and-conquer technique to solve the MDVSP. Performances of the proposed model and the heuristic algorithm have been shown to be very efficient based on real case data provided by the KBA. Solution results are also reasonably close to the theoretical optimal solution. Computer running time can be reduced more than 98.16% compared to the theoretical optimal solution. The problem-solving algorithm also provides better capability to deal with lager scale problems (representing more depots and trips). Sensitivity analyses also show that the lower bound of vehicle depreciation cost leading to the minimum number of required vehicles seems to exist in instances of MDVSP. The analyses also show that the greater the vehicle depreciation cost, the closer the solutions obtained by the heuristic algorithm are to the theoretical optimal solution, but the greater the deadhead trip cost, the further the solutions obtained by the heuristic algorithm are from the theoretical optimal solution conversely. Excellent computation efficiency, good larger scale problem solving capability, and good quality solution, all suggest that the findings in this study probably shorten the distance between a theoretical study and practical application in dealing with real world problems.

The transportation industry, while considering the vehicle scheduling problem, needs to combine the arrangement needs of both vehicles and crew. Crew assignment is not discussed in this study at this time. Exploring this issue in the future would be interesting. Fortunately, compared to traditional problem definitions, the proposed model formulation and problem-solving algorithm provide excellent computation capability. The model also provides a better chance to solve this type of problem. Techniques for solving the MDVSP

Table 4 Sensitivity analysis of deadhead trips cost for the KBA problem

The exchange rate was NT $1 = US $0.03051

Depreciation trips cost per minute (dollars/ min)	Total cost				Vehicle idle time				Vehicle depreciation cost				Changing routes cost				Deadhead trips cost			
	H1	H2	H3	E.O.[a]	H1	H2	H3	E.O.[a]	H1	H2	H3	E.O.[a]	H1	H2	H3	E.O.[a]	H1	H2	H3	E.O.[a]
4.67	59,396 (2.23%)	59,434 (2.29%)	59,354 (2.16%)	58,072	13,905	14,004	13,904	12,797	42,744 [26]	42,744 [26]	42,744 [26]	44,388 [27]	760	760	780	780	1,987	1,926	1,926	107
5.14	59,306 (1.85%)	59,117 (1.54%)	59,546 (2.25%)	58,140	13,905	14,004	13,904	12,854	42,744 [26]	42,744 [26]	42,744 [26]	44,388 [27]	760	760	780	780	1,897	1,609	2,118	118
5.60	60,033 (3.15%)	59,190 (1.77%)	59,738 (2.68%)	58,208	13,905	14,004	13,904	12,911	42,744 [26]	42,744 [26]	42,744 [26]	44,388 [27]	760	760	780	780	2,624	1,682	2,310	129
6.07	59,571 (2.17%)	59,330 (1.78%)	60,610 (3.85%)	58,276	13,905	14,004	13,904	12,968	42,744 [26]	42,744 [26]	42,744 [26]	44,388 [27]	760	760	780	780	2,162	1,822	3,182	140
6.54	59,089 (1.26%)	59,554 (2.03%)	59,880 (2.57%)	58,343	13,905	14,004	15,732	13,025	42,744 [26]	42,744 [26]	41,100 [25]	44,388 [27]	760	760	720	780	1,680	2,046	2,328	150
7.01	59,901 (2.49%)	59,608 (2.01%)	60,704 (3.78%)	58,410	13,905	14,004	13,904	13,081	42,744 [26]	42,744 [26]	42,744 [26]	44,388 [27]	760	760	780	780	2,492	2,100	3,276	161
7.47	60,485 (3.32%)	59,748 (2.13%)	60,922 (4.01%)	58,478	13,905	14,004	13,904	13,138	42,744 [26]	42,744 [26]	42,744 [26]	44,388 [27]	760	760	780	780	3,076	2,240	3,494	172
7.94	59,892 (2.25%)	60,332 (2.96%)	60,252 (2.83%)	58,546	13,905	14,004	13,904	13,195	42,744 [26]	42,744 [26]	42,744 [26]	44,388 [27]	760	760	780	780	2,483	2,824	2,824	183
8.41	60,399 (2.96%)	60,028 (2.36%)	61,358 (4.47%)	58,613	13,905	14,004	13,904	13,252	42,744 [26]	42,744 [26]	42,744 [26]	44,388 [27]	760	760	780	780	2,990	2,520	3,930	193
8.87	60,067 (2.31%)	59,900 (2.04%)	61,202 (4.12%)	58,680	13,905	14,004	15,732	13,308	42,744 [26]	42,744 [26]	41,100 [25]	44,388 [27]	760	760	720	780	2,658	2,392	3,650	204

[a] : Exact Optimization;

( ) : The percentage in parentheses is the gap with optimal solution;

[ ] : The value in braces is the number of vehicles used.

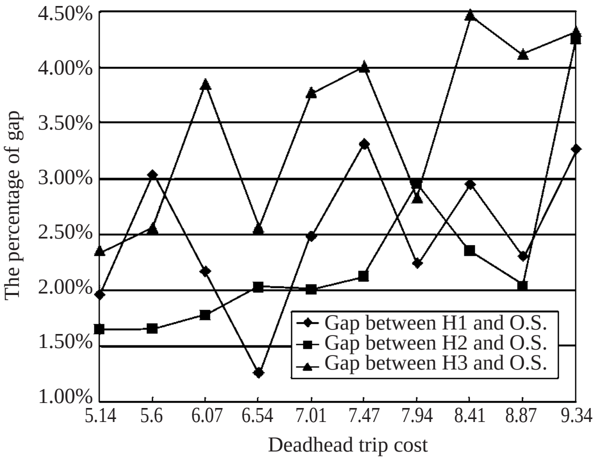


Fig. 4 The percentage of gap between the heuristic algorithms and the theoretial optimal solutions (o.s.) vs. deadhead trip cost

are still looking for a method that improves the gap of deadhead trip cost between the heuristic algorithm and the theoretical optimal solution in order to enhance solution quality. Finally, we hope that the findings in this study can be applied to develop a decision supporting system to help solve real-world vehicle scheduling problems, not just for the local public transit industry, but also for the global transportation industry (including airline and maritime industries).

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NOMENCLATURE

A	arcs in graph
c_{ij}	arc cost
e_i	arrival depot
G	graph representation
i, j	specific trip in known trip set
I	feasible set of pairs of trips
l_i	length of service time, or duration
n	number of trips
N	trip set
Oc_i	cost of serving the trip
p	specific depot in known depot set
P	depot set
s_i	departure depot
U	temporary set of depot set
V	vertexes in graph
x_{ij}^p	flow for type p through arc $(i, j) \in A^p$

Greek Symbols

α	cost for vehicle idle time (dollars/min)
β	cost for half of the depreciation per vehicle (dollars/vehicle)
γ_i	specific route number
λ	cost/penalty for every time changing route in a block (dollars/per change)
τ_i	departure time
ξ_{ij}	travel time from the end depot of trip i to the start depot of trip j
ω	cost of deadhead trips per minute (dollars/min)

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