Chapter 5

Conclusions and Recommendations

5.1. Conclusions

Process capability indices (PCIs), including C_p, C_{PU}, C_{PL}, C_{pk}, C_{pm} and C_{pmk}, the purpose of which is to provide numerical measures of whether or not the ability of a manufacturing process meets a predetermined level of production tolerance, have received considerable research attention and increased usage in process assessments and purchasing decisions. Proper understanding and accurate estimation of the capability index is essential for the company to maintain a capable supplier. The usual practice of judging process capability by evaluating the point estimates of process capability indices, have flaws since there is no assessment of the sampling errors. As the use of the capability indices grows more widespread, users are becoming educated and sensitive to the impact of the estimators and their sampling distributions, learning that capability measures must be reported in confidence intervals or via capability testing. Most of existing research works on capability analysis have focused on the traditional frequency approaches. However, the sampling distributions of PCI estimators are usually so complicated, this makes establishing the exact confidence interval very difficult.

Bayesian statistical techniques are an alternative to the frequency approach. These techniques specify a prior distribution for the parameter of interest, in order to obtain the posterior distribution for the parameter. We then could infer about the parameter by using its posterior distribution given the sample data. Cheng and Spiring (1989) proposed a Bayesian procedure for assessing process capability index C_p. Chan *et al.* (1988) applied a similar Bayesian approach to index C_{pm} under the assumption that the process mean μ is equal to the target value T. Shiau *et al.* (1999b) derived the posterior distributions for C_p^2, C_{pm}^2 under the restriction that process mean μ equals to the target value T, and C_{pk}^2 under the restriction that the process mean μ equals to the midpoint of the two specification limits, M, with respect to the two priors (a non-informative and a Gamma prior). However, the restriction of $\mu = T$ or $\mu = M$ is not a practical assumption for many industrial applications. A nice Bayesian procedure for assessing process capability index C_{pm} without the restriction on the process mean was proposed by Shiau *et al.* (1999a). They also applied Bayesian method for testing the index C_{pk} but under the restriction $\mu = M$. We note that in this case C_{pk} reduces to C_p. In this dissertation, we first consider testing the most popular capability index C_{pk} for processes with bilateral specifications relaxing the restriction on process mean and the indices C_{PU} and C_{PL} for processes with unilateral specifications based on Bayesian approach. The posterior probability, p, for which the process under investigation is capable, is derived. For processes with unilateral specifications, an accordingly Bayesian procedure for capability testing based on the one-sided indices C_{PU} and C_{PL}, is obtained.

In practice, manufacturing information regarding product quality characteristic is often derived from multiple samples rather than single sample, particularly, when a daily-based or weekly-based production control plan is implemented for monitoring process stability. To use estimators based on multiple samples and then interpret the results as if they were based on a single sample may result in incorrect conclusions. In order to use past in-control data from subsamples to make correct decisions regarding process capability, the distribution of the estimated PCIs based on multiple subsamples should be taken into account. Therefore, we further consider the problem of estimating and testing C_p, C_{pk}, C_{PU}, C_{PL} and C_{pm} with multiple samples based on Bayesian approach. The results obtained for C_p and C_{pm} with multiple samples in the dissertation, are generalizations of those obtained in Cheng and Spiring (1989) and Shiau *et al.* (1999a) from one single sample case to multiple samples case based on control chart data. To make this Bayesian procedure practical for in-plant applications, we tabulated the minimum values of $C^*(p)$ for which the posterior probability p reaches various desirable confidence levels with various commonly used capability requirement levels. The manufacturers can use the presented approach to perform quality testing and determine whether their processes are capable for reproducing product items satisfying customers' stringent quality requirements.

5.2. Recommendations

We remark again that these indices presented in the dissertation, are designed to monitor the performance for only normal and near-normal processes with symmetric tolerances, which are shown to be inappropriate for cases with asymmetric tolerances. For normal distributions, these PCI estimators based on $\bar{x} = \sum_{i=1}^{n} x_i / n$ and $s^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 / (n-1)$ are quite stable and reliable. However, for non-normal distributions, they become highly unstable since the distribution of the sample variance s^2 is sensitive to departures from normality. Somerville and Montgomery (1996) presented an extensive study to illustrate how poorly the normally based capability indices perform as a predictor of process fallout when the process is non-normally distributed. If the normally based capability indices are still used to deal with non-normal process data, the values of the capability indices are incorrect and might misrepresent the actual product quality. Therefore, normally-based process capability indices such as C_p, C_{pk}, C_{pm} and C_{pmk} are inappropriate to measure processes with non-normal distributions.

For non-normality of the distribution of X, Clements (1989) suggested that "6σ" be replaced by the length of the interval between the upper and lower 0.135 percentage points of the distribution and considered fitting a Pearson system distribution for X in order to obtain the required percentiles. Pearn *et al.* (1992) suggested replacing "6σ" in the denominator of C_p by "6θ", where θ is chosen so that the "capability" is not greatly affected by the shape of the distribution. English and Taylor (1993) examined the effect of the non-normality assumption on PCIs and concluded that C_{pk} is more sensitive to departures from normality than C_p. Kotz and Johnson (1993) provided a survey of works on the properties of PCIs and their estimators when the distribution is non-normal. Johnson *et al.* (1994) introduced a "flexible" PCI, which takes into account possible differences in variability above and below the target value. Vännman (1995) proposed a new family of indices $C_p(u,v)$, parametrized by (u,v), that includes many other indices as its special cases. Deleryd (1996) investigated the suitable u and v values of $C_p(u,v)$ when the process distribution is skewed. It is recommended that $C_p(1,1)$, which is equivalent to C_{pmk}, is most suited to handle non-normality in PCIs. Pearn and Chen (1997a) considered the generalization of $C_p(u,v)$ defined in the following, called $C_{Np}(u,v)$, which can be applied to processes with arbitrary distributions. Castagliola (1996) introduced a non-normal PCI calculation method by estimating the proportion of nonconforming items using Burr's distribution. A new index C_S proposed by Wright (1995) incorporates an additional skewness correction factor in the denominator of C_{pmk}. Shore (1998) proposed a new approach to analyzing non-normal quality data and demonstrated for process capability analysis. Chang *et al.* (2002) proposed a heuristic weighted standard deviation method to adjust the value of PCIs according to the degree of skewness by considering the standard deviations above and below the process mean separately. Tang and Than (1999) reviewed several methods and provided a comprehensive evaluation and comparison in their ability to handle non-normality.

Moreover, PCIs can be used only after it has been established that the manufacturing process is under statistical control. For applications where a routine-based data collection plans are implemented, a common practice on process control is to estimate the process capability by analyzing past "in control" data. To estimate σ we typically use either the sample standard deviation or the sample range. The control chart can be used as monitoring device or logbook to show the effect of changes in the process performance. We note that a process may be in control but not necessarily operating at an acceptance level. Thus, management intervention will be required either to improve the process capability, or to change the manufacturing requirements to ensure that the products meet at least the minimum acceptable level. If the process is out of control in the early stages of process capability analysis, it will be unreliable and meaningless to estimate process capability. In these cases the priority is to find and eliminate the assignable causes of variability in order to bring the process in-control. On the whole, capability indices are very powerful, but, like many powerful tools, can inflict heavy damage if used incorrectly. Properly calculated, they provide a whale of vital information concerning how the current output of a process satisfies customer requirements. Incorrectly applied and/or interpreted, these indices can generate an abundance of misinformation that will confuse practitioners, waste resources, and lead to make decision incorrectly.