

# Chapter 1

## Introduction

### 1.1. Background

As rapid advancement of the manufacturing technology, suppliers require their products be high quality with very low fraction of nonconformities. This is true, particularly, for high technology product requiring very low fraction of nonconformities, often measured in parts per million (PPM). Traditional methods for measuring fraction of nonconformities become inapplicable for those high quality processes since any manufacturing sample of reasonable size likely contains no defective product items. For this reason, recently developed process capability indices (PCIs), including  $C_p$ ,  $C_{PU}$ ,  $C_{PL}$ ,  $C_{pk}$ ,  $C_{pm}$  and  $C_{pmk}$ , have received substantial attention in the manufacturing industries, particularly, for companies making microelectronics devices and accessories demanding stringent quality requirements. Those indices have been widely used to monitor the actual process information with respect to the manufacturing specifications, and become the common language for process quality between the customer and the supplier, both internally within the organization and externally.

### 1.2. Motivation

Understanding process and quantifying process performance are essential for any successful quality improvement initiative. Process capability analysis has become an important and integrated part in the applications of statistical process control to the continuous improvement of quality and productivity. The relationship between the actual process performance and the specification limits or tolerance may be quantified using appropriate process capability indices.

The formulae of those capability indices are easy to understand and straightforward to apply. But, in practice, the process mean  $\mu$  and the process variance  $\sigma^2$  are usually unknown. In order to calculate the index value, sample data must be collected and a great degree of uncertainty may be introduced into capability assessments due to sampling errors. The approach by simply looking at the calculated values of the estimated indices and then make a conclusion on whether the given process is capable, is highly unreliable since the sampling errors have been ignored. As the use of the capability indices grows more widespread, users are becoming educated and sensitive to the impact of the estimators and their sampling distributions, learning that capability measures must be reported in confidence intervals or via capability testing. A reliable approach for testing process capability is to establish an interval estimate, for which we can assert, with a reasonable degree of certainty, that it contains the true PCI value. However, the construction of such an interval estimate is not trivial, since the distributions of the commonly used PCI estimators are usually quite complicated (see

Chan *et al.* (1988), Chou *et al.* (1990), Bissell (1990), Li *et al.* (1990), Zhang *et al.* (1990), Boyles (1991), Kirmani *et al.* (1991), Franklin and Wasserman (1992a), Kushler and Hurley (1992), Kotz *et al.* (1993), Nagata and Nagahata (1994), Chen and Hsu (1995), Tang *et al.* (1997), Zimmer and Hubele (1997), Vännman (1997), Pearn *et al.* (1998), Hoffman (2001), Zimmer *et al.* (2001), Pearn and Lin (2002, 2003), Pearn and Shu (2003a) for more details).

### 1.3. Previous Efforts

The use of process capability indices in industry initially began in the United States during early 1980s. Soon after, this explosion of use expanded into other industries such as automated, semiconductor, and IC assembly manufacturing industries, to determine product quality in order to meet stringent customers' specifications. Those capability indices provide the manufacturers a feasible method for measuring process quality. Based on those capability indices, the production department can trace and improve poor processes to meet customers' needs.

In the literature, several authors have promoted the use of various process capability indices and examined with different degrees of completeness. Examples include Chan *et al.* (1988), Chou *et al.* (1990), Boyles (1991), Pearn *et al.* (1992), Kushler and Hurley (1992), Rodriguez (1992), Kotz and Johnson (1993), Vännman and Kotz (1995), Bothe (1997), Spiring (1997), Vännman (1997), Kotz and Lovelace (1998), Franklin (1999), Palmer and Tsui (1999), Wright (2000), Jessenberger and Weihs (2000), Pearn and Shu (2003b), Vännman and Hubele (2003), and references therein. Applications of these indices include the manufacturing of semiconductor products (Hoskins *et al.* (1988)), head/gimbals assembly for memory storage systems (Rado (1989)), jet-turbine engine components (Hubele *et al.* (1991)), flip-chips and chip-on-board (Noguera and Nielsen (1992)), rubber edge (Pearn and Kotz (1994)), wood products (Lyth and Rabiej (1995)), aluminum electrolytic capacitors (Pearn and Chen (1997a)), audio-speaker drivers (Chen and Pearn (1997)), Pulux surround (Pearn and Chang (1998)), liquid crystal display module (Chen and Pearn (2002)), couplers and wavelength division multiplexers (Wu and Pearn (2003)). Other applications include performance measures on process with toolwear problem (Spiring (1989, 1991)), production process monitoring (McCoy (1991)), MPPAC (Multi-process Performance Analysis Chart) for in-plant applications to control the defectives and provide product quality assurance for factories with multiple processes or multiple quality characteristics (Singhal (1990, 1991), Pearn and Chen (1997b), Chen *et al.* (2001), Pearn and Shu (2003a)), supplier selections (Tseng and Wu (1991), Chou (1994)), capability measures for multiple manufacturing streams (Bothe (1999)), and many others.

In analyzing process performance for products with multiple quality characteristics, Chen (1994), Boyles (1996) and others have presented multivariate capability indices for assessing capability. Wang and Chen (1998) and Wang and Du (2000) proposed multivariate equivalents for  $C_p$ ,  $C_{pk}$ ,  $C_{pm}$ , and  $C_{pmk}$  based on the principal component analysis, which transforms numbers of original related measurement

variables into a set of uncorrected linear functions. Moreover, a comparison of three recently proposed multivariate methodologies for assessing capability are illustrated and their usefulness are discussed in Wang *et al.* (2000). Kotz and Johnson (2002) presented a thorough review for the development of process capability indices in the past ten years. Spiring *et al.* (2003) consolidated the research findings of process capability analysis for the period 1990–2002.

#### 1.4. Research Objectives

Most of existing research works for capability testing have focused on the traditional frequency approaches. However, the sampling distributions are usually so complicated, this makes establishing the exact confidence interval very difficult. Bayesian statistical techniques are an alternative to the frequency approach. These techniques specify a prior distribution for the parameter of interest, in order to obtain the posterior distribution for the parameter. We then could infer about the parameter by using its posterior distribution given the sample data. It is not difficult to obtain the posterior distribution when a prior distribution is given, even in the case where the form of the posterior distribution is complicated, as one can always use numerical methods or Monte Carlo methods (Berger (1985), Kalos and Whitlock (1986)) to obtain an approximate but quite accurate interval estimate. This is the advantage of the Bayesian approach over the traditional distribution frequency approach.

Cheng and Spiring (1989) proposed a Bayesian procedure for assessing process capability index  $C_p$ . Shiau *et al.* (1999a) applied a similar Bayesian approach to index  $C_{pm}$  and index  $C_{pk}$  but under the restriction that the process mean  $\mu$  equals to the midpoint of the two specification limits,  $M$ . We note that in this case  $C_{pk}$  reduces to  $C_p$ . In this dissertation, we first consider testing the most popular capability index  $C_{pk}$  for processes with bilateral specifications and the indices  $C_{PU}$  and  $C_{PL}$  for processes with unilateral specifications based on Bayesian approach.

A common practice of process capability estimation in the manufacturing industry is to first implement a daily-based or weekly-based sample data collection plan for monitoring/controlling the process stability, then to analyze the past “in control” data. However, statistical properties of the estimated PCIs based on one single sample, have been investigated extensively. The properties of the estimated PCIs based on multiple samples have been comparatively neglected. To use estimators based on several small subsamples and then interpret the results as if they were based on a single sample may result in incorrect conclusions. Therefore, it is more practical to develop a procedure for assessing process capability for cases with multiple samples. In the dissertation we further consider the problem of estimating and testing  $C_p$ ,  $C_{PU}$ ,  $C_{PL}$ ,  $C_{pk}$ ,  $C_{pm}$  based on multiple samples collected over time for an in-control process, and propose accordingly a Bayesian procedure for testing those capability indices. Practitioners can use the results obtained to determine whether their manufacturing processes are capable of reproducing products satisfying the preset process capability requirement when a daily-based or weekly-based production control plan is implemented for monitoring

process stability.

## 1.5. Organization

This dissertation is organized as follows. We first give a brief introduction of our dissertation in Chapter 1. In Chapter 2, a comprehensive literature review of the process capability analysis and some distributional and inferential properties of the estimated PCIs are provided. The calculations of critical value,  $p$ -value and lower confidence bound for testing process quality are also included. In Chapter 3, we consider testing the most popular capability index  $C_{pk}$  and the one-sided indices  $C_{PU}$  and  $C_{PL}$  using Bayesian approach based on single sample. The posterior probability  $p$  for which the process under investigation is capable is derived. Subsequently, we consider the problem of estimating and testing indices  $C_p$ ,  $C_{PU}$ ,  $C_{PL}$ ,  $C_{pk}$  and  $C_{pm}$  with multiple samples rather than single sample, particularly, when a daily-based or weekly-based production control plan is implemented for monitoring process stability. For illustrative purpose, several real-world application examples are presented. Finally, some conclusions and recommendations are made in Chapter 5.

