Chapter 1

Introduction

1.1 Motivation

Process capability indices, which establish the relationships between the actual process performance and the manufacturing specifications, have been the focus of recent research in quality assurance and process capability analysis. Those capability indices, quantifying process potential and performance, are important for any successful quality improvement activities and quality program implementation. Montgomery & Runger [29] noted that quality of the collected data relies very much on the gauge accuracy. Any variation in the measurement process has a direct impact on the ability to make sound judgment about the manufacturing process. However, most capability research works have assumed no measurement errors, such assumption is not realistic even the measurement is conducted using highly sophisticated advanced measuring instruments, conclusions drawn regarding process capability are not reliable. Since literatures ([1], [22], [23], [27]) of analyzing the effects of measurement errors on process capability indices are few, the problem to consider process capability analysis with measurement errors is needful.

1.2 Research Objective

Process Capability Indices

Process capability indices, C_P, C_{PL}, C_{PU}, C_{PK}, C_{PM}, and C_{PMK} (see Juran *et al.* [15], Kane [16], Chan *et al.* [4, 5], Pearn *et al.* [38]) have been defined as:

$$C_P = \frac{USL - LSL}{6\sigma}, \tag{1.1}$$

$$C_{PL} = \frac{\mu - LSL}{3\sigma}, \quad C_{PU} = \frac{USL - \mu}{3\sigma}, \tag{1.2}$$

$$C_{PK} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} = \frac{d - |\mu - m|}{3\sigma}, \tag{1.3}$$

$$C_{PM} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \tag{1.4}$$

$$C_{PMK} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\}, \tag{1.5}$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation, T is the target value, $d = (USL - LSL)/2$, and $m = (USL + LSL)/2$.

C_P, C_{PK}, C_{PM}, C_{PMK} are indices that measure the capability of processes with two specification limits, while C_{PL} and C_{PU} are indices that measure the capability of processes with only one specification limit. The index C_P measures only distribution spread (process consistency or process precision), and does not consider process mean. The indices C_{PK}, C_{PM} and C_{PMK} consider process location and common cause variability, which offset some of the weaknesses in C_P. The principal distinction between C_{PK} and C_{PM} indices is in the relative importance attached to the specification limits USL and LSL as opposed to the target T. A high C_{PK} (C_{PL}, C_{PU}) value implies high process yield, a high C_{PM} value implies low process expected loss, and C_{PMK} is a combination of C_{PK} and C_{PM}, which is more restrictive with regard to process means deviation from the target value than other indices.

Several authors have promoted the use of various process capability indices and examined with differing degrees of completeness. Examples include Chou & Owen [6], Chou *et al.* [7], Franklin & Wasserman [11, 12], Kushler & Hurley [21], Kotz *et al.* [20], Vännman & Kotz [45], Vännman [46], Kotz & Lovelace [19], Hoffman [14], and references therein. Kotz & Johnson [18] presented a thorough review for the development of process capability indices in the past ten years and Spiring *et al.* [42] consolidated the research papers in process capability analysis for the period 1990-2002.

The Estimators of C_P

Kane [16], Chou *et al.* [7], Chou & Owen [6] and Li *et al.* [24] have investigated the distribution of the natural estimator, $\hat{C}_P$, of C_P. Kotz & Johnson [18] have derived the various moments of $\hat{C}_P$. Kane [16] derived the confidence interval bounds and critical values for testing hypothesis. Since $\hat{C}_P$ is a biased estimator of C_P, by adding the correction factor b_{n-1}, such as $\tilde{C}_P = b_{n-1}\hat{C}_P$, Chou & Owen [6] derived the pdf of $\tilde{C}_P$, and Pearn *et al.* [36] showed that $\tilde{C}_P$ is the UMVUE. Moreover, Pearn *et al.* [36] presented the confidence interval bounds and critical values by the estimator $\tilde{C}_P$.

The Estimators of C_{PK}

Under the assumption of normality, Kotz & Johnson [18] obtained the r-th moment, and the first two moments as well as the mean and the variance of $\hat{C}_{PK}$. In addition, numerous methods for constructing approximate confidence intervals of C_{PK} have been proposed in the literature. Examples include Chou *et al.* [7], Zhang *et al.* [50], Franklin & Wasserman [11], Kushler & Hurley [21], Nagata & Nagahata [32], Tang *et al.* [43], Hoffman [14], and many others.

Since the distribution of $\hat{C}_{PK}$ is the joint distribution of folded-normal and chi-square random variables (see Pearn *et al.* [38]), the probability density function of $\hat{C}_{PK}$ can be obtained as Pearn *et al.* [34]. Using the integration technique similar to that presented in Vännman [46], Pearn & Lin [37] first obtain an exact and explicit form of the cumulative distribution function of the natural estimator. Based on the cumulative distribution function of $\hat{C}_{PK}$, Pearn & Lin [37] implemented the statistical theory of the hypotheses testing, and developed a simple but practical procedure accompanied with convenient tabulated critical values, for practitioners to use for decisions making in their factory applications. Pearn & Shu [39] further developed an efficient algorithm with Matlab computer program to find the exact lower confidence bounds conveying critical information regarding the true process capability.

The Estimators of C_I

Chou & Owen [6] showed that under normality assumption, the estimators $\hat{C}_{PU}$ and $\hat{C}_{PL}$ are distributed as $ct_{n-1}(\delta)$, where $c = (3\sqrt{n})^{-1}$, and $t_{n-1}(\delta)$ is a non-central t distribution with n-1 degrees of freedom and non-centrality parameter $\delta = 3\sqrt{n}C_{PU}$ and $\delta = 3\sqrt{n}C_{PL}$ respectively. By adding the well-known correction factor, b_{n-1}, to $\hat{C}_{PU}$ and $\hat{C}_{PL}$, such as $\tilde{C}_{PU} = b_{n-1}\hat{C}_{PU}$ and $\tilde{C}_{PL} = b_{n-1}\hat{C}_{PL}$, Pearn & Chen [35] showed that $\tilde{C}_{PU}$ and $\tilde{C}_{PL}$ are uniformly minimum variance unbiased estimators (UMVUEs) of C_{PU} and C_{PL}. Based on the two UMVUEs, $\tilde{C}_{PU}$ and $\tilde{C}_{PL}$, Pearn & Chen [35] implemented the statistical theory of the hypotheses testing, and developed a simple but practical procedure accompanied with convenient tabulated critical values. Lin & Pearn [25] developed effieient SAS computer programs to calculate the critical values and *p*-values needed based on the cumulative distribution function for the capability testing. Pearn & Shu [40] further developed an efficient algorithm with Matlab computer program to find the lower confidence bounds conveying critical information regarding the minimal true process capability. A summary of literatures about the estimators of C_P, C_{PK}, and C_I, is listed in Table 1.

Table 1. Literatures about the estimators of C_P, C_{PK} and C_I.

Index	Estimator	Moments or pdf or cdf	Confidence intervals	Hypothesis testing
C_P	$\hat{C}_P$	[6], [7], [16], [18], [24]	[16]	[16]
	$\tilde{C}_P$	[6], [36]	[36]	[36]
C_{PK}	$\hat{C}_{PK}$	[18], [34], [37], [38]	[7], [11], [14], [21], [32], [39], [43], [50]	[37]
C_I	$\hat{C}_I$	[6]	[6]	[6]
	$\tilde{C}_I$	[35]	[40]	[25], [35]

Measurement Errors

Gauge repeatability and reproducibility (GR&R) studies focus on quantifying the measurement errors. Common approaches to GR&R studies, such as the Range method [29] and the ANOVA method [26, 29] assume that the distribution of the measurement errors is normally distributed with a mean error of zero. Suppose that the measurement errors are described by a random variable M ~ Normal (0, σ_M^2), Montgomery & Runger [29] presents the gauge capability by

$$\lambda = \frac{6\sigma_M}{USL - LSL} \times 100\% . \tag{1.6}$$

For the measurement system to be deemed acceptable, the variability in the measurements due to the measurement system must be less than a predetermined percentage of the engineering tolerance. The Automotive Industry Action Group recommends the guidelines in Table 2 for gauge acceptance.

Burdick & Larson [3], Floyd & Laurent [10], Hamada & Weerahandi [13], Rocke & Lorenzato [41], Vardeman & Vanvalkenburg [47], Wang & Iyer [48], and Wilson *et al.* [49] have further studies of conducting the confidence bounds of measurement errors.

Table 2. Guidelines for gauge capabilities.

Gauge Capability	Result
$\lambda < 10\%$	Gauge system O.K.
$10\% < \lambda < 30\%$	May be acceptable based on importance of application, cost of gauge, cost of repair, and so on.
$30\% < \lambda$	Gauge system needs improvement; make every effort to identify the problems and have them corrected.

Process Capability Analysis with Measurement Errors

Suppose that X ~ Normal (μ, σ^2) represents the relevant quality characteristic of a manufacturing process, Mittag [27] provided some very definitive techniques for quantifying the percentage error in process capability estimation. With stochastic measurement errors, the measurement system adds additional variability to the process measurement, so that a variable $Y = X + M$, which is distributed as Normal (μ, $\sigma_Y^2 = \sigma^2 + \sigma_M^2$) results. Mittag [27] found that the presence of stochastic measurement error always results in a decrease in the estimates of C_P, C_{PK}, C_{PM}, C_{PMK}. He compared the ratio of the true and empirical process capability indices by introducing the degree of error contamination,

$$\tau = \frac{\sigma_M}{\sigma} . \tag{1.7}$$

And he showed how the proportionate ratio for C_P and C_{PK} decreases as τ increases.

Levinson [22, 23] discussed the behavior of theoretical process capability indices in the presence of measurement errors. He provides formulae to evaluate the chance of shipping a bad piece to the customer. Bordignon & Scagliarini [1] performed some statistical analysis in estimating C_P and C_{PK}. He discussed the performance of the bias and the MSE (mean square error) of the estimates $\hat{C}_P$ and $\hat{C}_{PK}$, which are evaluated by the empirical data.

Mittag [27] and Levinson [22, 23] had paid attention to the effects of measurement errors in process capability analysis, but they did not take sampling errors into account. Many efforts dedicated in the literature to the statistical properties of the estimators of process capability indices (examples as those in Table 2), but except for Bordignon & Scagliarini [1], they assumed that there is no measurement error.

In our research, we assume that measurement errors exist, and we consider sampling errors in process capability analysis. We notate the estimates, which evaluated by the empirical data, of the estimators, $\tilde{C}_P$, $\hat{C}_{PK}$ and $\tilde{C}_I$ as $\tilde{C}_P^Y$, $\hat{C}_{PK}^Y$ and $\tilde{C}_I^Y$. We will compare the bias and MSE of $\tilde{C}_P^Y$, $\hat{C}_{PK}^Y$, $\tilde{C}_I^Y$ with those of $\tilde{C}_P$, $\hat{C}_{PK}$, $\tilde{C}_I$. With no modification of the confidence interval bounds and critical values which are presented by Pearn *et al.* [36], Pearn & Shu [39], Pearn & Lin [37], Pearn & Chen [35], and Pearn & Shu [40], and use $\tilde{C}_P^Y$, $\hat{C}_{PK}^Y$, $\tilde{C}_I^Y$ to estimate the true process capability or to do a statistical testing, we will show the behavior of the confidence coefficient, the α-risk and the power. Since measurement error is unavoidable in most cases, the major objective in our research is to derive revised confidence bounds and critical values for practitioners.

1.3 Organization

In Chapter 1, we review some important literatures about process capability analysis and some about the discussion of measurement errors on process capability indices. And, we point out our research objectives. In Chapter 2, we first consider the sensitivity of C_P with measurement errors, and compare the MSE of $\tilde{C}_P^Y$ with that of $\tilde{C}_P$. Second, we show the behavior of confidence coefficient, α-risk and power by using the confidence bounds and critical values, which are presented by Pearn *et al.* [36], with empirical data. Third, we derive revised confidence bounds and critical values while we use $\tilde{C}_P^Y$ to estimate C_P. In Chapter 3, we first consider the sensitivity of C_{PK} with measurement errors, and compare the MSE of $\hat{C}_{PK}^Y$ with that of $\hat{C}_{PK}$. Second, we show the behavior of confidence coefficient, α-risk and power by using the confidence bounds and critical values, which are presented by Pearn & Shu [39], Pearn & Lin [37], with empirical data. Third, we derive revised confidence bounds and critical values while we use $\hat{C}_{PK}^{Y}$ to estimate C_{PK}. An example is in the final section. In Chapter 4, we first consider the sensitivity of C_I with measurement errors, and compare the MSE of $\tilde{C}_I^Y$ with that of $\tilde{C}_I$. Second, we show the behavior of confidence coefficient, α-risk and power by using the confidence bounds and critical values, which are presented by Pearn & Chen [35], and Pearn & Shu [40], with empirical data. Third, we derive revised confidence bounds and critical values while we use $\hat{C}_{PK}^{Y}$ to estimate C_{PK}. An example is in the final section. Finally, in Chapter 5, we have some conclusions.