Chapter 2

Estimating and Testing Process Precision with Presence of Gauge Measurement Errors

The first, and the original, process capability index was C_p, which was introduced outside of Japan by Juran *et al.* [15], but did not gain considerable acceptance until the early 1980s. The numerator of C_p gives the size of the range over which the process measurements can vary, and the denominator gives the size of the range over which the process is actually varying. Obviously, it is desirable to have a C_p as large as possible. Small values of C_p would not be acceptable, since this indicates that the natural range of variation of the process does not fit within the tolerance band.

Table 3. Minimum proportion NC associated with various values of C_p.

Amount of process data within specification range	C_p	Minimum % NC
6σ	1.00	0.27×10^{-2}
8σ	1.33	0.6334×10^{-4}
10σ	1.67	0.5733×10^{-6}
12σ	2.00	0.1973×10^{-8}

Table 4. Appropriate responses to C_p values.

Process Capability	Assessment	Response
$1.33\le C_p$	Pass	Sufficient to inspect at start of operations. Can consider speeding up process or otherwise increasing load.
$1\le C_p \le1.33$	Needs watching	Danger of producing defects. Needs watching.
$C_p <1$	Fail	Need to consider changing procedures, changing equipment and changing tolerance. Inspect total output.

Under the assumption of that process data are normal, independent, and in control, Kocherlakota [17] developed a general guideline for the percentage NC (non-conforming units) associated with C_p, assuming that the process is perfectly centered at the midpoint of the specification range (see Table 3). Mizuno [28] presented detailed criteria for C_p, which had been widely used in U.S. industries. These criteria provide guidelines for management response to specific ranges of C_p values (see Table 4).

In section 2.1, we discuss the relationship between the empirical process capability C_P^Y and the true process capability C_P. In section 2.2, we obtain the pdf, the expected value, the variance and the MSE of $\tilde{C}_P^Y$. And, we compare the MSE of $\tilde{C}_P^Y$ with that of $\tilde{C}_P$. In section 2.3, we use the confidence interval bounds in Pearn *et al.* [36] to estimate the true capability C_P by $\tilde{C}_P^Y$, and we show that the confidence coefficient becomes decrease with measurement errors. In section 2.4, we use the critical values in Pearn *et al.* [36] to test whether the process capability meets the requirement, and we show that the α-risk and the power both become decrease with measurement error. In section 2.5, we present our modified confidence interval bounds and critical values for the cases that measurement errors are unavoidable.

2.1 Empirical Process Capability C_P^Y

Suppose that X ~ Normal (μ , σ^2) represents the relevant quality characteristic of a manufacturing process. Because of measurement errors, the observed variable Y ~ Normal (μ , $\sigma_Y^2 = \sigma^2 + \sigma_M^2$) is measured by the assumption that X and M are stochastically independent, instead of measuring the true variable X. The empirical process capability index C_P^Y is obtained after substituting σ_Y for σ, and we have the relationship between the true process capability C_P and the empirical process capability C_P^Y as

$$C_P^Y = \frac{C_P}{\sqrt{1+\lambda^2 C_P^2}}. \tag{2.1}$$

Since the variation of data we observed is larger than the variation of the original data, the denominator of the index C_P becomes larger, and we will understate the true capability of the process if we calculate process capability index with variable Y.

Table 5. Process capability with λ = 0.05(0.05)0.50 for various C_P.

	λ									
C_P	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.49	0.49	0.49	0.49	0.49
1.00	1.00	1.00	0.99	0.98	0.97	0.96	0.94	0.93	0.91	0.89
1.33	1.33	1.32	1.30	1.29	1.26	1.24	1.21	1.17	1.14	1.11
1.50	1.50	1.48	1.46	1.44	1.40	1.37	1.33	1.29	1.24	1.20
1.67	1.66	1.65	1.62	1.58	1.54	1.49	1.44	1.39	1.34	1.28
2.00	1.99	1.96	1.92	1.86	1.79	1.71	1.64	1.56	1.49	1.41
2.50	2.48	2.43	2.34	2.24	2.12	2.00	1.88	1.77	1.66	1.56

In Table 5, we list some process capabilities with λ = 0.05(0.05)0.50 for various true process capability index C_P = 0.50, 1.00, 1.33, 1.50, 1.67, 2.00, and 2.50. It is obviously that the gauge becomes more important as the true capability improves (Levinson [22]). If λ = 0.50 (50%), C_P^Y = 0.49 with the true process capability $C_P = 0.50$, and C_P^Y = 1.56 with the true process capability $C_P = 2.50$. Substituting a perfect measuring instrument will help much for processes with higher capability.

2.2 Sampling Distribution of $\tilde{C}_P^Y$

Suppose that $\{X, i = 1, 2, \ldots, n\}$ denote the random sample of size n from the quality characteristics X. To estimate the precision index C_P, we consider the natural estimator $\hat{C}_P$ defined below, where S = $[\sum_{i=1}^{n}(X_i - \bar{X})/(n-1)]^{1/2}$ is the conventional estimator of σ, which may be obtained from a stable process,

$$\hat{C}_P = \frac{USL - LSL}{6S}. \tag{2.2}$$

On the assumption of normality, the statistic $K = (n-1)S^2/\sigma^2$ is distributed as χ_{n-1}^2, a chi-square with n-1 degrees of freedom. The pdf (probability density function) of $\hat{C}_P$ can be expressed as (Chou & Owen [6])

$$f(x) = 2\frac{(\sqrt{(n-1)/2}C_p)^{n-1}}{\Gamma[(n-1)/2]}(x)^{-n}\exp\left[-(n-1)C_p^2(2x^2)^{-1}\right]. \tag{2.3}$$

By adding the well-known correction factor

$$b_{n-1} = \sqrt{\frac{2}{n-1}}\Gamma(\frac{n-1}{2})\Gamma(\frac{n-2}{2})^{-1}. \tag{2.4}$$

to $\hat{C}_P$, such as $\tilde{C}_P = b_{n-1}\hat{C}_P$, Pearn *et al.* [36] showed that $\tilde{C}_P$ is the UMVUE of C_P. The expected value and the variance of the estimator $\tilde{C}_P$ are

$$\mathrm{E}(\tilde{C}_P) = C_P, \tag{2.5}$$

$$\mathrm{Var}(\tilde{C}_P) = (\frac{n-1}{n-3}b_{n-1}^2 - 1)C_P^2. \tag{2.6}$$

However, the sample observations are not X but Y. The estimator of estimating C_P is

$$\tilde{C}_P^Y = b_{n-1}(\frac{USL - LSL}{6S_Y}), \tag{2.7}$$

while we use $\tilde{C}_P$ to estimate C_P, where $S_Y = [\sum_{i=1}^{n}(Y_i - \bar{Y})/(n-1)]^{1/2}$. Based on the same arguments used in Chou & Owen [6] and Pearn *et al.* [36], we obtain the pdf of $\tilde{C}_P^Y$ as

$$f(y) = 2\frac{(\sqrt{(n-1)/2}C_p/\sqrt{1+\lambda^2 C_p^2})^{n-1}}{\Gamma\left[(n-1)/2\right]}(y)^{-n}\exp\left[\frac{-(n-1)C_p^2(2y^2)^{-1}}{1+\lambda^2 C_p^2}\right], \qquad (2.8)$$

The expected value and the variance of the estimator $\tilde{C}_P^Y$ are

$$\mathrm{E}(\tilde{C}_P^Y) = \frac{C_P}{\sqrt{1+\lambda^2 C_P^2}}, \qquad (2.9)$$

$$\mathrm{Var}(\tilde{C}_P^Y) = (\frac{n-1}{n-3}b_{n-1}^2 - 1)\frac{C_P^2}{1+\lambda^2 C_P^2}. \qquad (2.10)$$

For $\lambda > 0$, it is obviously that $\tilde{C}_P^Y$ is a biased estimator of C_P, and the bias is $(1/\sqrt{1+\lambda^2 C_P^2} - 1)C_P$ which decreases in λ. Since n is a finite positive integer, $[(n-1)/(n-3)](b_{n-1})^2 - 1$ is positive, so we have $\mathrm{Var}(\tilde{C}_P^Y) < \mathrm{Var}(\tilde{C}_P)$. Taking into account both the bias and the variance, we consider the MSEs of the two estimators $\tilde{C}_P$ and $\tilde{C}_P^Y$. The MSEs of $\tilde{C}_P$ and $\tilde{C}_P^Y$, which we denote as $\mathrm{MSE}(\tilde{C}_P)$ and $\mathrm{MSE}(\tilde{C}_P^Y)$ respectively, are

$$\mathrm{MSE}(\tilde{C}_P) = (\frac{n-1}{n-3}b_{n-1}^2 - 1)C_P^2, \qquad (2.11)$$

$$\mathrm{MSE}(\tilde{C}_P^Y) = [\frac{n-1}{n-3}(\frac{b_{n-1}^2}{1+\lambda^2 C_P^2}) - \frac{2}{\sqrt{1+\lambda^2 C_P^2}} + 1]C_P^2. \qquad (2.12)$$

Compare $\mathrm{MSE}(\tilde{C}_P^Y)$ with $\mathrm{MSE}(\tilde{C}_P)$, we consider the function $f(C_P, n, \lambda) = \mathrm{MSE}(\tilde{C}_P^Y) / \mathrm{MSE}(\tilde{C}_P)$. By some reduction, we have $f(C_P, n, \lambda) = 1$ if and only if

$$\lambda = \frac{2\sqrt{(n-1)b_{n-1}^2/(n-3) - 1}}{2-(n-1)b_{n-1}^2/(n-3)}C_P^{-1}, \qquad (2.13)$$

or $\lambda = 0$.

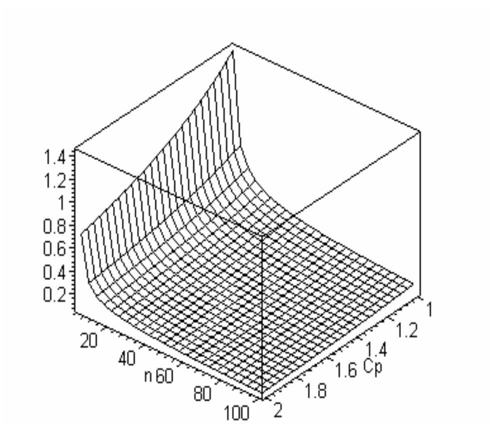

Figure 1(a). Surface plot of λ_0 for various $n = 5$ (1) 100 and C_P = [1.00, 2.00].

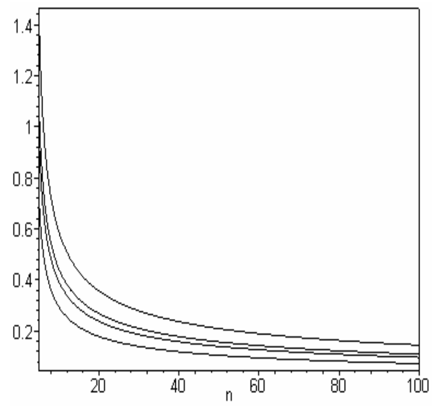

Figure 1(b). Plots of λ_0 versus $n = 5$ (1) 100 for C_P = 1.00, 1.33, 1.50, 2.00 (from top to bottom).

As we denote the right side of the equal sign in the above formula as λ_0, we have $f(C_P, n, \lambda) > 1$ if $\lambda > \lambda_0$ and $f(C_P, n, \lambda) < 1$ if $\lambda < \lambda_0$ exclusive of 0. It represents that $\text{MSE}(\tilde{C}_P^Y) > \text{MSE}(\tilde{C}_P)$ if $\lambda > \lambda_0$, $\text{MSE}(\tilde{C}_P^Y) < \text{MSE}(\tilde{C}_P)$ if $\lambda < \lambda_0$ exclusive of 0, and $\text{MSE}(\tilde{C}_P^Y) = \text{MSE}(\tilde{C}_P)$ if $\lambda = \lambda_0$ or 0. Figure 1(a) shows the surface plot of λ_0 values for $n = 5(1)100$ and C_P in [1.00, 2.00]. Figure 1(b) plots λ_0 versus $n = 5(1)100$ for $C_P = 1.00$, 1.33, 1.50, 2.00. By those figures, we see that λ_0 value decreases if n or C_P increases. The maximum value of λ_0 is 1.439 which occurs at $(n, C_P) = (5, 1.00)$, and the minimum value of λ_0 is 0.072 which occurs at $(n, C_P) = (100, 2.00)$.

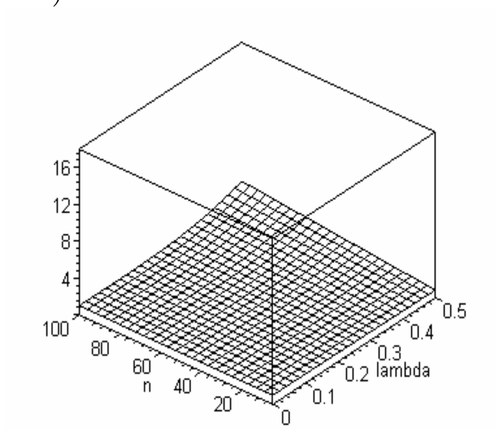

Figure 2(a). Surface plot of γ_1 with $n = 5(1)100$ and λ in [0, 0.5] for $C_P = 1.00$.

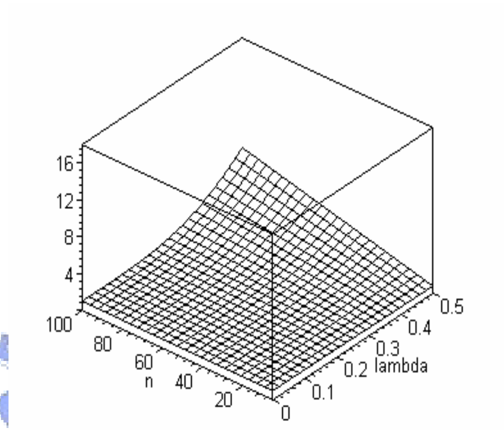

Figure 2(b). Surface plot of γ_1 with $n = 5(1)100$ and λ in [0, 0.5] for $C_P = 1.33$.

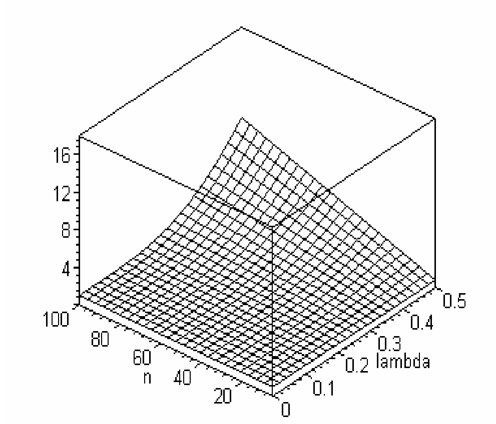

Figure 2(c). Surface plot of γ_1 with $n = 5(1)100$ and λ in [0, 0.5] for $C_P = 1.50$.

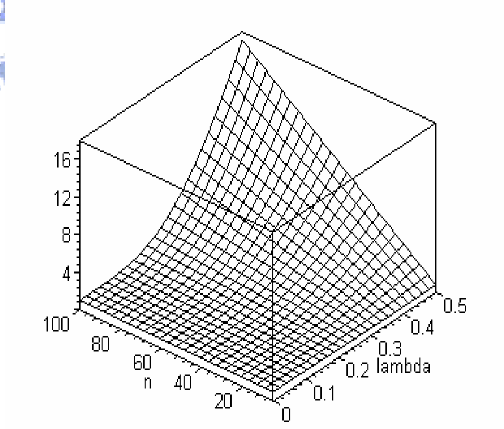

Figure 2(d). Surface plot of γ_1 with $n = 5(1)100$ and λ in [0, 0.5] for $C_P = 2.00$.

Figures 2(a)-2(d) display the surface plots of the ratios $\gamma_1 = f(C_P, n, \lambda)$ with $n = 5(1)100$ and λ in [0, 0.5] for $C_P = 1.00$, 1.33, 1.50, and 2.00. γ_1 varies with n or λ, the variation is more noticeable in higher capability case. For large n, γ_1 is greater than 1 for almost every value of λ, and γ_1 increases in λ. The maximum values of γ_1 in Figures 2(a)-2(d) are 2.957, 6.110, 8.380, and 17.100, which occur at $(n, \lambda) = (100, 0.50)$, (100, 0.50), (100, 0.50), and (100, 0.50) respectively, and the minimum values of γ_1 in Figures 2(a)-(d) are

0.841 (1/1.189), 0.796 (1/1.256), 0.786 (1/1.272), and 0.785 (1/1.274), which occur at (n, λ) = (5, 0.50), (5, 0.50), (5, 0.50), and (5, 0.39) respectively. The difference between MSE($\tilde{C}_P^Y$) and MSE($\tilde{C}_P$) with γ_1 > 1 is more significant than that with γ_1 < 1.

2.3 Confidence Bounds Based on $\tilde{C}_P^Y$

Under normality assumption, the (1-α)% confidence interval of C_p with confidence bounds L and U, can be established as

$$P\left(L \le C_P \le U\right) = \mathrm{P}\left(L \le \frac{\tilde{C}_P}{b_{n-1}\sqrt{n-1}} K^{1/2} \le U \right),$$

$$= \mathrm{P}\left(L^2 [\frac{b_{n-1}\sqrt{n-1}}{\tilde{C}_P}]^2 \le K \le U^2 [\frac{b_{n-1}\sqrt{n-1}}{\tilde{C}_P}]^2 \right) = 1-\alpha \,. \tag{2.14}$$

$$L^2 \left[\frac{b_{n-1}\sqrt{n-1}}{\tilde{C}_P} \right]^2 = \chi^2_{n-1,1-\alpha/2}, \quad U^2 \left[\frac{b_{n-1}\sqrt{n-1}}{\tilde{C}_P} \right]^2 = \chi^2_{n-1,\alpha/2}, \tag{2.15}$$

where $\chi^2_{n-1,\alpha}$ is the upper α th quantile of the χ^2_{n-1} distribution, and we can obtain the confidence bounds L and U of C_p as

$$L = \frac{\sqrt{\chi^2_{n-1,1-\alpha/2}}\,\tilde{C}_P}{\sqrt{n-1}\,b_{n-1}}, \quad U = \frac{\sqrt{\chi^2_{n-1,\alpha/2}}\,\tilde{C}_P}{\sqrt{n-1}\,b_{n-1}}. \tag{2.16}$$

However, as a result of the measurement errors, we take $\tilde{C}_P^Y$ as an estimator of C_p, thus the confidence bounds we calculated are

$$L^Y = \frac{\sqrt{\chi^2_{n-1,1-\alpha/2}}\,\tilde{C}_P^Y}{\sqrt{n-1}\,b_{n-1}}, \quad U^Y = \frac{\sqrt{\chi^2_{n-1,\alpha/2}}\,\tilde{C}_P^Y}{\sqrt{n-1}\,b_{n-1}}, \tag{2.17}$$

and the confidence coefficient θ (the probability that the confidence interval contains the actual C_p value) is

$$\theta = P\left(\frac{\sqrt{\chi^2_{n-1,1-\alpha/2}}\,\tilde{C}_P^Y}{\sqrt{n-1}\,b_{n-1}} \le C_p \le \frac{\sqrt{\chi^2_{n-1,\alpha/2}}\,\tilde{C}_P^Y}{\sqrt{n-1}\,b_{n-1}} \right)$$

$$= \mathrm{P}\left(\frac{\chi^2_{n-1,1-\alpha/2}(USL-LSL)^2}{(n-1)36S_Y^2} \le C_P^2 \le \frac{\chi^2_{n-1,\alpha/2}(USL-LSL)^2}{(n-1)36S_Y^2} \right)$$

$$= \mathrm{P}\left(\frac{1}{1+\lambda^2 C_P^2}\chi^2_{\text{n-1, 1-}\alpha/2} \le \chi^2_{n-1} \le \frac{1}{1+\lambda^2 C_P^2}\chi^2_{\text{n-1, }\alpha/2} \right), \tag{2.18}$$

where $K^Y = (n-1)S_Y^2/\sigma_Y^2$ is distributed as χ^2_{n-1}.

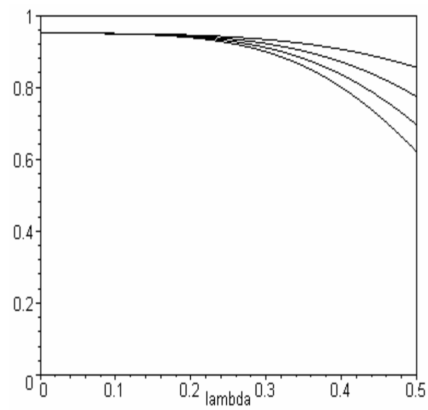

Figure 3(a). Plots of θ versus λ with C_P = 1.00 and n = 25(25)100 (from top to bottom) for 95% confidence intervals.

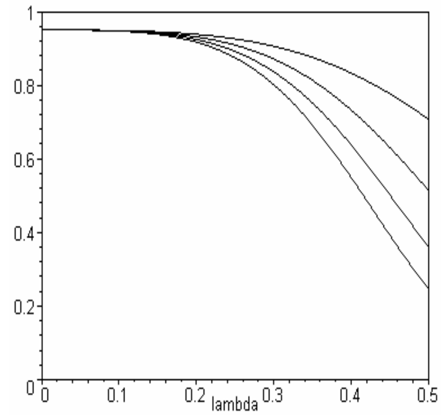

Figure 3(b). Plots of θ versus λ with C_P = 1.33 and n = 25(25)100 (from top to bottom) for 95% confidence intervals.

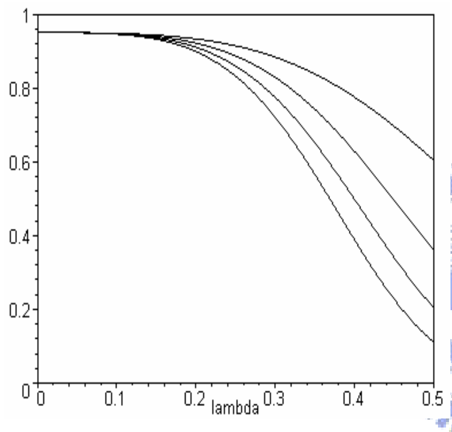

Figure 3(c). Plots of θ versus λ with C_P = 1.50 and n = 25(25)100 (from top to bottom) for 95% confidence intervals.

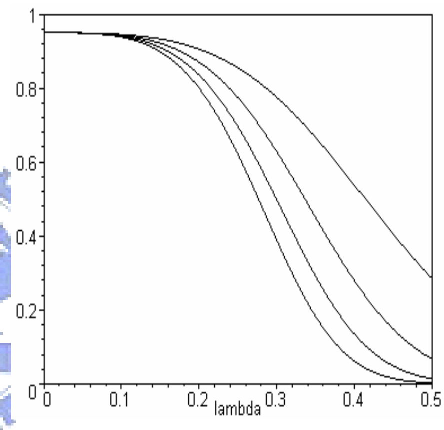

Figure 3(d). Plots of θ versus λ with C_P = 2.00 and n = 25(25)100 (from top to bottom) for 95% confidence intervals

Figures 3(a)-3(d) present plots of θ versus λ with $C_P = 1.00$, 1.33, 1.50, 2.00 and n = 25(25)100 (from top to bottom) for 95% confidence intervals. Obviously, those intervals do not maintain the stated confidence coefficient. The θ value decreases in measurement errors, and larger sample size or higher capability has more significant decrements. Because of the measurement errors, the confidence coefficients may become very small. For instance, when C_P = 2.00, n = 100, and λ = 0.50 (see Figure 3(d)), the confidence coefficient is only 0.26%, which is much smaller than the stated confidence coefficient 95%.

2.4 Capability Testing Based on $\tilde{C}_P^Y$

To determine whether a given process meets the present capability requirement and runs under the desired quality condition. We can consider the following statistical testing hypothesis, H_0: $C_P \le c$ versus H_1: $C_P > c$. Process fails to meet the capability requirement if $C_P \le c$, and meets the capability requirement if $C_P > c$. The critical value c_0 can be determined by the following with α-risk $\alpha(c_0) = \alpha$ (the chance of incorrectly judging an incapable process as capable),

$$P\left(\tilde{C}_P \ge c_0 \mid C_P = c\right) = \alpha \,, \tag{2.19}$$

and we can obtain c_0 is

$$c_0 = \frac{b_{n-1}\sqrt{n-1}\; c}{\sqrt{\chi^2_{n-1,1-\alpha}}} \,. \tag{2.20}$$

Meanwhile, the power of the test (the chance of correctly judging a capable process as capable) can be computed as

$$\pi(C_P) = P\left(\tilde{C}_P > c_0 \mid C_P\right) = P\left(b^2_{n-1}\frac{(USL-LSL)^2}{36S^2} > c_0^2 \mid C_P \right)$$

$$= P\left(K < \frac{b^2_{n-1}(n-1)C_P^2}{c_0^2} \right) = P\left(\chi^2_{n-1} < \frac{C_P^2}{c^2}\chi^2_{n-1,1-\alpha} \right). \tag{2.21}$$

In the presence of measurement errors, however, the α-risk (denoted by α^Y) and the power of the test (denoted by π^Y) are

$$\alpha^Y = P\left(\tilde{C}_P^Y \ge c_0 \mid C_P = c\right)$$

$$= P\left(\frac{b_{n-1}\sqrt{n-1}C_P^Y}{\sqrt{K^Y}} \ge c_0 \mid C_P = c \right) = P\left(\frac{\sqrt{\chi^2_{n-1,1-\alpha}}}{\sqrt{1+\lambda^2 C_P^2}} \ge \sqrt{K^Y} \right)$$

$$= P\left(K^Y \le \frac{1}{1+\lambda^2 C_P^2}\chi^2_{n-1,1-\alpha} \right) = P\left(\chi^2_{n-1} \le \frac{1}{1+\lambda^2 C_P^2}\chi^2_{n-1,1-\alpha} \right). \tag{2.22}$$

$$\pi^Y(C_P) = P\left(\tilde{C}_P^Y > c_0 \mid C_P\right)$$

$$= P\left(\frac{b_{n-1}\sqrt{n-1}C_P^Y}{\sqrt{K^Y}} > c_0 \mid C_P \right) = P\left(\frac{C_P}{\sqrt{1+\lambda^2 C_P^2}} > \frac{c}{\sqrt{\chi^2_{n-1,1-\alpha}}}\sqrt{K^Y} \right)$$

$$= P\left(\frac{C_P^2\chi^2_{n-1,1-\alpha}}{c^2(1+\lambda^2 C_P^2)} > K^Y \right) = P\left(\chi^2_{n-1} < \frac{C_P^2}{c^2(1+\lambda^2 C_P^2)}\chi^2_{n-1,1-\alpha} \right). \tag{2.23}$$

Since we underestimate the true capability of the process when we calculate process capability index using $\tilde{C}_P^Y$ instead of $\tilde{C}_P$, the probability that $\tilde{C}_P^Y$ is greater than c_0 will be less than the probability of that using $\tilde{C}_P$. Thus, the α-risk using $\tilde{C}_P^Y$ to estimate C_P is less than the α-risk using $\tilde{C}_P$ to estimate C_P ($\alpha^Y \le \alpha$), and the power using $\tilde{C}_P^Y$ to estimate C_P is also less than the power using $\tilde{C}_p$ to estimate C_p ($\pi^Y \le \pi$).

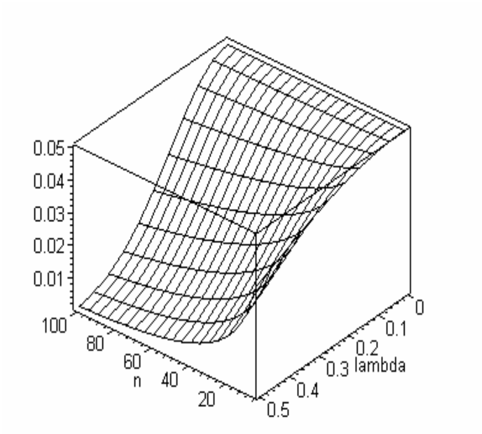

Figure 4(a). Surface plot of α^Y with n = 5(1)100 and λ in [0, 0.5] for c = 1.00 and α = 0.05.

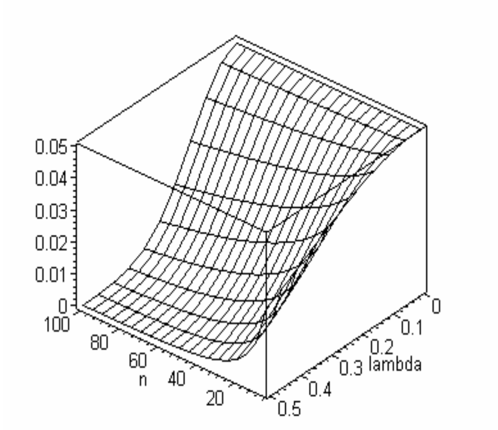

Figure 4(b). Surface plot of α^Y with n = 5(1)100 and λ in [0, 0.5] for c = 1.33 and α = 0.05.

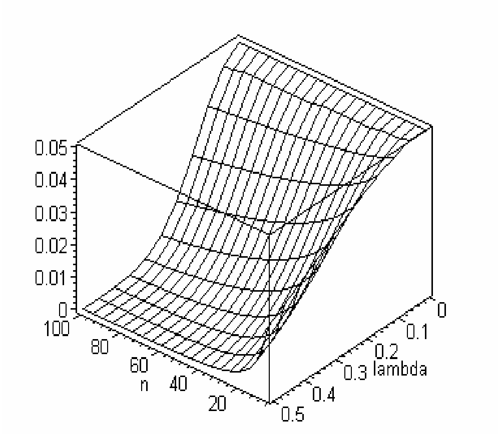

Figure 4(c). Surface plot of α^Y with n = 5(1)100 and λ in [0, 0.5] for c = 1.50 and α = 0.05.

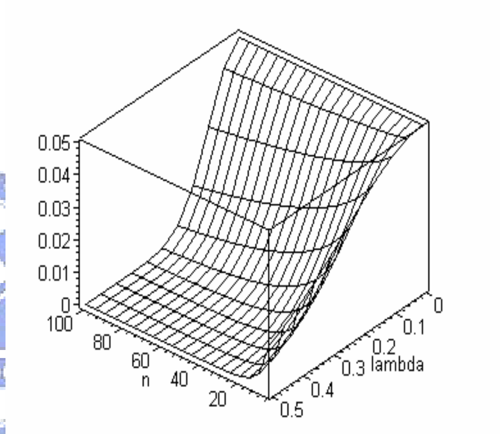

Figure 4(d). Surface plot of α^Y with n = 5(1)100 and λ in [0, 0.5] for c = 2.00 and α = 0.05

Figures 4(a)-4(d) are the surface plots of α^Y with n = 5(1)100 and λ in [0, 0.5] for c = 1.00, 1.33, 1.50, 2.00, and α = 0.05. Figures 5(a)-5(d) are plots of π^Y versus λ with n = 50 and α = 0.05 for c = 1.00, 1.33, 1.50, 2.00 and C_p = c(0.20)c+1. Note that we have $\alpha^Y = \alpha$ and $\pi^Y = \pi$ when λ = 0 in those figures. In Figures 4(a)-4(d), α^Y decreases if λ or n increases, and the decrements are significant with large c values. In addition, we find that large λ values may result α^Y smaller than 1×10^{-4} (such as λ = 0.50, c = 2.00, and $n \geq 50$), an α-risk may be very imperceptible because of measurement errors. In Figures 5(a)-5(d), π^Y decreases with λ, but increases with n. The decrements of power by λ are more significant with higher capability. Because of measurement errors, π^Y may decrease with significant decrements. For instance, we consider the π^Y values in Figure 5(b) (c = 1.33, n = 50) for C_p = 1.93, π^Y = 0.980 if there is no measurement error (λ = 0), but when λ = 0.50, π^Y decreases to 0.104, the decrement of power is about 0.88.

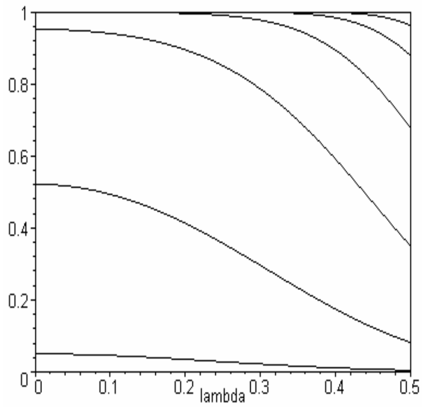

Figure 5(a). Plots of π^Y versus λ with n = 50 and α = 0.05 for c = 1.00 and C_p = 1.00(0.20)2.00 (from bottom to top).

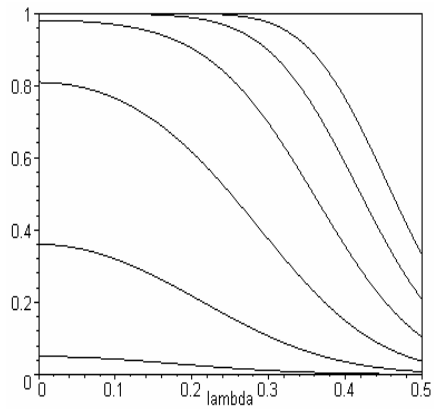

Figure 5(b). Plots of π^Y versus λ with n = 50 and α = 0.05 for c = 1.33 and C_p = 1.33(0.20)2.33 (from bottom to top).

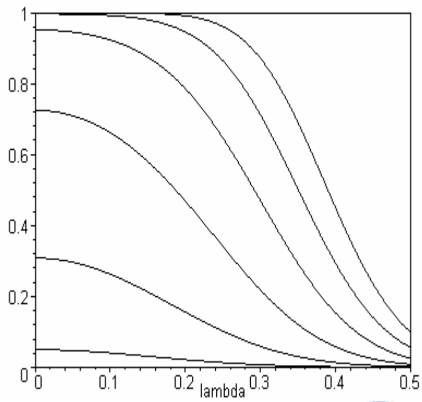

Figure 5(c). Plots of π^Y versus λ with n = 50 and α = 0.05 for c = 1.50 and C_p = 1.50(0.20)2.50 (from bottom to top).

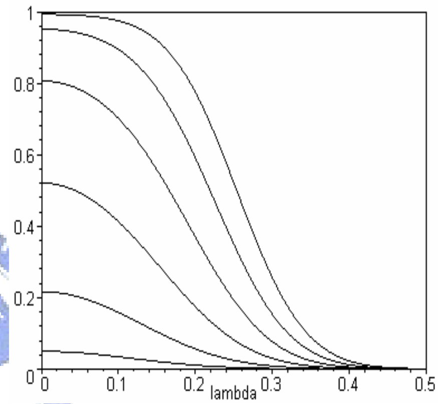

Figure 5(d). Plots of π^Y versus λ with n = 50 and α = 0.05 for c = 2.00 and C_p = 2.00(0.20)3.00 (from bottom to top).

2.5 Modified Confidence Bounds and Critical Values

We showed earlier that the confidence intervals do not maintain the stated confidence coefficients. We also showed that both the α-risk and the power of the test decrease when the gauge measurement error increases. If the producers do not take account of the effects of the gauge capability in process capability estimation and testing, it may result in serious loss. In that case, the producers cannot anymore affirm that their processes to be meet the capability requirement even if their processes are sufficiently capable. The producers may pay for a lot of cost because quantities of qualified product units are incorrectly rejected. Improving the gauge measurement accuracy and training the operators by proper education are essential for reducing the measurement errors. Nevertheless, measurement errors may be unavoidable in most manufacturing processes. In the following, we adjust the confidence intervals and critical values in order to ensure the intervals have the desired confidence coefficients and improve the power of the test with appropriate α-risk. Suppose that the desired confidence coefficient is 1-α, the adjusted confidence interval of C_p with confidence interval bounds L^* and U^*, can be established as

$$P\left(L^* \le C_P \le U^*\right) = \mathrm{P}\left(L^* \le \frac{\tilde{C}_P^Y}{\sqrt{(n-1)b_{n-1}^2 (K^Y)^{-1} - (\lambda \tilde{C}_P^Y)^2}} \le U^* \right)$$

$$= \mathrm{P}\left(L^{*2}\left[\frac{(n-1)b_{n-1}^2 (K^Y)^{-1}}{(\tilde{C}_P^Y)^2}\right] \le 1 + L^{*2}\lambda^2 \right) + \mathrm{P}\left(1 + U^{*2}\lambda^2 \le U^{*2}\left[\frac{(n-1)b_{n-1}^2 (K^Y)^{-1}}{(\tilde{C}_P^Y)^2}\right] \right)$$

$$= \mathrm{P}\left(L^{*2}\left[\frac{(n-1)b_{n-1}^2}{(\tilde{C}_P^Y)^2(1+L^{*2}\lambda^2)}\right] \le K^Y \le U^{*2}\left[\frac{(n-1)b_{n-1}^2}{(\tilde{C}_P^Y)^2(1+U^{*2}\lambda^2)}\right] \right) = 1-\alpha\,. \qquad (2.24)$$

$$L^{*2}\left[\frac{(n-1)b_{n-1}^2}{(\tilde{C}_P^Y)^2(1+L^{*2}\lambda^2)}\right] = \chi^2_{n-1,1-\alpha/2}, \quad U^{*2}\left[\frac{(n-1)b_{n-1}^2}{(\tilde{C}_P^Y)^2(1+U^{*2}\lambda^2)}\right] = \chi^2_{n-1,\alpha/2}\,. \qquad (2.25)$$

By some simplification, the adjusted (1-α)% confidence interval bound can be written as

$$L^* = \frac{\sqrt{\chi^2_{n-1,1-\alpha/2}}\,\tilde{C}_P^Y}{\sqrt{(n-1)b_{n-1}^2 - (\lambda\tilde{C}_P^Y)^2\,\chi^2_{n-1,1-\alpha/2}}}\,, \quad U^* = \frac{\sqrt{\chi^2_{n-1,\alpha/2}}\,\tilde{C}_P^Y}{\sqrt{(n-1)b_{n-1}^2 - (\lambda\tilde{C}_P^Y)^2\,\chi^2_{n-1,\alpha/2}}}\,. \qquad (2.26)$$

With our revised confidence interval bounds, we can ensure the interval would have the desired confidence coefficient. Moreover, in order to improve the power of the test, we let the critical values (denoted by c_0^*) we proposed to be satisfied c_0^* < c_0. Since c_0^* < c_0, the probability that $\tilde{C}_P^Y$ is greater than c_0^* will be more than the probability of that $\tilde{C}_P^Y$ is greater than c_0. And, both the α-risk and the power increase when we take c_0^* to be critical value for testing hypothesis. Suppose that the α-risk by our revised critical values c_0^* is α^*, the revised critical c_0^* can be introduced by

$$\alpha^* = P\left(\tilde{C}_P^Y \ge c_0^* \mid C_P = c\right) = P\left(\frac{b_{n-1}\sqrt{n-1}C_P^Y}{\sqrt{K^Y}} \ge c_0^* \mid C_P = c \right)$$

$$= P\left(\frac{b_{n-1}\sqrt{n-1}C_P}{c_0^*\sqrt{1+\lambda^2 C_P^2}} \ge \sqrt{K^Y} \mid C_P = c \right) = P\left(\frac{b_{n-1}^2 (n-1)c^2}{c_0^{*2}(1+\lambda^2 c^2)} \ge K^Y \right)$$

$$= P\left(\chi^2_{n-1} \le \frac{b_{n-1}^2 (n-1)c^2}{c_0^{*2}(1+\lambda^2 c^2)} \right). \qquad (2.27)$$

To ensure that the α-risk is within the preset magnitude, we let $\alpha^* = \alpha$, thus c_0^* and the power (denoted by π^*) can be obtained as

$$c_0^* = \frac{b_{n-1}\sqrt{n-1}c}{\sqrt{(1+\lambda^2 c^2)\chi^2_{n-1,1-\alpha}}} \qquad (2.28)$$

$$\pi^*(C_P) = P\left(\tilde{C}_P^Y > c_0^* \mid C_P\right) = P\left(\frac{b_{n-1}\sqrt{n-1}C_P^Y}{\sqrt{K^Y}} > c_0^* \mid C_P\right)$$

$$= P\left(\frac{C_P\sqrt{(1+\lambda^2c^2)\chi^2_{n-1,1-\alpha}}}{c\sqrt{1+\lambda^2C_P^2}} > \sqrt{K^Y}\right)$$

$$= P\left(\chi^2_{n-1} < (\frac{C_P}{c})^2(\frac{1+\lambda^2c^2}{1+\lambda^2C_P^2})\chi^2_{n-1,1-\alpha}\right). \qquad (2.29)$$

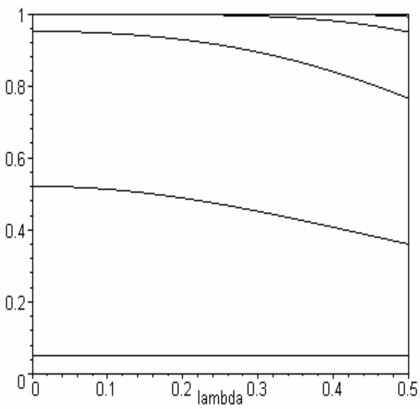

Figure 6(a). Plots of π^* versus λ with n = 50 and α = 0.05 for c = 1.00 and C_P = 1.00(0.20)2.00 (from bottom to top).

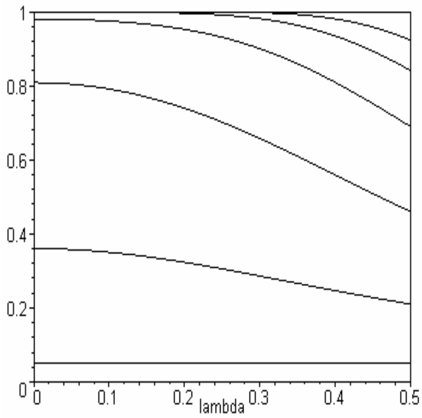

Figure 6(b). Plots of π^* versus λ with n = 50 and α = 0.05 for c = 1.33 and C_p = 1.33(0.20)2.33 (from bottom to top).

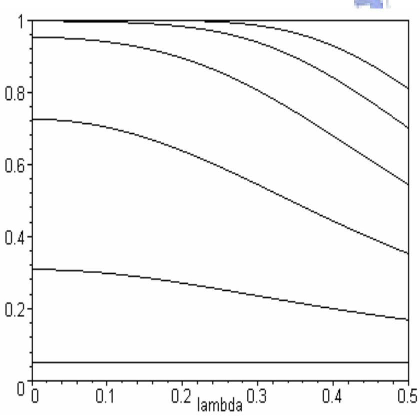

Figure 6(c). Plots of π^* versus λ with n = 50 and α = 0.05 for c = 1.50 and C_P = 1.50(0.20)2.50 (from bottom to top).

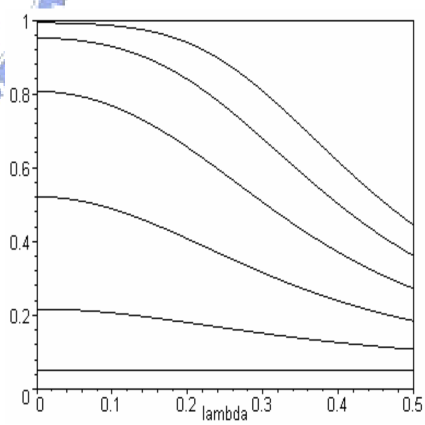

Figure 6(d). Plots of π^* versus λ with n = 50 and α = 0.05 for c = 2.00 and C_P = 2.00(0.20)3.00 (from bottom to top).

Figures 6(a)-6(d) are plots of π^* versus λ with n = 50 and α = 0.05 for c = 1.00, 1.33, 1.50, 2.00 and C_p = $c(0.20)c+1$. From those figures, we see that the powers corresponding to our adjusted critical values c_0 remain decreasing in measurement error, but the decrements originated in our adjusted critical values c_0 is smaller than those originated in the critical values with no correction. For instance, when we compare the π^Y values in Figure 5(b) (c = 1.33, n = 50) for C_p = 1.93 to the π^* values in Figure 6(b) (c = 1.33, n = 50) for C_p = 1.93, we obtain that π^Y = 0.104 and π^* = 0.690 with λ =

0.50. In this case, by our adjusted critical values c_0, the power we improved is about 0.60. With our revised critical values, we ensure the α-risk within the preset magnitude and we have improved a certain degree of power. For our results to be practical, we provide the tables of our revised critical values for some commonly used capability requirements in Tables 12-15 in the Appendix. Using those tables, the practitioner may skip the complex calculation and directly select the proper critical values for capability testing.