Channel-Aware Decision Fusion With Unknown Local Sensor Detection Probability

Jwo-Yuh Wu, Chan-Wei Wu, Tsang-Yi Wang, and Ta-Sung Lee

***Abstract*—Existing channel-aware decision fusion schemes assume that the local detection probability is known at the fusion center (FC). However, this paradigm ignores the possibility of unknown sensor alarm responses to the occurrence of events. Accordingly, this correspondence examines the binary decision fusion problem under the assumption that the local detection probability is unknown. Treating the communication links between the nodes and the FC as binary symmetric channels and assuming that the sensor nodes transmit simple one-bit reports to the FC, the global fusion rule is formulated initially in terms of the generalized likelihood ratio test (GLRT). Adopting the assumption of a high SNR regime, an approximate maximum likelihood (ML) estimate is derived for the unknown parameter required to implement the GLRT that is affine in the received data. The GLRT-based formulation is intuitively straightforward, but does not permit a tractable performance analysis. Therefore, motivated by the affine nature of the approximate ML solution, a simple alternative fusion rule is proposed in which the test statistic remains affine in the received data. It is shown that the proposed fusion rule facilitates the analytic characterization of the channel effect on the global detection performance. In addition, given a reasonable range of the local detection probability, it is shown that the global detection probability can be improved by reducing the total link error. Thus, a sensor power allocation scheme is proposed for enhancing the detection performance by improving the link quality. Simulation results show that: 1) the alternative fusion rule outperforms the GLRT; and 2) the detection performance of the fusion rule is further improved when the proposed power loading method is applied.**

***Index Terms*—Communication channels, distributed detection, power allocation, sensor networks.**

I. Introduction

The problem of distributed signal/event detection and decision fusion in wireless sensor networks has attracted significant attention in the literature [1], [12], [13]. However, most previous studies are based on the idealized assumption that the sensor reports are received at the fusion center (FC) without error [13], [16]. Recently, there have been several proposals that further take into account the communication channel impairments [2], [3], [6], [7], [11]; see [4] for a tutorial introduction to distributed detection in the presence of nonideal channel links. In general, these channel-aware schemes assume that the local sensor detection performance, characterized by the detection probability and the false-alarm probability, is known to the FC. However, this paradigm ignores the possibility of unknown sensor alarm responses to the occurrence of events within the sensing field. For example, consider a sensor network designed to monitor the rise in temperature within a room in order to detect the potential outbreak of a fire. In practice, the characteristics of a fire are uncertain, e.g., the mean temperature may vary from 100° to 1000° depending on the severity of the fire or the type of fire. Moreover, the characteristics of the fire may vary over time. As a result, the local detection probability (under a fixed threshold) could be unknown due to the response to the uncertain temperature of fire events. To accommodate such variations in the sensing field conditions, one conceivable approach is to simply model the local detection probability as an unknown parameter and to design suitable global decision rules for tackling such uncertainty.

This correspondence proposes a channel-aware decision fusion scheme tailored for the above-mentioned scenario. The communication links between the sensor nodes and the FC are modeled as binary symmetric channels. Each sensor, when triggered, sends a single bit to the FC to inform it of its local decision. Note that since the FC treats the local detection probability as an unknown parameter, the nodes do not need to send an additional message regarding the current local detection performance, and thus the communication overhead is reduced. Based solely on the received sensor reports, the global decision rule is formulated intuitively as a generalized likelihood ratio test (GLRT) [9]. The implementation of this test calls for the maximum likelihood (ML) estimate of the unknown parameter, which, in the current case, does not allow for a closed-form solution. Thus, under a high signal-to-noise ratio (SNR) assumption, an approximate ML estimate is derived that is *affine* in the received data. However, even when adopting this approximation, the detection performance of the GLRT decision rule remains difficult to characterize. Therefore, based on the approximated ML scheme, a simple alternative fusion rule is proposed in which the test statistic remains affine in the received data. The proposed fusion rule enables the derivation of a closed-form expression for the detection performance and, therefore, facilitates the analytic characterization of the channel effect. In addition, it is shown that, for reasonable ranges of the local detection and false alarm probabilities, the global detection performance can be improved by enhancing the communication-link quality, specifically, reducing the total link bit-error rate (BER). Hence, an optimal power allocation scheme is proposed to minimize the total BER subject to a total power budget. Simulations show that the proposed fusion rule outperforms the GLRT; in addition, the detection performances of both the proposed fusion rule and the GLRT decision rule are seen to be further improved via the application of the optimal sensor power loading scheme. The remainder of this correspondence is organized as follows. Section II formulates the problem, while Section III presents the GLRT based detection scheme and derives the approximate ML solution. Section IV introduces the proposed fusion rule and derives the associated analytic performance results. Section V formulates an algorithm for improving the channel link quality in order to improve the detection performance. Section VI presents the simulation results. Finally, Section VII provides some brief concluding remarks.

II. Problem Statement

Consider a sensor network with N identical binary nodes designed to monitor the occurrence of a certain event. Each sensor exists in one of two different states, namely active (e.g., when the measurement is above a certain threshold) or silent (e.g., the measurement is below the threshold, and the sensor simply remains quiet to conserve energy). Assume that each node is subject to a *known* false-alarm probability

Manuscript received June 05, 2009; accepted October 08, 2009. First published November 06, 2009; current version published February 10, 2010. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Pramod K. Varshney. This work was supported by the National Science Council of Taiwan under grants NSC 97-2221-E-009-101-MY3 and NSC 98-2221-E-110-043-MY3, by the Ministry of Education of Taiwan under the MoE ATU Program, by the Academia-Research Cooperation Program of MoEA under Grant 98-EC-17-A-02-S2-0050, and by MediaTek Research Center at National Chiao Tung University, Taiwan.

J.-Y. Wu, C.-W. Wu, and T.-S. Lee are with the Department of Electrical Engineering, National Chiao Tung University, Hsinchu, Taiwan (e-mail: jywu@cc.nctu.edu.tw; xl3505@gmail.com; tslee@mail.nctu.edu.tw).

T.-Y. Wang is with Institute of Communications Engineering, National Sun Yat-sen University, Kaohsiung, Taiwan (e-mail: tcwang@faculty.nsysu.edu.tw).

Color versions of one or more of the figures in this correspondence are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSP.2009.2036065

π_0 due to small ambient perturbations.[1] When triggered by the occurrence of the event of interest, the local detection probability across the sensors, π_1, falls within the interval $(\pi_0, 1)$. However, the exact value of π_1 is assumed to be *unknown*. The status of the ith sensor can thus be represented using a binary random variable $s_i \in \{0, 1\}$, with $\Pr\{s_i = 1\} = \pi_0$ in the absence of the event of interest, and $\Pr\{s_i = 1\} = \pi_1$ otherwise. Each sensor records its status using a single bit and then transmits this bit to the FC. Assume that the communication link between the ith sensor node and the FC is nonideal and is modeled by a binary-symmetric channel with a crossover probability ε_i, $1 \leq i \leq N$. At the FC, the bit received from the ith sensor is decoded, and the resultant message $r_i \in \{0, 1\}$ is a Bernoulli random variable

$$\Pr\{r_i = 1\} = \begin{cases} \pi_0(1-\varepsilon_i)+(1-\pi_0)\varepsilon_i, & \text{(event is absent)} \\ \pi_1(1-\varepsilon_i)+(1-\pi_1)\varepsilon_i, & \text{(event is present).} \end{cases} \quad (2.1)$$

Based on a single snapshot of the observed sensor reports r_i, $1 \leq i \leq N$, the FC utilizes a predefined decision rule in order to make a final decision regarding the occurrence (or absence) of the event of interest within the sensing field. The main objectives of this study include: 1) to formulate a suitable fusion rule for the case in which the local detection probability is unknown; and 2) to devise a technique for mitigating the channel effect on the global detection performance.

III. GLRT-Based Detection Scheme

A. GLRT Scheme

Assuming that the set of Bernoulli random variables $\{r_i\}$ are conditionally independent given the event under test, the joint probability mass functions of $\mathbf{r} := [r_1 \cdots r_N]$ under either π_0 and π_1 are given by

$$p(\mathbf{r}; \pi_m) = \prod_{i=1}^{N} [(1-2\varepsilon_i)\pi_m + \varepsilon_i]^{r_i} \times [-(1-2\varepsilon_i)\pi_m + (1-\varepsilon_i)]^{1-r_i}, \quad m = 0, 1. \quad (3.1)$$

Since π_1 is unknown, the detection problem can be formulated intuitively as the following binary composite hypothesis test:

$$\begin{cases} \mathcal{H}_0 : p(\mathbf{r}; \pi_0), & \text{(event is absent)} \\ \mathcal{H}_1 : p(\mathbf{r}; \pi_1), \pi_1 > \pi_0, & \text{(event is present).} \end{cases} \quad (3.2)$$

The GLRT [9, Ch. 6] is a typical decision rule for problems of this type and decides $\mathcal{H}_1$ if

$$\ln \frac{p(\mathbf{r}; \widehat{\pi}_{1,\mathrm{ML}})}{p(\mathbf{r}; \pi_0)} = \sum_{i=1}^{N} r_i \log\left[\frac{(1-2\varepsilon_i)\widehat{\pi}_{1,\mathrm{ML}} + \varepsilon_i}{(1-2\varepsilon_i)\pi_0 + \varepsilon_i}\right] + \sum_{i=1}^{N} (1-r_i) \log\left[\frac{(1-2\varepsilon_i)\widehat{\pi}_{1,\mathrm{ML}} - (1-\varepsilon_i)}{(1-2\varepsilon_i)\pi_0 - (1-\varepsilon_i)}\right] \geq \gamma \quad (3.3)$$

where $\hat{\pi}_{1,\mathrm{ML}}$ is the ML estimate of π_1 and the threshold γ is determined from the prescribed false-alarm probability. However, to implement (3.3), it is first necessary to find $\hat{\pi}_{1,\mathrm{ML}}$. This can be achieved by solving the equation $\partial \ln p(\mathbf{r}; \pi_1)/\partial \pi_1 = 0$, which based on (3.1) can determined directly as

$$\sum_{i=1}^{N} \frac{r_i}{\pi_1 + [\varepsilon_i/(1-2\varepsilon_i)]} + \sum_{i=1}^{N} \frac{1-r_i}{\pi_1 - [(1-\varepsilon_i)/(1-2\varepsilon_i)]} = 0. \quad (3.4)$$

[1]Note that knowledge about π_0 can be acquired in a training process conducted in the absence of the event of interest.

In other words, finding the unknown parameter $\hat{\pi}_{1,\mathrm{ML}}$ involves solving certain roots of the polynomial given in (3.4), which is of the order $N - 1$. While this can be achieved using numerical techniques, an analytic solution does not exist. Accordingly, given the assumption of a high SNR regime, the following section derives a simple closed-form approximate ML solution.

B. Approximate ML Estimate

Crucially, if the link error probability ε_i is small, it follows that

$$\frac{\varepsilon_i}{1-2\varepsilon_i} = \left(1 + 2\varepsilon_i + 4\varepsilon_i^2 + \cdots\right)\varepsilon_i \approx \varepsilon_i$$

and

$$\frac{1-\varepsilon_i}{1-2\varepsilon_i} = \left(1 + 2\varepsilon_i + 4\varepsilon_i^2 + \cdots\right)(1-\varepsilon_i) \approx 1 + \varepsilon_i \quad (3.5)$$

by neglecting the higher order terms. In accordance with (3.5), (3.4) can be well approximated by

$$\sum_{i=1}^{N} \frac{r_i}{\pi_1 + \varepsilon_i} + \sum_{i=1}^{N} \frac{1-r_i}{\pi_1 - (1+\varepsilon_i)} = \sum_{i=1}^{N} \frac{r_i(\pi_1 - 1 - \varepsilon_i) + (1-r_i)(\pi_1 + \varepsilon_i)}{\pi_1^2 - \pi_1 - \varepsilon_i(1-\varepsilon_i)} = 0. \quad (3.6)$$

Retaining only the first-order term in the denominator in each summand and rearranging, (3.6) becomes

$$\sum_{i=1}^{N} \frac{\pi_1 + \varepsilon_i - (2\varepsilon_i + 1)r_i}{\pi_1^2 - \pi_1 - \varepsilon_i} = 0. \quad (3.7)$$

Given the assumption that ε_i is small, $\pi_1^2 - \pi_1 - \varepsilon_i \approx \pi_1^2 - \pi_1$. Therefore, (3.7) can be further reduced to

$$\frac{1}{\pi_1^2 - \pi_1} \sum_{i=1}^{N} [\pi_1 + \varepsilon_i - (2\varepsilon_i + 1)r_i] = 0. \quad (3.8)$$

Hence, provided that $\pi_1 \neq \{0, 1\}$, $\hat{\pi}_1$ (hereafter denoting the approximate ML estimate of π_1) can be found by solving

$$\sum_{i=1}^{N} [\pi_1 + \varepsilon_i - (2\varepsilon_i + 1)r_i] = 0. \quad (3.9)$$

Therefore, the following approximate ML scheme is obtained

$$\hat{\pi}_1 = \frac{1}{N} \sum_{i=1}^{N} [(1 + 2\varepsilon_i)r_i - \varepsilon_i]. \quad (3.10)$$

C. Discussions

1) Although the estimate given in (3.10) is only an approximation to the true ML solution, it is nevertheless attractive since it is *affine* in the received data samples, r_i, and is therefore potentially amenable to analysis. Furthermore, extensive simulations reveal that the detection performances of the GLRT decision rule based on $\hat{\pi}_1$ and the true ML solution, respectively, are very similar (see Section VI).
2) Even with the approximate ML estimate $\hat{\pi}_1$ given in (3.10), the achievable detection performance of the GLRT (3.3), in particular, the impact due to channel uncertainty, remains quite difficult to characterize especially when the number of sensors is finite. Thus, the following section proposes an alternative fusion test that exploits the affine nature of $\hat{\pi}_1$ to facilitate the analytic characterization of the link error effect.

□

IV. Proposed Detection Scheme

A. Proposed Approach

It is widely known that the GLRT is merely a heuristic approach and does not take account of any specific optimality criteria regarding the detection performance [9], [10]. As suggested by [10, p. 204], an alternative (yet simple and intuitive) strategy is to simply compare the ML estimate against the known parameter π_0 and decide in favor of the null hypothesis whenever the resultant difference (measured using some appropriate metric) is less than a certain threshold. Utilizing this approach, and in order to further exploit the affine nature of the approximate ML estimate in (3.10), the following alternative test criterion is proposed:

$$\begin{cases} \mathcal{H}_0 : \hat{\pi}_1 - \pi_0 \leq \gamma \\ \mathcal{H}_1 : \hat{\pi}_1 - \pi_0 > \gamma. \end{cases} \tag{4.1}$$

The main advantage of this decision rule is that, unlike the GLRT in (3.3), the test statistic in (4.1) is affine in the estimate $\hat{\pi}_1$ and is therefore also affine in the received data samples r_i. This attractive feature enables the resultant decision performance to be analytically characterized, as shown below.

B. Analytic Performance

To proceed, let us write

$$\hat{\pi}_1 = \frac{1}{N}\sum_{i=1}^{N}[(1+2\varepsilon_i)r_i - \varepsilon_i] = \underbrace{\frac{1}{N}\sum_{i=1}^{N}(1+2\varepsilon_i)r_i}_{:=T} - \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i, \tag{4.2}$$

where T denotes the equivalent test statistic. Since $r_i \in \{0,1\}$, T assumes a finite number of alphabets, which are to be specified first. Thus, for each $0 \leq k \leq N$, let $I^{(k)} := \{I_1^{(k)}, I_2^{(k)} \cdots, I_{C_k^N}^{(k)}\}$ be the collection of all the distinct k-element subsets of $\{1, \cdots, N\}$, where $C_k^N := N!/[k!(n-k)!]$ and $I^{(0)} = \{\phi\}$. Also, for each $0 \leq k \leq N$, let $S^{(k)}$ be a set consisting of all possible values of T when k sensors are active, i.e.,

$$\begin{aligned} S^{(k)} &:= \{T | k \text{ sensors are active}\} \\ &= \left\{S_1^{(k)}, S_2^{(k)}, \cdots, S_{C_k^N}^{(k)}\right\} \\ \text{where} \quad S_l^{(k)} &:= N^{-1}\left(k + 2\sum_{i\in I_l^{(k)}}\varepsilon_i\right). \end{aligned} \tag{4.3}$$

As a result, it follows that

$$T \in \bigcup_{k=0}^{N} S^{(k)}, \quad \text{where} \quad S^{(0)} = \{0\}. \tag{4.4}$$

It is noted from (4.4) that there are a total of $C_0^N + C_1^N + \cdots + C_N^N = (1+1)^N = 2^N$ possible levels of T. To assess the performance of the proposed decision rule given in (4.1), assume without loss of generality that for each $1 \leq k \leq N$, the elements in $S^{(k)}$ are arranged as $S_1^{(k)} \leq S_2^{(k)} \leq \cdots \leq S_{C_k^N}^{(k)}$, i.e.,

$$N^{-1}\left(k + 2\sum_{i\in I_1^{(k)}}\varepsilon_i\right) \leq N^{-1}\left(k + 2\sum_{i\in I_2^{(k)}}\varepsilon_i\right) \leq \cdots \leq N^{-1}\left(k + 2\sum_{i\in I_{C_k^N}^{(k)}}\varepsilon_i\right). \tag{4.5}$$

Also, let $1 \leq k_l \leq N$ be such that

$$N^{-1}\left(k_l + 2\sum_{i\in I_1^{(k_l)}}\varepsilon_i\right) \leq \pi_0 + \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i + \gamma < N^{-1}\left(k_l + 1 + 2\sum_{i\in I_1^{(k_l+1)}}\varepsilon_i\right). \tag{4.6}$$

In accordance with the definition of the detection probability, $P_d = \Pr\{T \geq \pi_0 + (1/N)\sum_{i=1}^{N}\varepsilon_i + \gamma | \mathcal{H}_1\}$, the following lower bound for P_d can be derived:

$$P_d \geq P_d^{(L)} \tag{4.7}$$

in which based on (4.6) and (4.3)

$$\begin{aligned} P_d^{(L)} &= \Pr\left\{S^{(k_l+1)} \cup S^{(k_l+2)} \cup \ldots \cup S^{(N)} | \pi_1 \text{ is true}\right\} \\ &= \sum_{k=k_l+1}^{N}\sum_{l=1}^{C_k^N}\left\{\prod_{i\in I_l^{(k)}}[(1-2\varepsilon_i)\pi_1 + \varepsilon_i] \times \prod_{i\notin I_l^{(k)}}[-(1-2\varepsilon_i)\pi_1 + (1-\varepsilon_i)]\right\}. \end{aligned} \tag{4.8}$$

Similarly, for the false-alarm probability $P_f = \Pr\{T \geq \pi_0 + (1/N)\sum_{i=1}^{N}\varepsilon_i + \gamma | \mathcal{H}_0\}$, the associated lower bound is obtained as[2]

$$P_f \geq P_f^{(L)} \tag{4.9}$$

where

$$\begin{aligned} P_f^{(L)} &= \Pr\left\{S^{(k_l+1)} \cup S^{(k_l+2)} \cup \ldots \cup S^{(N)} | \pi_0 \text{ is true}\right\} \\ &= \sum_{k=k_l+1}^{N}\sum_{l=1}^{C_k^N}\left\{\prod_{i\in I_l^{(k)}}[(1-2\varepsilon_i)\pi_0 + \varepsilon_i] \times \prod_{i\notin I_l^{(k)}}[-(1-2\varepsilon_i)\pi_0 + (1-\varepsilon_i)]\right\}. \end{aligned} \tag{4.10}$$

It is observed that the performance bounds in (4.8) and (4.10) depend on the link error probability ε_i. Thus, the performance bounds provide a convenient means of evaluating the effect of channel link uncertainties on the detection performance, as discussed in the following section.

V. Impact of Channel Link Uncertainty on Detection Performance

A. Tractable Approximation of Performance Bounds

The performance bounds in (4.8) and (4.10) vary as a nonlinear function of the link error probability ε_i. Therefore, the original forms of (4.8) and (4.10) do not permit a straightforward analysis of the impact of nonideal communication channels on the detection performance. It will be recalled that the approximate ML estimate given in (3.10) is

[2]Note that the upper bounds for both P_d and P_f can be directly obtained by replacing the lower summation indexes in (4.8) and (4.10) by k_l. Simulations indicate that in both cases the gap between the lower and upper bounds is small.

based on the assumption of a high SNR regime (i.e., the link error probability has a low value). Adopting the same assumption here, the lower bounds given in (4.8) and (4.10) can be simplified considerably, as established in the following lemma.

Lemma 5.1: For a small value of ε_i, it can be shown that

$$P_d^{(L)} = \sum_{k=k_l+1}^{N} A_k + \left(\sum_{i=1}^{N} \varepsilon_i\right) \sum_{k=k_l+1}^{N} B_k \tag{5.1}$$

where

$$\begin{aligned} A_k &:= C_k^N (1-\pi_1)^{N-k} \pi_1^k \\ B_k &:= \pi_1^{k-1}(1-2\pi_1)(1-\pi_1)^{N-k-1} \\ &\quad \times \left(C_{k-1}^{N-1}(1-\pi_1) - C_k^{N-1}\pi_1\right). \end{aligned} \tag{5.2}$$

Similarly

$$P_f^{(L)} = \sum_{k=k_l+1}^{N} C_k + \left(\sum_{i=1}^{N} \varepsilon_i\right) \sum_{k=k_l+1}^{N} D_k \tag{5.3}$$

where

$$\begin{aligned} C_k &:= C_k^N (1-\pi_0)^{N-k} \pi_0^k \\ D_k &:= \pi_0^{k-1}(1-2\pi_0)(1-\pi_0)^{N-k-1} \\ &\quad \times \left(C_{k-1}^{N-1}(1-\pi_0) - C_k^{N-1}\pi_0\right). \end{aligned} \tag{5.4}$$

Proof: Based on (4.8) and (4.10), the results can be obtained by neglecting the high-order terms of ε_i and performing some direct manipulations. □

B. Characterization of Channel Effects

Lemma 5.1 shows that while (4.8) and (4.10) are complicated functions of ε_i, in the high SNR regime the detection performance is closely related to the total link error rate, namely $\sum_{i=1}^{N} \varepsilon_i$. Although $\sum_{i=1}^{N} \varepsilon_i$ provides a measure of the aggregate end-to-end communication link quality, it does not necessarily directly reflect the overall detection performance. However, given certain assumptions regarding the sensor alarm rates, the lower bounds of the global detection probability can be enlarged provided that $\sum_{i=1}^{N} \varepsilon_i$ is kept small. More precisely, the following theorem applies.

Theorem 5.2: Assume that $\pi_0 < 0.5 < \pi_1$. With a fixed false-alarm probability P_f, let $P_d^{(L)}$ and $P_d'^{(L)}$ be the detection probability lower bounds associated with two different summed link errors, i.e., $E = \sum_{i=1}^{N} \varepsilon_i$ and $E' = \sum_{i=1}^{N} \varepsilon_i'$, respectively. If $E' < E$, then it follows that $P_d'^{(L)} > P_d^{(L)}$. □

Proof: See Appendix. □

Theorem 5.2 suggests that, when the condition $\pi_0 < 0.5 < \pi_1$ is satisfied, the global detection probability tends to improve as the value of the total link error rate is reduced. Note that the assumption $\pi_0 < 0.5 < \pi_1$ is not too restricted for any reasonable detectors. Inspired by Theorem 5.2, the following section develops a sensor power allocation scheme designed to enhance the global detection performance by reducing the total link error.

C. Optimal Sensor Power Allocation Scheme

Assume that each sensor utilizes an ON–OFF signaling technique to transmit its one-bit reports to the FC. Assume also that the node-to-FC communications take place over flat fading channels. Thus, the following discrete-time baseband channel model can be applied:

$$y_i = h_i p_i s_i + v_i, \quad 1 \le i \le N \tag{5.5}$$

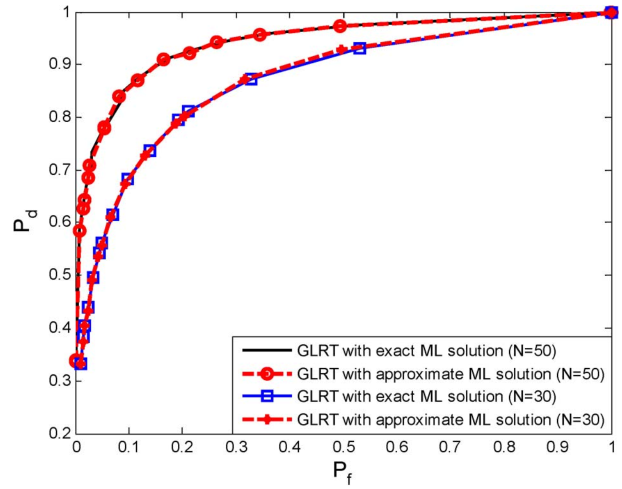

Fig. 1. ROC curves of GLRT (3.3) with exact and approximate ML solutions.

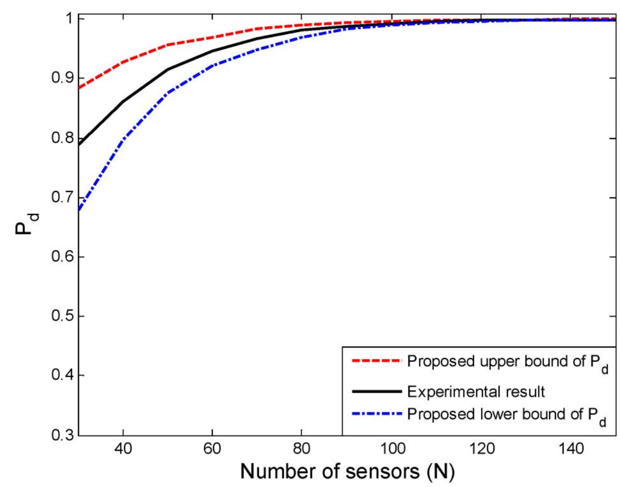

Fig. 2. Detection probability and theoretical performance bounds ($P_f = 0.1$).

where y_i is the data received at the FC from the ith sensor, h_i is the instantaneous channel gain of the ith link (assumed to be perfectly known to the FC), p_i^2 is the power allocation factor for the ith node, and v_i is zero-mean Gaussian measurement noise with variance σ_v^2. From (5.5), the crossover probability is given by $\varepsilon_i = Q(|h_i p_i|/\sigma_v)$, where $Q(t) := (\sqrt{2\pi})^{-1}\int_t^{\infty} \exp[-u^2/2]du$ is the Q-function. Given a fixed total power budget $\mathcal{P}$, the optimal sensor power allocation problem can be formulated as

$$\text{Minimize} \sum_{i=1}^{N} Q\left(|h_i p_i|/\sigma_v\right), \quad \text{subject to} \sum_{i=1}^{N} p_i^2 = \mathcal{P}. \tag{5.6}$$

Note that the optimization problem given in (5.6) has been addressed in the context of MIMO wireless communications in [14] and [15]. Thus, the algorithm proposed therein is used directly here to establish the optimal sensor power allocation factor for each of the nodes within the network.

VI. Simulation Results

In this section, the performance of the proposed scheme is investigated by means of numerical simulation. The channel gains of the communication paths are assumed to be complex Gaussian with zero mean and unit variance and are i.i.d. across sensors. Throughout the simulation, the noise variance is set to be $\sigma_v^2 = 0.05$; in Figs. 1–6, the local sensor alarm probabilities are $(\pi_0, \pi_1) = (0.4, 0.6)$. Fig. 1 presents the ROC curves of the GLRT test (3.3) implemented using the exact

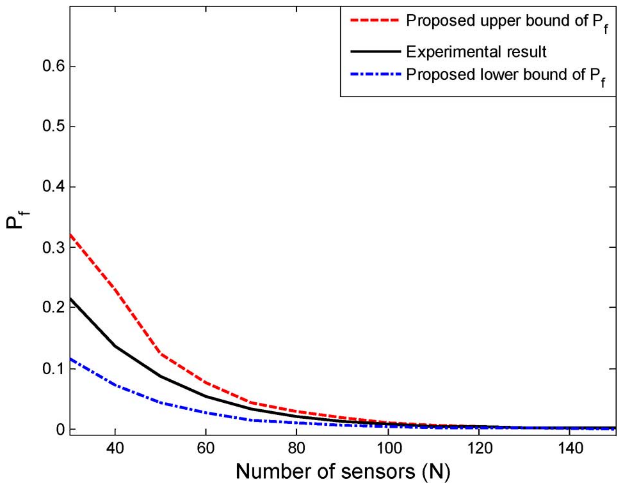

Fig. 3. False-alarm probability and theoretical performance bounds ($P_d = 0.9$).

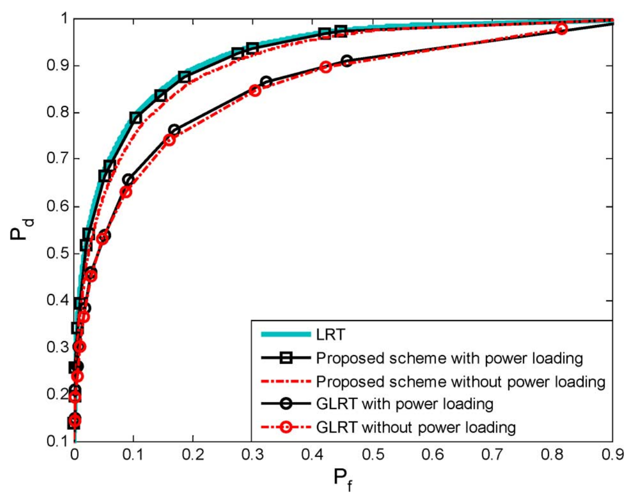

Fig. 4. ROC curves of GLRT (3.3) and proposed scheme (4.1) ($N = 30$).

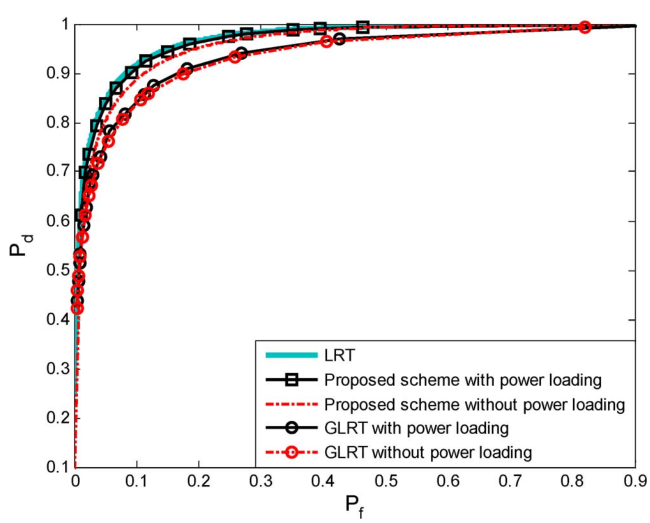

Fig. 5. ROC curves of GLRT (3.3) and proposed scheme (4.1) ($N = 50$).

ML solution [via solving the polynomial (3.4)] and the proposed approximate ML solution ($\hat{\pi}_1$ in (3.10)), respectively. It is seen that the detection performances achieved using $\hat{\pi}_1$ and the true ML solution are almost identical. Figs. 2 and 3 examine the tightness of the performance

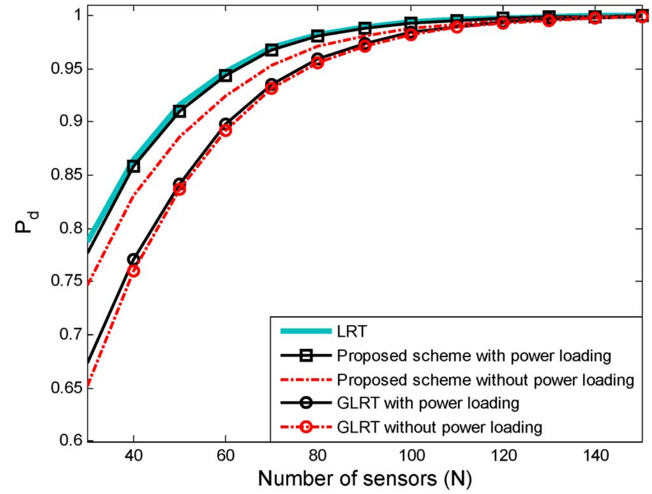

Fig. 6. Detection probabilities for different numbers of sensors ($P_f = 0.1$).

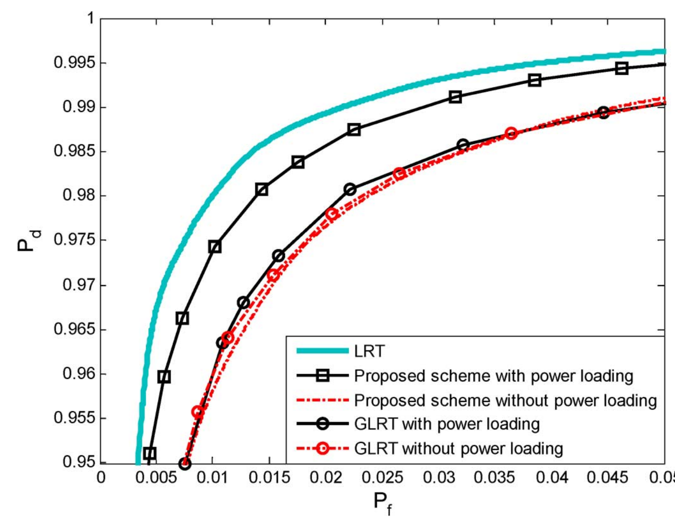

Fig. 7. ROC curves of GLRT (3.3) and proposed scheme (4.1) ($\pi_0 = 0.2$).

bounds for P_d and P_f derived in Section IV-B for different number of sensors N. As we can see, the theoretical bounds well predict the experimental results, especially when N is large. Figs. 4 and 5 compare the GLRT (3.3) (with the exact ML solution) and the alternative test (4.1) for, respectively, $N = 30$ and $N = 50$. Both methods with and without power allocation are considered. Note that by "without power allocation" we mean $p_i = 1$ for all i in (5.5), and the channel gains h_i's are drawn from the standard Gaussian distribution in each Monte Carlo run. Also, the ROC curves obtained by the likelihood ratio test (LRT) assuming that π_1 is exactly known are also included as the benchmark. It is seen that the detection performance of the proposed alternative rule (4.1) is discriminably improved via the application of the sensor power allocation scheme (especially when the network size is small) and is almost identical to the ideal LRT. For the GLRT, the performance improvement attained via power loading is only slight. This is reasonable since the proposed power allocation scheme is specifically aimed at enhancing the detection probability of the alternative test (4.1) but not for the GLRT. With fixed $P_f = 0.1$, Fig. 6 further depicts the detection probabilities of all methods for $30 \leq N \leq 150$. The results show that as the number of sensors increases, the performances of all methods improve and converge to the ideal LRT solution. The proposed test (4.1), however, performs quite close to the ideal LRT, irrespective of the network size. Finally, Fig. 7 shows the ROC curve when π_0 is set instead as $\pi_0 = 0.2$ ($N = 30$). Compared to Fig. 4, the detection

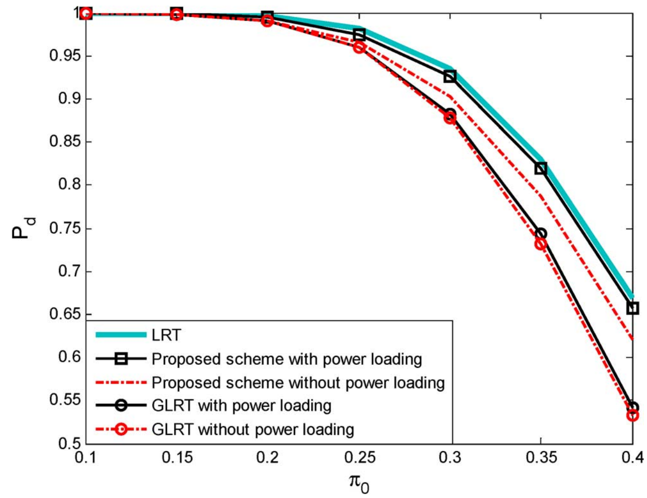

Fig. 8. Detection probabilities for different π_0 ($P_f = 0.05$).

performances of all methods improve. This is reasonable since when π_0 is small, the occurrence of change is potentially more discernible. With $P_f = 0.05$ and $N = 30$, Fig. 8 shows the detection probability for $0.1 \leq \pi_0 \leq 0.4$. As we can see, the proposed test (4.1) with power loading still outperforms the GLRT-based solution.

VII. Conclusions

This correspondence has presented an original contribution to binary decision fusion with identical sensors when the local detection probability is unknown. It has been shown that while the global fusion rule can be formulated in terms of the GLRT, the need for an ML estimate of the unknown parameter when implementing the GLRT decision rule prevents an analytic evaluation of the detection performance, even when a closed-form approximation of the ML solution is used. Thus, exploiting the affine nature of the approximate ML solution, a simple alternative fusion statistic has been proposed, which remains affine in the received sensor reports. Such an alternative scheme not only facilitates a tractable performance analysis, but also enables the analytic characterization of the effect of channel impairments on the global decision performance. Given a reasonable range of the local detection and false alarm probabilities, it has been shown that a higher aggregate link quality leads to an improved global detection probability. Therefore, a sensor power allocation scheme has been proposed to minimize the summed link errors. The simulation study has shown that the proposed alternative fusion rule outperforms the GLRT and yields a further improved performance when combined with the proposed power loading method. In the future, we will extend the current study to the scenario with nonidentical sensors and investigate the problem with the Bayesian approach.

Appendix
Proof of Theorem 5.2

To prove the theorem, let us define

$$S_k^A := \sum_{i=k}^{N} A_i,\ S_k^B := \sum_{i=k}^{N} B_i,\ S_k^C := \sum_{i=k}^{N} C_i,\ S_k^D := \sum_{i=k}^{N} D_i \tag{A.1}$$

where A_i and B_i are defined in (5.2) and C_i and D_i are defined in (5.4). The theorem is proven using the following two technical lemmas.

Lemma A.1: Assume that $\pi_0 < 0.5 < \pi_1$. The following results hold:

1) Both $S_k^A > 0$ and $S_k^C > 0$ are monotonically decreasing in k.
2) $S_k^B \leq 0$ and $S_k^D \geq 0$ for all k.

□

Proof of Lemma A.1: We note that 1) follows immediately by definition, and thus it remains to prove 2). Let us write

$$B_k = \underbrace{\pi_1^{k-1}(1-2\pi_1)(1-\pi_1)^{N-k}C_{k-1}^{N-1}}_{:=Q_k} - \underbrace{\pi_1^{k}(1-2\pi_1)(1-\pi_1)^{N-k-1}C_{k}^{N-1}}_{:=R_k}. \tag{A.2}$$

Note that $Q_0 = R_N = 0$ since $C_{-1}^{N-1} = 0$ and $C_N^{N-1} = 0$. Since

$$\sum_{k=1}^{N} Q_k = (1-2\pi_1)\sum_{k=1}^{N}\pi_1^{k-1}(1-\pi_1)^{N-k}C_{k-1}^{N-1} = (1-2\pi_1)(\pi_1+1-\pi_1)^{N-1} = (1-2\pi_1) \tag{A.3}$$

and

$$\sum_{k=0}^{N-1} R_k = (1-2\pi_1)\sum_{k=0}^{N-1}\pi_1^{k}(1-\pi_1)^{N-k-1}C_{k}^{N-1} = (1-2\pi_1)(\pi_1+1-\pi_1)^{N-1} = (1-2\pi_1) \tag{A.4}$$

we have

$$S_0^B = \sum_{i=0}^{N} B_i = \sum_{i=0}^{N} Q_i - \sum_{i=0}^{N} R_i = \sum_{i=1}^{N} Q_i - \sum_{i=0}^{N-1} R_i = 0. \tag{A.5}$$

Furthermore, since $\pi_1 > 0.5$, it follows that

$$S_N^B = \pi_1^{N-1}(1-2\pi_1) \to 0^- \text{ as } N \text{ gets large.} \tag{A.6}$$

We further observe that

$$\begin{aligned} C_{k-1}^{N-1}&(1-\pi_1) - C_k^{N-1}\pi_1 \\ &= \frac{(1-\pi_1)(N-1)!}{(k-1)!(N-k)!} - \frac{\pi_1(N-1)!}{(k)!(N-k-1)!} \\ &= (N-1)!\left[\frac{k-N\pi_1}{(k)!(N-k)!}\right]. \end{aligned} \tag{A.7}$$

From (A.7), and by the definition of B_k in (5.2), it follows immediately that

$$B_k > 0 \text{ for } k < N\pi_1 \quad \text{and} \quad B_k < 0 \text{ for } k > N\pi_1. \tag{A.8}$$

From (A.8), S_k^B decreases when $0 \leq k < N\pi_1$ and increases for $N\pi_1 < k \leq N$. This result, together with (A.5) and (A.6), implies that $S_k^B \leq 0$. Following a similar approach, it can be shown that $S_k^D \geq 0$. □

Lemma A.2: The following results hold:

1) If $\sum_{i=1}^N \varepsilon_i \leq (\pi_1/(2\pi_1 - 1))$, then $S_k^A + S_k^B(\sum_{i=1}^N \varepsilon_i)$ is monotonically decreasing.
2) If $\sum_{i=1}^N \varepsilon_i \leq (\pi_0/(1 - 2\pi_0))$, then $S_k^C + S_k^D(\sum_{i=1}^N \varepsilon_i)$ is monotonically decreasing.

□

Proof: We shall only prove 1) since 2) can be verified using a similar approach. To proceed, let us first focus on the case in which $k > N\pi_1$. By assumption we have

$$\begin{aligned} \sum_{i=1}^{N}\varepsilon_i &\leq \frac{\pi_1}{2\pi_1 - 1} = \frac{\pi_1}{2\pi_1 - 1} \times \frac{C_k^N(1-\pi_1)}{C_k^N(1-\pi_1)} \\ &= \frac{C_k^N}{2\pi_1 - 1} \times \frac{\pi_1(1-\pi_1)}{C_k^N - \pi_1 C_k^N} \\ &\leq \frac{C_k^N}{2\pi_1 - 1} \times \frac{\pi_1(1-\pi_1)}{C_{k-1}^{N-1} - \pi_1 C_k^N} \end{aligned} \tag{A.9}$$

where the last equality follows since $C_{k-1}^{N-1} < C_k^N$ and $k > N\pi_1$. From (A.9), we immediately have

$$\frac{C_{k-1}^{N-1} - \pi_1 C_k^N}{\pi_1(1-\pi_1)} \cdot \left(\sum_{i=1}^{N} \varepsilon_i\right) \leq \frac{C_k^N}{2\pi_1 - 1}. \tag{A.10}$$

Since $C_k^N = C_{k-1}^{N-1} + C_k^{N-1}$, we can rewrite (A.10) as

$$\begin{aligned} &\frac{C_{k-1}^{N-1} - \pi_1\left(C_{k-1}^{N-1} + C_k^{N-1}\right)}{\pi_1(1-\pi_1)} \cdot \left(\sum_{i=1}^{N} \varepsilon_i\right) \\ &= \frac{(1-\pi_1)C_{k-1}^{N-1} - \pi_1 C_k^{N-1}}{\pi_1(1-\pi_1)} \cdot \left(\sum_{i=1}^{N} \varepsilon_i\right) \\ &= \left[\frac{C_{k-1}^{N-1}}{\pi_1} - \frac{C_k^{N-1}}{(1-\pi_1)}\right] \cdot \left(\sum_{i=1}^{N} \varepsilon_i\right) \\ &\leq \frac{C_k^N}{2\pi_1 - 1}. \end{aligned} \tag{A.11}$$

The last inequality in (A.11) is equivalent to

$$C_k^N + \left(C_{k-1}^{N-1}\frac{1-2\pi_1}{\pi_1} - C_k^{N-1}\frac{1-2\pi_1}{1-\pi_1}\right)\left(\sum_{i=1}^{N} \varepsilon_i\right) \geq 0. \tag{A.12}$$

Multiplying both sides of (A.12) by $(1-\pi_1)^{N-k}\pi_1^k$ and rearranging, we obtain

$$\begin{aligned} &C_k^N(1-\pi_1)^{N-k}\pi_1^k + \pi_1^{k-1}(1-2\pi_1)(1-\pi_1)^{N-k-1} \times \\ &\qquad \left(C_{k-1}^{N-1}(1-\pi_1) - C_k^{N-1}\pi_1\right)\left(\sum_{i=1}^{N} \varepsilon_i\right) \geq 0. \end{aligned} \tag{A.13}$$

From the definition of the sequences A_k and B_k in (5.2), inequality (A.13) essentially asserts that

$$A_k + B_k\left(\sum_{i=1}^{N} \varepsilon_i\right) \geq 0. \tag{A.14}$$

Since $S_k^A - S_{k+1}^A = A_k$ and $S_k^B - S_{k+1}^B = B_k$, (A.14) implies that

$$S_k^A + S_k^B\left(\sum_{i=1}^{N} \varepsilon_i\right) \geq S_{k+1}^A + S_{k+1}^B\left(\sum_{i=1}^{N} \varepsilon_i\right) \tag{A.15}$$

which proves 1) for $k > N\pi_1$. If $k < N\pi_1$, we have $[(C_{k-1}^{N-1}/\pi_1) - (C_k^{N-1}/(1-\pi_1))] < 0$, and hence the last inequality in (A.11) still holds. Repeating the procedures shown in (A.12)–(A.14), the relation given in (A.15) is obtained. The proof is thus completed. □

Proof of Theorem 5.2: Associated with the total error rate E, let $(S_k^A, S_k^B, S_k^C, S_k^D)$ be accordingly defined as in (A.1). For a given threshold γ, and with the given E, we can then express the performance bounds in (5.1) and (5.3) as

$$P_d^{(L)} = S_{k_l+1}^A + S_{k_l+1}^B E \quad \text{and} \quad P_f^{(L)} = S_{k_l}^C + S_{k_l}^D E \tag{A.16}$$

where k_l is some positive integer. If E is reduced to $E' < E$, it follows from part 2) of Lemma A.1 that

$$S_{k_l}^A + S_{k_l}^B E < S_{k_l}^A + S_{k_l}^B E' \text{ and } S_{k_l}^C + S_{k_l}^D E > S_{k_l}^C + S_{k_l}^D E'. \tag{A.17}$$

Since $\pi_0 < 0.5 < \pi_1$, we have $(\pi_1/(2\pi_1 - 1)) > 0$ and $(\pi_0/(1-2\pi_0)) > 0$. Under the assumptions of Lemma A.2, $S_k^C + S_k^D E'$ is monotonically decreasing. Let $k_l' < k_l$ be such that

$$k_l' = \min\left\{k | S_k^C + S_k^D E' \leq S_{k_l}^C + S_{k_l}^D E\right\}. \tag{A.18}$$

For such k_l', it follows that $P_f^{(L)}(k_l') := S_{k_l'}^C + S_{k_l'}^D E' \leq S_{k_l}^C + S_{k_l}^D E = P_f^{(L)} \leq P_f$. The corresponding detection probability lower bound satisfies

$$P_d^{(L)}\left(k_l'\right) := S_{k_l'}^A + S_{k_l'}^B E' \overset{(a)}{>} S_{k_l}^A + S_{k_l}^B E' \overset{(b)}{>} S_{k_l}^A + S_{k_l}^B E = P_d^{(L)} \tag{A.19}$$

where (a) holds since $S_k^A + S_k^B E$ is also monotonically decreasing (see Lemma A.2) and (b) follows from the first inequality in (A.17). Hence, as E is reduced to E', we have $P_d^{(L)}(k_l') > P_d^{(L)}$ whenever $P_f^{(L)}(k_l') \leq P_f$. This implies that the detection probability lower bound $P_d^{(L)\prime}$ corresponding to P_f must exceed $P_d^{(L)}$. □

REFERENCES

[1] J. F. Chamberland and V. V. Veeravalli, "Wireless sensors in distributed detection applications," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 16–25, May 2007.

[2] B. Chen, R. Jiang, T. Kasetkasem, and P. K. Varshney, "Channel aware decision fusion in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 52, no. 12, pp. 3454–3458, Dec. 2004.

[3] Q. Cheng, B. Chen, and P. K. Varshney, "Detection performance limits for distributed sensor networks in the presence of non-ideal channels," *IEEE Trans. Wireless Commun.*, vol. 5, no. 11, pp. 3034–3038, Nov. 2006.

[4] B. Chen, T. Lang, and P. K. Varshney, "Channel aware distributed detection in wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 23, no. 4, pp. 16–26, Jul. 2006.

[5] R. V. Hogg, J. W. McKean, and A. T. Craig, *Introduction to Mathematical Statistics*, 6th ed. Englewood Cliffs, NJ: Prentice-Hall, 2005.

[6] S. K. Jayaweera, "Large system decentralized detection performance under communication constraints," *IEEE Commun. Lett.*, vol. 9, no. 9, pp. 769–771, Sep. 2005.

[7] V. R. Kanchumarthy, R. Viswanathan, and M. Madishetty, "Impact of channel errors on decentralized detection performance of wireless sensor networks: A study of binary modulations, Rayleigh-fading and nonfading channels, and fusion-combiners," *IEEE Trans. Signal Process.*, vol. 56, no. 5, pp. 1761–1769, May 2008.

[8] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cliffs, NJ: Prentice-Hall PTR, 1993.

[9] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*. Englewood Cliffs, NJ: Prentice-Hall PTR, 1998.

[10] B. C. Levy, *Principles of Signal Detection and Parameter Estimation*. New York: Springer, 2008.

[11] R. Niu, B. Chen, and P. K. Vahshney, "Fusion of decisions over Rayleigh fading channels in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 54, no. 3, pp. 1018–1027, Mar. 2006.

[12] A. Swami, Q. Zhao, Y. W. Hong, and L. Tong, *Wireless Sensor Networks: Signal Processing and Communications Perspectives*. Hoboken, NJ: Wiley, 2007.

[13] P. K. Varshney, *Distributed Detection and Data Fusion*. New York: Springer, 1997.

[14] N. Wang and S. D. Blostein, "Minimum BER power allocation for MIMO spatial multiplexing systems," in *Proc. ICC*, 2005, vol. 4, pp. 2282–2286.

[15] Z. Yan, K. M. Wong, and Z. Q. Luo, "Optimal diagonal precoder for multiantenna communication systems," *IEEE Trans. Signal Process.*, vol. 53, no. 6, pp. 2089–2100, Jun. 2005.

[16] Q. Zhao, A. Swami, and L. Tong, "The interplay between signal processing and networking in sensor networks," *IEEE Signal Process. Mag.*, vol. 23, no. 4, pp. 84–93, Jul. 2006.