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Decision Support

A joint measurement of efficiency and effectiveness for non-storable commodities: Integrated data envelopment analysis approaches

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ABSTRACT

Efficiency and effectiveness for non-storable commodities represent two distinct dimensions and a joint measurement of both is necessary to fully capture the overall performance. This paper proposes two novel integrated data envelopment analysis (IDEA) approaches, the integrated Charnes, Cooper and Rhodes (ICCR) and integrated Banker, Charnes and Cooper (IBCC) models, to jointly analyze the overall performance of non-storable commodities under constant and variable returns to scale technologies. The core logic of the proposed models is simultaneously determining the virtual multipliers associated with inputs, outputs, and consumption by additive specifications for technical efficiency and service effectiveness terms with equal weights. We show that both ICCR and IBCC models possess the essential properties of rationality, uniqueness, and benchmarking power. A case analysis also demonstrates that the proposed novel IDEA approaches have higher benchmarking power than the conventional separate DEA approaches. More generalized specifications of IDEA models with unequal weights are also elaborated.

1. Introduction

Data envelopment analysis (DEA) is a technique for measuring the relative efficiency of decision making units (DMUs) which produce similar products. Measures of both technical efficiency (a transformation of factors to production) and service effectiveness (consumption of production) for storable commodities are essentially the same because the commodities, once produced, can be stockpiled until consumed. Nothing will be lost throughout the transformation from production to consumption if one assumes that all the stockpiles are eventually sold, there is no storage cost, and there is no loss incurred. Namely, conventional measures for storable commodities assume perfect sale and no storage cost effectiveness. However, technical efficiency and service effectiveness for non-storable commodities, such as transport services, represent two distinct measurements because one can never store the surplus service during periods of low demand (off-peak hours) for use during periods of high demand (peak hours). When such non-storable commodities are produced and a portion of which are not concurrently consumed, the technical effectiveness (a joint effect of both technical efficiency and service effectiveness) would be less than the technical efficiency. To explain this concept, Fielding (1987) first introduced three performance measures for a public transit system by defining technical efficiency as the ratio of production to factors, service effectiveness as the ratio of consumption to production, and technical effectiveness as the ratio of consumption to factors as depicted in Fig. 1. As shown in Fig. 1, once the transport production (e.g. seat-miles) is transformed from such factors as labor, vehicle, and fuel, seat-miles must be consumed immediately by the passengers; otherwise they are exhausted and wasted. Thus, both technical efficiency and service effectiveness should be jointly evaluated to account for the portion of seat-miles not utilized in practice. The technical effectiveness depends not only on how well the production (seat-miles) is transformed from the factors but also on how well the consumption (passenger-miles) is transformed from the production. Any poor performance of transport services can be attributed to either poor technical efficiency or poor service effectiveness or a combination of both. Without separation of technical efficiency and service effectiveness measurements, it is difficult to scrutinize the sources of poor performance. In other words, to assess the system performance for non-storable commodities, it would become more informative if one could have jointly analyzed the efficiency and effectiveness measurements.

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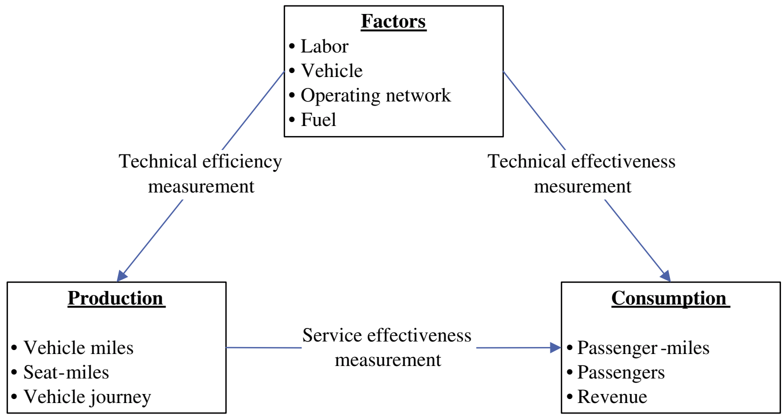

Fig. 1. Distinctive performance measurements for transport service. Source: Revised from Fielding (1987).

Over the past three decades, various DEA models have been widely used to evaluate the technical efficiency or technical effectiveness of decision making units (DMUs) in different organizations or industries. In transport performance evaluation, numerous applications of DEA have also been found in various fields, including airline (e.g. Schefczyk, 1993; Charnes et al., 1996; Sengupta, 1999; Alder and Golany, 2001), airport (e.g. Salazar de La Cruz, 1999; Joseph, 2000; Martin and Roman, 2001; Adler and Berechman, 2001), maritime (e.g. Tongzon, 2001; Cullinane et al., 2006), transit (e.g. Nolan, 1996; Kerstens, 1996; Viton, 1998; Cowie and Asenova, 1999; Odeck and Alkadi, 2001; Nolan et al., 2002; Karlaftis, 2003, 2004; Boame, 2004; Sheth et al., 2007; Margari et al., 2007), and railway (e.g. Oum and Yu, 1994; Cowie, 1999). Most of these works, however, merely evaluated the performance from the perspective of technical efficiency or technical effectiveness.

In order to completely and fairly evaluate the relative performance of non-storable transport services, several recent works have employed various DEA approaches to evaluating the efficiency and effectiveness. In general, they can be divided into four categories: separate DEA model (hereinafter, SDEA; e.g. Karlaftis, 2004; Chiou and Chen, 2006), separate two-stage DEA model (hereinafter, STDEA; e.g. Rousseau and Rousseau, 1997; Lan and Lin, 2003; Keh et al., 2006), network DEA model (hereinafter, NDEA; e.g. Yu and Lin, 2008; Yu, 2008; Kao, 2009), and integrated two-stage DEA model (hereinafter, ITDEA; e.g. Kao and Hwang, 2008; Chen et al., in press; Chen et al., 2009). The SDEA employs independent DEA models to measure technical efficiency, service effectiveness, and technical effectiveness separately. Hence, paradoxical improvement strategies were usually generated based on the results of these independent DEA models. To overcome this shortcoming, the STDEA uses an input-oriented DEA model to evaluate the technical efficiency and an output-oriented DEA model to assess the service effectiveness, holding the output level unchanged. Although the STDEA model will not generate conflicting improvement strategies, it suggests the organization be divided into two independent departments: production and sale, such that the performance of one department is not interrelated with that of the other department. This is of course not exactly true from the organizational perspective. The lack of interrelated performance among different departments may be solved by the NDEA or ITDEA modeling. However, due to the complexity of the modeling, the scale economy and slack values for each DMU are hard to compute by the NDEA model, proposed by Yu and Lin (2008) and Yu (2008), which is only applicable to the case of constant returns to scale. The ITDEA model proposed by Chen et al. (in press) can be applied to both technologies of constant and variable returns to scale, and the scale economy and slack values can easily be computed as well. However, for the ease of transforming the objective function into a linear form, the ITDEA model sets rather restricted weights proportional to the relative contributions of inputs, outputs and consumption in association with their corresponding virtue multipliers. This would lead to difficulties provided that the organization would value the weights differently across the departments. Strictly speaking, the weights should represent the relative importance of efficiency and effectiveness valued by the evaluator or the decision maker, and they should remain unchanged in evaluating all DMUs. To further rectify this shortcoming, this paper develops integrated DEA (IDEA) models which jointly evaluate the non-storable commodities' efficiency and effectiveness. In fact, the pioneering IDEA concept was proposed and tested in our early work (Chiou et al., 2007). The present study further extends the IDEA models to generalized IDEA models. Moreover, important underlying properties of the proposed models are also proven. In this paper, we also demonstrate the applicability and superiority of our proposed IDEA models and generalized IDEA models.

The rest of the paper is organized as follows. The formulation of the IDEA models under constant and variable returns to scale contexts is proposed in Section 2. The essential properties—rationality, uniqueness, benchmarking power—are proven in Section 3. To demonstrate the applicability of the proposed IDEA models and to compare the benchmarking power with the conventional SDEA models, a case analysis is presented in Section 4. Generalized IDEA approaches with unequal weights are further elaborated and analyzed in Section 5. Finally, concluding remarks and suggestions for future studies are addressed.

2. Model formulation

DEA is a method for measuring the relative efficiency of DMUs that perform similar tasks. A DEA model under constant returns to scale (CRS) context was developed by Charnes et al. (1978, CCR model hereinafter). A DEA model under variable returns to scale (VRS) context was later developed by Banker et al. (1984, BCC model hereinafter) based on the CCR model by adding the convexity constraint. To simultaneously measure the efficiency and effectiveness for non-storable commodities with avoidance of the above mentioned shortcomings, this paper proposes two integrated DEA approaches under CRS and VRS contexts, which are termed as integrated CCR (ICCR) model and integrated BCC (IBCC) model, respectively. The formulation of the proposed ICCR and IBCC models is given below.

2.1. Integrated CCR model

The proposed integrated CCR model [ICCR] aims to maximize the technical efficiency and service effectiveness by simultaneously solving for virtual multipliers corresponding to factor, production, and consumption variables under CRS assumptions. The model is formulated as follow:

$$[\text{ICCR}] \quad \underset{u,v,w}{Max} \quad H_k = \left(\frac{\sum_{r=1}^{R} u_r y_{kr}}{\sum_{j=1}^{J} v_j x_{kj}}\right) + \left(\frac{\sum_{s=1}^{S} w_s z_{ks}}{\sum_{r=1}^{R} u_r y_{kr}}\right) \tag{1}$$

$$s.t. \quad \sum_{r=1}^{R} u_r y_{ir} \leqslant \sum_{j=1}^{J} v_j x_{ij}, \quad i = 1,2,\ldots,I, \tag{2}$$

$$\sum_{s=1}^{S} w_s z_{is} \leqslant \sum_{r=1}^{R} u_r y_{ir}, \quad i = 1,2,\ldots,I, \tag{3}$$

$$v_j \geqslant 0, \quad j = 1,2,\ldots,J, \tag{4}$$

$$w_s \geqslant 0, \quad s = 1,2,\ldots,S, \tag{5}$$

$$u_r \geqslant 0, \quad r = 1,2,\ldots,R, \tag{6}$$

where $H_k \in [0,2]$ represents the overall efficiency score of DMU k. If H_k equals to two, the DMU is defined relatively efficient; otherwise the DMU is relatively inefficient. x_{kj} represents the jth input of DMU k. y_{kr} denotes the rth output of DMU k. z_{ks} represents the sth consumption of the DMU k. The variables v_j, u_r, w_s are corresponding virtual multipliers of the jth input, the rth output, and the sth consumption. I, J, S, R are the number of DMUs, inputs, outputs, and consumption, respectively.

Adding slack to each constraint, [ICCR] can then be reformulated as [ICCR-S]:

$$[\text{ICCR-S}] \quad \underset{u,v,w}{Max} \quad H_k = \left(\frac{\sum_{r=1}^{R} u_r y_{kr}}{\sum_{j=1}^{J} v_j x_{kj}}\right) + \left(\frac{\sum_{s=1}^{S} w_s z_{ks}}{\sum_{r=1}^{R} u_r y_{kr}}\right) \tag{7}$$

$$s.t. \quad \sum_{r=1}^{R} u_r y_{ir} = \sum_{j=1}^{J} v_j (x_{ij} - s_{ij}), \quad i = 1,2,\ldots,I, \tag{8}$$

$$\sum_{s=1}^{S} w_s (z_{is} + s_{is}) = \sum_{r=1}^{R} u_r y_{ir}, \quad i = 1,2,\ldots,I, \tag{9}$$

$$v_j \geqslant 0, \quad j = 1,2,\ldots,J, \tag{10}$$

$$w_s \geqslant 0, \quad s = 1,2,\ldots,S, \tag{11}$$

$$u_r \geqslant 0, \quad r = 1,2,\ldots,R. \tag{12}$$

For a given production (y_{ir}), [ICCR-S] model can be expressed as:

$$[\text{ICCR-S}] \quad \underset{u,v,w}{Max} \quad H_k = \left(\sum_{r=1}^{R} u_r y_{kr}\right)\left(\sum_{r=1}^{R} u_r y_{kr}\right) + \left(\sum_{s=1}^{S} w_s z_{ks}\right)\left(\sum_{j=1}^{J} v_j x_{kj}\right) \tag{13}$$

$$s.t. \quad \left(\sum_{j=1}^{J} v_j x_{kj}\right)\left(\sum_{r=1}^{R} u_r y_{kr}\right) = 1, \tag{14}$$

$$\sum_{r=1}^{R} u_r y_{ir} = \sum_{j=1}^{J} v_j (x_{ij} - s_{ij}), \quad i = 1,2,\ldots,I, \tag{15}$$

$$\sum_{s=1}^{S} w_s (z_{is} + s_{is}) = \sum_{r=1}^{R} u_r y_{ir}, \quad i = 1,2,\ldots,I, \tag{16}$$

$$v_j \geqslant 0, \quad j = 1,2,\ldots,J, \tag{17}$$

$$w_s \geqslant 0, \quad s = 1,2,\ldots,S, \tag{18}$$

$$u_r \geqslant 0, \quad r = 1,2,\ldots,R. \tag{19}$$

With the optimal virtual multipliers, the technical efficiency, service effectiveness and technical effectiveness of DMU k can be, respectively, calculated as follows:

$$h_k = \left(\frac{\sum_{r=1}^{R} u_r^* y_{kr}}{\sum_{j=1}^{J} v_j^* x_{kj}}\right), \quad g_k = \left(\frac{\sum_{s=1}^{S} w_s^* z_{ks}}{\sum_{r=1}^{R} u_r^* y_{kr}}\right), \quad \text{and} \quad o_k = \left(\frac{\sum_{s=1}^{s} w_s^* z_{ks}}{\sum_{j=1}^{J} v_j^* x_{kj}}\right).$$

Based upon the optimal slack values, we can determine the amount of inputs to be curtailed or the amount of consumption to be added or promoted to achieve efficiency.

2.2. Integrated BCC model

The above ICCR model can be easily extended to an integrated BCC model [IBCC] by simply adding the convexity constraint, which is expressed as:

$$[\text{IBCC}] \quad \underset{u,v,w}{Max} \quad \left(\frac{\sum_{r=1}^{R} u_r y_{kr} - u_0}{\sum_{j=1}^{J} v_j x_{kj}}\right) + \left(\frac{\sum_{s=1}^{S} w_s z_{ks} - u_1}{\sum_{r=1}^{R} u_r y_{kr} - u_0}\right) \tag{20}$$

$$s.t. \quad \sum_{r=1}^{R} u_r y_{ir} - u_0 \leqslant \sum_{j=1}^{J} v_j (x_{ij} - s_{ij}), \quad i = 1,2,\ldots,I, \tag{21}$$

$$\sum_{s=1}^{S} w_s (z_{is} + s_{is}) - u_1 \leqslant \sum_{r=1}^{R} u_r y_{ir} - u_0, \quad i = 1,2,\ldots,I, \tag{22}$$

$$v_j \geqslant 0, \quad j = 1,2,\ldots,J, \tag{23}$$

$$w_s \geqslant 0, \quad s = 1,2,\ldots,S, \tag{24}$$

$$u_r \geqslant 0, \quad r = 1,2,\ldots,R. \tag{25}$$

3. Properties of the proposed models

In what follows we prove the ICCR and IBCC models exhibiting three essential properties: rationality, uniqueness and benchmarking power.

3.1. Rationality property

3.1.1. Rationality of ICCR

According to Charnes et al. (1978), the efficiency of any DMU is obtained as the maximum of a ratio of weighted outputs, subject to the condition that the similar ratio for every DMU be less than or equal to unity. Since the proposed ICCR model is to maximize two aspects of efficiency values, the overall efficiency value should be less than or equal to two. The measure of the efficiency of any DMU can also be obtained in a similar way. Mathematically,

$$[\text{ICCR}'] \quad \underset{u,v,w}{Max} \quad \left(\frac{\sum_{r=1}^{R} u_r y_{kr}}{\sum_{j=1}^{J} v_j x_{kj}}\right) + \left(\frac{\sum_{s=1}^{S} w_s l_{ks}}{\sum_{r=1}^{R} u_r y_{kr}}\right) \tag{26}$$

$$s.t. \quad \frac{\sum_{r=1}^{R} u_r y_{ir}}{\sum_{j=1}^{J} v_j x_{ij}} \leqslant 1, \quad i = 1,2,\ldots,I, \tag{27}$$

$$\frac{\sum_{s=1}^{S} w_s l_{is}}{\sum_{r=1}^{R} u_r y_{ir}} \leqslant 1, \quad i = 1,2,\ldots,I, \tag{28}$$

$$v_j \geqslant 0, \quad j = 1,2,\ldots,J, \tag{29}$$

$$w_s \geqslant 0, \quad s = 1,2,\ldots,S, \tag{30}$$

$$u_r \geqslant 0, \quad r = 1,2,\ldots,R. \tag{31}$$

Let $E'_r = \frac{x_R}{x_r}$ and $E''_r = \frac{l_r}{l_R}$ respectively represent the technical efficiency (ratios of inputs at a given output) and service effectiveness (ratios of consumption at a given output), where x_R is the minimum input that can produce the given output and x_r is the actual input being rated from the same output. Likewise, l_R is the maximum consumption that can be generated from the given output and l_r is the actual consumption being rated from the same output. The overall efficiency can be calculated as $E_r = E'_r + E''_r = \frac{x_R}{x_r} + \frac{l_r}{l_R}$. Essentially, $0 \leqslant E_r \leqslant 2$.

Alternately, we can also derive the overall efficiency, E_r, of our proposed ICCR as follows. For any given output y,

$$\underset{u,v,w}{Max} \quad h_r = \frac{uy_r}{vx_r} + \frac{wl_r}{uy_r} \tag{32}$$

$$s.t. \quad \frac{uy_R}{vx_R} \leqslant 1, \tag{33}$$

$$\frac{uy_r}{vx_r} \leqslant 1, \tag{34}$$

$$\frac{wl_R}{uy_R} \leqslant 1, \tag{35}$$

$$\frac{wl_r}{uy_r} \leqslant 1, \tag{36}$$

$$u, v, w \geqslant 0. \tag{37}$$

Let u^*, v^*, w^* represent the optimal triplet of corresponding values. As $x_R \leqslant x_r, l_R \geqslant l_r$ and $y_R = y_r = y$ imply $u^* y_r = u^* y_R = v^* x_R$ and $u^* y_r = u^* y_R = w^* l_R$, we then have the following results and relationships:

$$\text{Technical efficiency} = \frac{u^* y_r}{v^* x_r} = \frac{u^* y_R}{v^* x_r} = \frac{v^* x_R}{v^* x_r} = E'_r,$$

$$\text{Service effectiveness} = \frac{w^* l_r}{u^* y_r} = \frac{w^* l_r}{u^* y_R} = \frac{w^* l_r}{w^* l_R} = E''_r.$$

Thus, $h_o = \frac{u^* y_r}{v^* x_r} + \frac{w^* l_r}{u^* y_r} = E'_r + E''_r = E_r$.

No matter which alternatives are adopted, the efficiency scores determined by the ICCR model are proven to possess an essential property of rationality $0 \leqslant E_r \leqslant 2$, because the optimal values of ICCR model have satisfied the definition of efficiency.

3.1.2. Rationality of IBCC

The proposed IBCC model can be expressed as follow:

$$[\mathrm{IBCC}'] \quad \underset{u,v,w}{Max} \quad \left(\frac{\sum_{r=1}^{R} u_r y_{kr} - u_0}{\sum_{j=1}^{J} v_j x_{kj}}\right) + \left(\frac{\sum_{s=1}^{S} w_s l_{ks} - u_1}{\sum_{r=1}^{R} u_r y_{kr} - u_0}\right) \tag{38}$$

$$s.t. \quad \frac{\sum_{r=1}^{R} u_r y_{ir} - u_0}{\sum_{j=1}^{J} v_j x_{ij}} \leqslant 1, \quad i = 1,2,\ldots,I, \tag{39}$$

$$\frac{\sum_{s=1}^{S} w_s l_{is} - u_1}{\sum_{r=1}^{R} u_r y_{ir} - u_0} \leqslant 1, \quad i = 1,2,\ldots,I, \tag{40}$$

$$v_j \geqslant 0, \quad j = 1,2,\ldots,J, \tag{41}$$

$$w_s \geqslant 0, \quad s = 1,2,\ldots,S, \tag{42}$$

$$u_r \geqslant 0, \quad r = 1,2,\ldots,R. \tag{43}$$

Similar to the measure of efficiency in the ICCR model, we can derive the overall efficiency from the IBCC model as follows:

$$\underset{u,v}{Max} \quad h_r = \frac{uy_r - u_0}{vx_r} + \frac{wl_r - u_1}{uy_r - u_0} \tag{44}$$

$$s.t. \quad \frac{uy_R - u_0}{vx_R} \leqslant 1, \tag{45}$$

$$\frac{uy_r - u_0}{vx_r} \leqslant 1, \tag{46}$$

$$\frac{wl_R - u_1}{uy_R - u_0} \leqslant 1, \tag{47}$$

$$\frac{wl_r - u_1}{uy_r - u_0} \leqslant 1, \tag{48}$$

$$u, v, w \geqslant 0. \tag{49}$$

Let u^*, v^*, w^*, u_0, u_1 represent the optimal set of corresponding values. As $x_R \leqslant x_r, l_R \geqslant l_r$ and $y_R = y_r = y$ imply $u^* y_r - u_0 = u^* y_R - u_0 = v^* x_R$ and $u^* y_r - u_0 = u^* y_R - u_0 = w^* l_R - u_1$, we can then obtain the following relationships:

$$\text{Technical efficiency} = \frac{u^* y_r - u_0}{v^* x_r} = \frac{u^* y_R - u_0}{v^* x_r} = \frac{v^* x_R}{v^* x_r} = E'_r,$$

$$\text{Service effectiveness} = \frac{w^* l_r - u_1}{u^* y_r - u_0} = \frac{w^* l_r - u_1}{u^* y_R - u_0} = \frac{w^* l_r - u_1}{w^* l_R - u_1} = \frac{w^* \left(l_r - \frac{u_1}{w^*}\right)}{w^* \left(l_R - \frac{u_1}{w^*}\right)} = \frac{l_r - \frac{u_1}{w^*}}{l_R - \frac{u_1}{w^*}},$$

$$0 < \text{Service effectiveness} = \frac{l_r - \frac{u_1}{w^*}}{l_R - \frac{u_1}{w^*}} < 1, \text{ where } u_1 \text{ is a scale variable.}$$

When $u_1 > 0$, the result of $l_r > l_r - \frac{u_1}{w^*}$ can be obtained, suggesting that the inputs for DMU r should be reduced to reach its optimal scale. When $u_1 = 0$, the result of $l_r = l_r - \frac{u_1}{w^*}$ can be obtained, suggesting that DMU r is already at its optimal scale. When $u_1 < 0$, the result of $l_r < l_r - \frac{u_1}{w^*}$ can be obtained, suggesting that DMU r needs to expand to reach its optimal scale.

In conclusion, the efficiency scores determined by the IBCC model have been proven to exhibit an essential property of rationality. Additionally, the IBCC model can further indicate the improving direction of each DMU to reach its optimal scale.

3.2. Uniqueness property

3.2.1. Uniqueness of ICCR

To show the uniqueness of the joint efficiency measurement of the ICCR model, we have to prove that the virtual multipliers of $\boldsymbol{u}$, $\boldsymbol{v}$, and $\boldsymbol{w}$ determined by the ICCR model are a global optimum, not a local optimum. Technically, we examine the concavity or convexity of the objective function as well as of the feasible region for this nonlinear programming problem. Concavity and convexity establish necessary conditions for optimality and the Karush–Kuhn–Tucker (KKT) conditions establish sufficient conditions.

For simplicity, without loss of generality, the mathematical model of [ICCR-S] is examined under the case of a single variable for each of the three stages of the model: input, output and service. Since all the constraints in [ICCR-S] are linear, the feasible set defined by these constraints is definitely convex. The bordered Hessian matrix of objective function of [ICCR-S] can be derived as:

$$H = \begin{vmatrix} 0 & 0 & -y^{-1} l u^{-2} \\ 0 & 2x^{-1} y v^{-3} u & -x^{-1} y v^{-2} \\ -y^{-1} l u^{-2} & -x^{-1} y v^{-2} & 2y^{-1} l u^{-3} w \end{vmatrix},$$

where the signs of the first, second and third leading principal minors of $\boldsymbol{H}$ are $|H_1| \leqslant 0, |H_2| \geqslant 0$ and $|H_3| \leqslant 0$, indicating that the bordered Hessian is negative semi-definite and the objective function is a concave function. In other words, the sufficient conditions for a global maximum are proven.

3.2.2. Uniqueness of IBCC

For simplicity and without loss of generality, the mathematical model of [IBCC-S] is also examined under the case of single variable in three aspects of input, output and service. Likewise, the bordered Hessian matrix of objective function of [IBCC-S] can be derived as:

$$H = \begin{vmatrix} 0 & 0 & 0 & l(uy-u_0)^{-2} & -yl(uy-u_0)^{-2} \\ 0 & 2x^{-1}v^{-3}(uy-u_0) & 0 & x(vx)^{-2} & -xy(vx)^{-2} \\ 0 & 0 & 0 & -(uy-u_0)^{-2} & y(uy-u_0)^{-2} \\ l(uy-u_0)^{-2} & x(vx)^{-2} & -(uy-u_0)^{-2} & 2(wl-u_1)(uy-u_0)^{-3} & -2y(wl-u_1)(uy-u_0)^{-3} \\ -yl(uy-u_0)^{-2} & -xy(vx)^{-2} & y(uy-u_0)^{-2} & -2y(wl-u_1)(uy-u_0)^{-3} & 2y^2(wl-u_1)(uy-u_0)^{-3} \end{vmatrix},$$

where, the signs of principal minors of $\boldsymbol{H}$ are $|H_1| \leqslant 0, |H_2| \geqslant 0, |H_3| \leqslant 0, |H_4| \geqslant 0$ and $|H_5| \leqslant 0$, indicating that the bordered Hessian is negative semi-definite and the objective function is a concave function. In other words, the sufficient conditions for a global maximum are proven.

3.3. Benchmarking power property

The benchmarking power of DEA models, in this study, is defined as "the fewer number of efficient DMUs, the higher the benchmarking power of the model." To show the benchmarking power of the IDEA model, we have to prove that the performance score evaluated by the IDEA model is lower than or equal to that evaluated by the conventional DEA model. On the other hand, if the IDEA model rates a DMU as overall efficient, the conventional DEA model should also rate the DMU as both "technically efficient" and "service effective."

Let u_T^*, v_T^* and u_S^*, w_S^* represent the optimal set of virtual multipliers determined by the conventional DEA models in aspects of technical efficiency and service effectiveness, respectively. Assuming that DMU R is evaluated as technical efficiency and service effectiveness by the conventional DEA models, implying that $h_{TR} = \frac{u_T^* y_R}{v_T^* x_R} = 1$ and $o_{SR} = \frac{w_S^* l_R}{u_S^* y_R} = 1$. Two cases are discussed. First, if $u_T^* = u_S^*$, then DMU R will be also evaluated as overall efficient by optimally setting $v_I^* = v_T^*, u_I^* = u_T^* = u_S^*$, and $w_I^* = w_S^*$, then $H_R = \frac{u_I^* y_R}{v_I^* x_R} + \frac{w_I^* l_R}{u_I^* y_R} = \frac{u_T^* y_R}{v_T^* x_R} + \frac{w_S^* l_R}{u_S^* y_R} = 2$. If $u_T^* \neq u_S^*$, due to the uniqueness property of the proposed IDEA and conventional DEA model, $\frac{u_T^* y_R}{v_T^* x_R} > \frac{u_I^* y_R}{v_T^* x_R}$, if $u_I^* \neq u_T^*$ and $\frac{w_S^* l_R}{u_S^* y_R} > \frac{w_S^* l_R}{u_I^* y_R}$, if $w_I^* \neq w_S^*$. Thus, for the case of $v_I^* = v_T^*, u_I^* = u_T^*, w_I^* = w_S^*, H_R = \frac{u_T^* y_R}{v_T^* x_R} + \frac{w_S^* l_R}{u_T^* y_R} < 2$ and for the case of $v_I^* = v_T^*, u_I^* = u_S^*, w_I^* = w_S^*, H_R = \frac{u_S^* y_R}{v_T^* x_R} + \frac{w_S^* l_R}{u_S^* y_R} < 2$. It can be concluded that the proposed IDEA model exhibits higher benchmarking power than the conventional DEA models.

4. Application

The main contribution of this study is to develop the novel IDEA approaches and to prove the theoretical properties exhibited in the proposed ICCR and IBCC models. To further demonstrate the applicability and superiority of our proposed IDEA models and to compare the benchmarking power with the conventional SDEA models (more specifically, SCCR and SBCC models associated with CRS and VRS technologies), a real case analysis from Taiwanese intercity bus companies is conducted.

4.1. Data

Currently, there are 39 intercity bus companies in Taiwan. We take these bus companies as our case analysis. Potential variables of two factor variables (number of buses and operating network), two production variables (number of bus runs and bus-km) and four consumption variables (operating revenue, number of passengers, passenger-km and average number of on-board passengers per run) are considered; all of these data are available from the annual report published by Ministry of Transportation and Communications. Table 1 presents the descriptive data of these variables. To select important and relevant variables, regression analyses are further conducted by respectively regressing production variables on factor variables and consumption variables on production variables, respectively. The results are presented in Table 2. Note that all the explanatory variables have shown positive and significant effects on at least one of the associated dependent variables, suggesting the appropriateness of the above variables selected.

4.2. Efficiency scores

The optimal virtual multipliers corresponding to all variables are determined by the proposed IDEA approaches, ICCR and IBCC models, which jointly measure the overall efficiency scores of each bus company under CRS and VRS respectively, and by the SDEA approaches,

Table 1
Summary of descriptive data for the case of 39 Taiwanese intercity bus companies in 2007.

Item	Factor variable		Production variable		Consumption variable			
	Number of buses	Operating network (km)	Number of bus runs	Vehicle-km	Operating revenue (NT)	Number of passengers	Passenger-km	Average number of on-board passengers per run
Median	24	92	37,526	1,852,906	40,062,094	441,581	19,311,871	13.30
Std. Dev.	208	1,666	243,305	39,782,472	700,333,679	4,530,141	588,420,076	4.44
Max	1,083	7,810	1,266,527	126,078,237	3,574,792,434	26,330,194	2,217,682,256	17.59
Min	4	65	3,866	297,875	2,272,480	26,056	2,114,183	2.33

SCCR and SBCC models, which separately measure the efficiency scores of each company under CRS and VRS respectively. Table 3 compares the efficiency scores under CRS by ICCR and SCCR models; whereas Table 4 compares the scores under VRS by IBCC and SBCC models.

Table 2
Regression results from technical efficiency and service effectiveness perspectives.

Perspective	Dependent variable	Independent variable			
		Number of bus	Operating network	Number of bus run (in thousand)	Bus-km (in thousand)
Technical efficiency	Number of bus runs (in thousand)	0.04 (0.00)	0.24 (4.72) $R^2 = 0.92$		
	Bus-km (in thousand)	266.91 (18.07)	10.02 (5.42) $R^2 = 0.99$		
Service effectiveness	Operating revenue (in thousands)			521.24 (5.71)	14.64 (26.25) $R^2 = 0.99$
	Number of passengers (in thousand)			25.46 (36.29)	0.05 (11.73) $R^2 = 0.99$
	Passenger-km (in thousand)			678.53 (9.73)	10.93 (25.62) $R^2 = 0.99$
	Average number of on-board passengers per run			0.01 (3.15)	0.00 (1.27) $R^2 = 0.35$

Note: t values in parentheses.

Table 3
Scores of overall and individual efficiencies for each company under constant returns to scale.

DMU	ICCR model				SCCR model		
	Overall efficiency	Technical efficiency	Service effectiveness	Technical effectiveness	Technical efficiency	Service effectiveness	Technical effectiveness
1	1.464	0.572	0.892	0.510	0.573	0.916	0.579
2	1.555	0.570	0.985	0.561	0.699	1.000*	0.993
3	1.410	0.767	0.643	0.493	0.799	0.809	0.587
4	1.531	0.830	0.701	0.582	0.830	0.726	0.671
5	1.174	0.393	0.781	0.307	0.414	0.818	0.440
6	0.823	0.153	0.670	0.102	0.574	0.704	0.570
7	1.597	0.636	0.961	0.611	0.673	1.000*	1.000*
8	1.045	0.105	0.940	0.099	0.569	1.000*	0.328
9	0.679	0.266	0.461	0.121	0.285	0.456	0.175
10	1.976	1.000*	0.976	0.976	1.000*	0.976	1.000*
11	1.666	0.754	0.912	0.688	0.754	1.000*	1.000*
12	1.644	0.683	0.961	0.656	0.696	1.000*	0.891
13	1.933	1.000*	0.933	0.933	1.000*	1.000*	1.000*
14	1.634	0.634	1.000*	0.634	0.668	1.000*	0.884
15	1.518	0.771	0.747	0.576	0.772	0.851	1.000*
16	1.627	0.761	0.866	0.659	0.761	0.869	0.695
17	1.157	0.505	0.652	0.329	0.539	0.808	0.574
18	1.423	0.764	0.659	0.504	0.764	0.659	0.685
19	1.832	1.000*	0.832	0.832	1.000*	0.832	1.000*
20	1.836	0.890	0.946	0.842	0.902	1.000*	0.971
21	1.362	0.623	0.738	0.460	0.651	0.820	0.642
22	1.776	0.985	0.791	0.779	1.000*	0.948	0.925
23	2.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
24	1.613	0.655	0.957	0.628	0.656	1.000*	0.917
25	1.258	0.469	0.789	0.370	0.502	0.798	0.592
26	1.261	0.570	0.691	0.394	0.627	0.693	0.747
27	1.281	0.478	0.803	0.384	0.478	0.804	0.471
28	0.765	0.212	0.553	0.117	0.213	0.653	0.260
29	1.569	1.000*	0.569	0.569	1.000*	0.738	0.983
30	1.393	0.625	0.768	0.480	0.668	0.826	0.720
31	1.240	0.605	0.635	0.384	0.634	0.712	0.609
32	1.151	0.375	0.776	0.291	0.407	0.781	0.411
33	1.443	0.443	1.000*	0.443	0.446	1.000*	0.922
34	2.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
35	1.456	0.456	1.000*	0.456	0.456	1.000*	0.909
36	1.274	0.683	0.591	0.403	0.684	0.757	0.691
37	0.836	0.192	0.644	0.124	0.192	0.768	0.251
38	0.973	0.531	0.442	0.235	0.531	0.506	0.395
39	1.467	0.468	0.999	0.468	0.468	1.000*	0.909

Note: *Denotes the DMU achieving corresponding efficiency (effectiveness).

Table 4
Scores of overall and individual efficiencies for each company under variable returns to scale.

DMU	IBCC model				SBCC model		
	Overall efficiency	Technical efficiency	Service effectiveness	Technical effectiveness	Technical efficiency	Service effectiveness	Technical effectiveness
1	1.490	0.579	0.911	0.528	0.579	0.916	0.599
2	1.230	1.000*	0.230	0.230	1.000*	1.000*	1.000*
3	1.132	0.533	0.599	0.319	0.835	0.823	0.589
4	1.535	0.826	0.709	0.586	0.831	0.731	0.707
5	1.315	0.489	0.826	0.404	0.489	0.831	0.475
6	0.977	0.248	0.729	0.180	0.803	0.729	0.719
7	1.880	1.000*	0.880	0.880	1.000*	1.000*	1.000*
8	0.903	0.138	0.765	0.106	0.579	1.000*	0.332
9	0.712	0.261	0.451	0.118	0.430	0.705	0.365
10	1.977	1.000*	0.977	0.977	1.000*	0.977	1.000*
11	1.751	0.754	0.997	0.752	0.754	1.000*	1.000*
12	1.710	0.710	1.000*	0.710	0.748	1.000*	0.893
13	1.439	1.000*	0.439	0.439	1.000*	1.000*	1.000*
14	1.998	1.000*	0.998	0.998	1.000*	1.000*	1.000*
15	1.539	0.771	0.769	0.592	1.000*	0.859	1.000*
16	1.652	0.798	0.853	0.681	0.798	0.874	0.696
17	1.204	0.204	1.000*	0.204	1.000*	1.000*	1.000*
18	1.355	0.690	0.665	0.459	0.850	0.665	0.715
19	1.836	1.000*	0.836	0.836	1.000*	0.836	1.000*
20	1.691	0.699	0.992	0.693	0.904	1.000*	0.973
21	1.446	0.610	0.836	0.510	0.673	0.836	0.643
22	1.823	0.996	0.828	0.824	1.000*	0.951	0.951
23	2.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
24	1.659	0.659	1.000*	0.659	0.659	1.000*	0.917
25	1.313	0.512	0.801	0.410	0.519	0.801	0.594
26	1.389	0.686	0.703	0.483	0.697	0.703	0.763
27	1.282	0.478	0.804	0.385	0.482	0.807	0.475
28	1.572	0.571	1.000*	0.571	0.571	1.000*	0.571
29	1.768	1.000*	0.768	0.768	1.000*	0.768	1.000*
30	1.786	0.887	0.899	0.797	1.000*	1.000*	0.895
31	1.361	0.683	0.678	0.463	0.725	0.786	0.644
32	1.016	0.220	0.796	0.175	0.415	0.796	0.419
33	1.547	0.547	1.000*	0.547	0.575	1.000*	0.946
34	2.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
35	2.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*
36	1.608	0.818	0.790	0.646	0.823	0.790	0.816
37	0.985	0.188	0.797	0.150	0.260	0.797	0.266
38	1.000	0.444	0.556	0.247	0.554	0.556	0.504
39	2.000*	1.000*	1.000*	1.000*	1.000*	1.000*	1.000*

Note: *Denotes the DMU achieving corresponding efficiency (effectiveness).

Note from Table 3 that only two bus companies (DMU 23 and DMU 34) are benchmarked as overall efficient by the proposed ICCR model. In contrast, three bus companies (DMU 13, DMU 23 and DMU 34) are evaluated as overall efficient by the SCCR models. Namely, the proposed ICCR model has higher benchmarking power than the conventional SCCR models. By definition, the overall efficient score of the proposed ICCR model is equal to the sum of scores of technical efficiency and service effectiveness. However, the SCCR models do not possess this essential relationship. Also note from Table 4 that four companies are benchmarked as overall efficient by the proposed IBCC model, whereas nine companies have been assessed as overall efficient by the SBCC models. Once again, our proposed IBCC model demonstrates superior benchmarking power over the conventional SBCC model. In sum, the proposed IDEA approach is superior to conventional SDEA approach in terms of the benchmarking power.

Using the proposed IBCC model, we further examine the signs of $u_0(u_1)$ to identify the scale property for technical efficiency (service effectiveness). The DMU is characterized with increasing returns to scale (IRS) if $u_0^* < 0(u_1^* < 0)$. If $u_0^* > 0(u_1^* > 0)$, the DMU is decreasing returns to scale (DRS). If $u_0^* = 0(u_1^* = 0)$, the DMU is constant returns to scale (CRS). The results are summarized in Table 5. Note that most DMUs are characterized with IRS in their production or consumption, implying enlarging the scale may be required for most of the Taiwanese intercity bus companies to become more technical efficiency and service effective.

4.3. Slack analysis

To propose improvement strategies for the inefficient companies, slack values for each of the factor and consumption variables are computed according to [ICCR-S] models. The results are reported in Table 6. Except for two efficient companies (DMU23 and DMU34), most of the inefficient companies require either reducing factor amounts or raising consumption amounts. Taking DMU 9 as an example, decreasing 6.46% of buses, 9.08% of operating network, or increasing operating revenue by 11.05% would achieve efficiency frontier. Note that contradictory improvement suggestions are likely to emerge on the basis of slack analysis, provided that SDEA approaches are employed for the same case analysis.

Table 5
Returns to scale for each company.

DMU	Technical efficiency		Service effectiveness	
	u_0^*	RTS	u_1^*	RTS
1	−0.101	IRS	−0.123	IRS
2	2.120	DRS	2.760	DRS
3	−0.035	IRS	0.004	DRS
4	−0.075	IRS	−0.088	IRS
5	−0.484	IRS	−0.522	IRS
6	−0.568	IRS	−0.617	IRS
7	−0.740	IRS	−0.804	IRS
8	−0.022	IRS	0.011	DRS
9	−0.996	IRS	−1.000	IRS
10	−0.383	IRS	2.940	DRS
11	0.002	DRS	2.352	DRS
12	−0.191	IRS	−0.170	IRS
13	0.154	DRS	0.159	DRS
14	0.107	DRS	4.315	DRS
15	−1.000	IRS	−1.000	IRS
16	0.107	DRS	0.103	DRS
17	−0.999	IRS	−1.000	IRS
18	−0.360	IRS	−0.390	IRS
19	−0.139	IRS	−0.154	IRS
20	−0.063	IRS	−0.093	IRS
21	−0.389	IRS	−0.425	IRS
22	−0.110	IRS	−0.133	IRS
23	0.000	CRS	0.000	CRS
24	0.065	DRS	0.270	DRS
25	−0.149	IRS	−0.124	IRS
26	−0.264	IRS	−0.350	IRS
27	0.001	DRS	−0.001	IRS
28	−0.999	IRS	−1.000	IRS
29	−0.787	IRS	−0.816	IRS
30	−0.632	IRS	−0.672	IRS
31	−0.159	IRS	−0.200	IRS
32	−0.967	IRS	−0.981	IRS
33	−0.249	IRS	−0.307	IRS
34	0.000	CRS	0.000	CRS
35	−1.000	IRS	−1.000	IRS
36	−0.622	IRS	−0.676	IRS
37	−1.000	IRS	−1.000	IRS
38	−1.000	IRS	−1.000	IRS
39	3.924	DRS	0.704	DRS

5. Generalized IDEA approaches

5.1. Models

Both of the abovementioned two novel IDEA approaches have adopted an additive form of technical efficiency and service effectiveness terms with equal weights. More generalized specifications of the proposed IDEA approaches can be reformulated by introducing unequal weights for both terms. The generalized ICCR model [GICCR] can thus be formulated as:

$$[\text{GICCR}] \quad \underset{u,v,w}{Max} \quad H_k = \alpha\left(\frac{\sum_{r=1}^{R} u_r y_{kr}}{\sum_{j=1}^{J} v_j x_{kj}}\right) + (1-\alpha)\left(\frac{\sum_{s=1}^{S} w_q l_{kq}}{\sum_{r=1}^{R} u_r y_{kr}}\right) \tag{50}$$

$$s.t. \quad \sum_{r=1}^{R} u_r y_{ir} = \sum_{j=1}^{J} v_j (x_{ij} - s_{ij}), \quad i = 1,2,\ldots,I, \tag{51}$$

$$\sum_{s=1}^{S} w_s (l_{is} + s_{is}) = \sum_{r=1}^{R} u_r y_{ir}, \quad i = 1,2,\ldots,I, \tag{52}$$

$$v_j \geqslant 0, \quad j = 1,2,\ldots,J, \tag{53}$$

$$w_s \geqslant 0, \quad s = 1,2,\ldots,S, \tag{54}$$

$$u_r \geqslant 0, \quad r = 1,2,\ldots,R, \tag{55}$$

where α is the weight for technical efficiency and $(1-\alpha)$ is the weight for service effectiveness. If, for instance, the decision maker wishes to place more emphasis on service effectiveness, then α should be set less than 0.5.

Similarly, the generalized IBCC model [GIBCC] can be formulated by introducing α as the weight for technical efficiency and $(1-\alpha)$ as the weight for service effectiveness.

Table 6
Slack values of factors and consumption under constant returns to scale (in percentage).

DMU	Factor variable			Consumption variable		
	Number of buses	Operating network	Operating revenue	Number of passengers	Passenger-km	Average number of passengers on board per run
1	0.00	−8.01	1.23	1.23	1.23	0.00
2	0.00	−1.23	1.23	0.00	0.00	1.23
3	−27.97	−1.30	1.23	1.23	1.23	28.38
4	−1.23	−42.00	1.23	4.95	1.23	1.23
5	−2.11	−68.93	0.00	0.00	0.00	4.75
6	−1.94	0.00	0.00	0.00	0.00	6.03
7	−12.35	−1.23	1.23	1.23	1.23	1.23
8	−19.72	0.00	0.00	1.23	0.00	1.43
9	−6.46	−9.08	0.00	0.00	0.00	11.05
10	−1.24	−17.08	1.24	0.00	0.00	1.69
11	−15.13	−3.67	23.77	0.00	1.23	0.00
12	−4.50	−9.10	8.67	0.00	0.00	0.00
13	−16.91	−7.05	1.00	7.05	0.00	1.17
14	−0.01	−47.70	0.00	10.81	4.31	0.00
15	−4.27	0.00	0.00	0.00	0.00	5.55
16	−43.44	0.00	18.35	0.00	0.00	1.59
17	−11.53	0.00	1.23	0.00	43.02	0.00
18	−30.13	−75.25	0.00	0.00	0.00	11.46
19	−26.82	−6.25	0.00	26.51	0.00	0.00
20	−17.73	0.00	0.50	0.00	0.00	0.00
21	0.00	−12.58	0.00	0.00	0.00	26.24
22	−8.15	0.00	0.00	0.00	0.00	1.23
23	0.00	0.00	0.00	0.00	0.00	0.00
24	0.00	−27.42	0.00	35.89	0.00	0.00
25	−12.94	0.00	21.21	0.00	0.00	0.00
26	−2.86	0.00	1.76	0.00	0.00	0.00
27	−59.77	0.00	0.00	0.00	0.00	12.04
28	0.00	−11.47	0.00	0.00	0.00	1.20
29	−3.10	0.00	0.00	0.00	0.00	0.22
30	−4.04	−6.00	1.24	1.21	0.00	0.00
31	0.00	−9.46	0.00	0.00	23.43	1.27
32	−24.93	0.00	0.00	0.00	0.00	0.89
33	−24.16	−0.59	0.00	0.00	0.00	0.10
34	0.00	0.00	0.00	0.00	0.00	0.00
35	−2.35	0.00	0.00	0.00	0.00	6.20
36	−2.94	0.00	0.00	0.00	1.23	7.33
37	−23.76	−31.25	0.00	31.25	0.00	0.00
38	−1.65	0.00	0.00	0.00	0.00	2.32
39	−1.58	0.00	1.23	0.00	0.00	0.00

Table 7
Technical efficiency and service effectiveness for DMU 6 under various weights.

α	Technical efficiency	Service effectiveness
0.1	0.128	0.702
0.2	0.130	0.701
0.3	0.148	0.681
0.4	0.153	0.670
0.5	0.153	0.670
0.6	0.159	0.637
0.7	0.160	0.619
0.8	0.160	0.619
0.9	0.160	0.619

$$[\text{GIBCC}] \quad \underset{u,v,w}{Max} \quad \alpha\left(\frac{\sum_{r=1}^{R} u_r y_{kr} - u_0}{\sum_{j=1}^{J} v_j x_{kj}}\right) + (1-\alpha)\left(\frac{\sum_{s=1}^{S} w_s l_{ks} - u_1}{\sum_{r=1}^{R} u_r y_{kr} - u_0}\right) \tag{56}$$

$$s.t. \quad \frac{\sum_{r=1}^{R} u_r y_{ir} - u_0}{\sum_{j=1}^{J} v_j x_{ij}} \leqslant 1, \quad i = 1, 2, \ldots, I, \tag{57}$$

$$\frac{\sum_{s=1}^{S} w_s l_{is} - u_1}{\sum_{r=1}^{R} u_r y_{ir} - u_0} \leqslant 1, \quad i = 1, 2, \ldots, I, \tag{58}$$

$$v_j \geqslant 0, \quad j = 1, 2, \ldots, J, \tag{59}$$

$$w_s \geqslant 0, \quad s = 1, 2, \ldots, S, \tag{60}$$

$$u_r \geqslant 0, \quad r = 1, 2, \ldots, R. \tag{61}$$

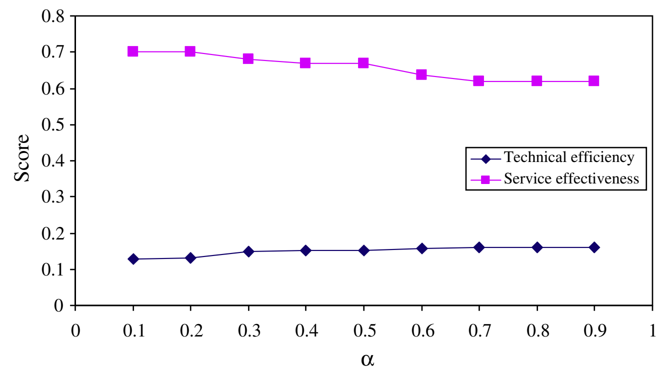

Fig. 2. Technical efficiency and service effectiveness for DMU 6 under various weights.

Table 8
Slack values of factors and consumption for DMU6 under various weights (in percentage).

α	Factor variable		Consumption variable			
	Number of buses	Operating network	Operating revenue	Number of passengers	Passenger-km	Average number of passengers on board per run
0.1	−7.53	0.00	0.00	0.00	0.00	6.14
0.2	−2.31	0.00	0.00	1.23	0.00	4.88
0.3	−2.29	−0.04	2.79	1.29	1.99	4.89
0.4	−2.12	−0.04	2.37	1.10	1.69	5.17
0.5	−1.94	0.00	0.00	0.00	0.00	6.03
0.6	−9.38	0.00	0.00	1.29	0.00	4.87
0.7	−5.12	0.00	0.00	1.23	0.00	1.52
0.8	−5.13	0.00	0.00	1.23	0.00	6.27
0.9	−1.90	0.00	0.00	0.00	0.00	5.23

Table 9
Technical efficiency for each DMU under various weights.

DMU	α (weight)								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.490	0.542	0.546	0.572	0.572	0.572	0.572	0.572	0.572
2	0.570	0.570	0.569	0.570	0.570	0.570	0.570	0.570	0.570
3	0.537	0.538	0.538	0.572	0.767	0.799	0.799	0.799	0.799
4	0.751	0.777	0.805	0.830	0.830	0.830	0.830	0.830	0.830
5	0.259	0.377	0.379	0.393	0.393	0.393	0.411	0.411	0.414
6	0.128	0.130	0.148	0.153	0.153	0.159	0.160	0.160	0.160
7	0.584	0.584	0.584	0.584	0.636	0.636	0.673	0.673	0.673
8	0.105	0.105	0.105	0.105	0.105	0.516	0.517	0.550	0.568
9	0.266	0.266	0.266	0.266	0.266	0.275	0.275	0.285	0.285
10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
11	0.637	0.637	0.637	0.637	0.754	0.754	0.754	0.754	0.754
12	0.683	0.683	0.683	0.683	0.683	0.683	0.683	0.683	0.683
13	0.315	0.929	0.951	0.951	1.000	1.000	1.000	1.000	1.000
14	0.634	0.634	0.634	0.634	0.634	0.634	0.634	0.634	0.642
15	0.520	0.630	0.771	0.771	0.771	0.771	0.771	0.772	0.772
16	0.748	0.761	0.761	0.761	0.761	0.761	0.761	0.761	0.761
17	0.499	0.499	0.505	0.505	0.505	0.505	0.505	0.505	0.505
18	0.763	0.763	0.763	0.763	0.764	0.764	0.764	0.764	0.764
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.628
20	0.633	0.843	0.855	0.890	0.890	0.890	0.892	0.902	0.902
21	0.381	0.567	0.567	0.567	0.623	0.651	0.651	0.651	0.651
22	0.651	0.651	0.900	0.984	0.985	1.000	1.000	1.000	1.000
23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
24	0.634	0.634	0.634	0.634	0.655	0.655	0.656	0.656	0.656
25	0.462	0.462	0.462	0.469	0.469	0.469	0.502	0.502	0.502
26	0.570	0.570	0.570	0.570	0.570	0.627	0.627	0.627	0.627
27	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478	0.478
28	0.163	0.163	0.163	0.163	0.212	0.212	0.213	0.213	0.213
29	0.706	0.706	0.799	1.000	1.000	1.000	1.000	1.000	1.000
30	0.247	0.604	0.604	0.625	0.625	0.625	0.646	0.646	0.663
31	0.222	0.564	0.587	0.605	0.605	0.605	0.605	0.613	0.626
32	0.370	0.370	0.370	0.375	0.375	0.407	0.407	0.407	0.407
33	0.443	0.443	0.443	0.443	0.443	0.443	0.443	0.443	0.443
34	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
35	0.456	0.456	0.456	0.456	0.456	0.456	0.456	0.456	0.456
36	0.477	0.477	0.477	0.543	0.683	0.684	0.684	0.684	0.684
37	0.150	0.150	0.192	0.192	0.192	0.192	0.192	0.192	0.192
38	0.383	0.431	0.447	0.531	0.531	0.531	0.531	0.531	0.531
39	0.462	0.468	0.468	0.468	0.468	0.468	0.468	0.468	0.468

Table 10
Service effectiveness for each DMU under various weights.

DMU	α (weight)								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.915	0.906	0.904	0.892	0.892	0.892	0.892	0.892	0.890
2	0.984	0.986	0.984	0.986	0.985	0.985	0.985	0.984	0.985
3	0.810	0.809	0.809	0.788	0.643	0.610	0.610	0.610	0.610
4	0.726	0.721	0.713	0.701	0.701	0.701	0.701	0.701	0.701
5	0.818	0.790	0.789	0.781	0.781	0.781	0.745	0.745	0.725
6	0.702	0.701	0.681	0.670	0.670	0.637	0.619	0.619	0.619
7	1.000	1.000	1.000	1.000	0.961	0.961	0.898	0.898	0.900
8	0.940	0.940	0.940	0.940	0.940	0.283	0.282	0.420	0.304
9	0.046	0.046	0.461	0.461	0.461	0.451	0.451	0.425	0.425
10	0.976	0.976	0.976	0.976	0.976	0.976	0.976	0.977	0.976
11	1.000	1.000	1.000	1.000	0.912	0.912	0.912	0.912	0.912
12	0.961	0.961	0.961	0.961	0.961	0.961	0.961	0.961	0.955
13	0.788	0.954	0.948	0.948	0.933	0.933	0.933	0.933	0.933
14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	0.817	0.792	0.747	0.747	0.747	0.747	0.747	0.742	0.742
16	0.869	0.866	0.866	0.866	0.866	0.866	0.866	0.866	0.866
17	0.701	0.701	0.652	0.652	0.652	0.652	0.652	0.652	0.652
18	0.659	0.659	0.659	0.659	0.659	0.658	0.661	0.658	0.658
19	0.833	0.833	0.832	0.832	0.832	0.834	0.832	0.835	0.882
20	0.999	0.966	0.962	0.946	0.946	0.946	0.946	0.912	0.912
21	0.822	0.788	0.787	0.788	0.738	0.707	0.707	0.707	0.707
22	0.950	0.949	0.843	0.792	0.791	0.773	0.773	0.772	0.772
23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
24	0.907	0.907	0.907	0.907	0.957	0.957	0.961	0.961	0.961
25	0.793	0.793	0.793	0.789	0.789	0.790	0.720	0.720	0.720
26	0.691	0.691	0.691	0.691	0.691	0.620	0.620	0.620	0.620
27	0.803	0.803	0.803	0.803	0.803	0.803	0.803	0.803	0.803
28	0.588	0.588	0.588	0.588	0.553	0.553	0.550	0.551	0.550
29	0.736	0.736	0.699	0.569	0.569	0.569	0.569	0.569	0.569
30	0.846	0.779	0.779	0.768	0.768	0.768	0.726	0.726	0.595
31	0.713	0.655	0.646	0.635	0.635	0.635	0.635	0.606	0.518
32	0.779	0.779	0.779	0.776	0.776	0.735	0.735	0.734	0.734
33	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
34	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
36	0.761	0.761	0.761	0.722	0.591	0.589	0.589	0.589	0.589
37	0.647	0.647	0.644	0.644	0.644	0.644	0.644	0.644	0.644
38	0.499	0.493	0.488	0.442	0.442	0.442	0.442	0.442	0.442
39	1.000	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999

5.2. Case analysis

Various weights are attempted to DMU 6, for instance, for the same case analysis as above and the detailed results are presented in Table 7 and Fig. 2. These results show that technical efficiency increases and service effectiveness decreases as α increases. Moreover, slack values for factor and consumption variables associated with various weights are also detailed in Table 8. Looking into the slack values under the weights $\alpha = 0.1, 0.5$ and 0.9, for example, one can see the improvement pressure to reduce the number of buses and to increase the average number of on-board passengers per run so as to achieve both efficiency and effectiveness are relieved as α increases. It indicates that DMU 6 is an efficiency-emphasis company—the preference of efficiency over effectiveness will lessen the improvement pressure for this company. Such a case analysis demonstrates that change in weights can not only alter the performance measures but also influence the improvement strategies.

The technical efficiency and service effectiveness for each of 39 bus companies under various weights are further detailed in Tables 9 and 10, respectively. Note from Table 9 that six DMUs are evaluated as technical efficient by ICCR model ($\alpha = 0.5$), but only four remain efficient by GICCR model (α ranging from 0.1 to 0.9). Also note from Table 10 that five DMUs are evaluated as service effective by ICCR model ($\alpha = 0.5$) and these DMUs remain effective by GICCR model (α ranging from 0.1 to 0.9). It is interesting to note that only two companies, DMU 23 and DMU 34, are robustly overall efficiency because these two companies are originally benchmarked as both technical efficient and service effective by ICCR model (Table 3) and they remain technical efficient and service effective by GICCR model.

6. Concluding remarks

To more correctly evaluate the overall performance and to fully capture the insights of lacking efficiency or effectiveness for non-storable commodities, it is imperative to measure the efficiency and effectiveness simultaneously because both terms represent distinct aspects of performance. As transport services are typically non-storable commodities, conventional measurement of technical efficiency or technical effectiveness only represents one aspect of the performance. The managers may also need to know the service effectiveness to understand how much consumption (passenger-miles or ton-miles) would be generated from the output (vehicle-miles). This paper proposes two novel integrated data envelopment analysis (IDEA) approaches, including the ICCR and IBCC models, to jointly measure the overall performance for non-storable commodities from two aspects: technical efficiency and service effectiveness. We prove that the proposed

ICCR and IBCC models possess the essential properties of rationality, uniqueness, and benchmarking power. In addition to this theoretical contribution to the DEA literature, we also demonstrate that the proposed IDEA approaches have revealed higher benchmarking power than the conventional separate DEA approaches for practical applications. We therefore recommended our proposed IDEA approaches (with equal weights) or generalized IDEA approaches (with unequal weights) be used for non-storable commodities' overall efficiency measurement.

Some directions for future studies can be identified. The IDEA models are specified in an additive form in the present paper, other specification forms of IDEA models or even multi-objective IDEA models deserve further exploration. The present paper only demonstrates the overall efficiency measurement for bus transit services with two departments—production (technical efficiency) and sale (service effectiveness). It is a challenging issue to extend our proposed IDEA models to evaluate the overall performance for an enterprise with more than two departments vertically and/or horizontally interrelated, e.g., the supply chain systems within an enterprise, the postal mail pickup, processing and delivery, the air-express courier's ground operation (pickup/delivery and processing) and air transport and hubs, among others. Although the proposed IDEA models have been proven and demonstrated with higher benchmarking power than the conventional SDEA models, further comparisons with other DEA models aiming at enhancement of benchmarking power, such as super-efficiency and cross-efficiency models (e.g. Banker et al., 1989; Chen, 2002), are worthy of investigation.

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