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全球商學院之排序與分群

Ranking and Grouping on World Business Schools

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指導教授：黎漢林 教授

中 華 民 國 九十三 年 六 月

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全球商學院之排序與分群

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摘　　　　要

時下許多雜誌如 Time、U.S. News 和 Financial Times 等，利用不同數學公式出版全美或全球大學評鑑刊物，然而卻被許多人在評比方法之確切性及資料正確性上遭受批評。本論文所提出的方法能利用決策者所提供之偏好自行計算出各評比指標之權重，並以表格及 3D Ball 之視覺工具呈現排序與分群結果。以此結果為參考，決策者能再次加入偏好或是在決策單位〔DMU〕間作修正，以獲得想要之結果。此方法能根據決策者多次加入之偏好來計算與其邏輯相似之評比結果，以達成協助決策者做明確的決策選擇。

關鍵字：排序、分群、商學院、階層分析法、偏好

Ranking and Grouping on World Business Schools

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ABSTRACT

Companies like Time, U.S. News, and Financial Times use different ranking models to publish university ranking guides. However, many critics say the ranking formulas are constantly changing and the data is highly manipulable. In the proposed model, the decision makers can rank universities based on their preferences. Based on the preferences, this model will automatically generate a set of weightings for criteria in the ranking process. The ranking and the grouping result will be displayed using both tables and 3D ball visualization tool. The decision makers can further specify the relationships between DMUs or add more preferences to obtain desired outcome. Providing decision makers various chances and means to add their opinions through out the ranking process, this model can ensure that the result are consistent with what decision makers had in mind and can ,hence, help them in the decision making process.

Keyword: Business School, Ranking, Grouping, Pairwise Comparison,
AHP, Preference

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Ping-Chung Yang
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1. Introduction

Background

Every year, many high school graduates and university graduates purchase University Ranking Guides to help them select the right undergraduate program or graduate program that is best suited for them. Although among the quarter million freshmen who participated in the survey done by the Higher Education Research Institute, only 8.6% responded that the rankings were very important to them when selecting colleges or universities (Crissey, 1997). The reasons may lie on the question of ranking methodology. How do we know these rankings are right for the students and rank universities in the way the students needed? How do we know the criteria participated in the ranking system are what the ones students consider important? These are some of the key concerns which should be solved.

Currently, there are many publishers which release various kinds of ranking each year. US News and World Report, for example, started releasing university ranking in with the October issue in late 1980's. They have realized that in the subsequent years, the October issue had sold many more copies than any other issues. Hence, they decided to start publishing an independent issue for university ranking. In the 1990's, many other publishers like Time, Newsweek, Money Magazine, and many more have also realized that the market for university ranking is enormous and have started to create their own rankings and publish them. Similarly, Canada, Asia, and Europe all have magazines that do rankings for universities in different regions.

Objective

The ranking guides currently in the market are heavily criticized by many people ranging from educational field to people in the publishing industry. Some of these criticisms are as follow:

(1) To increase the sales, publishers may introduce new measures or change the weightings of measures from year to the next (Gater, 2003).

(2) Some of the factors are highly manipulable, and, as a result, the ranking outcome is meaningless (Leiter, 2003).

(3) Ranking formula and factors participated in the ranking process are constantly changing, so the results are high in variation (Levin, 1997).

In this study, we propose a new ranking method that can help the Decision Makers (DM) rank Decision Making Units (DMUs). The characteristics are listed below:

(1) The model can automatically generate weightings with minimal human influence.

(2) Ranking can still be done with minimum information from Decision Makers, i.e. preferences.

(3) 3D ball representation gives clear view on the correlations.

(4) This model allows DM to add preferences through out the ranking process.

(5) DM can specify groupings for DMUs.

Organization of Study

Chapter 2 reviews related literatures. The discussed area will include review on current ranking methods, Data Envelopment Analysis, Analytic Hierarchy Process, Transitivity, and Clustering.

Chapter 3 explains the whole ranking and grouping model by using a small data set. The concept of each mathematical model used in the ranking and grouping process will be explained in detail.

Chapter 4 ranks schools using the hard data from Financial Times using the new model and compare with the original ranking.

Conclusion drawn from the experiment and discussions will be presented in chapter 5, along with the recommendations for future works.

Scope and limitations

This study will focus on the mathematical model, which will try to generate a set of optimal weightings without the needs of Decision Makers to specify the weightings manually

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2. Literature Review

2.1 Ranking Methodology

There are several rankings published in the market. Each of them has different methodology to rank universities. They vary in criteria selection, assignment of weightings, and raw data, just to name a few. Let us look at few of the more popular ranking systems and their methodology.

U.S. News and World Report

Source: www.usnews.com

U.S. News ranks business colleges in United States in 2004 and listed 82 of them. They have used three major sections with total of eight criteria for the entire ranking process. These criteria are listed below with their weightings and descriptions.

(1) Quality Assessment (total 40%):

I. *Peer Assessment (25%)* – Deans and directors from business schools of accredited programs were asked to rate programs from marginal (1) to outstanding (5). Notice that 56% of them have returned the survey.

II. *Recruiter Assessment (15%)* – Corporate recruiters were also asked to rank the programs which they have hired employee from in the previous year. However, only 32% of them replied the survey.

(2) Placement Success (total 35%):

I. *Average Starting Salary and Bonus (14%)* – This is the mean of starting salary and bonus.

II. *Percentage of Graduates Employed at Graduation (7%)* – The percentage of emplacement rate is measure before the students actually graduate from full-time MBA program.

III. *Percentage of Graduates Employed 3 Months after Grad (14%)* – The percentage of employed graduates three months after completing the full-time MBA program.

(3) Student Selectivity (total 25%):

I. *Average Undergrad GPA (7.5%)* – The average GPA of new students.

II. *Average GMAT (16.25%)* – Average GMAT score of new students who are accepted to the full-time MBA program.

III. *Acceptance Rate (1.25%)* – Percentage of accepted applications.

From their hard data, we have tried to duplicate their ranking formula and have found a very similar ranking result with identical overall scores. The formula should be very close to $Score = \sum_{k=1}^{n} \left(w_k * \frac{C_k - \underline{C_k}}{\overline{C_k} - \underline{C_k}} \right)$ where n is the total number of criteria and C_k is the value of k^{th} criterion and $\overline{C_k}$ and $\underline{C_k}$ are the maximum and minimum values of k^{th} criteria.

Financial Times

Source: www.ft.com

Unlike U.S. News & World Report, Financial Times (FT) has ranked business schools from all over the world and has listed 100 of them. FT has also selected twenty criteria for the ranking process. The following are those criteria and their weightings.

(1) *Weighted Salary (20%)* – This is the average salary today with adjustment for different industries. Also, this figure is the average salary three years after graduation. (in US dollars)

(2) *Salary Percentage Increase (20%)* – The percentage increase in salary from beginning of MBA program to three years after graduation.

(3) *Value for Money (3%)* – This is calculated by the salary earned by MBA graduates three years after graduation with the course costs and the opportunity cost, while still in school and not employed.

(4) *Career Progress (3%)* – The degree to which alumni have moved up the career ladder three years after graduating. Progression is measured through changes in level of seniority and the size of company in which they are employed.

(5) *Aims Achieved (3%)* – The extent ot which alumni fulfilled their goals or reasons for doing an MBA. This is measured as a percentage of total returns for a school and presented as a rank.

(6) *Placement Success (2%)* – The percentage of 2000 alumni that gained employment with the help of career advice. The data is presented as rank.

(7) *Alumni Recommendation (2%)* – Alumni of 2000 were asked to name three business schools from which they would recruit MBA graduates. The figure represents the number of votes received by each school. The data is presented as a rank.

(8) *International Mobility (6%)* – A rating system that measures the degree of international mobility based on the employment movements of alumni between graduation and today.

(9) *Employed at Three Months (2%)* – the percentage of the most recent graduating class that had gained employment within three months.

(10) *Women Faculty (2%)* – Percentage of female faculty.

(11) *Women Students (2%)* – Percentage of female students.

(12) *Women Board (1%)* – Percentage of female members in the advisory board.

(13) *International faculty (4%)* – The percentage of international students.

(14) *International Students (4%)* – Percentage of the board whose nationality differs from their country of employment.

(15) *International board (2%)* – Percentage of the board whose nationality differs from their country of employment.

(16) *International Experience (2%)* – Weighted average of three criteria that measure international exposure during the course.

(17) *Languages (2%)* – Number of additional languages required on completion of the MBA. Where a proportion of students required another language due to an additional diploma or degree chosen that figure is included in the calculations but not presented in the final table.

(18) *Faculty with Doctorates (5%)* – Percentage of faculty with a doctoral degree.

(19) *FT Doctoral Rating (5%)* – Number of doctoral graduates from the last three academic years with additional weighting for those graduates taking up a faculty position at one of the top 50 school in this year's ranking.

(20) *FT Research Rating (10%)* – a rating of faculty publications in 40 international academic and practitioner journals. Points are accrued by the business school at which the author is presently employed. Adjustment is made for faculty size.

The results and hard data of both U.S. News and World Report and Financial

Times are attached in the Appendix section. Both publishers have worked with other companies for data collection. However, they did not explain how the weightings for the criteria were decided. Moreover, perhaps because U.S. News and World Report is the most recognized publisher in university ranking, it receives many criticisms on both the changes on weightings from year to year and the correctness of hard data. On the contrary, Financial Times has fixed their weightings. However the way hard data is presented has been modified from year to year. For example, the criterion “value for money” was a score ranging from 1 to 5 in year 2002 and 2003 ranking. In 2004, this criterion has been changed into “value for money rank”. When it was a score from 1 to 5, there can be only 50 different scores and is unlikely that all the variation of the score will be assigned. Hence there are many schools with the same scores. When it changed to rank, only few schools are being ranked as the same, so the variation is larger. This problem arises on more than one criterion in Financial Times’ ranking.

2.2 Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a method for evaluating the activity performance, especially for organizations such as business firms, government agencies, hospitals, educational institutions, and etc (Cooper etc. 1999). A commonly used measure for efficiency is the output-input ratio. Number of items sold in a store will be an example of the output; number of sales clerk in the store will be the input. Hence, the efficiency of this store, basing on only these two criteria, will simply be NumberOfGoodsSold / NumberOfClerk. These comparable entities are often called Decision Making Units (DMUs).

The purpose of DEA is to empirically estimate the efficient frontier based on the set of available DMUs and assumes that each performance measure can be categorized as either an input or an output (Schrage, 1997). It provides the user information about both efficient and inefficient units along with the efficiency scores and reference sets for inefficient units (Halme etc, 1999). An Efficient Frontier is a line that has at least one DMU point touching it. The DMUs, who touch the EF line, are the most efficient DMUs. The idea of Production Frontier is first discussed by Farrell in 1975 which has three assumptions. The attractive feature of DEA is that it produces efficiency score between 0 and 1.

In 1978, Charnes, Cooper, and Rhodes proposed a DEA model called the CCR model basing on Farrell's single input-output model in 1975. CCR model is designed to measure the cases of multi input and multi output. The following is the pseudo-code for the CCR model. U_r represents the weighting for r^{th} output criterion and V_i represents the weighting for i^{th} input criterion. They are automatically generated when the score of k^{th} DMU is maximized. Y_r and X_i are the output and input criteria.

For each DMU *k*

$$MAX \quad Score_k = \frac{\sum_{r=1}^{s} U_r Y_r}{\sum_{i=1}^{m} V_i X_i}$$

$$such \ \ that$$

$$Score_k \leq 1$$

$$U_r > 0$$

$$V_i > 0$$

Where

Y_r is the r^{th} output of DMU

X_i is the i^{th} input of DMU

U_r is the weighting for r^{th} output

V_i is the weighting for i^{th} input

In this CCR model, it will calculate the score of each DMU based on the weightings that can maximize the score of current DMU, which means that the n^{th} DMU can obtain the best score with n^{th} set of weightings. Hence, if there are n numbers of DMUs, then there will have n set of weightings. k^{th} set of weighting is determined under the condition that they can maximize the $Score_k$. All the scores have to be between 0 and 1. Once score of each DMU is determined, it then compares all of them again with their score. The DMU with highest score is the most efficient one.

2.3 Analytic Hierarchy Process

The Analytical Hierarchy Process (AHP) was proposed by Saaty in 1980 and his collaborators as a method for establishing priorities in multi-criteria decision making contexts based on variables that do not have exact numerical consequences (Genest, 1996). It also helps people set priorities and make the best decision when both qualitative and quantitative aspects of a decision need to be considered. AHP not only helps decision makers arrive at the best decision, but also provides a clear rationale that it is the best.

AHP can be conducted in three steps:

Setp 1: *Perform pairwise comparisons between each DMU on every criterion*

In this step, the goal is to obtain the priorities between DMUs for each criterion. To do so, a pairwise comparison has to take place between each DMU with respect to each criterion. For each criterion, a m by m matrix, where m is the number of DMUs, will be generated and the priority vector will be calculated from this matrix. Priority vector displays the preference orders for each DMU with respect to criteria. Since there are n numbers of criteria, n number of priority vector will be generated at the end.

Step 2: *Perform pairwise comparison between each criterion*

In the decision making process, not every criterion is quantitatively measurable, so a pairwise comparison between each criterion has to take place in order to specify the importance between each criterion. From the comparison, a set of weightings can be found for score calculation at the last step.

Step 3: *Compute final scores for DMUs*

With the priority vectors and the weightings for criteria, DM can now calculate the score for each DMU. DMU with the higher score should be the better alternative for the Decision Maker.

Following is an illustration of an example of a student, John, wanting to purchase a car. Due to his financial limitation, John can only buy a second hand car, and only has few things that he really car. He wants to buy a car that is cheap, nice out look, and comfortable. However, among the three cars he has in mind, none of them has best score on each of these criteria. He has decided to use AHP to help him select a car from these three. Table 2.1 lists all the data he gathered about these three cars.

Table 2.1 Hard data provided by John on cars.

	Price	Look	Comfort
Car 1	13100	Good	Very good
Car 2	12000	Fair	Good
Car 3	9800	Good	Fair

To perform pairwise comparison between each car with respect to each criterion, a priority score has to be assigned to each comparison. The scores can range from 1 to 9, where 9 is the most satisfactory score. Notice that if a DM compare A_1 to A_2 and assigns a score of 4, then the score between comparison of A_2 and A_1 will be the inverse of A_1 and A_2's, which will be 1/4. This property can ensure the logical consistency for each comparison.

1 Choice i and j are equally important
3 Choice i is weakly more important than j
5 Choice i is strongly more important than j
7 Choice i is very strongly more important than j
9 Choice i is absolutely more important than j
2, 4, 6, 8 are intermediate values

After finishing pairwise comparisons, matrixes with these priority scores will be generated (Table 2.2).

Table 2.2 Comparison score for each car with respect to each criterion

Criteria	**Price**			**Look**			**Comfort**		
	Car1	**Car2**	**Car3**	**Car1**	**Car2**	**Car3**	**Car1**	**Car2**	**Car3**
Car1	1	1/3	1/8	1	3	1	1	3	6
Car2	3	1	1/6	1/3	1	1/4	1/3	1	4
Car3	8	6	1	1	4	1	1/6	1/4	1

From these matrixes, normalization has to be done before the priority vectors can be calculated (Table 2.3). Normalization is simply divides each value by the sum of corresponding column. For example, the normalized value between car2 and car3 with respect to price is calculated by

$$(1/6) / (1/8 + 1/6 + 1) = 0.1290.$$

Table 2.3 Normalized comparison table

Criteria	**Price**			**Look**			**Comfort**		
	Car1	**Car2**	**Car3**	**Car1**	**Car2**	**Car3**	**Car1**	**Car2**	**Car3**
Car1	0.0833	0.0454	0.0967	0.4286	0.375	0.4444	0.6666	0.7059	0.5454
Car2	0.250	0.1363	0.1290	0.1428	0.125	0.1111	0.2222	0.2352	0.3636
Car3	0.6666	0.8182	0.7742	0.4286	0.5	0.4444	0.1111	0.0588	0.0909

Each criterion has its own priority vector and the values in the vector can be seen as the score of each DMU on corresponding criterion. The values in the priority vectors are the sum of rows from the normalized pairwise comparison matrix and divided by the number of DMUs, as in Table 2.4. The values in priority vector for price is calculated as follow:

$$(0.0833 + 0.0454 + 0.0976) / 3 = 0.2254$$
$$(0.2500 + 0.1363 + 0.1290) / 3 = 0.5153$$
$$(0.6666 + 0.8182 + 0.7742) / 3 = 2.2590$$

Table 2.4 Priority vectors with respect to each criterion

	Priority Vector for Price	**Priority Vector for Look**	**Priority Vector for Comfort**
Car1	0.0751	0.4160	0.6393
Car2	0.1717	0.1263	0.2736
Car3	0.7530	0.4576	0.0869

After the values of priority vector is calculated, pairwise comparison has to perform on criteria to obtain the weightings for each criterion. Similar to previous steps, a 3 by 3 matrix, with criteria on both row and column, will be created. Using the same calculation method for priority vector, the weighting for each criterion can also be found (Table 2.5).

Table 2.5 Comparison tables and weightings for criteria

	Comparison Matrix			Normalized Comparison Matrix			
	Price	Look	Comfort	Price	Look	Comfort	Weighting
Price	1	1/5	3	0.1579	0.1489	0.2727	0.1931
Look	5	1	7	0.7894	0.7447	0.6363	0.7234
Comfort	1/3	1/7	1	0.0526	0.1064	0.0909	0.0833

The weightings on Table 2.5 suggest that Look is the most important criterion for John. Price is the next concern and comfort is the last. With the weightings on the criteria and the priority vectors on each criterion, the score for each car can now be calculated as follow:

Car 1: (0.0751 * 0.1931)+(0.4160 * 0.7234)+(0.6393 * 0.0833) = 0.3687
Car 2: (0.1717 * 0.1931)+(0.1263 * 0.7234)+(0.2736 * 0.0833) = 0.1473
Car 3: (0.7530 * 0.1931)+(0.4576 * 0.7234)+(0.0896 * 0.0833) = 0.4839

From the calculation, Car 3 has the highest score and should be the best choice for John to consider.

2.4 Intransitivity

When Decision Makers are making decisions, some do a pairwise comparison with AHP before they make the actual decision. However, AHP does not have a means for detecting an intransitivity situation. An intransitivity is when A > B, B > C, but C > A. This situation is also called logically inconsistent. When there is a cycle exists in the decision process and is not very logical. Hence, the intransitivity detection is a very important process before the any decision is made.

In Gass' study (1998), he presented a way to detect the intransitivity with simple matrix operation.

Theorem:

Let **P** be the preference matrix of a preference diagram **D**. Then in $\mathbf{P}^k$**, the (i,j) entry, denoted by** $\mathbf{P}_{i,j}^{(k)}$, is the number of sequences in **D** of length k from node v_i to node v_j. ($\mathbf{P}^k$ is the k^{th} power of **P**)

The theorem states that $P_{i,j}^k$ denotes the number of cycles, with different sequence. Take a preference graph shown in Figure 2.1 as an example. We can generate a tournament matrix from this preference graph. The preference matrix P, Table 2.11, has values of 0 or 1. $P_{i,j}$ is set to 1 if i is smaller than j.

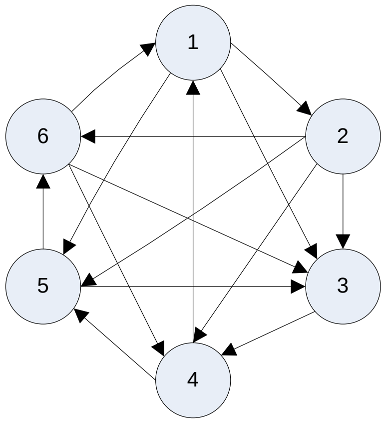

Figure 2.1 Preference Graph of six nodes

Table 2.6 Preference matrix on six nodes

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	0	0	0	1	0	1
P_2	1	0	0	0	0	0
P_3	1	1	0	0	1	1
P_4	0	1	1	0	0	1
P_5	1	1	0	1	0	0
P_6	0	1	0	0	1	0

From this preference matrix, we can apply the theorem to this matrix and look for the cycles. Since the theorem said that the value of P_{ij}^k means there are the same numbers of combinations of sequences in the preference graph of length k from node i to node j. Similarly, if we look at P_{ii}^k, then this will mean the sequence start at node i and come back to node i with the length of k. Hence, we can simply check the diagonal of each P^k for k = 3 up to k = n, where n is the number of nodes.

Table 2.7a to Table 2.7d are the power of preference matrix from P^3 to P^6. In Table 2.7a, we can see that the diagonal has nonzero values. P_{11}^3 is 4, so there are four cycles with the length of 3 and the starting and ending node is P_1. The cycles are (P_1, P_2, P_4, P_1), (P_1, P_3, P_4, P_1), (P_1, P_2, P_6, P_1), and (P_1, P_5, P_6, P_1). With the same technique, it is very easy to find the existence of cycles for any given preference graph. From Table 2.7b to Table 2.7d, it is clear that there are cycles with the length of 4, 5, and 6.

Table 2.7 Preference Matrixes

(a) P^3 of Preference Matrix

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	4	3	0	1	2	1
P_2	0	2	1	0	1	1
P_3	3	4	2	3	1	4
P_4	4	3	0	4	1	2
P_5	2	4	1	1	3	3
P_6	1	1	1	2	0	3

(b) P^4 of Preference Matrix

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	5	4	1	6	1	5
P_2	4	3	0	1	2	1
P_3	7	10	3	4	6	8
P_4	4	7	4	5	2	8
P_5	8	8	1	5	4	4
P_6	2	6	2	1	4	4

(c) P^5 of Preference Matrix

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	6	13	6	6	6	12
P_2	5	4	1	6	1	5
P_3	19	21	4	13	11	14
P_4	13	19	5	6	12	13
P_5	13	14	5	12	5	14
P_6	12	11	1	6	6	5

(d) P^6 of Preference Matrix

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	25	30	6	12	18	18
P_2	6	13	6	6	6	12
P_3	36	42	13	30	18	36
P_4	36	36	6	25	18	24
P_5	24	36	12	18	19	30
P_6	18	18	6	18	6	19

2.5 Clustering

Clustering involves dividing a set of data points into non-overlapping into groups, where points in each group are more similar to each other than to points in other groups (Faber, 1994). When a set of data is clustered, every point is assigned to a group and every group can be characterized by a single reference point, normally the average of points in the same group.

There are several techniques in the field of clustering. General clustering techniques are Hierarchical clustering, K-Mean clustering, Incremental clustering, and Probability-based clustering. K-mean clustering is also called Iterative Distance-based clustering. The character “k” in the name of K-mean is the number of groups, or clusters, DM wants to make. The basic idea for K-mean is randomly start with k number of points and assign each data point to one of the reference point in k by calculating the minimal total distance. Once the groups are determined, it then tries to adjust the position of the reference points so that it will locate in the center of corresponding group. The algorithm for the k-mean clustering is shown below.

Algorithm for K-mean Clustering:

(1) Choose k centroid points.
(2) Calculate the distance of each point to all centroids.
(3) Get the minimum distance. This data is said belong to the cluster that has minimum distance from this data
(4) Adjust the centroid location based on the current data updated data.
(5) Assign all the data to this new centroid.
(6) Repeat until no data is moving to another cluster anymore.

In this study, the proposed model will be able to generate a set of weightings for criteria based on the preferences given by the decision makers. The model has applied similar idea from Data Envelopment Analysis. In DEA, it is trying to measure the efficiency based on maximizing the score of DMU. However, in the proposed model, it will try to maximize the rank for each DMU instead of score. The concept from Analytic Hierarchy Process is also used to create tournament matrix for ranking by doing pairwise comparison. Gass' technique is also used to ensure the non-existence of intransitivity. Last but not least, the concept from K-mean clustering will be modified to help this ranking method to present the data points on a 3D ball to help DM make decisions.

3. Ranking and Grouping Models

In this chapter, the ranking and grouping process can be break down into two major parts. First part will deal with the actual ranking and score calculation. The second part is mapping each school onto a 3D ball and clustering these data points. Figure 3.1 shows the entire process of proposed ranking and grouping model.

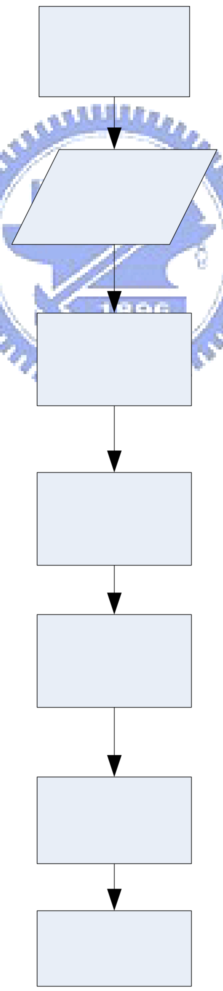

Figure 3.1 Flowchart

3.1 Common Weight Model

As discussed in chapter 2, DEA is mainly used for efficiency measurement. The concept of DEA is to calculate the ratio between inputs and outputs, and rank each DMU (Data Making Unit) by their maximized scores. In this ranking objective, however, DEA is not the perfect tool for the ranking process because the most efficient DMU might not be the best choice for DM (Decision Maker). Moreover, , sometimes criteria are hard to distinguish from input or output, the proposed method has modified the traditional DEA method to meet the DMs' requirement without the need to identify inputs and outputs for criteria. This model will automatically ranks and groups the DMUs based on the absolute dominance relationships found in the hard data, so the DMs do not need to worry about assigning weightings for each criterion. This is a big improvement from the traditional ranking systems, which often have controversy on weighting settings.

In the experiments, Lingo8.0 is used as the optimization tool. Given the correct model and inputs, the system will calculate the ideal weights for each criterion, which will allow us to rank the DMUs and map each DMU to a coordinate on 3D ball to help DM visualize the relationships between DMUs, as well as the correlation between DMUs. In this section, the mathematical model and the concept behind it will be discussed in detail and the model will be applied on an example of 20 universities. Before the mathematical model is being discussed, Table 3.1 lists and describes the variables, following is the model.

Table 3.1 Variables for Common Weight Model

Variables	Descriptions
m	Total number of DMUs
n	Total number of criteria
$t_{i,j}$	$t_{i,j} = 1$ if DMU *j* is better than DMU *i*, *else* $t_{i,j} = 0$
$\overline{C_k}$, $\underline{C_k}$	Maximum and minimum values of k^{th} criterion
$C_{i,k}$	The k^{th} criterion of i^{th} DMU
w_k	Weight for k^{th} criterion
M	A large constant number

Common Weight Model (Model 1):

$$Min \quad \sum_{i=1}^{m}\sum_{j\neq i}^{m} t_{i,j} \tag{3.1}$$

Subject to

$$\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{i,k}-\underline{C_k}}{\overline{C_k}-\underline{C_k}}\right)\right)+\left(M * t_{i,j}\right) \quad \geq \quad \sum_{k=1}^{n}\left(w_k * \left(\frac{C_{j,k}-\underline{C_k}}{\overline{C_k}-\underline{C_k}}\right)\right). \quad \forall \ i, j \ and \ j \neq i \tag{3.2}$$

$$\sum_{k=1}^{n} w_k \quad = \quad 1 \tag{3.3}$$

$$w_k \geq \varepsilon, \quad \forall \ k \tag{3.4}$$

$$t_{i,j} \in \{0,1\} \tag{3.5}$$

$$t_{i,j} + t_{j,i} \leq 1, \quad \forall \ i, j < i \tag{3.6}$$

In this model, Lingo will generate a set of weightings for the ranking process. This model ranks the DMUs without DMs worrying about the numbers (weightings). Moreover, these weightings could be more convincing for some DM because these numbers are generated by the system automatically based only on the absolute dominance relationships.

After this model is run by Lingo, Lingo will return a matrix with the size of m by m. This matrix will consist values of only 0 and 1. For t_{ij}, if $t_j > t_i$, then t_{ij} will be set to 1. The sum of each row will represent their rank correspondingly. The objective function (3.1) is trying to maximize the rank of each DMU by minimizing the sum of *t* for each row. Note that the DMU with lower the sum of *t*, the higher rank it will get. Constraint 3.2 is for determining the values of $t_{i,j}$. If $\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{i,k} - \underline{C_k}}{\overline{C_k} - \underline{C_k}}\right)\right)$ is greater than $\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{j,k} - \underline{C_k}}{\overline{C_k} - \underline{C_k}}\right)\right)$, then $t_{i,j}$ will be 0, since we are minimizing the sum of $t_{i,j}$. On the other hand, if $\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{i,k} - \underline{C_k}}{\overline{C_k} - \underline{C_k}}\right)\right)$ is smaller than $\sum_{k=1}^{n}\left(w_k * \left(\frac{C_{j,k} - \underline{C_k}}{\overline{C_k} - \underline{C_k}}\right)\right)$, in order to satisfy constraint 3.2, the value of $M * t_{i,j}$ must not be 0, so $t_{i,j}$ will be set to 1.

Constraint 3.3 is to make sure that the sum of weights of all the criteria will be equal to 1. Also, constraint 3.4 ensures that the weights are all non-zero, so every criterion will be taken into account in this ranking process. Constraint 3.5 specifies that $t_{i,j}$ is a binary variable, which can only be 0 or 1. The last constraint is to insure that if *i* is better than *j*, then j can not be better than *i* at the same time.

Once the weights for each criterion are automatically generated by the model, score of each DMU will be calculated by equation 3.7 for future ranking purposes.

This score function ensures that the scores are all between 0 and 1 by normalizing the hard data. This will help DM to see the differences in the scores.

$$SCORE_i = \sum_{k=1}^{n}\left(w_k * \frac{\left(C_{i,k} - \underline{C_k}\right)}{(\overline{C_k} - \underline{C_k})} \right) \qquad \textbf{(3.7)}$$

Table 3.2 shows the original hard data of the first twenty universities listed on the Financial Times' 2004 Global MBA Ranking. The data has been normalized so that 1 is the maximum score and 0 is the minimum score. Notice that we have only chosen six criteria that have the heaviest weightings.

Table 3.2 Hard data after normalization from Financial Times' 2004 Global MBA Ranking

Rank in 2004	School name	Weighted salary (US$)	Salary increase (%)	International mobility rank	Faculty with doctorates (%)	FT doctoral rank	FT research rank
1	University of Pennsylvania: Wharton	0.836865335	0.855670103	0.74157303	1	1	0.987805
2	Harvard Business School	1	0.525773196	0	0.888889	0.92	1
3	Columbia Business School	0.696863457	1	0.39325843	0.888889	0.88	0.939024
4	Insead	0.553465223	0.257731959	1	0.888889	0.373333	0.890244
4	London Business School	0.42117949	0.680412371	0.87640449	0.888889	0.56	0.780488
4	University of Chicago GSB	0.658188819	0.855670103	0.6741573	0.888889	0.773333	0.963415
7	Stanford University GSB	0.814405559	0.402061856	0.35955056	0.944444	0.866667	0.97561
8	New York University: Stern	0.408235773	0.886597938	0.4494382	0.944444	1	0.865854
9	MIT: Sloan	0.645918112	0.463917526	0.17977528	0.777778	0.973333	0.902439
10	Dartmouth College: Tuck	0.725693358	0.773195876	0.30337079	0.777778	0	0.829268
11	Northwestern University: Kellogg	0.640330558	0.494845361	0.78651685	0.833333	0.746667	0.95122
12	IMD	0.694437488	0	0.47191011	0.722222	0	0.097561
13	Iese Business School	0.018985162	0.907216495	0.97752809	0.944444	0.346667	0.146341
13	Yale School of Management	0.485553747	0.979381443	0.04494382	0.888889	0.12	0.560976
15	Instituto de Empresa	0	0.515463918	0.95505618	0	0	0.04878
16	Cornell University: Johnson	0.490624804	0.618556701	0.28089888	0.666667	0.16	0.743902
17	Georgetown Uni: McDonough	0.359716396	0.824742268	0.53932584	0.5	0	0.402439
17	Uni of N Carolina: Kenan-Flagler	0.303355663	0.659793814	0.69662921	0.555556	0.64	0.853659
19	University of Virginia: Darden	0.606570463	0.742268041	0.23595506	0.888889	0.12	0
20	Duke University: Fuqua	0.375430414	0.505154639	0.68539326	0.555556	0.453333	0.878049

After applying the hard data to the Common-Weight Model, Tables 3.3a and 3.3b displays the results. Table 3.3a shows the new score and the new rankings for these twenty universities along with the original rankings and Table 3.3b shows the new weightings. Please note that due the number of the original criteria, only five were selected from the original twenty criteria. Hence the result varied greatly.

Table 3.3 Results from Common-Weight Model

(a) New scores and rankings

Schools	score	Original Ranking	New Ranking	Change in Rankings
University of Pennsylvania: Wharton	0.845614	1	1	0
Harvard Business School	0.594397	2	11	-9
Columbia Business School	0.726394	3	3	0
Insead	0.668107	4	6	-2
London Business School	0.692608	5	4	0
University of Chicago GSB	0.758528	6	2	2
Stanford University GSB	0.615344	7	10	-3
New York University: Stern	0.6338	8	7	1
MIT: Sloan	0.50519	9	17	-8
Dartmouth College: Tuck	0.6338	10	8	2
Northwestern University: Kellogg	0.688126	11	5	6
IMD	0.435994	12	19	-7
Iese Business School	0.632058	13	9	4
Yale School of Management	0.543776	14	16	-3
Instituto de Empresa	0.390719	15	20	-5
Cornell University: Johnson	0.501724	16	18	-2
Georgetown Uni: McDonough	0.543776	17	13	4
Uni of N Carolina: Kenan-Flagler	0.562208	18	12	5
University of Virginia: Darden	0.543776	19	13	6
Duke University: Fuqua	0.543776	20	13	7

(b) New weightings obtained from Common-Weight Model

	Weighted salary (US$)	Salary increase (%)	International mobility rank	Faculty with doctorates (%)	FT research rank
Original Weightings	0.2	0.2	0.06	0.05	0.1
Normalized original weightings	0.303030303	0.303030303	0.09090909	0.07575758	0.151515
New weightings	0.291382783	0.243472234	0.27496259	0.13661036	0.053572
Change (%)	-1.16%	-5.96%	18.41%	6.09%	-9.80%

By studying both tables, it is clear that the criterion “International Mobility Rank” has increased its weighting by more than double of its original weightings and criteria other than “Weighted Salary” has changed about 6% to 10% each. These changes have effected the new extremely. In the new ranking, half of the universities have shifted their rankings for more than 4 spots. Harvard and MIT have shifted 9 spots and 8 spots accordingly. Harvard has dropped 9 spots in ranking due to the fact that it has the lowest value in “International Mobility Rank”, which is accounted for 27.50% of the total score. MIT has dropped 8 spots because it has the second lowest score on “International Mobility Rank” and fourth lowest score on “Salary Increase %”, which accounted for 24.35%.

After applying the statistical t-test, the P value was found to be 0.8919, which means the differences between the original rankings and the new rankings are considered to be not statistically significant. Hence the result from the Common-Weight Model is acceptable statistically.

3.2 3D Spherical Model

In last section, the weights for each criterion were generated by the model, as well as the rankings. The model will calculate the coordinates of each DMU based on the weightings and project them onto a 3D ball. To insure the correctness of the mapping and the correlations between each DMU, the concept of dissimilarity is used in the calculation of the coordinates. Dissimilarity is the degree of difference between subjects. The general calculation method for dissimilarity will be discussed later in this section.

Table 3.4 lists the variables used in 3D Spherical Model and their meanings. Note that all the radius of the 3D balls is set to 1, and an ideal solution will be projected onto the North Pole. Ideal solution is an imaginary DMU that has the maximum value for each of its criterion. The purpose of this ideal DMU, as the standard, is to help the comparison process.

Table 3.4 Variables and descriptions

Variables	**Descriptions**
m	Total number of DMUs
n	Total number of criteria
S_i	Score of i^{th} DMU
$D_{i,j}$	The dissimilarity between DMU *i* and DMU *j*
$\overline{C_k}$, $\underline{C_k}$	Maximum and minimum values of k^{th} criterion
$C_{i,k}$	The k^{th} criterion of i^{th} DMU
w_k	Weight for k^{th} criterion
X_i, Y_i, Z_i	The X,Y, and Z coordinates of DMU *i*

The X_i, Y_i, and Z_i are the actual coordinates of the DMUs on the 3D ball. Also, because the distances between DMUs on the 3D ball are not exactly the same as the values of dissimilarities, we minimize the error between these two values to obtain the closest solution (Equation 3.8). With this solution, the projection of the points on the ball will be able to represent the relationships of the DMUs.

3D Spherical Model (Model 2):

$$MIN \quad \sum_{i=1}^{m}\sum_{j>i}^{m}\left|(X_i - X_j)^2 + (Y_i - Y_j)^2 + (Z_i - Z_j)^2 - D_{i,j}^2\right| \qquad \textbf{(3.8)}$$

Subject to:

$$S_i = \sum_{k=1}^{n}\left(w_k * \left(\frac{C_{i,k} - \underline{C_k}}{\overline{C_k} - \underline{C_k}}\right)\right) \qquad \textbf{(3.9)}$$

$$D_{i,j} = \sqrt{2} * \sum_{k=1}^{n}\left(w_k * \left(\frac{\left|C_{i,k} - C_{j,k}\right|}{\overline{C_k} - \underline{C_k}}\right)\right) \qquad \textbf{(3.10)}$$

$$X_i^2 + Y_i^2 + Z_i^2 = 1 \ , \quad \forall \quad i \qquad \textbf{(3.11)}$$

$$Y_i = 2S_i - S_i^2 \ , \quad \forall \quad i \qquad \textbf{(3.12)}$$

The objective of this model is to let the dissimilarity between two DMUs represents the distance between two DMUs. This is accomplished by minimizing the difference between the straight line distance of two DMUs and their dissimilarity value.

Equation 3.9 is the function to calculate score, which is the same as equation 3.7. Equation 3.10 calculates the dissimilarity between DMU *i* and DMU *j*. The largest possible value for $D_{i,j}$ is $\sqrt{2}$, because when one DMU is the ideal solution, which have all the maximum value for each criterion, and the other DMU is the worst possible DMU, which must have minimum value for each criterion. Since the ideal solution will be at the North Pole and the worst possible solution will be on the equator. The straight line distance from the North Pole to the Equator on a ball with radius of 1 will be $\sqrt{2}$. Similarly, if two DMUs are exactly the same, thought it is not likely to happen, the numerator will become 0, and so the $D_{i,j}$ will be 0.

Equation 3.11 is to ensure that every point is on the surface of the ball. And equation 3.12 defines the relationship between the Y coordinates and the score. To explain this equation, there is a proposition to discuss, as stated below.

Proposition 1:

$$Y_i = 2 * S_i - S_i^2 \quad , \quad \forall \quad i \tag{3.13}$$

Proof:

$$(X_i - 0)^2 + (Y_i - 1)^2 + (Z_i - 0)^2 = (\sqrt{2} * D_{i,*})^2 = 2(1 - S_i)^2 \tag{3.14}$$

$$2 - 2Y_i = 2(1 - 2S_i + S_i^2) \tag{3.15}$$

$$Y_i = 2S_i - S_i^2 \tag{3.16}$$

In this proposition, $D_{i,*}$ in equation 3.14 represent the dissimilarity between DMU *i* and the ideal solution. The original equation that calculates the distance between two points was changed to the current form, $(X_i - 0)^2 + (Y_i - 1)^2 + (Z_i - 0)^2$, since the ideal solution has the coordinate of (0, 1, 0). Equation 3.14 can be verified with (ideal solution, worst possible solution) pair and (ideal solution, best possible solution) pair. When these two pairs of DMUs are plugged in 3.165, they both hold. Hence, equation 3.14 is further simplified to 3.15 and finally 3.16. The simplification processes are shown as below.

LHS:

$$(X_i - 0)^2 + (Y_i - 1)^2 + (Z_i - 0)^2$$
$$\Rightarrow X_i^2 + Y_i^2 - 2Y_i + 1 + Z_i^2$$
$$\Rightarrow (X_i^2 + Y_i^2 + Z_i^2) - 2Y_i + 1$$
$$\Rightarrow 1 - 2Y_i + 1$$
$$\Rightarrow 2 - 2Y_i$$

RHS:

$$2(1 - S_i)^2$$
$$\Rightarrow 2(1 - 2S_i + S_i^2)$$
$$\Rightarrow 2 - 4S_i + 2S_i^2$$

LHS = RHS:

$$(X_i - 0)^2 + (Y_i - 1)^2 + (Z_i - 0)^2 = 2(1 - S_i)^2$$

$$\Rightarrow \quad 2 - 2Y_i = 2 - 4S_i + 2S_i^2$$

$$\Rightarrow \quad Y_i = 2S_i - S_i^2$$

By applying the model to the example from section 3.1, we obtain the result shown in Table 3.5.

Table 3.5 Coordinates for each universities

Schools	score	New Ranking	x	y	z
Ideal Solution	1		0	1	0
University of Pennsylvania: Wharton	0.845613979	1	-0.21658096	0.97616496	0.013952
Harvard Business School	0.594397421	11	-0.41597168	0.83548655	0.359068
Columbia Business School	0.726394479	3	-0.36376656	0.92514002	0.108581
Insead	0.66810701	6	-0.32914496	0.88984704	-0.31597
London Business School	0.692608172	4	-0.32199465	0.90551026	-0.27635
University of Chicago GSB	0.758528351	2	-0.33056332	0.94169144	-0.06281
Stanford University GSB	0.615343904	10	-0.49832929	0.85203969	0.160301
New York University: Stern	0.633799985	7	-0.45945342	0.86589755	-0.1978
MIT: Sloan	0.505189925	17	-0.65293543	0.75516299	0.058345
Dartmouth College: Tuck	0.633799985	8	-0.49305753	0.86589755	0.084355
Northwestern University: Kellogg	0.688125846	5	-0.41447314	0.90273451	-0.11525
IMD	0.435994334	19	-0.73131613	0.68189761	0.0139
Iese Business School	0.63205834	9	-0.12768982	0.86461893	-0.48593
Yale School of Management	0.543776095	16	-0.58187571	0.79185975	0.185415
Instituto de Empresa	0.390719143	20	-0.36325638	0.62877684	-0.68752
Cornell University: Johnson	0.501723624	18	-0.65438051	0.75172065	-0.08187
Georgetown Uni: McDonough	0.543776095	13	-0.53405605	0.79185975	-0.29621
Uni of N Carolina: Kenan-Flagler	0.562207931	12	-0.49102552	0.8083381	-0.32478
University of Virginia: Darden	0.543776095	13	-0.60957381	0.79185975	0.037127
Duke University: Fuqua	0.543776095	13	-0.52498427	0.79185975	-0.31201

As previously mentioned, the ideal point is a point formed by setting the value of each of its criterion to the maximum value found from hard data. This point will lie on the North Pole with coordinates of (0, 1, 0) and score of 1. The worst point will be A_4, with coordinates of (0.99127, 0, 0) and score of 0. With this example, it is coincident that the ideal solution is same as A_1 and the worst point A_4 is lying on the equator. Despite these facts, the distances between each point are shown in Table 3.6. These numbers also represent the dissimilarity between each DMU.

Table 3.6 Dissimilarity matrix

	A0	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20
A0	0	0.22	0.57	0.39	0.47	0.43	0.34	0.54	0.52	0.7	0.52	0.44	0.8	0.52	0.65	0.86	0.7	0.65	0.62	0.65	0.65
A1	0.22	0	0.49	0.27	0.45	0.32	0.12	0.33	0.32	0.48	0.3	0.26	0.58	0.52	0.51	0.81	0.49	0.43	0.4	0.43	0.43
A2	0.57	0.49	0	0.45	0.67	0.65	0.52	0.27	0.56	0.27	0.35	0.48	0.59	0.99	0.42	1.03	0.41	0.7	0.68	0.4	0.6
A3	0.39	0.27	0.45	0	0.55	0.42	0.18	0.28	0.2	0.31	0.15	0.36	0.47	0.61	0.26	0.91	0.32	0.37	0.47	0.26	0.49
A4	0.47	0.45	0.67	0.55	0	0.26	0.38	0.42	0.5	0.45	0.55	0.22	0.44	0.52	0.67	0.57	0.48	0.57	0.43	0.55	0.35
A5	0.43	0.32	0.65	0.42	0.26	0	0.25	0.48	0.26	0.47	0.41	0.21	0.59	0.34	0.47	0.49	0.33	0.31	0.2	0.41	0.23
A6	0.34	0.12	0.52	0.18	0.38	0.25	0	0.35	0.22	0.36	0.23	0.19	0.49	0.47	0.39	0.74	0.36	0.3	0.3	0.3	0.31
A7	0.54	0.33	0.27	0.28	0.42	0.48	0.35	0	0.38	0.2	0.23	0.29	0.34	0.8	0.5	0.86	0.31	0.53	0.51	0.34	0.43
A8	0.52	0.32	0.56	0.2	0.5	0.26	0.22	0.38	0	0.38	0.26	0.39	0.53	0.43	0.25	0.74	0.25	0.2	0.29	0.29	0.31
A9	0.7	0.48	0.27	0.31	0.45	0.47	0.36	0.2	0.38	0	0.19	0.26	0.37	0.81	0.34	0.8	0.19	0.47	0.46	0.22	0.37
A10	0.52	0.3	0.35	0.15	0.55	0.41	0.23	0.23	0.26	0.19	0	0.34	0.41	0.68	0.31	0.85	0.19	0.35	0.41	0.17	0.43
A11	0.44	0.26	0.48	0.36	0.22	0.21	0.19	0.29	0.39	0.26	0.34	0	0.4	0.55	0.56	0.57	0.35	0.43	0.29	0.4	0.21
A12	0.8	0.58	0.59	0.47	0.44	0.59	0.49	0.34	0.53	0.37	0.41	0.4	0	0.83	0.66	0.79	0.43	0.51	0.57	0.42	0.48
A13	0.52	0.52	0.99	0.61	0.52	0.34	0.47	0.8	0.43	0.81	0.68	0.55	0.83	0	0.62	0.34	0.66	0.44	0.44	0.61	0.53
A14	0.65	0.51	0.42	0.26	0.67	0.47	0.39	0.5	0.25	0.34	0.31	0.56	0.66	0.62	0	0.92	0.27	0.38	0.53	0.25	0.55
A15	0.86	0.81	1.03	0.91	0.57	0.49	0.74	0.86	0.74	0.8	0.85	0.57	0.79	0.34	0.92	0	0.68	0.54	0.44	0.78	0.43
A16	0.7	0.49	0.41	0.32	0.48	0.33	0.36	0.31	0.25	0.19	0.19	0.35	0.43	0.66	0.27	0.68	0	0.28	0.28	0.21	0.28
A17	0.65	0.43	0.7	0.37	0.57	0.31	0.3	0.53	0.2	0.47	0.35	0.43	0.51	0.44	0.38	0.54	0.28	0	0.19	0.35	0.22
A18	0.62	0.4	0.68	0.47	0.43	0.2	0.3	0.51	0.29	0.46	0.41	0.29	0.57	0.44	0.53	0.44	0.28	0.19	0	0.46	0.09
A19	0.65	0.43	0.4	0.26	0.55	0.41	0.3	0.34	0.29	0.22	0.17	0.4	0.42	0.61	0.25	0.78	0.21	0.35	0.46	0	0.48
A20	0.65	0.43	0.6	0.49	0.35	0.23	0.31	0.43	0.31	0.37	0.43	0.21	0.48	0.53	0.55	0.43	0.28	0.22	0.09	0.48	0

The dissimilarity values represent the degree dissimilarity between any two DMUs. If the value is 1, then the DMUS are totally different. If the value is 0, then the two DMUs are exactly the same, so the coordinates of these two DMUs will be the same as well. The school name has been replaced by variables due to the size of the dissimilarity matrix. A0 represents the Ideal Solution, A1 represents UPenn, A2 represents Harvard, and so on. Figure 3.2 is the projection of these points on a 3D ball by using the coordinates in Table 3.5.

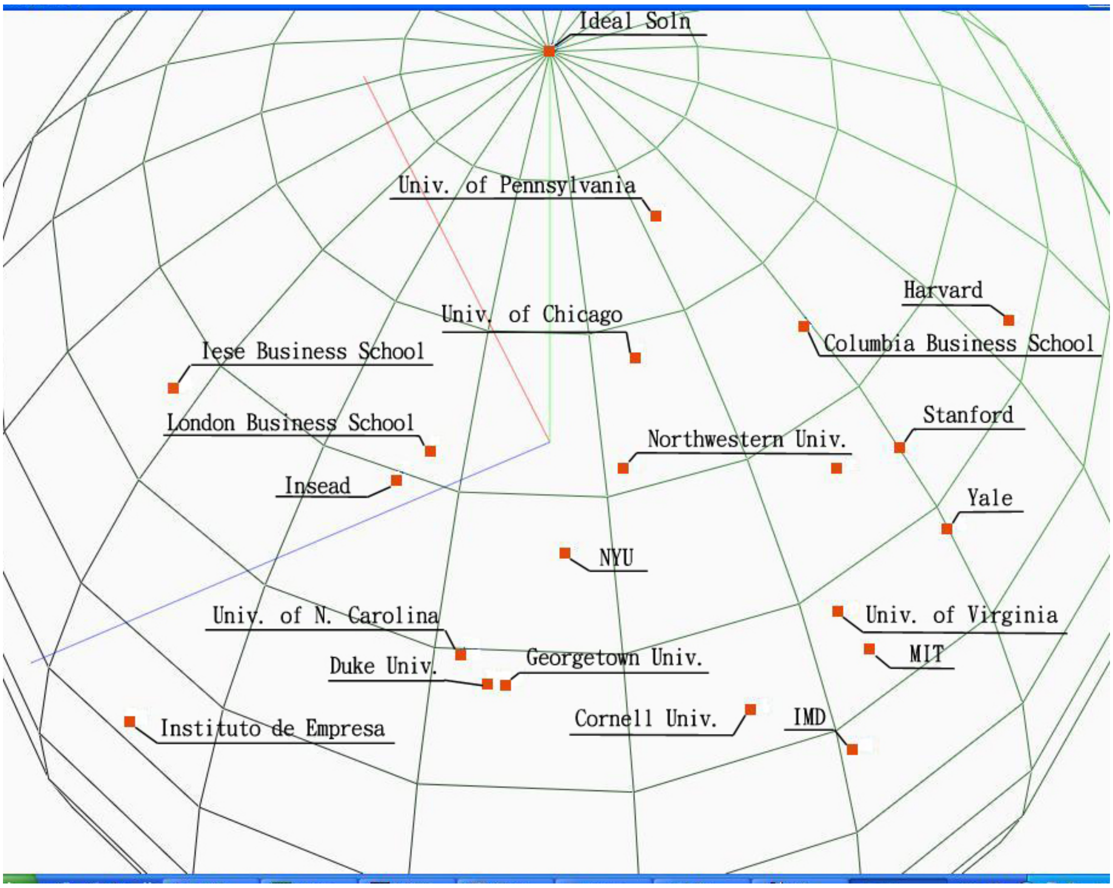

Figure 3.2 3D ball with DMUs projected on the surface

Notice that the North Pole is the ideal point. The points with higher altitudes are points with higher rankings. Universities that are closer to the equator are the ones with lower ranking and scores. Figure 3.2 clearly shows that Instituto de Empresa has the lowest ranking and IMD has the second lowest ranking, where University of Pennsylvania still has the best score.

3.3 Clustering

In this step, the Clustering Model will assign each data point to a best fitting group. The DM can specify the number of groups he/she wants. The model will make sure that every group will have at least one data points.

Table 3.7 Variables and descriptions for Clustering Model

Variables	**Descriptions**
m	Total number of DMUs
g	Total number of groups DM wants.
$Tdist_i$	Total distance between data points to their center point in a group
grp_{ij}	Binary variable. grp_{ij} = 1 if DMU i belongs to group j.
pt_{ij}	Coordinate of DMU i. j = x, y, or z.
$ctpt_{ij}$	Coordinate of Center Point i. j = x, y, z.

Clustering Model (Model 3):

$$Min\left(\sum_{i=1}^{g} tdist_i - \sum_{j=1}^{g}\sum_{k=1}^{g}\left((x_j - x_k)^2 + (y_j - y_k)^2 + (z_j - z_k)^2\right)\right) \quad \textbf{(3.17)}$$

Subject to:

$$tdist_i = \sum_{j=i}^{m}\left(grp_{ji} * \left((x_{pt_j} - x_{ctpt_i})^2 + (y_{pt_j} - y_{ctpt_i})^2 + (z_{pt_j} - z_{ctpt_i})^2\right)\right) \quad \textbf{(3.18)}$$

$$grp_{ij} \in \{0,1\} \quad \textbf{(3.19)}$$

$$\sum_{j=1}^{g} grp_{ij} = 1 \quad , \quad \forall\ i \quad \textbf{(3.20)}$$

$$\sum_{j=1}^{m} grp_{ij} \geq 1 \quad , \quad \forall\ i \quad \textbf{(3.21)}$$

$$(x_{ctpt_i})^2 + (y_{ctpt_i})^2 + (z_{ctpt_i})^2 = 1 \quad , \quad \forall\ i \quad \textbf{(3.22)}$$

$$(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 \leq \sqrt{2} \quad , \quad \forall\ i,\ j \quad \textbf{(3.23)}$$

Equation 3.17 is the objective function, which tries to minimize the sum of distance between center points and data points in their group. Also, the distance between each center point has to be maximized to ensure that the clusters will be as far from each other as possible. Equation 3.18 calculates the distance between data points and center points in each cluster for every group. Equation 3.20 limits each DMU to belong to only one cluster. Equation 3.21 is to ensure every group has at least one DMU. Equation 3.22 is to force the center point to fall on the surface of the 3D ball. Finally, Equation 3.23 is to ensure that the longest distance between any two center points will be $\sqrt{2}$.

From the 3D ball, we can group the DMUs by using the Clustering Model. The

By running the Clustering Model on this example, the grouping result is shown in Table 3.8. These twenty universities were grouped into three groups, where Harvard was grouped as the only member for group 1. Group 2 has 12 members and group 3 has 7. The number of members in a group was determined by the model automatically, but the user can specify the number of clustering groups.

Table 3.8 Groupings for universities

	Group 1	Group 2	Group 3
University of Pennsylvania: Wharton	0	0	1
Harvard Business School	1	0	0
Columbia Business School	0	0	1
Insead	0	1	0
London Business School	0	1	0
University of Chicago GSB	0	1	0
Stanford University GSB	0	0	1
New York University: Stern	0	1	0
MIT: Sloan	0	0	1
Dartmouth College: Tuck	0	0	1
Northwestern University: Kellogg	0	1	0
IMD	0	1	0
Iese Business School	0	1	0
Yale School of Management	0	0	1
Instituto de Empresa	0	1	0
Cornell University: Johnson	0	1	0
Georgetown Uni: McDonough	0	1	0
Uni of N Carolina: Kenan-Flagler	0	1	0
University of Virginia: Darden	0	0	1
Duke University: Fuqua	0	1	0

The grouping situation is shown as Figure 3.3.

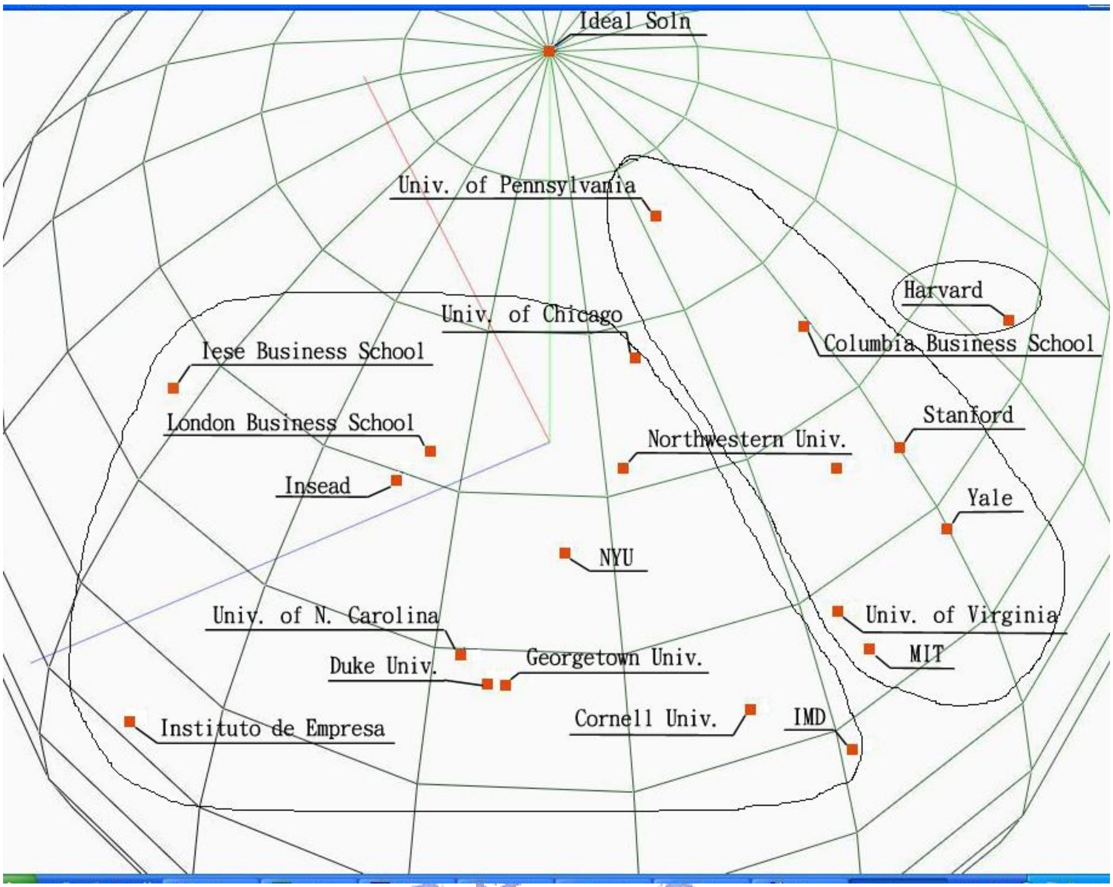

Figure 3.3 Groupings for twenty universities

4. Iterative Ranking and Grouping

In this chapter, the models presented in chapter 3 will be combined for the iterative ranking and grouping procedure. This new approach can be break into two parts and is give decision makers multiple chances to add preferences to the model. The first part of the iterative ranking and grouping model is the initial ranking process. This part is marked by the dotted line on the flow chart presented on the next page (Figure 4.1). This model will list the absolute dominance relationships in both table and preference graph format to the decision makers for them to add preferences. With the added preferences, the model will rank and group the DMUs and returns a set of weightings for criteria.

The second part of the iterative ranking and grouping model is shown by the shaded par of Figure 4.1. In this part, the DM will be presented with ranking result with grouping information. At this point, the DM can add preferences or change groupings for the DMUs. With the new input from the DM, the system will take the newly specified grouping information with preferences as input to recalculate the ranking and coordinates for the DMUs. The ranking is very likely to change due to DM's grouping demand, so the coordinates and the weightings will surly be different from the previous iteration. The DM can choose to run this iterative ranking and grouping process as many times as she/he wants until she/he obtains the desired result.

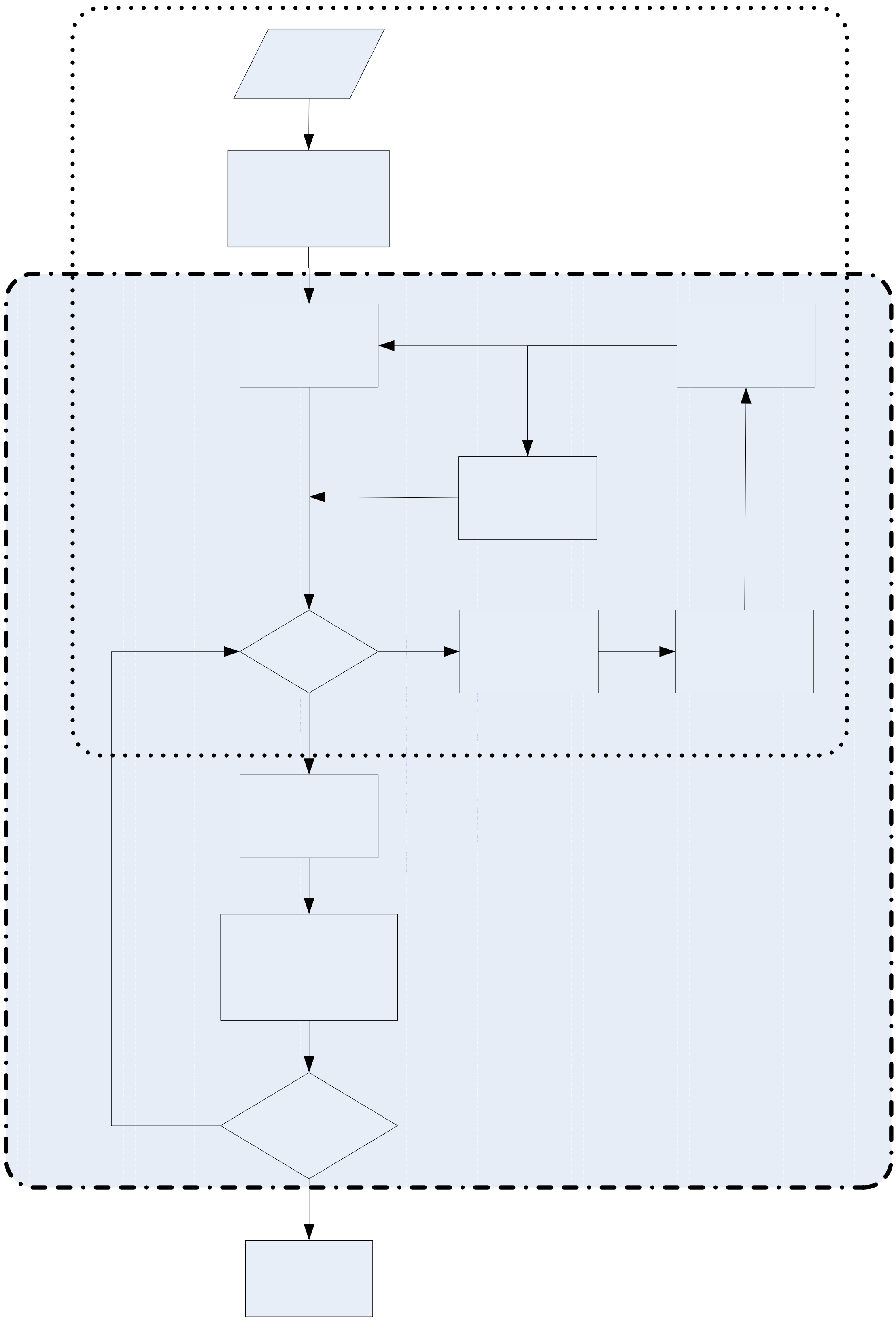

Figure 4.1 Flow chart for Iterative Ranking and Grouping Model

Suppose John is a university student who is going to apply for MBA program. He knows the university in the United States very well and has four schools that he wants to apply to. However, his aunt is asking him to consider attending London Business School in England. John has no knowledge about this school, so he has decided to apply this Iterative Ranking and Grouping model to see how this London Business School will rank among his other four universities. Table 4.1 is the list of schools and part of data John obtained from Financial Times.

Table 4.1 Hard data on five universities John selected

			C1	C2	C3	C4	C5	C6
	School name	**Country**	**Weighted salary (US$)**	**Salary increase (%)**	**International students (%)**	**Faculty with doctorates (%)**	**Faculty with doctorates (%)**	**International faculty (%)**
A1	University of Pennsylvania: Wharton	USA	1	0.965116279	0.197183099	0.741573034	1	0.176470588
A2	London Business School	UK	0.635739	0.76744186	0.887323944	0.876404494	0.8	0.694117647
A3	New York University: Stern	USA	0.6243965	1	0.042253521	0.449438202	0.9	0.294117647
A4	MIT: Sloan	USA	0.832675	0.523255814	0.112676056	0.179775281	0.6	0.070588235
A5	University of Arizona: Eller	USA	0	0.395348837	0.028169014	0	0.5	0

The very first step after obtaining hard data is to create a tournament matrix. The purpose of tournament matrix is to identify the absolute dominant relationships in the matrix. Absolute dominance is said to exist when every criterion for DMU (Decision Making Unit) *i* is better than every criterion for DMU *j*. No matter what weightings we assign to these criteria in the future, the absolute dominance will still exist. Notice that the data will have to be processed prior to the ranking and grouping process so that all the data is set to be the larger the better.

In the tournament matrix T, the value of T_{ij} is set to 1 if every criterion of T_i is smaller than T_j, which means T_j dominates T_i; otherwise a 0 will be recorded. Table 4.2 shows the variables used in Tournament Matrix Model, where it can automatically generate a tournament matrix to represent the absolute dominance relationships.

Table 4.2 Variables for Model 1

Variables	**Descriptions**
m	Total number of DMUs
n	Total number of criteria
$t_{i,j}$	$t_{i,j} = 1$ if DMU *j* is better than DMU *i*, *else* $t_{i,j} = 0$
$tc_{i,j,k}$	$tc_{i,j,k}$ is binary variable for pairwise comparison on criteria. $tc_{i,j,k} = 1$ if $tc_{i,k} < tc_{j,k}$, else $tc_{i,j,k} = 0$.
$\overline{C_k}$, $\underline{C_k}$	Maximum and minimum values of k^{th} criterion
$C_{i,k}$	The k^{th} criterion of i^{th} DMU
w_k	Weight for k^{th} criterion
M	A large constant number

Tournament Matrix Model:

$$MIN \quad \sum_{i=1}^{m}\sum_{j=1}^{m} t_{i,j} \tag{4.1}$$

Such that

$$\frac{(C_{i,k}-\underline{C_k})}{(\overline{C_k}-\underline{C_k})}+tc_{j,i,k} \quad \geq \quad \frac{(C_{j,k}-\underline{C_k})}{(\overline{C_k}-\underline{C_k})} \quad , \quad \forall\ i,j,k \tag{4.2}$$

$$n-\sum_{k=1}^{n} tc_{i,j,k}+t_{j,i} \quad \geq \quad 1 \quad , \quad \forall\ i,j \tag{4.3}$$

This model, will try to minimize the total number of t value, where t is a binary variable. The first constraint, equation 4.2, compares each criterion between every DMU. Since $tc_{i,j,k}$ is set to be a binary variable and the value of $\frac{(C_{i,k}-\underline{C_k})}{(\overline{C_k}-\underline{C_k})}$ is normalized to between 0 and 1, $tc_{i,j,k}$ will be set to one if $C_{i,k}$ is smaller than $C_{j,k}$. The purpose of the second constraint is to fill in the values into the tournament matrix.

Following is the tournament matrix generated by this model.

Table 4.3 Tournament Matrix for Universities

	A1	A2	A3	A4	A5	Sum
A1	0	0	0	0	0	0
A2	0	0	0	0	0	0
A3	0	0	0	0	0	0
A4	1	0	0	0	0	1
A5	1	1	1	1	0	4

From Table 4.3 we can see that A4 (MIT) is dominated by A1 (U Penn) and A5 (Univ. of Arizona) is dominated by all other universities. These relationships will not change no matter what weightings are returned by the model.

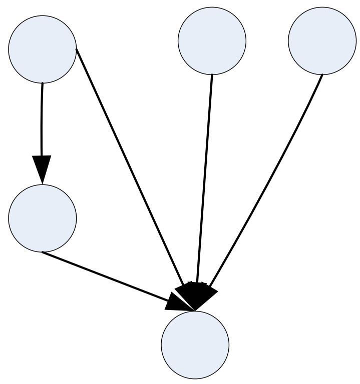

Figure 4.2 Preference graph from Table 4.3

Aside from the tournament matrix, a preference diagram (Figure 4.3) can also be drawn to help John see the relationships between the schools. As shown on Figure 4.2, there are arcs that are not necessary. This is because Table 4.3 has redundant information. When the number of DMU and criteria grows, the redundant information may increase and make the preference graph very confusing and difficult to read. Hence, the extra information in tournament matrix will need to be removed by following Algorithm 1.

Algorithm 1:

For each base DMU in matrix T, sum the values in each row, so that the least dominant DMU will have the highest value.

Take DMU with the lowest non-zero sum and call it R (the row number). If there are ties in the sum, pick the one with larger raw number.

For row R,
    For each non-zero value on row R,
        do
            if there exist “1” on location T_{RC},
            then if T_{XR} and T_{XC} both are 1,
                then change T_{XC} to 0
                else do nothing

Repeat with next DMU that has the next lowest sum or DMU that has the equal sum but next largest row number.

For example, the row with lowest non-zero sum will be row 4 in Table 4.3. Suppose we call this tournament matrix T, location T_{41} is set to be “1”, and row 5 has “1” at both T_{51} and T_{54}. Hence, T_{51} needs to be set to “0”. This procedure has to be performed on the whole matrix to eliminate the extra information. The reduced matrix is shown in Table 4.4, and the new preference graph is shown in Figure 4.3.

Table 4.4 Reduced tournament matrix

	A_1	A_2	A_3	A_4	A_5
A_1	0	0	0	0	0
A_2	0	0	0	0	0
A_3	0	0	0	0	0
A_4	1	0	0	0	0
A_5	0	1	1	1	0

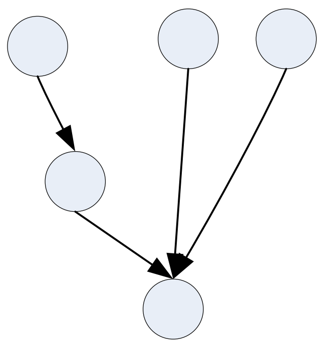

Figure 4.3 New preference graph

With the information reduced from tournament matrix, we can draw a preference graph (Figure 4.3). This graph can help the John to visualize the absolute relationships obtained from hard data and add preferences accordingly.

Figure 4.3 shows that $A_1 > A_4$, $A_2 > A_5$, $A_3 > A_5$, and $A_4 > A_5$. These dominance relationships are extracted from the hard data directly, so John can not change any of these relationships. There are still relationships that are either unclear or unable to determine from the hard data.

Now, John can add preference(s) by drawing two types of connecting lines on the preference graph for any two DMUs sets as the additional preference to help the model find optimal weightings for criteria. The first type is directed arrow, "→". The head of the arrow can implied the ">" sign in math, which means the starting node is superior to the ending node. The second type of connecting lines is the dotted line, "- - -". If John connects two nodes with dotted line, this means that he thinks these two nodes should have the same ranking. At this point, John has decided not to add any preferences. He wants to wait until the results are returned from the model before he specifies any preferences.

Notice that he can only add one preference at a time. After one insertion, the corresponding value in the tournament matrix will be changed. Since the tournament matrix will record only the "greater" relationships, all the "equality" relationships will be incorporated with the objective function of the model during the weightings for criteria.

In order to keep the preference diagram free from cycles, after every preference addition, the tournament matrix will need to undergo a “transitivity test”, which uses the technique presented by Gass. Since the matrix has size of m by m, all the diagonal values need be 0 for T^3 up to T^m, by the theory Gass proposed. If any of the T^x has one or more non-zero diagonal value, the last added preference will be removed in order to keep the transitivity. If the new preference passed the transitivity test, the extra information will need to be removed by applying Algorithm 1 from last section to this new tournament matrix. The complete preference adding procedure can be seen from the flow chart (Figure 4.4).

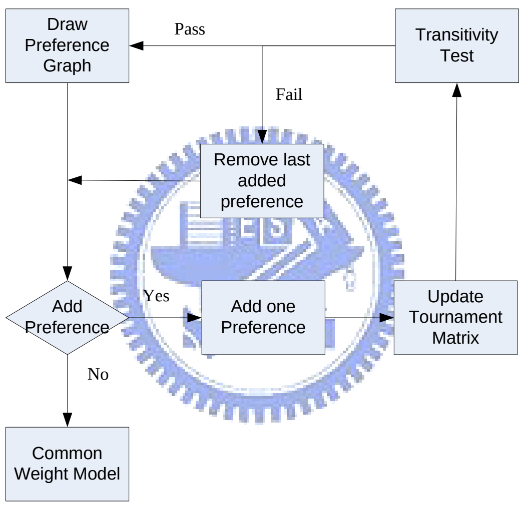

Figure 4.4 Flowchart for preference addition

After both the transitivity test and removal of extra information from the matrix, the Preference Diagram will be re-drawn and present to the John to add next preference. It is important that John adds preferences from the most important relationship to the least important ones, since with increasing number of preferences, the chance to get an intransitive relationship will increase as well.

This procedure will repeat until John has no more preferences to add. Given

John's preferences, the model can use these preferences as part of the constraints to automatically produce weightings for each criterion. Table 4.5a shows the ranking results and Table 4.5b shows the weightings for each criterion.

Table 4.5 Results from initial run of the model

(a) Ranking for five universities

	X	Y	Z	Score	Rank
A1	0.1298738	0.990961946	0.03357416	0.904931317	1
A2	0.1687944	0.855201207	0.490040133	0.619475634	3
A3	0.3260319	0.841850725	0.430105285	0.602320135	4
A4	-0.1002794	0.880834783	0.462681456	0.654796847	2
A5	0.1418996	0	0.989881053	0	5

(b) Weights for criteria

	C1	**C2**	**C3**	**C4**	**C5**	**C6**
Weight	0.74028	0.15465	0.001	0.10206843	0.001	0.001

The weight for C_1, which is the weighted salary, is weighted almost three-quarters of all attributes in the initial run of the model. The minimum value is not 0 because every criteria has to participate in the ranking process. However, due to the fact that every criterion has to be accounted in the score calculation, there are three criteria are with weights of 0.001.

Table 4.6 Grouping table for the initial result

	Group1	Group2
A1	0	1
A2	0	1
A3	0	1
A4	1	0
A5	1	0

Groupings can be seen from both Table 4.6 and Figure 4.5 shows the grouping situation for these five schools. MIT and University of Arizona are being ranked in the same group, which John thinks is not quite logical. He can change the groupings for these schools at the first iteration of the ranking and grouping process.

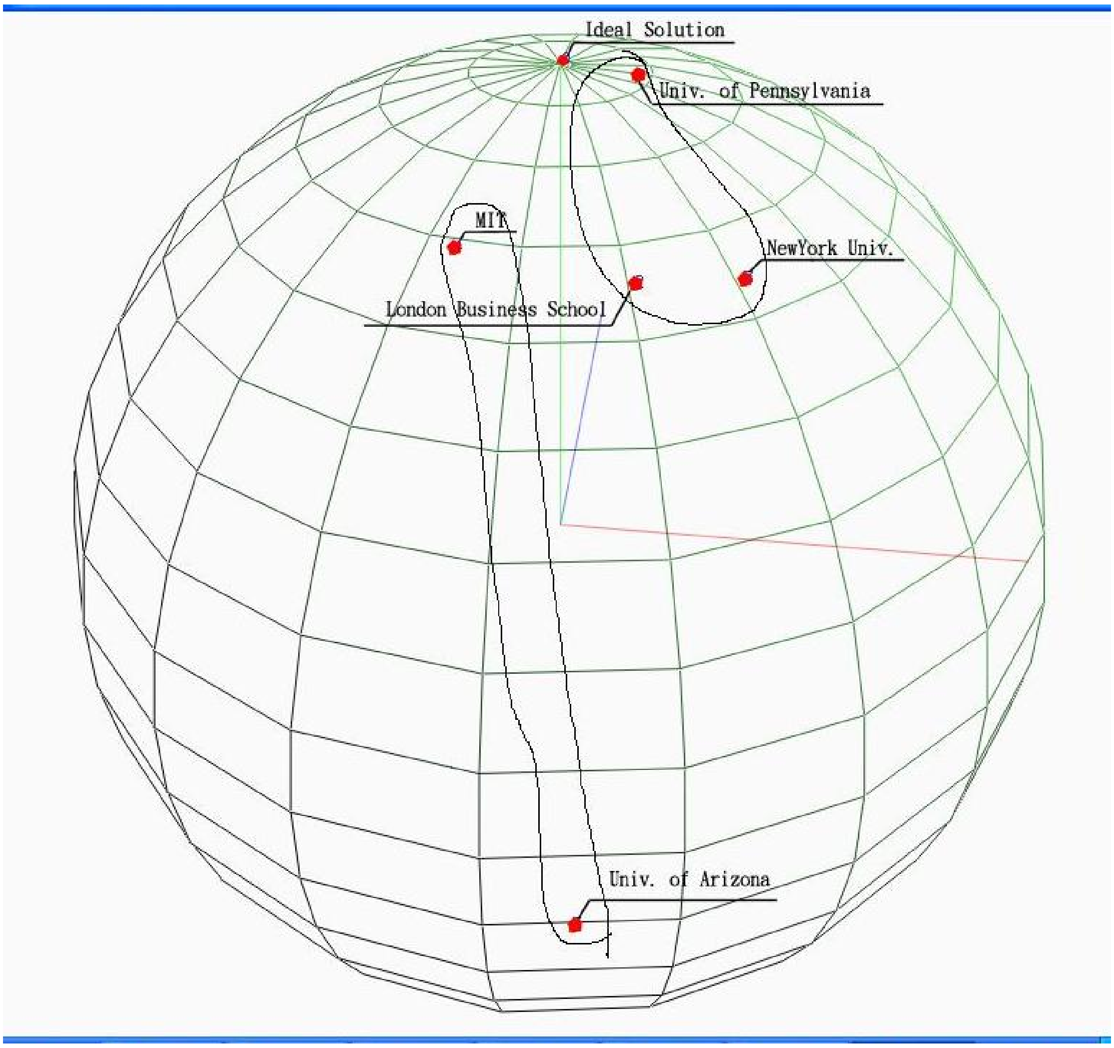

Figure 4.5 Initial grouping situation represented on 3D ball

To specify the desired grouping, John believes that MIT should be in the same group as University of Pennsylvania, though he is not sure about the London Business School. He also thinks that New York University should be grouped in the same groups as University of Arizona. Hence, he modified the grouping table and obtained a new grouping table as shown in Table 4.7.

Table 4.7 New grouping table specified by John

	Group1	Group2
A1	0	1
A2	0	1
A3	1	0
A4	0	1
A5	1	0

This table will then be one of the input values for the iterative ranking and grouping model to calculate the new rankings, weightings, and coordinates for the universities. Table 4.8a and Table 4.8b display the result from the first iteration. Although the ranking is still the same, other value have being changed. Criterion 6, Faculty with doctorates, is now weighted slightly than the previous run.

Table 4.8 Results form the groupings specified by John

(a) New score and rankings for universities

	X	Y	Z	Score	Rank
A1	0.1794657	0.983523545	0.021760375	0.871639354	1
A2	0.1817976	0.844442538	0.503851591	0.605592264	3
A3	0.3387656	0.835473124	0.432692181	0.594380873	4
A4	0.301143	0.878376667	0.371170188	0.65125463	2
A5	0.5546003	0	0.832116892	0	5

(b) New weighting for criteria

	C1	**C2**	**C3**	**C4**	**C5**	**C6**
Weight	0.74518	0.1581203	0.001	0.0336	0.001	0.0611

Now John can look at the 3D graph (Figure 4.6) that displays the groupings he specified.

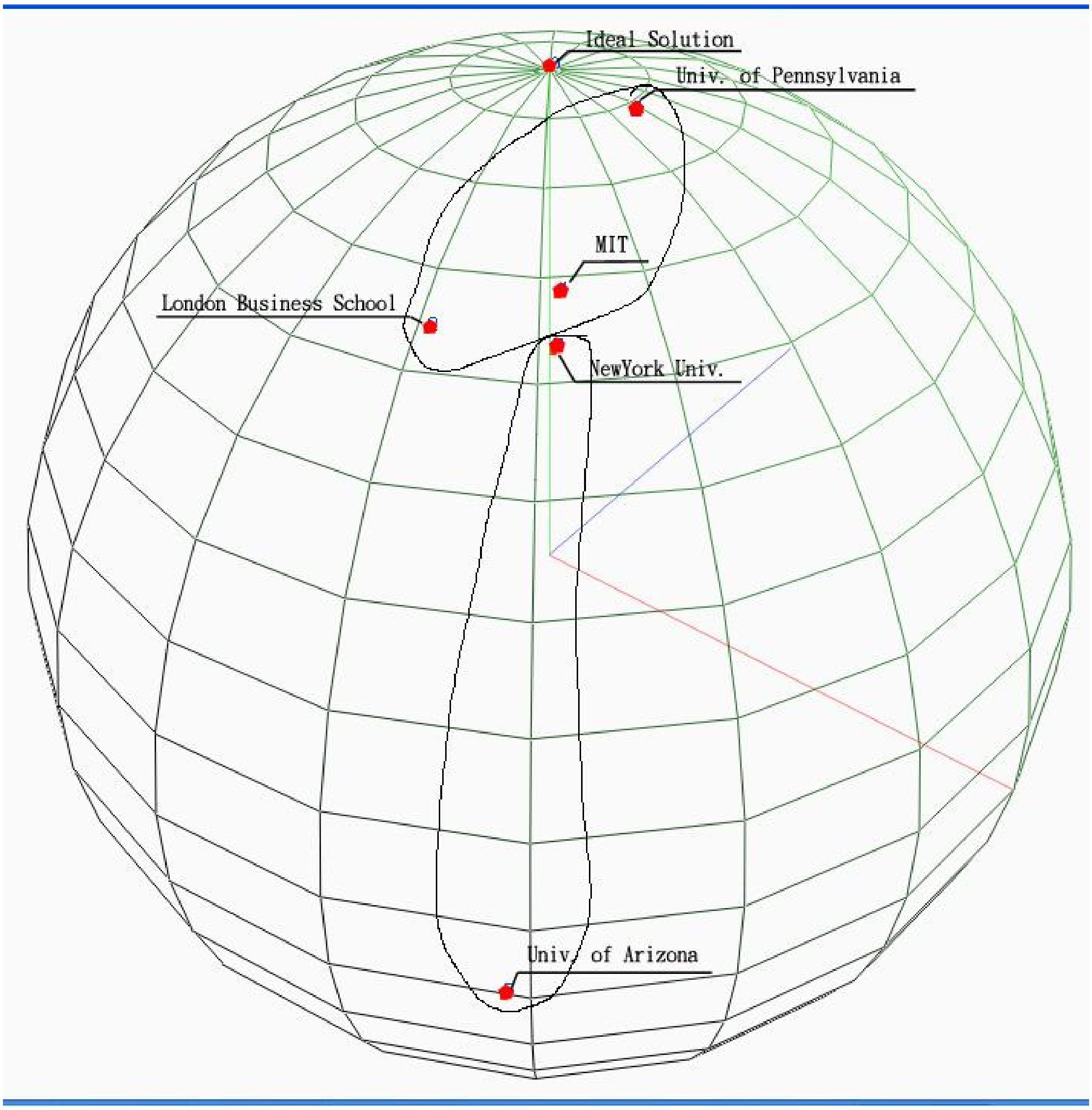

Figure 4.6 New coordinates and groupings for the universities

Because the points for New York University and University of Arizona are far apart, John is not quite satisfied with the current ranking and grouping result. He has noticed that New York University has lower score than MIT and he has heard some rumor form his friend which changed his perspective on MIT. So he now adds a preference which states that the score of New York University should be higher than the score of MIT.

Although he can still change the groupings again, but he is satisfied with the groupings she set. Hence the grouping table will still be the input value for the model. Under the constraint section, we need to add a constraint that says “score of NYU is higher than the score of MIT”. After this preference is added, Table 4.9 shows the results for second iteration.

Table 4.9 Results with preference

(a) New score and rankings for universities

	X	Y	Z	Score	Rank
A1	0.1361369	0.786711637	0.602122535	0.538168469	1
A2	0.127071	0.777934447	0.615362452	0.528761681	2
A3	0.5582307	0.761262615	0.329935869	0.511392401	3
A4	0.6453213	0.450878182	0.616659831	0.258972458	4
A5	0.8251283	0	0.564945379	0	5

(b) New weighting for criteria

	C1	**C2**	**C3**	**C4**	**C5**	**C6**
Weight	0.21127	0.4750437	0.001	0.001	0.001	0.310691

Notice that the score for A3 (NYU) is much higher than the score of A4 (MIT). By add this one constraint, not only the ranking has changed, but also the weightings has changed dramatically. Notice that the first criterion’s weighting was more than 0.7, and now has dropped to around 0.21. This means that C1 (weighted Salary) is a strong attribute for MIT. Now that NYU is better than MIT, the weighting for weighted salary is lowered. On the country, the weightings for salary increase (C2) and faculty with doctorates (C6) has increased in a great deal.

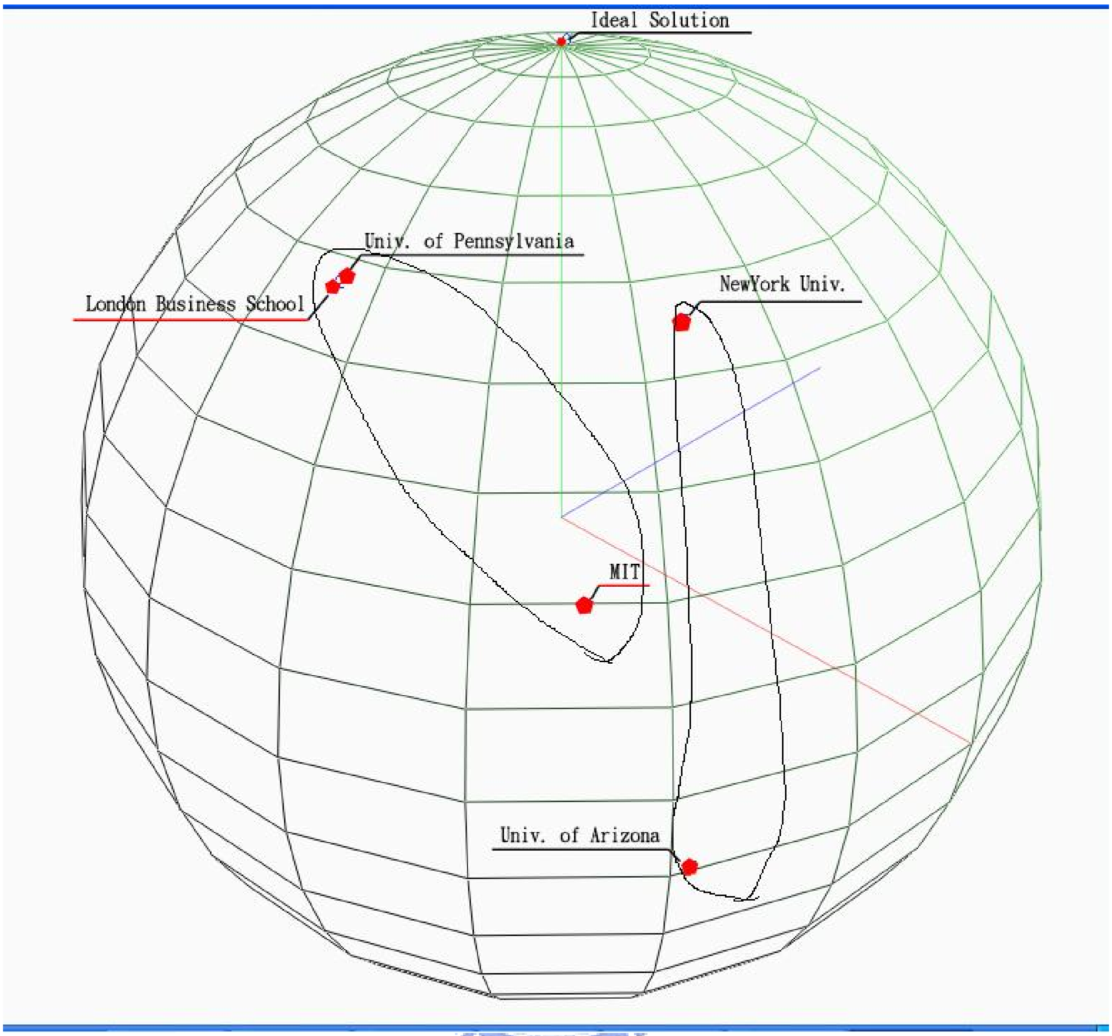

Figure 4.7 New coordinates for the university with John's preference.

With John's preference, the score of MIT has dropped considerably lower than U Penn, London Business School, and NYU.

If John has no more preference to add, this will be the final ranking for him to use as a reference to help him decide whether he should apply for London business School. If John admires students who are attending University of Pennsylvania, then perhaps John should really consider applying to London Business School. However, If John he can still adds more preferences and run the model iteratively.

Remark

This model provides many chances for the decision makers to add their opinions and runs it iteratively. By this approach, the final result will be logically consistent with what the decision makers have in mind, because this model ranks DMUs based on the preferences provided by decision makers. Hence, the result should be very useful for the DM in further decision making processes.

5. Conclusion

People have being ranking DMU to show their importance and priorities since long ago. There are many ways to rank and each method has their strengths and weaknesses. From this study, we have proposed a method to help Decision Makers rank DMUs with out the needs to specify weightings for each criteria, which often is the most controversy and difficult in the whole ranking process. Using the techniques from Linear Programming, this model can produce a set of weightings for DMUs based on the absolute dominances relationships and preferences relationships, given by the Decision Makers. The 3D Ball representation not only has given Decision Makers the views they can not have by only looking at the table, but also allows them to categorize the DMUs and change the groupings for DMUs.

This model has focused on the mathematical models. There are still many issues can be studied in this area. Following are some suggestions for future works:

- Efficiency and validity in data collection and criteria selection.
- Although this model provides the function of changing groupings for DMUs, the clustering function can be improved. Certain clustering technique could be applied and help the groupings to be more accurate.
- The mathematical model can be modified to produce a more profound model, which can reduce the computation time and returns globally optimized solution.

References

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Appendix

U.S. News and World Report 2005 Ranking

Rank	University	Score	Peer Assessment (5.0 highest)	Recruiter Assessment (5.0 highest)	Avg. Undergrad GPA	Av.g GMAT	Acceptance rate	average starting salary & bonus	Graduates employed at graduation	Employed 3 months after grad	out-of-state tuition and fees	total full-time enrollment
1	Harvard University (MA)	100	4.8	4.5	3.6	708	0.116	105896	0.776	0.874	33650	1802
2	Stanford University (CA)	99	4.8	4.4	3.57	713	0.092	107320	0.754	0.855	36252	761
3	University of Pennsylvania (Wharton)	97	4.8	4.6	3.48	713	0.159	101404	0.737	0.867	37323	1704
4	Massachusetts Institute of Technology	96	4.7	4.5	3.5	710	0.182	99539	0.77	0.883	34780	765
5	Northwestern University (Kellogg)	93	4.7	4.6	3.45	703	0.157	98358	0.727	0.83	34314	1066
6	Columbia University (NY)	89	4.5	4.2	3.4	709	0.125	98611	0.705	0.845	34788	1178
	University of Chicago	89	4.7	4.3	3.4	690	0.229	97872	0.672	0.848	34733	1046
8	University of California – Berkeley	87	4.6	4.1	3.5	700	0.128	91934	0.682	0.806	28020	516
9	Dartmouth College (Tuck) (NH)	86	4.3	4.3	3.4	696	0.19	96714	0.751	0.838	34500	482
10	University of Michigan – Ann Arbor	83	4.4	4.2	3.4	692	0.227	97039	0.616	0.75	34688	849
11	Duke University (Fuqua) (NC)	81	4.4	4.2	3.4	703	0.253	92459	0.562	0.707	34354	747
12	University of California – Los Angeles	80	4.2	4.1	3.5	701	0.206	90952	0.521	0.74	29218	673
	University of Virginia (Darden)	80	4.1	4.1	3.4	678	0.242	92855	0.667	0.842	33270	633
14	Cornell University (Johnson) (NY)	78	4.2	4.1	3.35	672	0.304	94818	0.626	0.77	33859	550
	New York University (Stern)	78	4.1	3.9	3.4	700	0.185	95896	0.624	0.731	34126	784
	Yale University (CT)	78	4.1	4.1	3.5	703	0.186	89526	0.474	0.726	33560	483
17	Carnegie Mellon University (PA)	72	4	3.8	3.35	680	0.273	85737	0.581	0.773	35220	485
18	University of Southern California	70	3.8	3.7	3.3	688	0.28	76670	0.694	0.84	33675	530
19	Emory University (Goizueta) (GA)	69	3.7	3.5	3.4	676	0.276	81131	0.608	0.823	30972	367
	Ohio State University (Fisher)	69	3.6	3.2	3.39	665	0.425	86153	0.709	0.883	25224	290
21	University of Minnesota – Twin Cities	68	3.5	3.6	3.28	654	0.417	84041	0.725	0.928	26900	237
	University of North Carolina	68	4	3.9	3.2	668	0.36	87819	0.551	0.71	31004	547

U.S. News and World Report 2005 Ranking (page 2)

Rank	University	overall Score	Peer Assessment (5.0 highest)	Recruiter Assessment (5.0 highest)	Average Undergrad GPA	Avg. GMAT	Acceptance rate	average starting salary & bonus	graduates employeed at graduation	employeed 3 months after grad	out-of-state tuition and fees	total full-time enrollment
23	Indiana University － Bloomington (Kelley)	67	3.9	4	3.31	650	0.318	82270	0.57	0.705	25361	506
	Texas A&M University － College Station (Mays)	67	3.2	3.2	3.3	639	0.25	91644	0.865	0.959	$16,800**	172
	University of Illinois － Urbana-Champaign	67	3.6	3.4	3.4	648	0.393	79610	0.667	0.871	24664	338
	University of Texas － Austin (McCombs)	67	3.9	4	3.34	676	0.4	82167	0.501	0.673	26083	795
27	Purdue University － West Lafayette (Krannert) (IN)	66	3.7	3.5	3.31	658	0.322	86577	0.64	0.746	25488	307
	University of Washington	66	3.4	3.3	3.4	669	0.401	70605	0.656	0.957	19857	243
29	Arizona State University － Main Campus (W. P. Carey)	63	3.5	3.1	3.44	651	0.353	72343	0.562	0.917	21902	263
	Michigan State University (Broad)	63	3.4	3.4	3.36	646	0.253	82898	0.632	0.805	21420	204
	University of California － Davis	63	3.1	3.5	3.4	679	0.31	73328	0.686	0.843	28912	122
	University of Notre Dame (Mendoza) (IN)	63	3.5	3.6	3.4	664	0.341	78047	0.476	0.78	28235	327
33	Georgetown University (McDonough) (DC)	62	3.4	3.3	3.3	661	0.272	83726	0.535	0.832	30888	521
	University of Maryland － College Park (Smith)	62	3.5	3.2	3.35	656	0.265	80085	0.5	0.843	26382	357
35	University of Arizona (Eller)	61	3.4	3.3	3.46	643	0.566	67645	0.622	0.844	22124	131
36	University of Rochester (Simon) (NY)	60	3.5	3.4	3.2	647	0.365	77844	0.558	0.831	32475	382
	University of Wisconsin － Madison	60	3.5	3.5	3.3	662	0.243	75002	0.622	0.7	24602	275
	Wake Forest University (Babcock) (NC)	60	3.3	3.6	3.2	643	0.459	74531	0.678	0.856	26625	229
39	Brigham Young University (Marriott) (UT)	59	3	3.4	3.58	640	0.479	68986	0.553	0.84	9826	266
	Vanderbilt University (Owen) (TN)	59	3.4	3.6	3.28	638	0.538	77770	0.548	0.802	30995	428
	Washington University in St. Louis (Olin)	59	3.7	3.5	3.25	651	0.407	76845	0.519	0.707	31810	295
42	Boston College (Carroll)	58	3.1	3.3	3.3	661	0.209	71674	0.568	0.865	$914*	200
	Georgia Institute of Technology (DuPree)	58	3.2	3.2	3.32	668	0.39	70245	0.56	0.867	21790	217
	University of Georgia (Terry)	58	3.3	3.3	3.26	659	0.319	73413	0.614	0.814	17820	170

U.S. News and World Report 2005 Ranking (Page 3)

Rank	University	overall Score	Peer Assessment (5.0 highest)	Recruiter Assessment (5.0 highest)	Average Undergrad GPA	Avg. GMAT	Acceptance rate	average starting salary & bonus	graduates employeed at graduation	employeed 3 months after grad	out-of-state tuition and fees	total full-time enrollment
45	Penn State University – University Park (Smeal)	56	3.5	3.5	3.3	653	0.369	75962	0.42	0.67	24192	189
	Rice University (Jones) (TX)	56	3.2	3.7	3.2	624	0.349	82357	0.556	0.742	29746	351
47	Boston University	55	2.9	3.3	3.17	666	0.375	71655	0.623	0.869	29831	361
	Tulane University (Freeman) (LA)	55	3.1	3.2	3.3	663	0.644	66076	0.583	0.854	$1,019*	221
	University of California – Irvine	55	3.2	3	3.37	670	0.349	67162	0.464	0.821	29183	239
	University of Iowa (Tippie)	55	3.2	3.3	3.3	637	0.53	66340	0.615	0.821	18996	160
51	University of Pittsburgh (Katz)	54	3.3	3.2	3.3	635	0.515	65867	0.507	0.836	$37,803**	147
52	Southern Methodist University (Cox) (TX)	53	3.1	3.2	3.26	665	0.456	73083	0.454	0.778	29462	194
53	Babson College (Olin) (MA)	52	3.3	3.5	3.06	644	0.432	71508	0.496	0.798	28344	417
	University of Florida (Warrington)	52	3.3	3.1	3.23	650	0.439	64200	0.344	0.875	$775*	135
55	University of Colorado – Boulder (Leeds)	50	3.1	3	3.17	650	0.513	66630	0.48	0.84	22469	124
	University of Connecticut	50	2.7	2.7	3.52	625	0.545	86916	0.5	0.679	18098	121
	Virginia Tech (Pamplin)	50	2.9	3	3.3	648	0.394	56390	0.68	0.84	11618	51
58	University of Oregon (Lundquist)	49	2.9	3.1	3.29	632	0.436	57892	0.542	0.896	15339	134
59	Case Western Reserve University (Weatherhead) (OH)	48	3.3	3.2	3.16	615	0.578	63540	0.481	0.779	30940	282
	Claremont Graduate School (Drucker) (CA)	48	3.1	3.1	3.22	607	0.653	75409	0.581	0.698	34217	202
61	University of Alabama – Tuscaloosa (Manderson)	47	2.8	2.8	3.4	615	0.382	56051	0.56	0.88	12894	107
62	University of Oklahoma (Price)	45	2.7	2.6	3.61	644	0.686	55229	0.524	0.69	$432*	86
	University of Texas – Dallas	45	2.5	2.9	3.5	638	0.496	72136	0.4	0.68	8211	73
64	North Carolina State University	44	2.5	2.9	3.3	615	0.559	60841	0.636	0.864	18184	72
	University of Missouri – Columbia	44	2.9	3	3.33	620	0.562	51206	0.39	0.847	$639*	201
	University of Utah (Eccles)	44	2.8	3.1	3.51	611	0.747	50376	0.404	0.807	15727	171

U.S. News and World Report 2005 Ranking (Page 4)

Rank	University	overall Score	Peer Assessment (5.0 highest)	Recruiter Assessment (5.0 highest)	Average Undergrad GPA	Avg. GMAT	Acceptance rate	average starting salary & bonus	graduates employeed at graduation	employeed 3 months after grad	out-of-state tuition and fees	total full-time enrollment
67	Iowa State University	43	2.6	2.9	3.45	592	0.55	45600	0.789	0.842	14332	88
	Temple University (Fox) (PA)	43	2.6	2.5	3.2	635	0.478	76435	0.425	0.792	12726	178
	University of South Carolina (Moore)	43	3	3.1	3.33	622	0.581	65004	0.467	0.582	22200	298
70	University of Massachusetts – Amherst (Isenberg)	41	2.7	2.8	3.3	624	0.269	60875	0.5	0.714	16498	64
71	DePaul University (Kellstadt) (IL)	40	2.8	2.5	3.22	573	0.547	60860	0.64	0.84	$650*	65
	George Washington University (DC)	40	2.9	3.2	3.18	630	0.287	58660	0.383	0.654	$834*	200
	Louisiana State University – Baton Rouge (Ourso)	40	2.7	2.8	3.38	603	0.268	48077	0.621	0.758	9364	146
	Rensselaer Polytechnic Institute (Lally) (NY)	40	2.7	3.1	3.28	643	0.556	55606	0.381	0.667	29137	62
	Texas Christian University (Neeley)	40	2.4	3.1	3.2	607	0.682	60041	0.5	0.86	17950	55
76	Santa Clara University (Leavey) (CA)	39	2.9	3.3	3.08	624	0.765	77367	0.343	0.486	$618*	154
77	Pepperdine University (Graziadio) (CA)	38	2.7	2.7	3.13	636	0.66	63551	0.42	0.7	28032	282
	Rutgers State University	38	2.6	2.9	3.2	634	0.44	78820	0.765	0.882	22077	234
	University of Central Florida	38	2.3	1.5	3.4	531	0.644	86000	0.818	0.818	$776*	64
80	University of Denver (Daniels)	37	2.5	3.4	3.26	560	0.837	55363	0.612	0.759	25188	314
	University of Miami (FL)	37	2.8	2.5	3.2	602	0.592	61501	0.546	0.694	$1,074*	311
	University of Tennessee – Knoxville	37	2.9	2.8	3.34	600	0.398	57961	0.308	0.631	$25,395**	76

** Tuition is reported for the entire program.

Sources: U.S. News and the schools.

Assessment data collected by Synovate

Financial Times 2004 Ranking

Following are the names of abbreviations used in the table.

WS	-	Weighted Salary
SI	-	Salary Increase (%)
VFMR	-	Value for Money Rank
CPR	-	Career Progress Rank
AAR	-	Aims Achieved Rank
PSR	-	Placement Success Rank
EATM	-	Employment at Three Months (%)
ARR	-	Alumni Recommend Rank
WF	-	Woman Faculty (%)
WS	-	Woman Student (%)
WB	-	Woman Board (%)
IF	-	International Faculty (%)
IS	-	International Students (%)
IB	-	International Board (%)
IER	-	International Experience Rank
IMR	-	International Mobility Rank
L	-	Language **
FWD	-	Faculty with Doctorates (%)
FTDR	-	Financial Times Doctoral Rank
FTRR	-	Financial Times Research Rank

2004 Rank	School name	WS	SI	VFMR	CPR	AAR	PSR	EATM	ARR	WF	WS	WB	IF	IS	IB	IE	IMR	L	FWD	FTDR	FTRR
1	University of Pennsylvania: Wharton	151,726	182	52	23	32	18	86	1	17	33	8	30	39	52	64	30	0*	100	3	2
2	Harvard Business School	162,149	150	71	26	47	19	87	2	24	35	14	35	33	21	53	96	0	98	9	1
3	Columbia Business School	142,781	196	55	75	31	13	87	8	14	30	9	51	31	36	47	61	0*	98	12	6
4	Insead	133,619	124	1	31	17	43	70	5	15	24	6	86	88	69	7	7	2	98	50	10
4	London Business School	125,167	165	85	33	16	55	65	7	11	23	6	74	88	60	3	18	1	98	36	19
4	University of Chicago GSB	140,310	182	81	40	39	8	87	6	13	29	16	42	27	12	58	36	0*	98	20	4
7	Stanford University GSB	150,291	138	94	17	12	25	85	3	16	35	13	35	35	18	63	64	0	99	13	3

Financial Times 2004 Global MBA Ranking (Page 2)

2004 Rank	School name	WS	SI	VFMR	CPR	AAR	PSR	EATM	ARR	WF	WS	WB	IF	IS	IB	IE	IMR	L	FWD	FTDR	FTRR
8	New York University: Stern	124,340	185	93	54	20	11	80	16	19	34	13	40	28	5	57	56	0	99	3	12
9	MIT: Sloan	139,526	144	79	34	8	9	88	9	16	26	10	21	33	25	37	80	0	96	5	9
10	Dartmouth College: Tuck	144,623	174	65	77	19	1	90	11	21	24	15	29	29	6	50	69	0	96	78	15
11	Northwestern University:Kellogg	139,169	147	100	39	15	7	86	4	21	28	10	24	28	6	51	26	0	97	22	5
12	IMD	142,626	99	2	3	2	17	91	17	10	16	2	100	96	77	2	54	0	95	78	75
13	Iese Business School	99,470	187	61	25	55	40	96	21	12	25	4	29	70	78	12	9	1	99	52	71
13	Yale School of Management	129,280	194	75	36	30	21	73	26	12	29	17	31	24	7	59	92	0	98	69	37
15	Instituto de Empresa	98,257	149	5	1	9	36	80	71	34	38	22	44	72	80	5	11	1	82	78	79
16	Cornell University: Johnson	129,604	159	89	87	53	14	77	25	29	27	15	27	35	40	56	71	0	94	66	22
17	Georgetown Uni: McDonough	121,240	179	91	38	28	69	83	41	28	30	19	27	38	9	48	48	0*	91	78	50
17	Uni of N Carolina	117,639	163	58	90	66	12	71	19	16	27	11	25	25	4	36	34	0	92	30	13
19	University of Virginia: Darden	137,012	171	63	68	14	6	65	13	23	27	13	8	25	7	66	75	0	98	69	83
20	Duke University: Fuqua	122,244	148	96	88	26	2	80	12	20	30	11	37	32	5	72	35	0	92	44	11
21	University of Toronto: Rotman	98,285	161	14	60	13	65	77	23	24	31	42	56	39	50	61	74	0	93	44	36
22	Emory University: Goizueta	116,310	152	72	48	18	22	81	30	32	24	16	24	29	6	79	60	0	95	78	7
22	Rotterdam School of Mgnt	107,305	142	36	13	96	84	79	42	11	17	9	31	97	27	10	2	0	96	34	56
22	UC Berkeley: Haas	120,379	126	70	22	34	35	81	14	24	24	17	31	32	11	43	41	0*	98	11	16
22	York University: Schulich	85,734	158	3	29	85	90	82	45	23	36	19	53	69	48	8	55	0*	99	58	44
26	University of Oxford: Said	122,098	122	4	27	23	47	81	44	15	21	12	40	88	40	9	37	0	90	68	59
27	University of Maryland: Smith	97,323	175	48	99	43	47	85	47	21	34	5	10	34	32	49	91	0	100	26	17
28	Carnegie Mellon University	118,604	155	87	70	50	5	80	24	14	22	8	33	25	8	76	84	0	90	7	25
29	University of Western Ontario	106,010	165	29	64	38	73	70	20	22	21	13	34	40	43	29	40	0	92	52	41
30	SDA Bocconi	92,411	164	11	80	35	30	80	47	30	22	47	17	44	27	30	13	2	80	10	87
30	University of Michigan	121,754	135	99	57	22	4	81	10	24	24	22	33	27	4	77	63	0	95	25	18
32	UCLA: Anderson	126,388	130	90	81	36	10	74	15	10	33	10	23	24	11	74	23	0	100	44	8
32	Warwick Business School	103,984	112	12	52	3	72	87	29	37	22	21	34	74	32	35	12	1	84	1	61
34	University of Cambridge: Judge	110,801	110	22	37	57	59	79	56	26	33	35	43	87	35	19	39	0	88	8	70
35	University of Rochester: Simon	104,661	164	92	61	61	31	83	62	14	24	9	36	46	35	40	76	0	94	55	40
36	University of South Carolina	96,071	183	59	73	93	88	65	55	15	29	8	14	29	3	41	8	0*	90	37	57
37	Manchester Business School	98,287	145	27	6	77	86	80	32	16	28	0	32	72	0	21	3	0	80	30	88
38	Uni of S California: Marshall	107,117	149	88	91	60	23	84	46	23	27	12	26	21	20	78	42	0	85	55	14
39	McGill University	82,243	136	50	16	51	75	66	36	29	30	10	71	61	30	16	25	0	96	58	49

Financial Times 2004 Global MBA Ranking (Page 3)

2004 Rank	School name	WS	SI	VFMR	CPR	AAR	PSR	EATM	ARR	WF	WS	WB	IF	IS	IB	IE	IMR	L	FWD	FTDR	FTRR
40	Ohio State University: Fisher	94,856	150	49	7	83	61	88	53	20	21	11	21	23	0	85	27	0	94	26	21
40	Uni of Illinois: Urbana-Champaign	83,198	157	54	100	69	62	85	70	24	28	15	30	57	0	96	58	0	100	2	29
42	City University: Cass	95,113	136	6	11	88	85	95	74	18	33	23	36	70	25	18	31	0	61	20	81
42	Washington University: Olin	96,569	156	84	30	90	37	71	39	17	24	9	45	37	0	67	70	0	90	65	20
44	Pennsylvania State: Smeal	93,408	168	39	72	41	46	67	62	22	29	11	16	34	0	62	81	0	84	43	35
44	Vanderbilt University: Owen	115,270	160	67	86	33	24	80	34	24	25	8	19	25	8	93	53	0	98	75	72
46	Purdue University: Krannert	96,968	151	43	76	49	15	75	36	11	21	14	17	38	2	69	65	0	95	17	31
46	Uni of Texas at Austin: McCombs	111,366	138	66	97	59	27	70	18	25	23	16	18	24	0	83	66	0*	90	15	23
48	Rice University: Jones	106,265	145	62	58	58	33	85	35	27	28	10	31	25	2	94	96	0	98	78	28
49	College of William and Mary	97,834	172	42	49	81	74	82	65	22	28	9	18	44	2	68	88	0	100	78	80
49	University of Iowa: Tippie	88,587	172	40	83	68	26	82	98	18	28	17	20	39	0	71	73	0	93	48	60
51	Michigan State University: Broad	87,462	157	38	98	42	3	80	52	21	28	16	5	29	4	79	93	0	98	13	34
52	Queen's School of Business	94,463	141	31	28	1	51	70	31	23	22	20	42	31	20	46	96	0	85	73	64
53	Australian Graduate School of Mgt	98,763	111	32	95	48	28	73	43	23	21	17	51	50	19	27	15	0	96	57	26
53	Ceibs	61,556	194	80	50	45	20	96	75	18	33	10	71	11	50	100	5	0	88	78	85
53	HEC Paris	94,701	121	25	9	63	57	54	40	15	18	11	22	78	74	15	29	2	75	42	84
56	Indiana University: Kelley	108,262	141	73	66	62	16	71	28	27	27	7	15	30	2	87	51	0	76	26	42
56	University of Pittsburgh: Katz	85,008	157	16	94	75	87	86	98	19	30	4	17	42	0	28	94	0*	93	37	53
56	University of Wisconsin - Madison	91,770	144	44	53	82	44	70	56	25	36	26	22	26	3	88	85	0	100	44	33
59	SMU: Cox	101,524	164	77	82	73	42	78	51	22	24	13	15	24	4	75	21	0	89	78	62
60	Arizona State University: Carey	91,308	145	60	42	67	56	92	33	15	30	20	15	23	5	73	78	0	92	26	32
60	University of California at Irvine	92,040	130	76	56	44	39	82	72	34	32	13	32	34	15	89	67	0	98	58	27
60	University of Minnesota: Carlson	88,822	146	74	69	56	41	93	53	23	24	11	25	27	2	60	59	0	91	50	30
63	Babson College: Olin	108,280	137	98	20	72	53	80	38	31	30	24	17	30	9	81	72	0	94	78	46
63	Boston University School of Mgt	97,743	133	97	4	91	79	87	67	25	33	15	27	47	15	44	45	0	80	49	43
63	Cranfield School of Management	120,661	107	9	21	11	54	62	22	22	19	42	13	62	25	33	90	0	58	16	94
63	Virginia Tech: Pamplin	75,557	172	30	65	7	63	84	96	21	20	8	2	45	0	69	96	0	92	37	68
67	Universiteit Nyenrode	88,156	111	21	15	5	82	78	78	20	34	0	27	80	63	4	14	0	80	72	95
67	University of British Columbia	78,257	121	19	14	64	81	89	56	17	29	16	68	59	8	34	20	0	90	30	51
69	Hong Kong UST Business School	62,089	84	69	79	6	52	89	77	14	48	33	88	82	94	52	1	1	100	61	24
69	Lancaster University Mgt School	73,164	113	17	45	29	80	91	82	21	53	45	24	78	36	31	24	0	82	6	82
71	Esade Business School Spain	144	83	5	46	32	74	47	16	25	22	17	70	89	25	6	2	71	71	92	71
72	Melbourne Business School	100,717	106	34	44	78	78	69	56	24	24	17	44	76	8	13	16	0	96	76	86

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2004 Rank	School name	WS	SI	VFMR	CPR	AAR	PSR	EATM	ARR	WF	WS	WB	IF	IS	IB	IE	IMR	L	FWD	FTDR	FTRR
73	Thunderbird	92,033	126	51	63	76	64	37	27	33	27	12	39	51	17	17	17	0*	94	78	76
73	Tulane University: Freeman	89,019	156	86	85	94	95	82	81	18	26	7	24	36	4	32	68	0*	89	61	54
75	Brigham Young University	85,624	181	18	84	21	45	84	47	7	14	7	2	12	1	99	82	0*	92	78	55
75	Imperial College London: Tanaka	102,313	106	15	59	25	58	44	65	19	32	36	40	56	45	39	86	0	83	37	78
75	University of Notre Dame	101,144	153	64	96	74	66	78	60	20	19	13	8	26	1	86	47	0	93	78	39
78	Ipade	67,112	223	37	2	52	70	70	76	7	20	15	10	10	19	82	19	1	30	78	96
79	Texas A&M University: Mays	82,118	137	20	93	40	38	96	62	25	19	7	9	24	0	54	43	0	88	22	47
80	University of Georgia: Terry	86,720	147	28	55	24	76	81	72	19	22	0	13	33	0	97	89	0	87	22	63
80	Wake Forest University: Babcock	91,365	159	68	24	71	68	86	67	10	30	12	5	21	4	95	83	0	95	78	58
82	Brisbane Graduate Sch of Bus	51,203	165	35	8	100	28	87	95	38	33	38	21	76	25	11	62	0	56	78	90
82	University of Durham Bus School	82,434	107	24	42	89	98	83	82	23	35	13	45	81	19	14	79	1	93	63	93
84	Case Western Reserve	86,945	123	95	71	95	47	78	67	16	28	17	37	41	4	90	38	0	97	35	38
84	University College Dublin	92,228	93	13	10	99	99	96	87	23	26	11	27	52	60	23	44	1	90	64	74
86	Bradford School of Mgt/Nimbas	79,728	92	23	35	27	83	92	89	34	31	36	30	80	27	26	32	0	68	17	73
86	Incae Costa	44,899	171	53	45	97	89	57	82	9	25	8	57	78	85	45	4	1	91	78	96
86	Trinity College Dublin	92,012	93	10	18	37	91	85	82	33	20	33	30	60	33	6	77	0	77	76	91
89	University of Tennessee	90,392	140	7	92	80	50	64	78	19	36	19	4	18	0	91	95	0	89	52	77
90	University of Arizona: Eller	78,814	133	56	78	79	60	84	98	26	17	20	15	27	3	98	96	0	95	19	52
91	University of California: Davis	93,735	97	82	19	4	70	86	87	19	32	11	38	15	0	84	46	0	100	78	48
92	IAE Management and Bus School	62,845	156	57	32	54	77	70	89	9	27	0	33	27	57	24	10	1	51	78	96
93	Edinburgh University Mgt School	76,395	93	26	62	70	94	84	89	19	30	25	31	75	50	42	50	1	82	41	69
94	Georgia Institute of Tech: DuPree	85,438	126	46	89	98	34	70	60	13	31	13	17	31	0	92	57	0	100	66	65
95	Ashridge	111,353	74	41	12	10	100	100	78	26	8	60	37	42	40	55	22	0	37	78	66
95	University of Bath School of Mgt	77,934	86	33	74	65	92	81	82	24	48	16	22	69	21	38	49	0	78	30	67
97	University of Alberta	65,135	103	45	51	92	93	93	96	14	39	24	61	47	17	65	52	0	93	73	45
98	ESCP - EAP	82,468	69	47	41	87	96	52	89	23	43	9	34	86	70	22	28	2	78	78	89
98	Theseus International Mgt Institute	86,701	101	8	67	84	97	71	89	14	25	20	86	62	80	20	87	0	14	78	96
100	ENPC MBA Paris	76,801	72	78	47	86	67	40	89	13	37	7	63	66	21	1	33	1	63	78	96

* KPMG reported on the results of obtaining evidence and applying specified audit procedures relating to selected data provided for the Financial Times 2004 MBA survey ranking for selected business schools. Inquiries in the process can be made by contact

* * These schools run additional courses for MBA students for which additional language skills are required. These figures are included in the calculations for the ranking but are not represented on the table to avoid confusion. Although the headline rank