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First-ply failure strength of laminated composite pressure vessels

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Strengths of laminated composite pressure vessels are studied via both analytical and experimental approaches. Experimental techniques are presented to determine the first-ply failure and burst strengths of laminated composite pressure vessels with different lamination arrangements. Different analytical methods, together with various failure criteria, are used to predict the first-ply failure strengths of the laminated pressure vessels. The accuracy of the theoretical prediction of first-ply failure strength is verified by the test data. The suitability of the failure criteria, as well as the limitations of the analytical methods are discussed. © 1997 Elsevier Science Ltd.

INTRODUCTION

The application of composites in pressure vessels and piping has drawn close attention in recent years [1–5]. Much work has been devoted to the manufacturing and design aspects of laminated composite pressure vessels [6–10]. In general, the design of laminated composite pressure vessels is achieved by the use of the first-ply failure approach, i.e. a suitable failure criterion is adopted to determine the first-ply failure load, and the classical lamination theory for stress analysis. The suitability of the adopted failure criterion and the classical lamination theory in determining the first-ply failure strength of laminated composite pressure vessels, however, has not been studied in detail nor validated by experimental data. For safety reasons, pressure vessels must be designed for high reliability. A meaningful reliability assessment of a laminated composite pressure vessel relies on the accurate prediction of the first-ply failure strength of the vessel. Therefore, more work must be devoted to the failure analysis of laminated composite pressure vessels if reliable as well as economical vessels are desired.

In this paper, first-ply failure of laminated composite pressure vessels is studied via both analytical and experimental approaches. Experiments are performed to determine the strengths of laminated composite pressure vessels with different lamination arrangements. The suitabilities of different failure criteria and analytical methods commonly used in determining first-ply failure strength of laminated composite pressure vessels are studied via the test data.

FIRST-PLY FAILURE ANALYSIS OF PRESSURE VESSELS

The pressure vessel is modeled as a symmetrically laminated cylindrical shell of thickness h, length L and radius R, where R refers to the radius of the middle surface. The shell is constructed of an even number of orthotropic layers of equal thickness, t. The fiber orientation θ is defined as the angle between the fiber direction and the longitudinal axis x. The stress resultants in the geometric coordinate axes are given by [11]

$$\underline{N} = \underline{A}\underline{\varepsilon} \tag{1}$$

where $\underline{N}$ is the vector of stress resultants, $\underline{A}$ is the matrix of extensional stiffnesses, $\underline{\varepsilon}$ is the vector of strains. The stress–strain relations for the kth orthotropic layer are given by

$$\underline{\sigma}^{(k)} = \underline{\bar{Q}}^{(k)}\underline{\varepsilon} \tag{2}$$

where $\underline{\sigma}^{(k)}$ is the vector of stresses for the kth ply, $\bar{\underline{Q}}$ is the matrix of the transformed material stiffness constants. According to the principle of the strength of materials, the stress resultants of the pressure vessel subjected to internal pressure p are given by

$$N_x = \frac{pR}{2}, N_\phi = pR, N_{x\phi} = 0 \tag{3}$$

where N_x, N_ϕ are stress resultants in the axial and circumferential directions, respectively; $N_{x\phi}$ is the shear stress resultant which is zero due to the symmetry of the lamination. The first-ply failure analysis of the laminated composite pressure vessel is performed via the use of a suitable failure criterion. Herein, a number of phenomenological failure criteria are adopted in the analysis. For comparison purpose, the laminated composite pressure vessel is also analyzed using the finite element method which is formulated on the basis of the first-order shear deformation theory [12].

EXPERIMENTAL INVESTIGATION

A number of laminated composite cylindrical pressure vessels made of graphite/epoxy prepreg tapes were subjected to burst strength test. The properties of the composite material determined from experiment are listed in Table 1. The lamination arrangements and the dimensional properties of the pressure vessels are tabulated in Table 2. The experimental apparatus consists of a test rig for supporting the pressure vessel, an acoustic emission (AE) system (AMS3) with two AE sensors for detecting sound waves emitted from the pressure vessel during failure process, a data acquisition system, a digital pressure meter used for measuring oil pressure in the vessel, and two strain gauges attached to the surface of the vessel. A schematic description of the experimental setup is shown in Fig. 1. During testing, oil was pumped into the pressure vessel at low speed and the pressures at which first-ply and ultimate failures of the pressure vessel occurred were recorded. The first-ply failure pressure of

Table 1. Properties of graphite/epoxy laminate

Material constant	Value	Strength parameter	Value
E_1	88.53 GPa	X_T	1560 MPa
E_2	6.72 GPa	X_c	1760 MPa
G_{12}	4.03 GPa	Y_T	35.75 MPa
G_{23}	1.022 GPa	Y_c	178 MPa
ν_{12}	0.28	S	61 72 MPa
		R	46.21 MPa
		$X_{\epsilon T}$	1.174×10^{-2}
		$Y_{\epsilon T}$	0.35×10^{-2}

Table 2. Properties of pressure vessels

Lamination arrangement	Dimensions
$[54°/-54°]_s$	Outer radius $r_0 = 4$ cm
$[45°/-45°]_s$	Length $L = 23$ cm
$[90/0°]_s$	lamina thickness $t = 0.15$ mm
$[54°/-54°/54°]_s$	
$90°/0°/90°]_s$	
$[90°/0°/90°/0°]_s$	

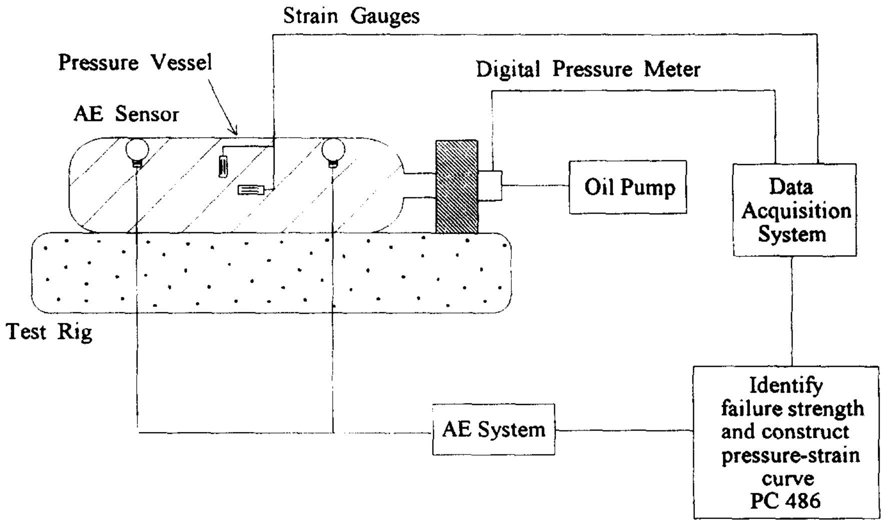

Fig. 1. A schematic description of the experimental setup.

the vessel was determined by identifying the first major energy rise in the energy–pressure diagram produced by the AE system. Figure 2, for instance, shows the energy–pressure relation of the $[54°/-54°/54°]_s$ pressure vessel generated by the AMS3 system. The ultimate burst strength of the vessel was identified from the measured pressure history. Deformation of the vessel was monitored via the strain gauges and the data acquisition system.

RESULTS AND DISCUSSION

Six laminated composite pressure vessels with different layups and various number of plies were tested to failure. The laminated composite pressure vessels subjected to burst test are also analyzed using the aforementioned analytical methods on the basis of different phenomenological failure criteria. The theoretical and experimental results are listed in Tables 3–8 for comparison. It is noted that when the number of plies in the laminated composite pressure vessels is equal to or less than 6, the first-ply failure pressures predicted by the analytical methods are in good agreements with the experimental ones. In particular, both the maximum stress and Hoffman criteria can yield very accurate theoretical first-ply failure pressures for the pressure vessels composed of 4 or 6 plies. As for the $[90°/0°/90°/0°]_s$ pressure vessel which is composed of 8 plies, there exists significant differences between the theoretical and experimental first-ply failure pressures which are greater than 20% as shown in Table 8. This implies that the present analytical methods are inadequate and more sophisticate methods such as those constructed on the basis of the higher order shear deformation theory are required for the stress analysis of moderately thick laminated composite pressure vessels. It is noted that for the laminated composite pressure vessels composed of same number of plies, the optimally designed pressure vessel possesses the highest first-ply failure strength as well as the ultimate burst strength. For instance, amongst the laminated composite pressure vessels composed of 4 plies (Tables 3–5) the $[54°/-54°]_s$ pressure vessel which has been opti-

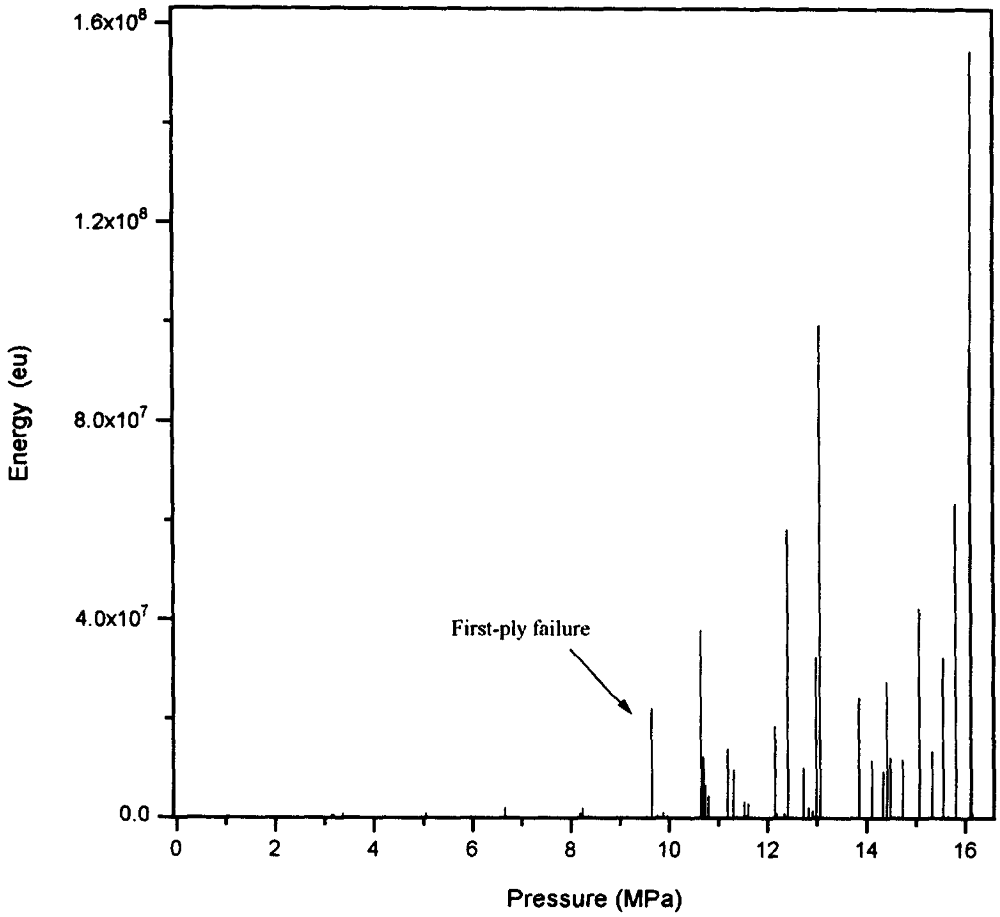

Fig. 2. Energy vs. pressure produced by tlle AE system for the $[54°/-54°/54°]_S$ pressure vessel.

Table 3. Failure strength of the [54°/ − 54°] pressure vessel

Failure criterion	Theoretical first-ply failure pressure P_t (MPa)	Experimental first-ply failure pressure P_f (MPa)	Experimental burst failure pressure P_b (MPa)	$\frac{\|P_f-P_t\|}{P_f}$ (%)	$\frac{P_f}{P_b}$ (%)
Maximum strain	9.32 (a)	7.18	14.32	29.81	50.14
	9.44 (b)			31.47	
Maximum stress	7.61			5.99	
	7.64			6.41	
Hoffman	7.62			6.13	
	7.65			6.55	
Tsai–Hill	7.75			7.94	
	7.78			8.36	
Tsai–Wu	7.97			11.00	
	8.03			11.83	

(a) first order shear deformation theory (finite element analysis), (b) classical plate theory.

mally designed yields the highest first-ply failure strength as well as the ultimate burst strength. It is also worth noting that for the same pressure vessel, the ultimate burst strength is usually much higher than the first-ply failure strength. Therefore, the pressure vessel will be safe enough if it is designed against the first-ply failure pressure.

Table 4. Failure strength of the [45°/ − 45°] pressure vessel

Failure criterion	Theoretical first-ply failure pressure P_t (MPa)	Experimental first-ply failure pressure P_f (MPa)	Experimental burst failure pressure P_b (MPa)	$\frac{\|P_f-P_t\|}{P_f}$ (%)	$\frac{P_f}{P_b}$ (%)
Maximum strain	5.21 (a)	3.47	10.36	50.14	33.49
	5.34 (b)			53.89	
Maximum stress	3.40			2.02	
	3.43			1.12	
Hoffman	3.42			1.44	
	3.45			0.58	
Tsai–Hill	4.35			25.36	
	4.39			26.51	
Tsai–Wu	4.03			16.14	
	4.06			17.00	

(a) first order shear deformation theory (finite element analysis), (b) classical plate theory.

Table 5. Failure strength of the $[90°/0°]_s$ pressure vessel

Failure criterion	Theoretical first-ply failure pressure P_t (MPa)	Experimental first-ply failure pressure P_f (MPa)	Experimental burst failure pressure P_b (MPa)	$\frac{\|P_f-P_t\|}{P_f}$ (%)	$\frac{P_f}{P_b}$ (%)
Maximum strain	7.07 (a)	5.68	11.57	24.47	49.09
	7.11 (b)			25.17	
Maximum stress	6.04			6.33	
	6.06			6.69	
Hoffman	6.05			6.51	
	6.07			6.87	
Tsai–Hill	6.21			9.33	
	6.24			9.86	
Tsai–Wu	6.42			13.03	
	6.46			13.73	

(a) first order shear deformation theory (finite element analysis), (b) classical plate theory.

Table 6. Failure strength of the [54°/−54°/54°] pressure vessel

Failure criterion	Theoretical first-ply failure pressure P_t (MPa)	Experimental first-ply failure pressure P_f (MPa)	Experimental burst failure pressure P_b (MPa)	$\frac{\|P_f-P_t\|}{P_f}$ (%)	$\frac{P_f}{P_b}$ (%)
Maximum strain	12.83 (a)	9.66	16.07	32.82	60.11
	12.91 (b)			33.64	
Maximum stress	10.52			8.90	
	10.56			9.32	
Hoffman	10.53			9.01	
	10.57			9.42	
Tsai–Hill	10.96			13.45	
	10.99			13.77	
Tsai–Wu	11.15			15.42	
	11.20			15.94	

(a) first order shear deformation theory (finite element analysis), (b) classical plate theory.

CONCLUSION

First-ply failure of laminated composite pressure vessels was studied via both theoretical and experimental approaches. Different methods were used to predict the first-ply failure pressures of the laminated composite pressure vessels on the basis of various failure criteria.

Table 7. Failure strength of the [90°/0°/90°] pressure vessel

Failure criterion	Theoretical first-ply failure pressure P_t (MPa)	Experimental first-ply failure pressure P_f (MPa)	Experimental burst failure pressure P_b (MPa)	$\frac{\|P_f-P_t\|}{P_f}$ (%)	$\frac{P_f}{P_b}$ (%)
Maximum strain	11.48 (a)	8.84	10.07	29.86	65.92
	11.52 (b)			30.32	
Maximum stress	9.71			9.84	
	9.76			10.41	
Hoffman	9.73			13.41	
	9.77			10.52	
Tsai–Hill	9.99			13.01	
	10.08			14.03	
Tsai–Wu	10.31			16.63	
	10.40			17.64	

(a) first order shear deformation theory (finite element analysis), (b) classical plate theory.

Table 8. Failure strength of the $[90°/0°/90°/0°]_s$ pressure vessel

Failure criterion	Theoretical first-ply failure pressure P_t (MPa)	Experimental first-ply failure pressure P_f (MPa)	Experimental burst failure pressure P_b (MPa)	$\frac{\|P_f-P_t\|}{P_f}$ (%)	$\frac{P_f}{P_b}$ (%)
Maximum strain	13.65 (a)	10.41	20.75	31.12	47.40
	13.73 (b)			31.89	
Maximum stress	12.55			20.56	
	12.64			21.42	
Hoffman	12.57			21.96	
	12.67			21.71	
Tsai–Hill	12.71			22.09	
	12.86			23.54	
Tsai-Wu	12.90			23.92	
	12.99			24.78	

(a) first order shear deformation theory (finite element analysis), (b) classical plate theory.

Experiments were carried out to verify the accuracy of the analytical methods. The analytical methods together with the maximum stress criterion or Hoffman failure criterion can predict accurate first-ply failure pressures for thin laminated composite pressure vessels. Both of the adopted analytical methods are inadequate for failure analysis of moderately thick laminated composite pressure vessels. Results on ultimate burst pressure for the pressure vessels were presented for comparison. Ultimate burst pressure is generally much higher than first-ply failure pressure of a laminated composite pressure vessel. It may be appropriate to use first-ply failure as a criterion for the design of laminated composite pressure vessels.

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