國立交通大學

資訊管理研究所

博 士 論 文

創新動態中的最佳化能力集合轉化

Optimal Transformation of Competence Set in Innovation Dynamics

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指導教授：游伯龍　教授

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中文摘要

我們的記憶、觀念、想法、判斷、反應（統稱為念頭和思路）雖然是動態的，但經過一段時間以後，如果沒有重大事件的刺激，會漸漸地穩定下來，而停在一個固定的範圍內。這些念頭和思路的綜合範圍，包括它們的動態和組織，就是我們的習慣領域（habitual domain, HD）。當我們面對一個決策問題時，相對應的存在一個能力集合（competence set, CS），包括能使我們得到滿意解答所須的想法、知識、技能、資源等等。因為能力集合是我們習慣領域針對某一決策問題的投影，如果習慣領域僵化了，我們的能力集合便無法擴展，進而會阻礙創新的展現。若沒有持續不斷地擴展、升級我們的習慣領域與能力集合，我們就很有可能走入決策陷阱或做出錯誤的決策而不自知。本論文基於習慣領域理論及能力集合分析，提出運作領域（working domain）的概念以及創新循環模式（innovation dynamics）。此模式可以使我們的運作領域能更加的靈活、有彈性。接著我們將著眼於創新循環模式中能力集合轉化的部份，討論最佳化能力集合調整問題。給定一個目標解，我們提出能力集合調整模式（competence set adjustment model, CSA model）求得生產參數的最佳調整，使得該目標解得以被達成。當目標解無法被達成時，我們透過二分法（bisection algorithm）或模糊線性規劃（fuzzy linear programming）的方法修正此目標解。最後，我們利用多準則多資源水準限制線性規劃模式（multiple criteria and multiple constraint levels linear programming, MC^2LP）探討可變生產參數的線性規劃模式。這些參數包括單位利潤、可用資源以及投入產出參數。在這些參數會隨著投資或時間而改變的情況下，我們將探討如何找到動態最佳解使得「接單時虧損，交貨時獲利」（紅色接單、黑色出貨）的目標得以實現。

關鍵詞：習慣領域、能力集合、創新動態、能力集合調整、多目標決策、多準則多資源水準限制數學規劃、紅色接單黑色出貨

Optimal Transformation of Competence Set in Innovation Dynamics

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Abstract

Habitual domain (HD) is a collection of ways of thinking, coupled with its formation, interaction and dynamics, which can gradually stabilize as time passes. Unless extraordinary events occur, our thinking process will reach a steady state. For a decision problem *E*, there exists a corresponding competence set (CS) consisting of ideas, knowledge, skills, and resources to successfully solve the problem. Being a projection of our habitual domains with respect to *E*, our competence set may not be expanded as our habitual domains get ossified. This can conceal our innovation. Therefore, without continuous expanding and upgrading our HD and CS, we may unconsciously step into decision traps and make wrong decisions. Based on HD theory and CS analysis, the dissertation presented here introduces the concepts of working domain and innovation dynamics. The model of innovation dynamics helps our working domain become more flexible and agile. We, in advance, focus on the transformation of competence set in the innovation dynamics to investigate the problem of optimal adjustment of competence set. The program is formulated into a linear programming model called competence set adjustment model (CSA model). By means of the CSA model, we study how to optimally adjust the relevant coefficients so that a given target solution could be attainable. In case the target is unattainable, we may either utilize the bisection method or the fuzzy linear programming techniques to revise the target as to make it a reachable one. Finally, we utilize multiple criteria and multiple constraint levels linear programming (MC^{2}LP) model and its extended techniques to explore the linear programming models with changeable parameters. The parameters include: unit profit, available resources and input-output coefficients of production function. With those parameters changed with capital investment and/or time, we study how to find dynamic best solutions to make "taking loss at the ordering time and making profit at the time of delivery" feasible. For more general cases we also sketch a generalized mathematical programming model with changeable parameters and control variables.

Keywords: Habitual Domains, Competence Set Analysis, Innovation Dynamics, Competence Set Adjustment, Multiple Criteria Decision Making, Multiple Criteria and Multiple Constraint Levels Linear Programming, Red in-Black out

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賴宗智

國立交通大學資訊管理研究所

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CHAPTER 1 INTRODUCTION

1.1 Research Background

In the last few decades, *habitual domains* (HDs) theory has attracted enormous managers and researchers to study it. HDs theory can be of help in developing leadership skills [1], generating innovative ideas [2], preventing from decision traps [3], and forming winning strategies [4]. Having also noticed the benefit of the HDs theory to fundamental education, the Ministry of Education in Taiwan funded the seed teacher training program of HDs in 2007 and 2008. More than 120 teachers, mostly from center of general education, have plunged into researching and/or teaching the HDs theory. Undoubtedly, the importance of HDs theory has increased day by day.

The concept of HDs was initiated by Yu [5]. Its main idea is that the set of ideas and knowledge which are stored in our mind can gradually stabilize over a period of time. Unless extraordinary events occur, our thinking process will reach some steady state [6]. This phenomenon continuously takes place in our daily life. For example, someone learns how to drive a car. At the beginning, he/she may feel that controlling a car is quite difficult. However, after times of practice, he/she may feel more comfortable to drive a car (a steady state).

Our habitual domains can be stabilized. This can be mathematically proved [7] based on commonly observed facts:

1. The more we learn, the less the likelihood that an arriving event or piece of information is new to us.

2. To interpret arriving events, we tend to relate them to past experiences.

3. We tend to look for rhythms in our lives and force arriving events to conform to those rhythms.

Our HDs go wherever we go and have great impact on our decision making both individual level and organizational level.

From the individual point of view, given a decision problem or event *E* we need a set of ideas, knowledge, skills, and resources that could be of help to obtain a satisfactory solution. The set is called a *competence set* (CS), denoted by *CS*(*E*). The concept of CS is also proposed by Yu [8], [9]. As an extension of HDs, a competence set is a projection of our habitual domains with respect to *E*. Once we have acquired the competence set, *CS*(*E*), it could be transformed into services or products to relieve pains and frustrations.

Each organization, in abstract, is a living entity and hence has its HD and CS. If the organization wants to be more competitive than its competitors, its CS must be adequately flexible and adaptable. That is, to be powerful, the organization should abidingly create new product or service which fulfills target customers' requirements by the fusion of innovative ideas and the required resources. The process must be flexible and adaptable so that the CS of the organization can be properly adjusted according to both internal and external environment changes.

Competence set analysis (CSA) contains two inherent domains: *competence domain* and *problem domain*. As shown in Figure 1-1, there are two kinds of short-term problems in CSA:

1. Given a problem or set of problems, what is the needed competence set, and how to acquire or obtain it?

2. Given a set of competence, what kind of problems can be solved as to maximize the value of the competence?

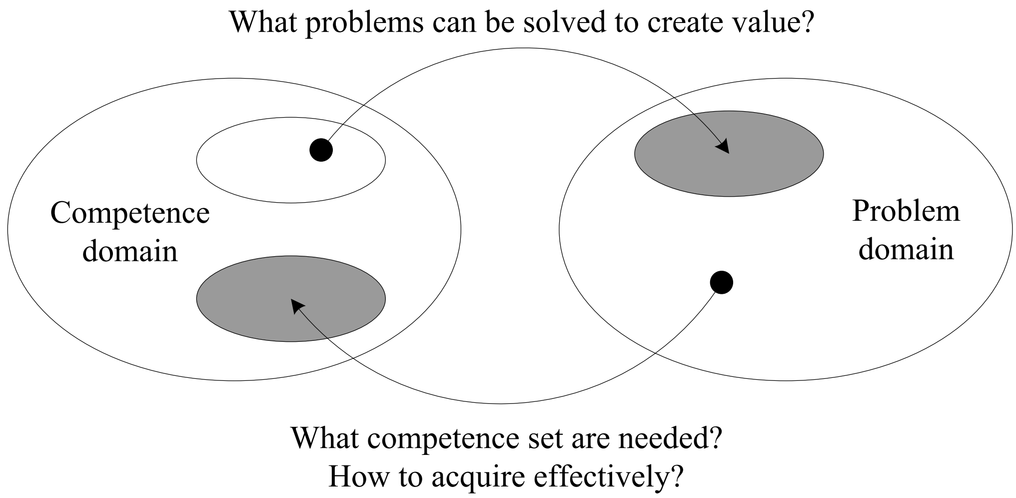

Figure 1-1 Two domains of competence set analysis [10].

The former problem is called the *problem-oriented competence set analysis*, and the latter is called the *skill-oriented competence set analysis*. In the long term, we want to expand our competence set over time as to maximize the value of our individual live, or maximize the value of the organization over its time of existence. Further discussion on competence set analysis can be found in [11], [12].

1.2 Motivation and Objective

With the advances of information technologies (ITs) including computer, network, etc. knowledge management (KM) has been rapidly innovated in recent years. KM is useless if it cannot help some people release their frustrations and pains, or if it cannot help them make better decisions. The challenging for KM nowadays is how it can be developed to maximize its value to help solve complex problems.

KM has helped many people in making decision and transactions. Nevertheless, being a projection of our HDs with respect to a given decision problem, our competence sets can not be expanded if our HDs get trapped. As a consequence, KM may lead us to decision traps and make wrong decisions if we do not continuously expand and upgrade our HD and CS.

In order to prevent us from stepping into decision traps, we introduce the concepts of HD and CS analysis in such a way that we could see where KM can commit decision traps and how to avoid them. Innovation dynamics, as an overall picture of continued enterprise innovation, is also introduced so that we could know the areas and directions in which KM can make maximum contributions and create value. As a result, KM empowered by HD can make KM even more powerful.

Providing services or products to relieve customers' pains and frustrations is fundamental to the value creation of the companies. Once a service or product has been chosen to be produced, companies have to efficiently transform its existent competence set, including skills, resources, and facilities, etc, so that the service or product could be realized. Some targets for producing the service or product may be set in the process of competence transformation. To reach the targets, management by objectives (MBO) is an effective way for enterprise management [15].

By setting the targets of the productivity the companies try their best, including adjustment of resource allocation and competence, to reach the targets. Within the same framework of productivity and of resources the targets may not be attainable. However, by stretching a little bit, human capacity, resources, the production coefficients, and other relevant parameters may be adjusted so as to make the target feasible. This dissertation proposed a linear programming model and studied how to optimally adjust the relevant coefficients so that the target solution can be attainable.

The companies' competence set not only can be actively adjusted through capital investment, but also be dynamically changed overtime, which can explain the phenomenon of "taking loss at the ordering time and making profit at the time of delivery". Such phenomenon has existed in practice for a long time, but there are no mathematical model that can explain it adequately. Therefore, we utilize multiple criteria and multiple constraint levels linear programming (MC^2LP) model and its extended techniques to explore the linear programming models with changeable parameters.

1.3 Overview of Dissertation

Chapter 2 contains the literature reviews which include the concept of habitual domains and that of competence set analysis.

In chapter 3, we introduce the concept of working domain (WD). We could see in this chapter how WD can commit decision traps and how to avoid them. Innovation dynamics, as an overall picture of continued enterprise innovation, is also introduced so that we could know the areas and directions in which KM can make maximum contributions and create value.

In chapter 4, we shall focus on the optimal adjustment of the competence set for reaching a target. It emphasizes on optimal adjustment of competence set to accomplish a target or fixed value. We formulate the program into linear programming model and study how to optimally adjust the relevant coefficients so that the target solution can be attainable. In case the target is unattainable, we may either utilize the bisection method or the fuzzy linear programming techniques to revise the target as to make it a reachable one.

In chapter 5, we shall explore the potential value of a given adjustable competence set over a time horizon. It emphasizes more on identifying potential value of the competence set than on the adjustment of the competence set to reach a target. We utilize multiple criteria and multiple constraint levels linear programming (MC^2LP) model and its extended techniques to explore the linear programming models with changeable parameters. The parameters include: unit profit, available resources and input-output coefficients of production function. With those parameters changed with capital investment and/or time, we study how to find dynamic best solutions to make "taking loss at the ordering time and making profit at the time of delivery" feasible. For more general cases we also sketch a generalized mathematical programming model with changeable parameters and control variables

Chapter 6 concludes this work and proposes the future work. Finally, the references and appendices are attached at the end of the dissertation.

Figure 1-2 depicts the organization of the dissertation as follows.

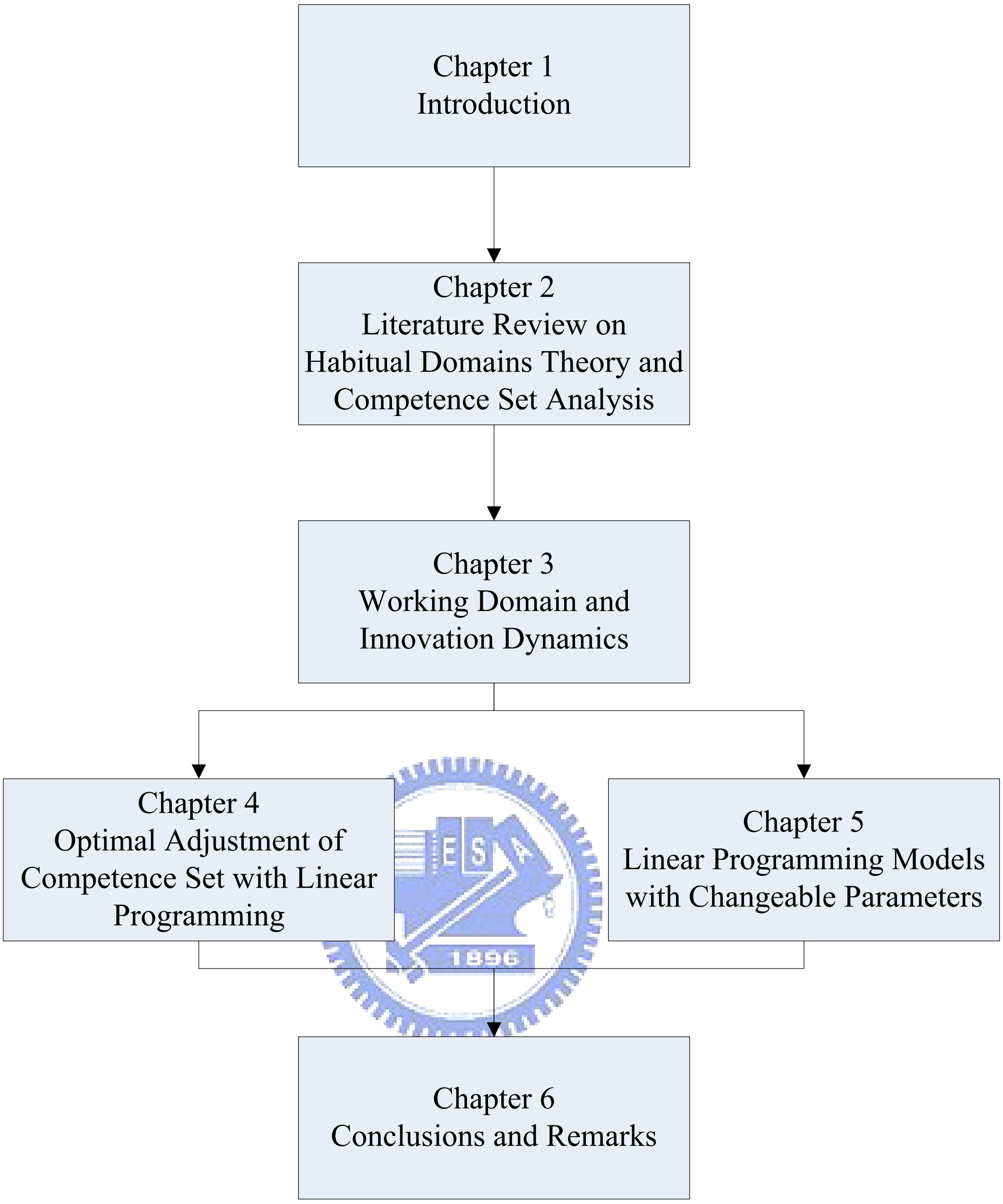

Figure 1-2 The organization of the dissertation.

CHAPTER 2 LITERATURE REVIEW ON HABITUAL DOMAINS THEORY AND COMPETENCE SET ANALYSIS

In order to facilitate the latter discussion, we shall in this chapter introduce the habitual domains theory and the competence set analysis.

2.1 Habitual Domains Theory

Each person has a unique set of behavioral patterns resulting from his/her ways of thinking, judging, responding, and handling problems, which gradually stabilized within a certain boundary over a period of time (see Yu [11], [12], [13]). This collection of ways of thinking, judging, etc., accompanied with its formation, interaction, and dynamics, is called habitual domain (HD). Let us take a look at an example.

Example 2.1. Chairman Ingenuity. A retiring corporate chairman invited to his ranch two finalists, say A and B, from whom he would select his replacement using a horse race. A and B, equally skillful in horseback riding, were given a black and white horse respectively. The chairman laid out the course for the horse race and said, “Starting at the same time now, whoever’s horse is slower in completing the course will be selected as the next Chairman!” After a puzzling period, A jumped on B’s horse and rode as fast as he could to the finish line while leaving his horse behind. When B realized what was going on, it was too late! Naturally, A was the new Chairman.

Most people consider that the faster horse will be the winner in the horse race (a habitual domain). When a problem is not in our HD, we are bewildered. The above example makes it clear that one's habitual domain can be helpful in solving problems but it also can come his or her way of thinking. Moreover, one may be distorting information in a different way.

2.1.1 Behavior Mechanism

Habitual domain is very closely related to our life goals, behavior, and decision making. To understand habitual domains theory in advance, it is necessary to know how our wonderful brain and mind work. Yu [11], [12], [13] has tried to capture the behavior mechanism through *eight basic hypotheses* based on the findings and observations of psychology and neuron science. The eight hypotheses are listed in the Appendix 1.

The basic concept of the behavior mechanism is originated from the abilities of internal information process and problem solving. With the help of these abilities, we can perform variety of activities and handle diversity of events. The brain is the internal information process center. Our memory and thought processes are summarized according to four basic hypotheses: circuit pattern hypothesis, unlimited capacity hypothesis, efficient restructuring hypothesis, and analogy/association hypothesis. Understanding each hypothesis thoroughly is essential to understanding human behavior. The remaining four hypotheses are related to how our mind works: goal setting and state evaluation hypothesis, charge structures and attention allocation hypothesis, discharge hypothesis, and information input hypothesis.

Let us briefly state five aspects of the dynamics of behavior mechanisms as follows. These five aspects continuously interact with each other, resulting in infinite wonderful human behavior patterns.

1. Experience, learning and memory are the bases for interpreting and judging arriving

events;

2. The dynamics of unfavorable discrepancies, between the ideal goal states (or equilibriums) and the perceived states, create the dynamic change of charge structure, which commands attention allocation and prompts actions, passively or actively; (the charge, a kind of mental force, is a precursor to drive or stress.)

3. Dynamic attention allocation, at any given point in time, to the events perceived as most significant (measured in terms of charges) is a fundamental element in human information processing;

4. The least resistance principle, which is the way that human beings release their charges, includes active problem solving or avoidance justification;

5. External information is necessary for human beings to achieve and maintain their ideal goals; unless attention is paid, the external information is not processed.

The above dynamics of human behavior can be depicted in Figure 2-1. The human brain

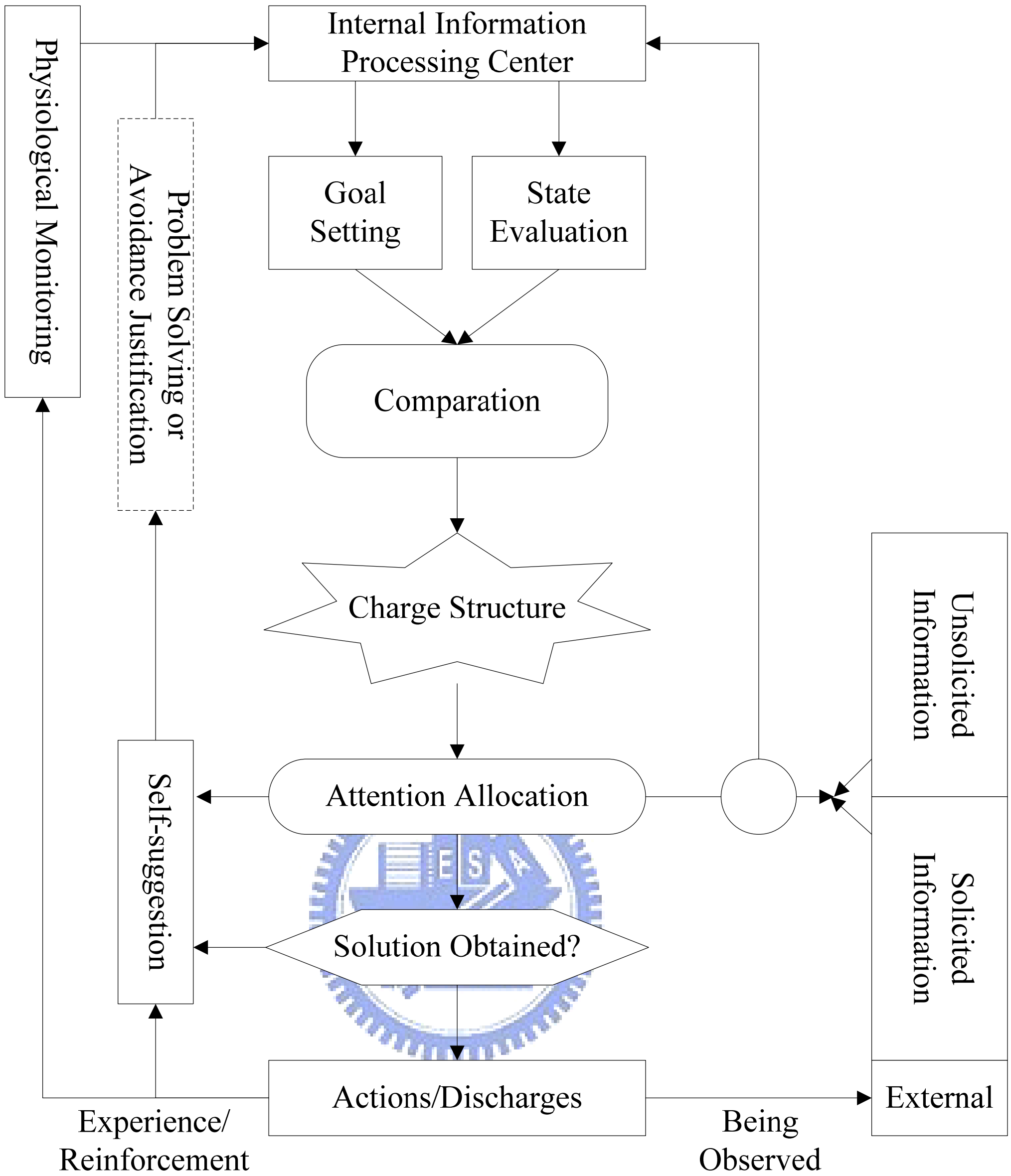

Figure 2-1 The behavior mechanism [11].

is the internal information-processing center, which receives all kinds of information from various sources. Each one of us has a set of living goals and for each living goal we have an ideal state or equilibrium point to reach and maintain (goal setting). Yu [12] classified living goals as a structure of goal functions comprised of seven goals which has shown in Table 2-1.

Table 2-1 A structure of goal functions [12].

(i) **Survival and Security:** physiological health (correct blood pressure, body temperature and balance of biochemical state); right level and quality of air, water, food, heat, clothes, shelter and mobility; safety; acquisition of money and other economic goods;

(ii) **Perpetuation of the Species:** sexual activities; giving birth to the next generation; family love; health and welfare;

(iii) **Feelings of Self-Importance:** self-respect and self-esteem; esteem and respect from others; power and dominance; recognition and prestige; achievement; creativity; superiority; accumulation of money and wealth; giving and accepting sympathy and protectiveness;

(iv) **Sensuous Gratification:** sexual; visual; auditory; smell; taste; tactile;

(v) **Cognitive Consistency and Curiosity:** consistency in thinking and opinions; exploring and acquiring knowledge, truth, beauty and religion;

(vi) **Self-Actualization:** ability to accept and depend on the self, to cease from identifying with others, to rely on one's own standard, to aspire to the ego-ideal and to detach oneself from social demands and customs when desirable.

We continuously monitor, consciously or subconsciously, where we are relative to the ideal state or equilibrium point (state evaluation). Goal setting and state evaluation are dynamic, interactive and are subject to physiological forces, self-suggestion, external information forces, current memory and information processing capacity. When there is an unfavorable discrepancy of the perceived value from the ideal, each living goal will produce various levels of charge. The totality of the charges by all living goals is called the *charge structure* and it can change dynamically. At any point in time, our attention will be paid to the event which has the most influence on our charge structure. To release charges, we tend to select the action (which belongs to either active problem solving or avoidance justification) which yields the lowest remaining charge (the remaining charge is the resistance to the total discharge) and this is called the *least resistance principle*. When we try to relieve pains or frustrations, external information inputs may not be processed unless attention is paid.

2.1.2 The Four Basic Elements of Habitual Domain

Our habitual domains go wherever we go and have great impact on our decision making. As our HD, over a period of time, will gradually become stabilized, unless there is an occurrence of extraordinary events or we purposely try to expand it, our thinking and behavior will reach some kind of steady state and predictable. Our habitual domains are comprised of the following four elements.

1. *Potential domain* (*PD*). This is the collection of all thoughts, concepts, ideas, and actions that can be potentially activated by one person or by one organization. The potential domain at time t is denoted by PD_t.

2. *Actual domain* (*AD*). This is the collection of all thoughts, concepts, ideas, and actions, which actually catch our attention and mind. The actual domain at time t is denoted by AD_t.

3. *Activation Probability* (*AP*). This represents the probability that the ideas, concepts and actions in the potential domain that can be actually activated. The activation probability at time t is denoted by AP_t. Furthermore, we denote the activation probability of an idea i at time t by $AP_t(i)$. That is, the activation probability of the ith element in the potential domain. Note that the activation probability of an idea will be strengthened by repeatedly activating the idea.

4. *Reachable domain* (*RD*). This is the collection of thoughts, concepts, ideas, actions and operators that can be generated from initial actual domain. The reachable domain that generated from an idea set I_t and the operator set O_t at time t is denoted by $RD_t(I_t, O_t)$.

At any point in time habitual domains, denoted by HD_t, will mean the collection of the above four subsets. That is, $HD_t = (PD_t, AD_t, AP_t, RD_t(I_t, O_t))$. In general, the actual domain is only a small portion of the reachable domain; in turn, the reachable domain is only a small portion of potential domain, and only a small portion of the actual domain is observable. That is, $AD_t \subset RD_t(I_t, O_t) \subset PD_t$. Note that HD_t changes with time. We will take an example to illustrate PD_t, AD_t, and RD_t.

Example 2.2. Assume we are taking an iceberg scenic trip. At the moment of seeing an iceberg, we can merely see the small part of the iceberg which is above sea level and faces us. We cannot see the part of iceberg under sea level, nor see the seal behind the back of iceberg (see Figure 2-2). Let us assume t is the point of time when we see the iceberg, the portion which we actually see may be considered as the actual domain (AD_t), in turn, the reachable domain (RD_t) could be the part of iceberg above sea level including the seal. The potential domain (PD_t) could be the whole of the iceberg including those under the sea level.

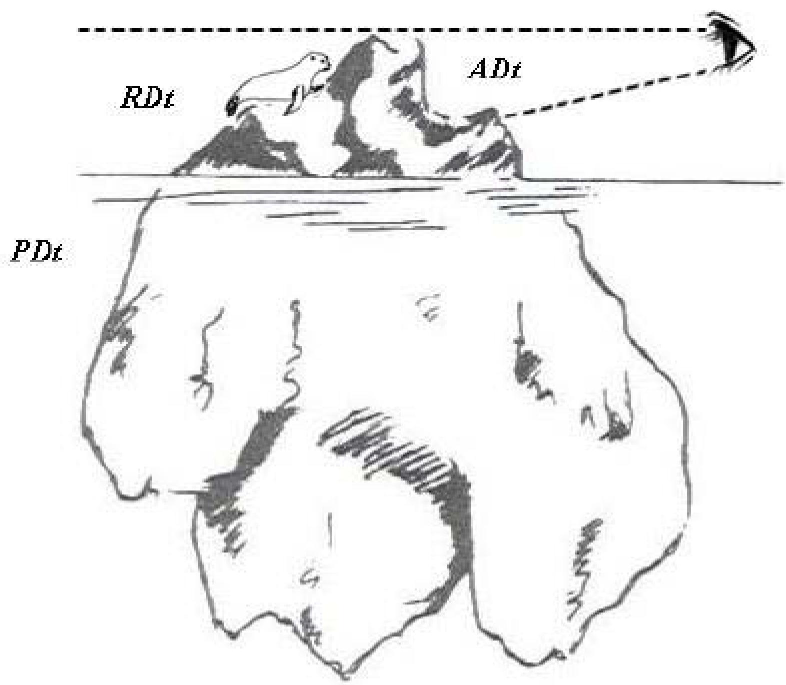

Figure 2-2 A vivid illustration of PD_t, RD_t, and AD_t.

At time *t*, if we do not pay attention to the backside of the iceberg, we will never find the seal. In addition to this, never can we see the spectacular iceberg if we do not dive into the sea. Some people might argue it is nothing special to see a seal on the iceberg. But, what if it is a box of jewelry rather than a live seal! This example illustrates that the actual domain can easily get trapped in a small domain resulting from concentrating our attention on solving certain problems. In doing so, we might overlook the tremendous power of the reachable domain and potential domain.

2.1.3 Degree of Habitual Domain Expansion

In studying the expansion of habitual domains, we shall focus only on how we expand the actual domains (AD_s) from its initial sets at an initial point of time, say *s* (starting time), to another time, *t*. Let AD_{st} be the actual domain accumulated from *s* to *t*.

There are three kinds of expansions of the actual domains as follows [12]:

1. Zero degree expansion

Starting from the original set AD_s, one can expand the actual domains to a subset of the reachable domains. Mathematically speaking, AD_{st} has a zero degree expansion if $AD_{st} \setminus AD_s \neq \emptyset$ and $RD_s \supset AD_{st}$. Note, RD_s is a function of AD_s. There are no extraordinary events within the time interval $[s, t]$ to trigger a new conception that is outside of the reachable domain RD_s.

2. First degree expansion

By expansion of first degree, we mean that the actual domain AD_{st} is not contained by the reachable domain RD_t, but is still contained in the potential domain PD_s. That is, $AD_{st} \setminus RD_t \neq \emptyset$ and $PD_s \supset AD_{st}$.

3. Second degree expansion

By second degree expansion we mean that through external information inputs or self-suggestion we acquire new concepts or operators which are not contained by our previous potential domains. Therefore, the actual domain AD_{st} is not contained by PD_s. That is, $AD_{st} \setminus PD_s \neq \emptyset$.

There are many methods for helping us to improve or expand our habitual domains and avoid decision traps. We list some of them in the following Table 2-2 and Table 2-3. In the Appendix 2 and 3, we briefly provide eight basic methods and nine principles for deep knowledge which are some mental operators (thinking procedure and attitudes). The interested reader is referred to [11], [12], and [13] for more detail.

Table 2-2 Eight basic methods for expanding habitual domains.

1. Learning Actively.
2. Take the Higher Position.
3. Active Association.
4. Changing the Relative Parameters.
5. Changing the Environment.
6. Brainstorming.
7. Retrieve in Order to Advance
8. Praying or Meditation.

Table 2-3 Nine principles for deep knowledge.

1. Deep and Down Principle.
2. Alternating Principle.
3. Contrasting and Complementing Principle.
4. Revolving and Cycling Principle.
5. Inner Connection Principle.
6. Changing and Transforming Principle.
7. Contradiction Principle.
8. Cracking and Ripping Principle.
9. Void Principle.

2.2 Competence Set Analysis

For each decision problem or event E, there is a competence set consisting of ideas, knowledge, skills, and resources for its effective solution [8], [9]. When the decision maker

(DM) thinks he/she has already acquired and mastered the competence set as perceived, he/she would feel comfortable making the decision. Note that conceptually, competence set of a problem may be regarded as a projection of a habitual domain on the problem. Thus, it also has potential domain, actual domain, reachable domain, and activation probability as described in subsection 2.1.2. Also note that through training, education, and experience, competence set can be expanded and enriched (i.e. its number of elements can be increased and their corresponding activation probability can become larger).

2.2.1 α-core Competence Set

Given an event or a decision problem E which catches our attention at time t, the probability or propensity for an idea I or element in our habitual domains that can be activated is denoted by $P_t(I, E)$. Like a conditional probability, we know that $0 \leq P_t(I, E) \leq 1$, that $P_t(I, E) = 0$ if I is unrelated to E or I is not an element of P_t (potential domain) at time t; and that $P_t(I, E) = 1$ if I is automatically activated in the thinking process whenever E is presented. Empirically, like probability functions, $P_t(I, E)$ may be estimated by determining its relative frequency. For instance, if I is activated 7 out of 10 times whenever E is presented, then $P_t(I, E)$ may be estimated at 0.7. Probability theory and statistics can then be used to estimate $P_t(I, E)$. The α-core of competence set at time t, denoted by $C_t(\alpha, E)$, is defined to be the collection of skills or elements of our habitual domains that can be activated with a propensity larger than or equal to α. That is, $C_t(\alpha, E)=\{I | P_t(I, E) \geq \alpha\}$ as depicted in Figure 2-3 for illustration.

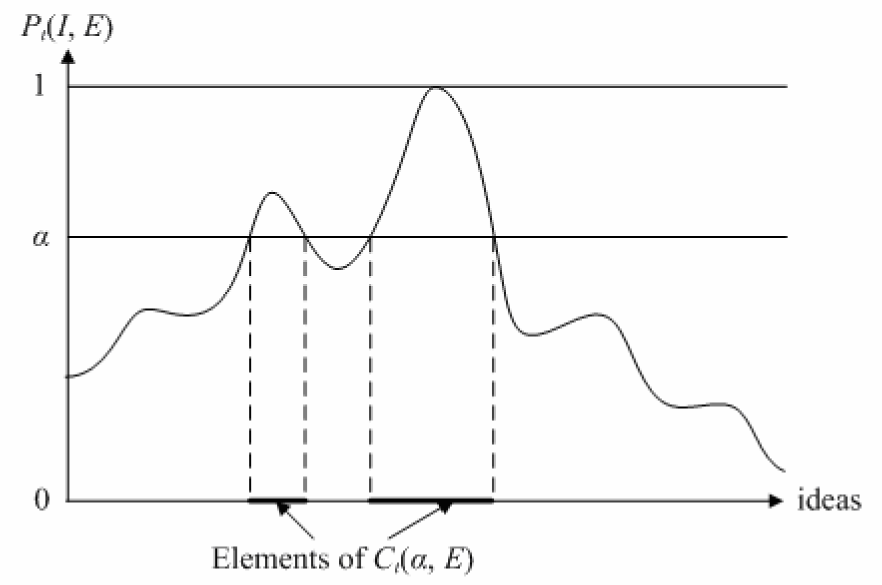

Figure 2-3 Illustration of α-core competence set [12].

2.2.2 Four Elements of Competence Set

For a given problem E there are four basic elements of competence set and they are interrelated as depicted in Figure 2-4.

1. The *true competence set* ($Tr(E)$): consists of ideas, knowledge, skills, attitudes, information and resources that are truly needed for solving problem E successfully;

2. The *perceived competence set* ($Tr^*(E)$): The true competence set as perceived by the decision maker (DM);

3. The DM's *acquired skill set* ($Sk(E)$): consists of ideas, knowledge, skills, attitudes, information and resources that have actually been acquired by the DM;

4. The *perceived acquired skill set* ($Sk^*(E)$): The acquired skill set as perceived by the DM.

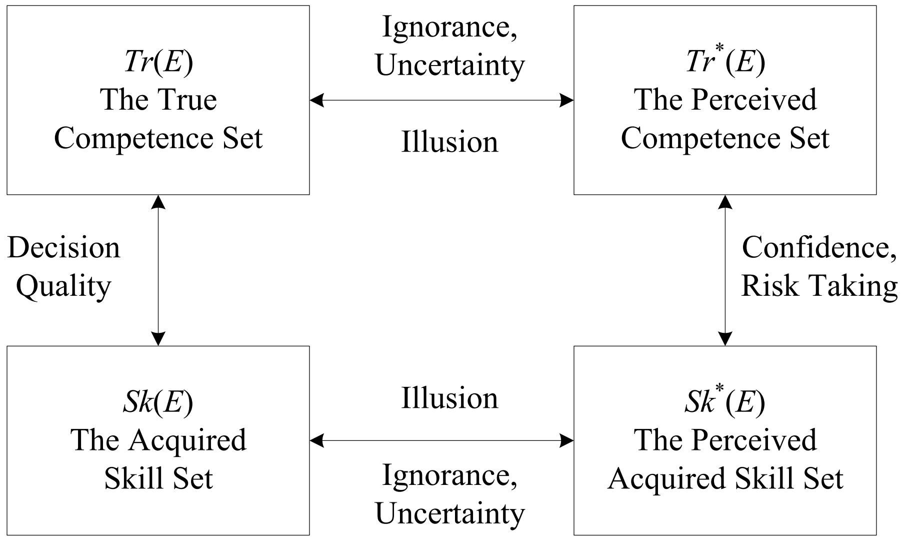

Figure 2-4 Four basic elements of competence set and their relationships [12].

Note that the above four elements are some special subsets of the habitual domain (HD) of a decision problem *E*, see [11] for details. For simplicity and without confusion, we shall drop *E* for the following discussion. The four elements are closely related. For instance:

1. The gaps between the true competence set (*Tr* or *Sk*) and perceived competence set (Tr^* or Sk^*) are due to ignorance, uncertainty, illusion and wishful thinking;

2. If Tr^* is much larger than Sk^* (i.e. $Tr^* \supset\supset Sk^*$), the DM would feel uncomfortable and lack confidence to make decisions; conversely, if Sk^* is much larger than Tr^* (i.e. $Sk^* \supset\supset Tr^*$), the DM would be fully confident in making decisions;

3. If *Sk* is much larger than Sk^* (i.e. $Sk \supset\supset Sk^*$), the DM underestimates his own competence; conversely, if Sk^* is much larger than Sk (i.e. $Sk^* \supset\supset Sk$), the DM overestimates his own competence;

4. If *Tr* is much larger than Tr^* (i.e. $Tr \supset\supset Tr^*$), the DM underestimates the difficulty of the problem; conversely, if Tr^* is much larger than *Tr* (i.e. $Tr^* \supset\supset Tr$), the DM overestimates

the difficulty of the problem;

5. If *Tr* is much larger than *Sk* (i.e. *Tr*⊃⊃*Sk*), and decision is based on *Sk*, then the decision can be expected to be of low quality; conversely, if *Sk* is much larger than *Tr* (i.e. *Sk*⊃⊃*Tr*), then the decision can be expected to be of high quality.

2.2.3 Classification of Decision Problems

Let the truly need competence set at time t, the acquired skill set at time t, and the α-core of an acquired skill set at time t be denoted by $Tr_t(E)$, $Sk_t(E)$, and $C_t(\alpha, E)$, respectively. Depending on $Tr_t(E)$, $Sk_t(E)$, and $C_t(\alpha, E)$, we may classify decision problems into following categories:

1. If $Tr_t(E)$ is well-known and $Tr_t(E) \subseteq C_t(\alpha, E)$ with high value of α or $\alpha \rightarrow 1$, as depicted in Figure 2-5, then the problem is a *routine problem*, for which satisfactory solutions are readily known and routinely used.

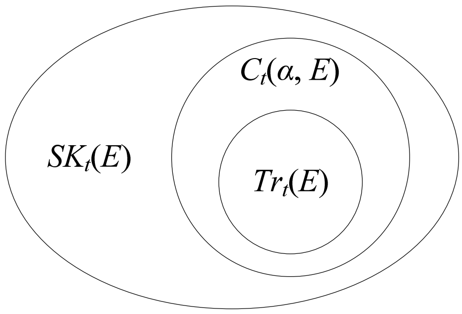

Figure 2-5 Routine problems.

2. *Mixed-routine problem* consists of a number of routine sub-problems, we may decompose it into a number of routine problems to be solved.

3. If $Tr_t(E)$ is only fuzzily known, the ideas, concepts and skills are elements of the

potential domain PD_t, even if they may not contained in $C_t(\alpha, E)$ with a high value of α, as depicted in Figure 2-6, then the problem is a *fuzzy problem*, for which solutions are fuzzily known. Note that once the $Tr_t(E)$ is gradually clarified and contained in α-core with a high value of α, the fuzzy problem may gradually become routine problem.

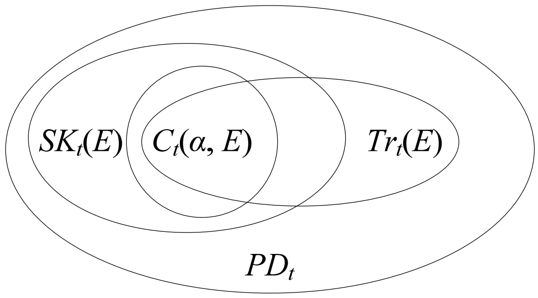

Figure 2-6 $Tr_t(E)$ is only fuzzily known.

4. If $Tr_t(E) \setminus C_t(\alpha, E)$ is very large relative $C_t(\alpha, E)$ no matter how small is α or $Tr_t(E)$ is unknown and difficult to know, which implies that $Tr_t(E)$ contains some elements outside of the existing potential domain, as depicted in Figure 2-7. Then the problem is a *challenging problem*. Because of the fact that the most part of $Tr_t(E)$ is unknown and there are many parameters, which can vary over certain ranges or domains, challenging decision problems are very complex. Yu and Chianglin (2006) described this kind of problems as decision problems with changeable spaces (parameters). A system scheme based on HDs theory has been introduced to help us reduce decision blinds and avoid decision traps so that we could make quality decisions.

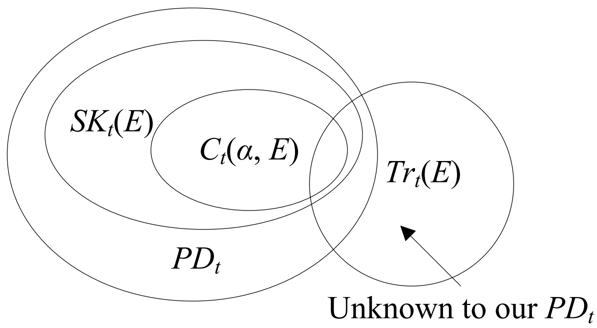

Figure 2-7 Competence set of challenging problems.

CHAPTER 3 WORKING DOMAIN AND INNOVATION DYNAMICS

Based on HD theory and CS analysis, we shall in this chapter introduce the concepts of working domain and innovation dynamics. We could see how our working domains can commit decision traps and how to avoid them. The model of innovation dynamics helps our working domains become more flexible and agile.

3.1 Property of the Activation Probability

In order to facilitate the later discussion, we shall introduce the *activation continuity assumption* [14]: let s and t be two distinct point of time, i.e., $s \neq t$, all activated ideas and operators for solving a given decision problem can be continuously activated again within the time interval $[s, t]$ if needed. In general, the activation continuity assumption is held when either of the following two cases occurs:

1. the time interval $[s, t]$ is sufficiently small;

2. all the activated ideas, concepts, and information for solving the decision problem can be properly recorded and saved.

If the activation continuity assumption does not hold, let $t_1, t_2, t_3 \in [s, t]$ and $s<t_1<t_2<t_3<t$. Suppose that the activation probability for each circuit pattern at t_1, t_2, and t_3, denoted by AP_{t_1}, AP_{t_2}, and AP_{t_3}, is shown in Figure 3-1, 3-2, and 3-3, where the actual domains with respect to t_1, t_2, and t_3 are $AD_{t_1} = \{a\}$, $AD_{t_2} = \{b\}$, and $AD_{t_3} = \{c\}$ respectively.

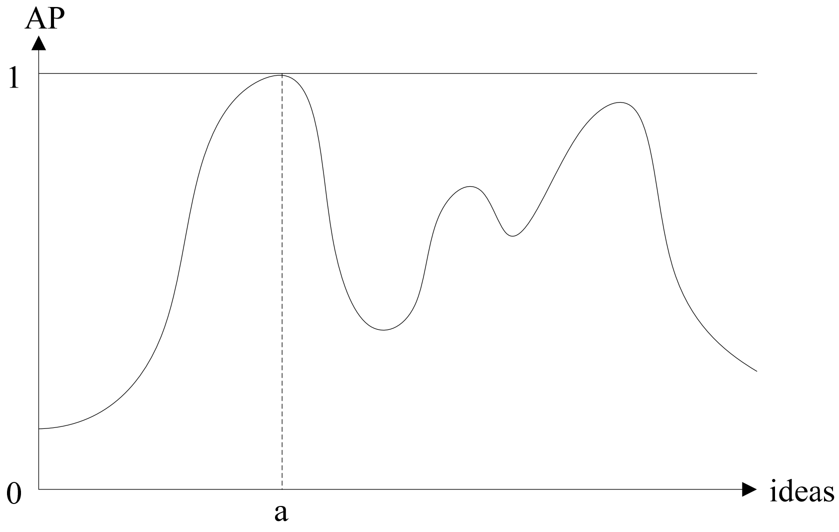

Figure 3-1 The activation probability for circuit patterns at t_1.

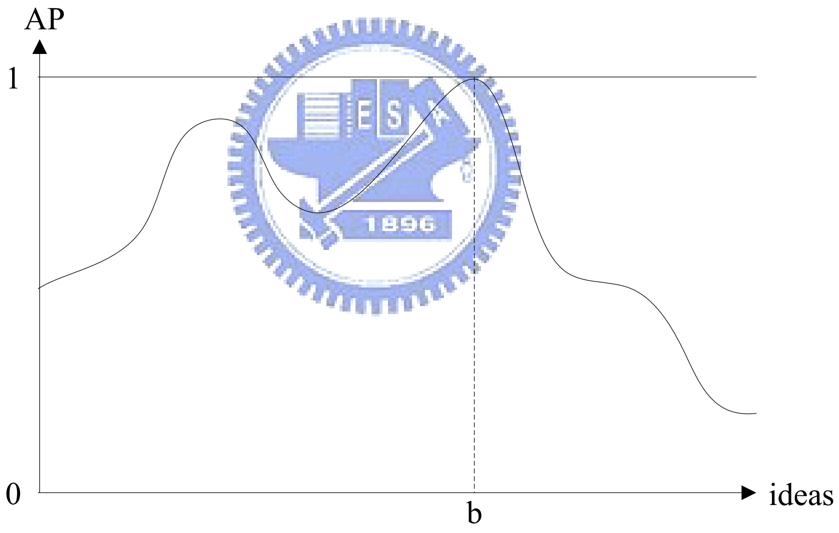

Figure 3-2 The activation probability for circuit patterns at t_2.

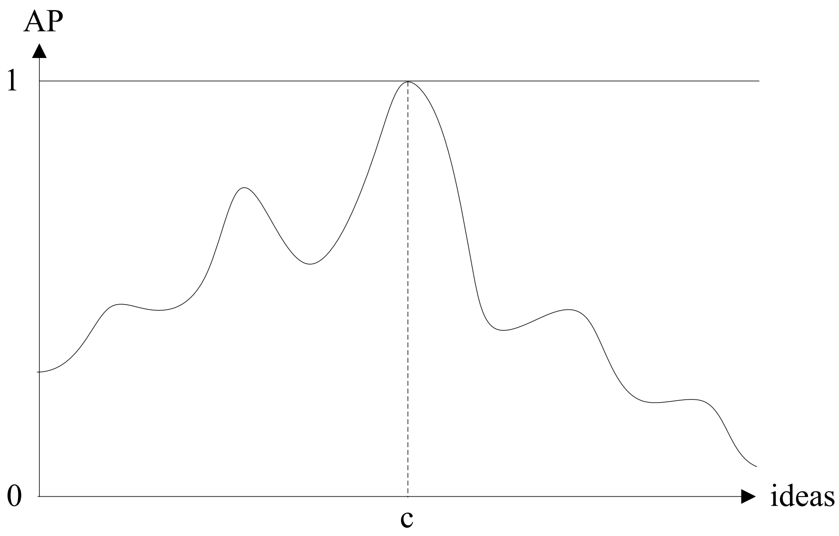

Figure 3-3 The activation probability for circuit patterns at t_3.

Suppose that the time interval [s, t] is sufficiently small (i.e., the activation continuity assumption is held). Figure 3-4 depicts the activation probability for circuit patterns at t_3. The corresponding actual domain is then $AD_{t_3} = \{a, b, c\}$ since all the activated ideas and operators could be retrieved if needed. By the activation continuity assumption, the activation probability for each circuit pattern at time t_2 and t_3 can be denoted by

$$AP_{t_2}(i) = \max\{AP_{t_1}(i), AP_{t_2}(i)\}, \quad (1)$$

and

$$AP_{t_3}(i) = \max\{AP_{t_1}(i), AP_{t_2}(i), AP_{t_3}(i)\}. \quad (2)$$

For generalization, let t_0 be the time that we encounter a decision problem. The activation probability for an idea i at time $t_{\text{now}} \geq t_0$ can be denoted by

$$AP_{t_{\text{now}}}(i) = \max_{\{t_0 \leq t \leq t_{\text{now}}\}} \{AP_t(i)\}.$$

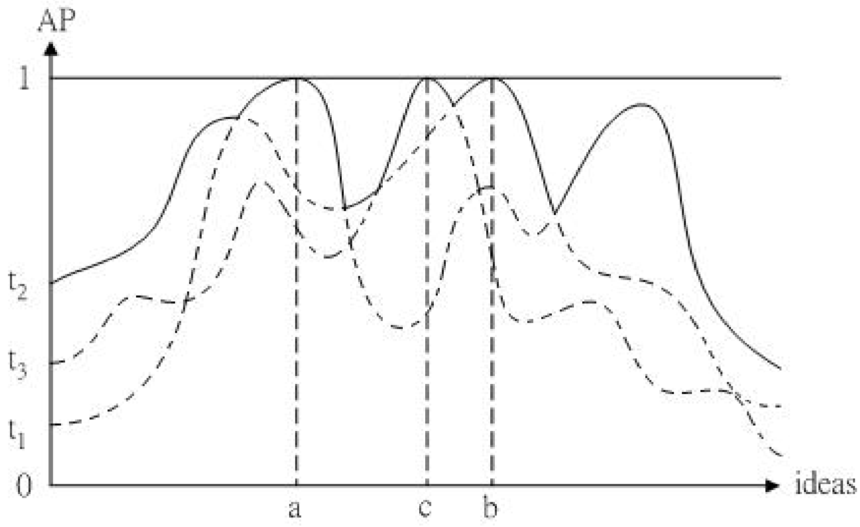

Figure 3-4 The activation probability for circuit patterns at t_3 when the activation continuity assumption is held.

By (1), (2) and Figure 3-4, if the activation continuity assumption is held, the activation probability would be increased when time passes. On the other hand, the ideas and operators in our brains would be increased and more easily to be retrieved when time passes. Therefore, how to make the activation continuity assumption be held is very important for solving a decision problem.

3.2 Working Domain

The memory of humanity is one of extensive research issues studied by psychologists. A number of classifications and models related to the memory are introduced. Working memory (WM) plays a vital role in our life especially when cognitive works such as learning, comprehending, and reasoning etc. [16], [17] are performed. WM will be used when we try to look up a telephone number and remember it to dial it without writing it down [18], [19]. Atkinson and Shiffrin [20], [21] proposed that short-term memory works as WM which allows us to retain useful information as to solve the problem to use it. Olton et al. [22], [23] proposed that WM is a system which is responsible for the maintenance of the information related to the works we have to do so that we could perform efficiently.

Baddeley and Hitch [24] gave the definition of WM which is a system used to temporarily store and manipulate information to help us in complicated cognitive work and developed the multiple-component model of WM, as shown in Figure 3-5.

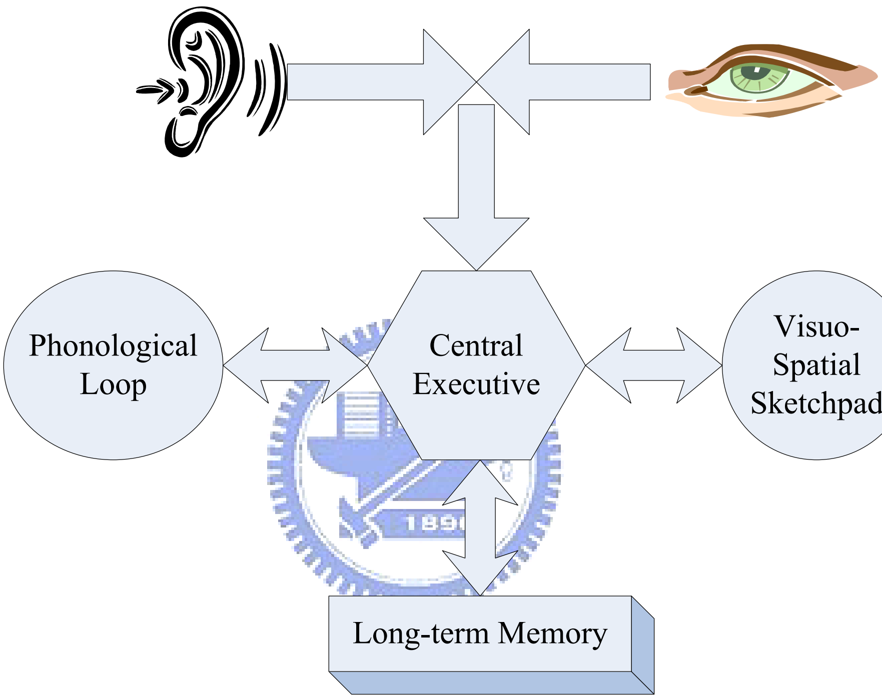

Figure 3-5 Multiple-component model of working memory [24].

The model consisted of two slave systems (phonological loop and visuo-spatial sketchpad) and a central executive. The former are responsible for short-term maintenance of information, and the latter is responsible for the supervision of information integration and for coordinating the slave systems. Subsequently, Baddeley [25] added episodic buffer, a fourth component, to the model. The episodic buffer is the third slave system which is responsible for linking information across domains to form integrated units of visual, spatial, and verbal information with time sequencing (or chronological ordering), such as the memory of a story.

Based on habitual domains theory, working domain (WD) refers to the set of all the ideas and the operators used to relieve our charge structure, which is inflicted by a decision problem. For example, when sharing the knowledge about the breeding of dogs, all the related special experiences, such as the name, the figure, and the barks of the dog, will be retrieved. Hence, we may portray the WD as the set of ideas which has been actually activated for solving the decision problem or the set of ideas whose activation probabilities are greater than α ($0 \le \alpha \le 1$). The former is denoted by

$$WD(E)=\{i \in HD_t \mid AP_t(i)=1, \forall t \in [t_1,t_2]\}, \tag{3}$$

and the latter is denoted by

$$WD_\alpha(E)=\{i \in HD_t \mid AP_t(i) \geqq \alpha, \forall t \in [t_1,t_2]\}, \tag{4}$$

where i denotes an idea; $AP_t(i)$ denotes the activation probability of the idea i at time t, t_1 is the point of time when a decision problem is constituted, and t_2 is the point of time when the decision problem is solved.

Note that the actual domains are emphasized in (3), and both the actual domains and the reachable domains are included in (4).

3.3 Classification of Decision Problems

Let the needed competence set, the potential domain, and the working domain with respect to a given decision problem or event E at time t are denoted by $CS_t(E)$, $PD_t(E)$, and $WD_t(\alpha, E)$, respectively. We may restate the four categories mentioned in subsection 2.2.2 into the following:

1. If $CS_t(E)$ is well-known and $CS_t(E) \subset WD_t(\alpha, E) \subset PD_t(E)$, $\alpha \to 1$, then the problem is a

routine problem. When encounter this class of problems, we know the truly needed competence set without ambiguous so that the problem could be efficiently solved. That is, our working domain can immediately react to solve it.

2. Mixed-routine problem consists of a number of routine sub-problems, we may decompose it into a number of routine problems to be solved.

3. If $CS_t(E)$ is only fuzzily known and may not contained in $WD_t(\alpha, E)$ with a high value of α, then the problem is a fuzzy problem, for which solutions are fuzzily known. That is, $CS_t(E)\backslash WD_t(\alpha, E)\neq\emptyset$, and $CS_t(E)\subset PD_t(E)$. Note that once the $CS_t(E)$ is gradually clarified and contained in α-core with a high value of α, the fuzzy problem may gradually become routine problem.

4. If $CS_t(E)$ is unknown or a small part of $CS_t(E)$ is known and the remaining part is not in $PD_t(E)$, then the problem is a challenge problem. That is, $CS_t(E)\backslash WD_t(\alpha, E)\neq\emptyset$, and $CS_t(E)\backslash PD_t(E)\neq\emptyset$. Note that no matter how low the degree of α is, $CS_t(E)$ can not be included by $WD_t(\alpha, E)$.

These four decision problems can be easily seen in our life. Information technologies (ITs) only can facilitate us to solving routine problems and mixed-routine problems other than challenge problems due to the unknown $CS_t(E)$. Even if a small part of $CS_t(E)$ is known, the remaining part is not in our habitual domains. Let us see the following example.

Example 3.1. Adopted from [26]. Archimedes, a great scientist, was summoned by the King of Greece to verify if his new crown was made of pure gold. Of course, in the verification process, the beautiful crown should not be damaged. The problem was a great challenge and created a very high level of charge on Archimedes. The scientist's curiosity was increased and his reputation was at stake. The burning desire to solve the problem kept Archimedes awake day and night. One day, when Archimedes was in his bathtub watching the water fill up and overflow, a solution suddenly struck him. He rushed out of the bathtub shouting “eureka” (means “I found it” in Greek) and in his excitement, he even forgot to put on his clothes. His discovery which is the well-known *displacement principle* states that the volume of the displaced water should be equal to the volume of his entire body in the water. Thus the crown, when immersed in the water, should displace its own volume. By comparing the weight of the crown with an equivalent weight of pure gold of the same volume, one should be able to verify if the crown is made of pure gold. What a relief to Archimedes!

The problem whether the new crown of the King of Greece was made of pure gold without damage is a challenge problem to Archimedes. When Archimedes was taking a bath (routine problem) in his bathtub, he noticed the water fill up and overflow (information input). His working domain was expanded through active association so that he came up with a simple but great solution for verifying if the new crown was made of pure gold.

3.4 How Working Domain Get Trapped

With rapid advancement of Information Technology (IT), Knowledge Management (KM) has enjoyed its rapid growth [27]. In the market, there are many software available to help people make decisions or transactions, such as supply chain management (SCM), enterprise resource planning (ERP), customer relationship management (CRM), accounting information system (AIS), etc. [28], [29], [30]. In the nutshell, KM is useful because it can help certain people to relieve the pains and frustrations for obtaining useful information to make certain decisions or transactions. For salesperson, KM could provide useful information as to close sales. For credit card companies, KM could provide useful information about card holders’ credibility. For supply chain management, KM can efficiently provide where to get needed materials, where to produce and how to transport the product and manage the cash flow, etc.

It seems, KM could do “almost everything” to help people make “any decision” with good results. Let us consider the following example.

Example 3.2. Breeding Mighty Horses. For centuries, many biologists paid their attention and worked hard to breed endurable mighty working horses so that the new horse could be durable, controllable and did not have to eat. To their great surprise, their dream was realized by mechanists, who invented a kind of “working horse”, tractors. The biologists’ decision trap and decision blind are obvious.

Biologists habitually thought that to produce the mighty horses, they had to use ”breeding methods”–a bio-tech, a decision trap in their mind. Certainly, they made progress. However, their dream could not be realized.

IT or KM, to certain degree, is similar to breeding, a biotech. One wonders: is it possible that IT or KM could create traps for people as to make wrong decision or transactions? If it is possible, how could we design a good KM that can minimize the possibility to have decision traps and maximize the benefits for the people who use it?

In the information era, even the advances of IT and KM can help solve people’s decision problems, our working domain could still easily get trapped, leading us to make wrong decision or action.

Example 3.3. Dog Food. A dog food company designed a special package that not only was nutritious, but also could reduce dogs’ weight. The statistical testing market was positive. The company started “mass production”. Its dog food supply was far short from meeting the overwhelming demand. Therefore, the company doubled its capacity. To their big surprise, after one to two months of excellent sales, the customers and the wholesalers began to return the dog food package, because the dogs did not like to eat it.

Clearly, a decision trap was committed by using statistics on buyers, not on the final users (dogs). The KM used statistical method on "wrong" subject and committed the trap. If the RD (reachable domain) of the KM could include the buyers and the users, the decision traps and wrong decisions might be avoided.

So far, KM with IT can accelerate decisions for the routine or mixed-routine problems. There still are many challenging problems, which cannot be easily solved by KM with IT. This is because the needed competence set ($Tr_t(E)$) of a challenging problem is unknown or only partially known, especially when humans are involved. The following illustrates this fact.

Example 3.4. Alinsky's Strategy. Adapted from [31]. In 1960 African Americans living in Chicago had little political power and were subject to discriminatory treatment in just about every aspect of their lives. Leaders of the black community invited Alinsky, a great social movement leader, to participate in their effort. Alinsky clearly was aware of deep knowledge principles. Working with black leaders he came up with a strategy so alien to city leaders that they would be powerless to anticipate it. He would mobilize a large number of people to legally occupy all the public restrooms of the O'Hare Airport. Imagine thousands of individuals visit the airport daily who were hydraulically loaded (very high level of charge) rushed for restroom but there would be no place for all these persons to relieve themselves.

How embarrassing when the newspaper and media around the world headlined and dramatized the situation. As it turned, the plan never was put into operation. City authorities found out about Alinsky's strategy and, realizing their inability to prevent its implementation and its potential for damaging the city's reputation, met with black leaders and promised to fulfill several of their key demands.

The above example shows us the importance of understanding one's potential domain. At the beginning, African Americans did not entirely know the habitual domain of city authorities (a challenging problem). Their campaigns, such as demonstration, hunger strike, etc., failed to reach their goal (an working domain). Alinsky observed a potentially high level of charge of the city authorities, the public opinion (potential domain), that could force them to act. As a result, the authorities agreed to meet the key demands of the black community, with both sides claiming a victory.

3.5 Innovation Dynamics

Without creative ideas and innovation, our lives will be bound in a certain domain and become stable. Similarly, without continuous innovation, our business will lose its vitality and competitive edge [32], [33]. Bill Gates indicated that Microsoft would collapse in about two years if they do not continue the innovation.

In this section, we are going to explore innovation dynamics based on Habitual Domains (HD) and Competence Set (CS) Analysis as to increase competitive edge. From HD theory and CS analysis, all things and humans can release pains and frustrations for certain group of people at certain situations and time. Thus all humans and things carry the competence (in broad sense, including skills, attitudes, resources, and functionalities). For instance, a cup is useful when we need a container to carry water as to release our pains and frustrations of having no cup.

The competitive edge of an organization or human can be defined as the capability to provide right services and products at right price to the target customers earlier than the competitors, as to release their pains and frustrations and make them satisfied and happy.

To be competitive, we therefore need to know what would be the customers' needs as to produce the right products or services at a lower cost and faster than the competitors. At the same time, given a product or service of certain competence or functionality, how to reach out the potential customers as to create value (the value is usually positively related to how much we could release the customers' pains and frustrations).

If we abstractly regard all humans and things as a set of different CS, then producing new products or services can be regarded as a transformation of the existent CS to a new form of CS. Based on this, we could draw clockwise innovation dynamics as in Figure 3-6:

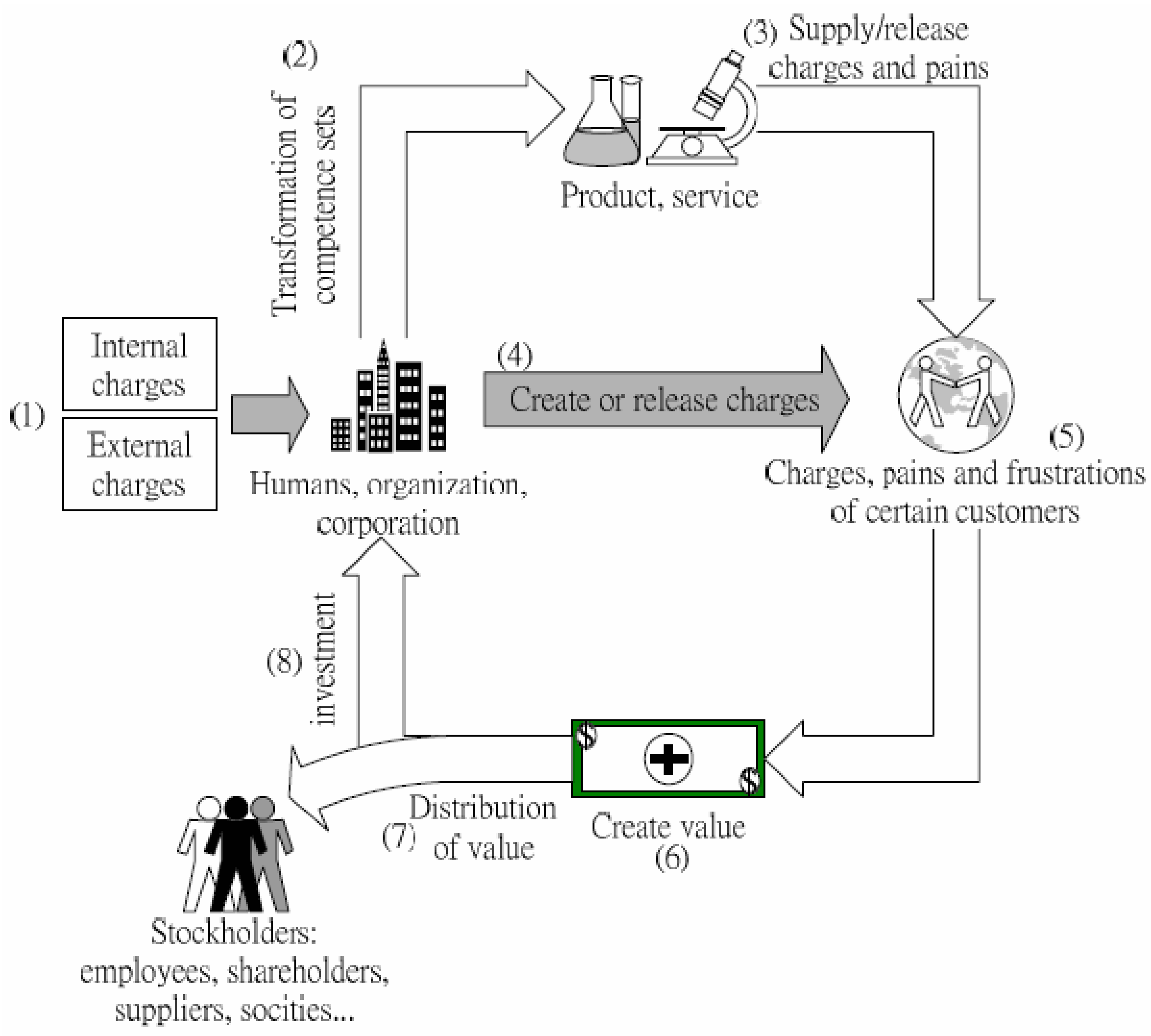

Figure 3-6 Clockwise Innovation Dynamics.

Although Figure 3-6 is self-explaining, the following are worth mentioning. The numbers are corresponding to that of the figure.

Note 1: According to HD Theory, when the current states and the ideal goals have unfavorable discrepancies (for instance losing money instead of making money, technologically behind, instead of ahead of, the competitors), these discrepancies will create mental charge which can prompt us to work harder to reach our ideal goals.

Note 2: Producing product and service is a matter of transforming CS from the existing one to a new form.

Note 3: Our product could release the charges and pains of certain group of people and make them satisfied and happy.

Note 4: The organization can create or release charges of certain group of people through advertising, marketing and selling.

Note 5: The target group of people will experience the change of charges. When their pains and frustrations, by buying our products or services, are relieved and become happy, the products and services can create value, which is Note 6.

Note 7 and Note 8 respectively are the distribution of the created value and reinvestment. To gain the competitive edge, products and services need to be continuously upgraded and changed. The reinvestment, Note 8, is needed for research and development for producing new product and service.

In a contrast, the innovation dynamics can be counter-clockwise. We could draw counter-clockwise innovation dynamics as in Figure 3-7:

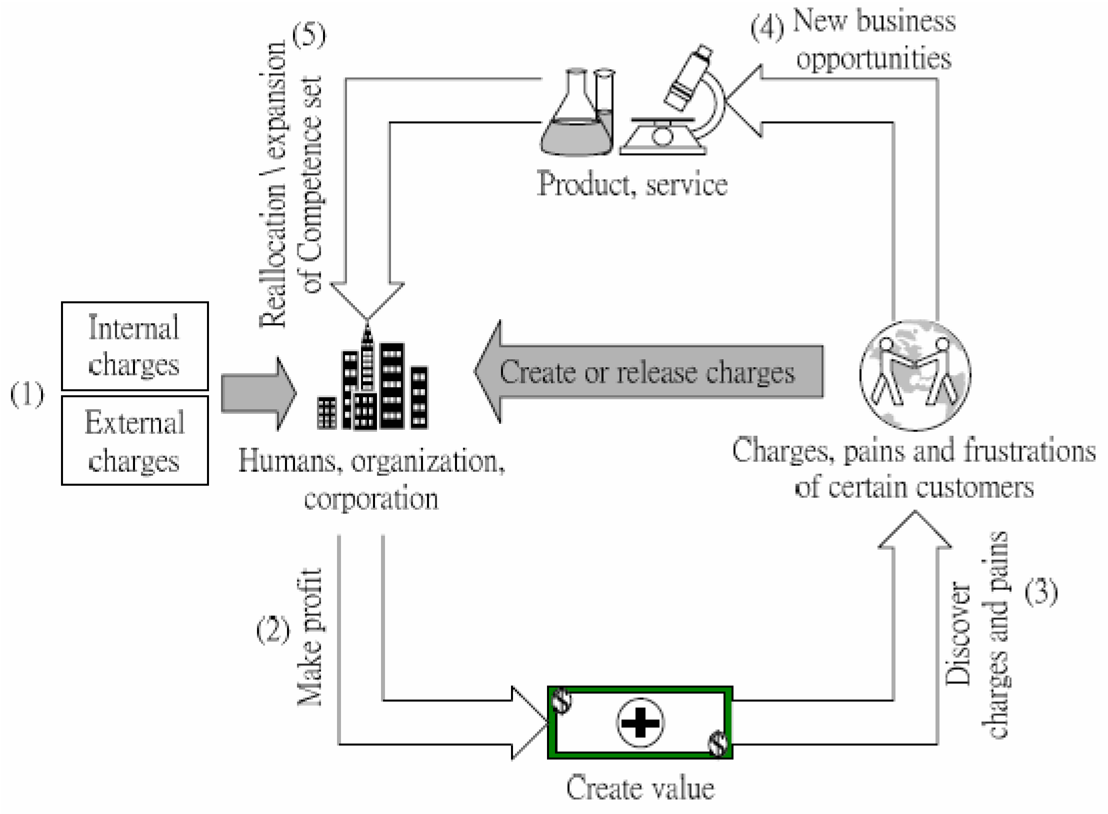

Figure 3-7 Counter-clockwise Innovation Dynamics.

Note 1: According to HD Theory, when the current states and the ideal goals have unfavorable discrepancies will create mental charge which can prompt us to work harder to reach our ideal goals.

Note 2: In order to make profit, organization must create value.

Note 3: According to CS analysis, all things carry competence which can release pains and frustrations for certain group of people at certain situations and time.

Note 4: New business opportunities could be found by understanding and analyzing the pains and frustrations of certain group of people.

Note 5: Reallocation or expansion of competence set is needed for innovating products or services to release people's pains and frustrations.

Innovation needs creative ideas, which are outside the existing HD and must be able to relieve the pains and frustrations of certain people. From this point of view, the method of expanding and upgrading our HDs becomes readily applicable. Innovation can be defined as the work and process to transform the creative ideas into reality as to create the value expected. It includes planning, executing (building structures, organization, processes, etc.), and adjustment. It could demand hard working, perseverance, persistence and competences. Innovation is, therefore, a process of transforming the existing CS toward a desired CS (product or service).

CHAPTER 4 OPTIMAL ADJUSTMENT OF COMPETENCE SET WITH LINEAR PROGRAMMING

The innovation dynamics could be portrayed as a process of transforming the existent competence set to a new form of competence set. It includes planning, executing (building structures, organization, processes, etc.), and adjustment. After having known the innovation dynamics, we shall in this chapter focus on the optimal adjustment of competence set so that an expected state could be achieved. To make our studies specific and mathematically precise we shall limit ourselves to linear models based on management by objectives (MBO).

4.1 Introduction to Optimal Competence Set Adjustment Problems

MBO is an efficient and effective managerial system [34]. Goal setting is the first crucial step in the system of MBO. At this step the participants identify the targets to be achieved. The company then mobilizes all resources and competence, including their reallocation, to reach the targets, or to move toward the targets as close as possible. Therefore, achieving the targets becomes one of the most important criteria in the system of MBO. In order to achieve the targets some relevant parameters, such as the constraint coefficients and the right hand sided resource level in linear programming (LP) problems, need to be adjusted and/or expanded.

One of the well-known researches on the adjustment of parameters is the *inverse LP optimization*. In the class of inverse LP problems, the parameters of the objective function with the minimum deviation from the original ones are sought so that a given feasible solution $\boldsymbol{x}'$ becomes an optimal one [35]. Zhang and Liu [36] studied inverse assignment and minimum cost flow problems under L_1-norm based on optimality conditions for LP problems. Zhang and Liu [37] further took L_∞-norm into account and investigated inverse 0-1 programming and network programming problems. Ahuja and Orlin [38] considered more general inverse LP problems under both L_1- and L_∞-norms. In addition, Troutt et al. [39] investigated a so-called *linear programming system identification problem* in which both objective function coefficients and constraint matrix are evaluated to best fit a set of historical decisions and its corresponding used resources.

A competence set is a collection of ideas, knowledge, information, resources, and skills for satisfactorily solving a given decision problem [11], [12]. By using mathematical programming, a number of researchers have focused on searching for the optimal expansion process from an already acquired competence set to a needed one [9], [40], [41]. Feng and Yu [42] proposed a minimum spanning table algorithm to find the optimal competence set expansion process without formulating the related mathematical program. However, the competence set so far has been assumed to be discrete and finite so as to represent its elements by nodes of a graph. This makes the applications of the competence set expansion in these studies somehow limited, because the number of feasible solutions of a linear system might not be discrete and finite.

In this chapter, we focus on linear systems. While the literature on inverse LP optimization treats only a feasible target, we intend to determine the optimal adjustment of constraint coefficients in a linear system so that a given target, originally unattainable, can be achieved. Given a target solution, we set up a *competence set adjustment model* (CSA model) to study the optimal adjustment of the related competence sets. The model will enable us to find the optimal adjustment whenever the target is reachable.

In case the target is unattainable, we utilize the bisection method or the fuzzy linear programming techniques to help the DM revise the target as to make it an achievable one. The former is to find a solution which is as close as possible to the target and the latter is to interactively select an achievable target. Then the optimal adjustment could be derived from the aforementioned CSA model with the revised target.

4.2 Problem Statement

Consider a standard LP model as follows.

$$\begin{aligned} \max \quad & z(\boldsymbol{x}) = \boldsymbol{c}\boldsymbol{x} \\ \text{s.t.} \quad & \boldsymbol{A}\boldsymbol{x} \le \boldsymbol{b}, \\ & \boldsymbol{x} \ge 0, \end{aligned} \tag{5}$$

where $\boldsymbol{c}=[c_i]$ is the $1\times n$ objective coefficient vector, $\boldsymbol{x}=[x_j]$ denotes the $n\times 1$ decision vector, $\boldsymbol{A}=[a_{ij}]$ is the $m\times n$ consumption (or productivity) matrix, and $\boldsymbol{b}=[b_i]$ is the $m\times 1$ resource availability vector. The sensitivity analysis helps us to investigate whether the optimal basis changes if c_i, a_{ij}, or b_i has been changed. It provides us insight into the ranges over which the parameters of a model can vary without changing the optimal basis. While the sensitivity analysis considers the changes on c_i, a_{ij}, or b_i, we consider the simultaneous changes on a_{ij} and b_i so that a given target can be reached.

Suppose that $\boldsymbol{x}^0$ is a target solution set by decision maker (DM). Let $\boldsymbol{D}$ be a parameter matrix whose element, δ_{ij}, denotes the deviation from a_{ij}, and $\boldsymbol{\gamma}$ be a parameter vector whose component, γ_i, denotes the deviation from b_i. By changing $\boldsymbol{D}$ and $\boldsymbol{\gamma}$, we tried to construct $X^0(\boldsymbol{D},\boldsymbol{\gamma})$, where $X^0(\boldsymbol{D},\boldsymbol{\gamma})=\{\boldsymbol{x}|(\boldsymbol{A}+\boldsymbol{D})\boldsymbol{x}\le\boldsymbol{b}+\boldsymbol{\gamma}\}$.

Since a_{ij}=0 implies that the resource i has no impact on the product j. Thus, a_{ij} is not subject to adjustment. Consequently, we have δ_{ij}=0 if a_{ij}=0.

Definition 4.1. Given a target $\boldsymbol{x}^0$, a *feasible adjustment* is a pair $(\boldsymbol{D},\boldsymbol{\gamma})$ such that $(\boldsymbol{A}+\boldsymbol{D})\boldsymbol{x}^0 \le \boldsymbol{b}+\boldsymbol{\gamma}$. Thus, $\boldsymbol{x}^0 \in X^0(\boldsymbol{D},\boldsymbol{\gamma})$.

Let $\Psi=\{(\boldsymbol{D},\boldsymbol{\gamma}) | \boldsymbol{x}^0 \in X^0(\boldsymbol{D},\boldsymbol{\gamma})\}$ be the set of all feasible adjustments, and $\Phi=\{(i,j) | a_{ij}\neq 0\}$,

Definition 4.2. Given a target solution $\boldsymbol{x}^0$, and $(\boldsymbol{D}^0, \boldsymbol{\gamma}^0)\in\Psi$, define the *relative adjustment measure* of $(\boldsymbol{D}^0, \boldsymbol{\gamma}^0)$ by

$$\Re(\boldsymbol{D}^0,\boldsymbol{\gamma}^0) = \sum_{(i,j)\in\Phi} r_{ij}(\boldsymbol{D}^0) + \sum_{i=1}^{m} s_i(\boldsymbol{\gamma}^0),$$

where

$$r_{ij}(\boldsymbol{D}^0) = |\delta_{ij}^0| / |a_{ij}|,\ a_{ij}\neq 0,$$

and

$$s_i(\boldsymbol{\gamma}^0) = |\gamma_i^0| / h_i,$$

where

$$h_i = \begin{cases} |b_i| & \text{if } b_i \neq 0, \\ |M_i| & \text{if } b_i = 0. \end{cases}$$

Note that $r_{ij}(\boldsymbol{D}^0)$, $a_{ij}\neq 0$, is a relative adjustment measure with respect to the parameter a_{ij}, while $s_i(\boldsymbol{\gamma}^0)$ is that with respect to b_i. Note, when b_i=0, $|\gamma_i^0| / |b_i|$ is not defined. The positive number M_i needs to be chosen properly to reflect the impact of the adjustment on b_i.

Remark 4.1. When needed, $\Re(\boldsymbol{D}^0, \boldsymbol{\gamma}^0)$, r_{ij} and s_i can be changed into other forms of cost functions to fit the cost of adjustment.

Definition 4.3. A *feasible adjustment alternative* ($\boldsymbol{D}^*$, $\boldsymbol{\gamma}^*$) is optimal if ($\boldsymbol{D}^*$, $\boldsymbol{\gamma}^*$) minimizes the relative adjustment measure over Ψ. That is,

$$\Re(\boldsymbol{D}^*, \boldsymbol{\gamma}^*) = \min\{\Re(\boldsymbol{D}, \boldsymbol{\gamma}) \mid (\boldsymbol{D}, \boldsymbol{\gamma}) \in \Psi\}.$$

The adjustment deviation measure as defined in Definition 4.2 is not a linear form because of "absolute value". To eliminate the sign of the absolute value in Definition 4.2, the following Lemma 4.1 is useful.

Lemma 4.1. Given $\boldsymbol{D}$=[δ_{ij}] and $\boldsymbol{\gamma}$=[γ_i], let $\bar{a}_{ij}$=a_{ij}+δ_{ij} and $\bar{b}_i$=b_i+γ_i. Define $\boldsymbol{D}^+ = [\delta_{ij}^+]$, $\boldsymbol{D}^- = [\delta_{ij}^-]$, $\boldsymbol{\gamma}^+ = (\gamma_1^+, \ldots, \gamma_m^+)$, and $\boldsymbol{\gamma}^- = (\gamma_1^-, \ldots, \gamma_m^-)$ with

$$\delta_{ij}^+ = \begin{cases} \bar{a}_{ij} - a_{ij} & \text{if } \bar{a}_{ij} > a_{ij}, \\ 0 & \text{otherwise;} \end{cases} \tag{6}$$

$$\delta_{ij}^- = \begin{cases} a_{ij} - \bar{a}_{ij} & \text{if } a_{ij} > \bar{a}_{ij}, \\ 0 & \text{otherwise;} \end{cases} \tag{7}$$

$$\gamma_i^+ = \begin{cases} \bar{b}_i - b_i & \text{if } \bar{b}_i > b_i, \\ 0 & \text{otherwise;} \end{cases} \tag{8}$$

$$\gamma_i^- = \begin{cases} b_i - \bar{b}_i & \text{if } b_i > \bar{b}_i, \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Then:

(i) $\delta_{ij} = \delta_{ij}^+ - \delta_{ij}^-$ and $\gamma_i = \gamma_i^+ - \gamma_i^-$, or $\boldsymbol{D} = \boldsymbol{D}^+ - \boldsymbol{D}^-$ and $\boldsymbol{\gamma} = \boldsymbol{\gamma}^+ - \boldsymbol{\gamma}^-$.

(ii) $|\delta_{ij}| = \delta_{ij}^+ + \delta_{ij}^-$ and $|\gamma_i| = \gamma_i^+ + \gamma_i^-$.

(iii) $\delta_{ij}^+, \delta_{ij}^-, \gamma_i^+, \gamma_i^- \geq 0$.

Proof.

(i) Since $\bar{a}_{ij} = a_{ij} + \delta_{ij}$, we have $\delta_{ij} = \bar{a}_{ij} - a_{ij}$. We may replace (7) by (10) as follows.

$$\delta_{ij}^{-} = \begin{cases} 0 & \text{if } \bar{a}_{ij} > a_{ij}, \\ a_{ij} - \bar{a}_{ij} & \text{otherwise.} \end{cases} \tag{10}$$

By subtracting (10) from (6) on both sides, we have

$$\delta_{ij}^{+} - \delta_{ij}^{-} = \begin{cases} \bar{a}_{ij} - a_{ij} & \text{if } \bar{a}_{ij} > a_{ij}, \\ \bar{a}_{ij} - a_{ij} & \text{otherwise.} \end{cases} \tag{11}$$

By (11), we have $\delta_{ij} = \delta_{ij}^{+} - \delta_{ij}^{-}$. That $\gamma_i = \gamma_i^{+} - \gamma_i^{-}$ could be proved in a similar way.

(ii) By definition,

$$\left|\delta_{ij}\right| = \begin{cases} \delta_{ij} & \text{if } \delta_{ij} > 0, \\ -\delta_{ij} & \text{otherwise.} \end{cases} \tag{12}$$

We may rewrite (12) as follows.

$$\left|\delta_{ij}\right| = \begin{cases} \bar{a}_{ij} - a_{ij} & \text{if } \bar{a}_{ij} > a_{ij}, \\ a_{ij} - \bar{a}_{ij} & \text{otherwise.} \end{cases} \tag{13}$$

Observe that (13) could be obtained by adding (10) to (6). Thus, $|\delta_{ij}| = \delta_{ij}^{+} + \delta_{ij}^{-}$. That $|\gamma_i| = \gamma_i^{+} + \gamma_i^{-}$ can be proved similarly.

(iii) It is obviously from (6)-(9). □

Note that δ_{ij}^{+} is the value of $\bar{a}_{ij}$ exceeding a_{ij} and δ_{ij}^{-} is that of $\bar{a}_{ij}$ below a_{ij}, while γ_i^{+} is the value of $\bar{b}_i$ exceeding b_i and γ_i^{+} is that of $\bar{b}_i$ below b_i.

4.3 Optimal Adjustment of Competence Set

Given a target $\boldsymbol{x}^0$, we try to identify the optimal adjustment alternative $(\boldsymbol{D}^*, \boldsymbol{\gamma}^*)$ by minimizing the relative adjustment measure $\Re(\boldsymbol{D}^*, \boldsymbol{\gamma}^*)$ over Ψ. By Definition 4.1-4.3, the optimal adjustment of competence set for reaching $\boldsymbol{x}^0$ can be formulated as the following *competence set adjustment model* (CSA model).

Program 4.1.

$$\begin{aligned} z^0 = \min \quad & \sum_{(i,j)\in\Phi} \left(|\delta_{ij}| / |a_{ij}|\right) + \sum_{i=1}^{m} \left(|\gamma_i| / h_i\right) \\ s.t. \quad & \sum_{(i,j)\in\Phi} (a_{ij} + \delta_{ij}) x_j^0 \le b_i + \gamma_i, i = 1, 2, \ldots, m, \end{aligned} \tag{14}$$

where δ_{ij} and γ_i are unrestricted in sign. After having applied Lemma 4.1, we may reformulate Program 4.1 as the following linear programming model.

Program 4.2.

$$\begin{aligned} z^0 = \min \quad & \sum_{(i,j)\in\Phi} \left\{\left(\delta_{ij}^+ + \delta_{ij}^-\right) / |a_{ij}|\right\} + \sum_{i=1}^{m} \left\{\left(\gamma_i^+ + \gamma_i^-\right) / h_i\right\} \\ s.t. \quad & \sum_{(i,j)\in\Phi} (a_{ij} + \delta_{ij}^+ - \delta_{ij}^-) x_j^0 \le b_i + \gamma_i^+ - \gamma_i^-, i = 1, 2, \ldots, m, \\ & \delta_{ij}^+ \ge 0, \delta_{ij}^- \ge 0, \gamma_i^+ \ge 0, \gamma_i^- \ge 0. \end{aligned} \tag{15}$$

Note that when z^0=0, there is no need for adjustment. That is, the original system can produce the target solution x^0.

Lemma 4.2. The optimal solution $(\boldsymbol{D}^{+*}, \boldsymbol{D}^{-*}, \boldsymbol{\gamma}^{+*}, \boldsymbol{\gamma}^{-*})$ to Program 4.2 has the property that $\delta_{ij}^{+*} \bullet \delta_{ij}^{-*}$=0, $\gamma_i^{+*} \bullet \gamma_i^{-*}$=0, for all i, j.

Proof. (i) if $\delta_{ij}^{+*} > \delta_{ij}^{-*} > 0$, set $\delta_{ij}^{+0} = \delta_{ij}^{+*} - \delta_{ij}^{-*}$ and $\delta_{ij}^{-0} = 0$;

(ii) if $\delta_{ij}^{-*} > \delta_{ij}^{+*} > 0$, set $\delta_{ij}^{-0} = \delta_{ij}^{-*} - \delta_{ij}^{+*}$ and $\delta_{ij}^{+0} = 0$;

(iii) if $\gamma_i^{+*} > \gamma_i^{-*} > 0$, set $\gamma_i^{+0} = \gamma_i^{+*} - \gamma_i^{-*}$ and $\gamma_i^{-0} = 0$;

(iv) if $\gamma_i^{-*} > \gamma_i^{+*} > 0$, set $\gamma_i^{-0} = \gamma_i^{-*} - \gamma_i^{+*}$ and $\gamma_i^{+0} = 0$.

Then, $(\boldsymbol{D}^{+0}, \boldsymbol{D}^{-0}, \boldsymbol{\gamma}^{+0}, \boldsymbol{\gamma}^{-0})$ is a better solution which leads to a contradiction. □

In order to make Program 4.2 more effective in computing, we derive the following proposition.

Proposition 4.1. Given a target solution $\boldsymbol{x}^0$, the optimal solution $(\boldsymbol{D}^{+*}, \boldsymbol{D}^{-*}, \boldsymbol{\gamma}^{+*}, \boldsymbol{\gamma}^{-*})$ to Program 4.2 has the property that $\boldsymbol{D}^{+*} = 0$, and $\boldsymbol{\gamma}^{-*} = 0$.

Proof. Given a target solution $\boldsymbol{x}^0$, consider two possible cases.

Case 1: $\sum_{j=1}^{n} a_{ij} x_j^0 \le b_i$, $\forall i \in \{1,2,\ldots,m\}$. Then $\delta_{ij}^{+*} = \delta_{ij}^{-*} = \gamma_i^{+*} = \gamma_i^{-*} = 0$, for all *i*, *j*, is the optimal solution. The property obviously holds.

Case 2: $\exists i \in \{1,2,\ldots,m\}$ such that

$$\sum_{j=1}^{n} a_{ij} x_j^0 > b_i. \tag{16}$$

(i) Assume $\delta_{ij}^{+*} > 0$. By Lemma 4.2, we have $\delta_{ij}^{-*} = 0$. Thus, from (16), we have $\sum_{j=1}^{n} \left(a_{ij} + \delta_{ij}^{+*}\right) x_j^0 > b_i$. In order to satisfy (15), $\gamma_i^{+*} > 0$, and $\gamma_i^{-*} = 0$ (by Lemma 4.2). Indeed,

$$\gamma_i^{+*} = \sum_{j=1}^{n} \left(a_{ij} + \delta_{ij}^{+*}\right) x_j^0 - b_i. \tag{17}$$

Choose $\delta_{ij}^{+0} = \delta_{ij}^{-0} = 0$,

$$\gamma_i^{+0} = \sum_{j=1}^{n} a_{ij} x_j^0 - b_i, \tag{18}$$

and $\gamma_i^{-0} = 0$. Note, (17)-(18) and $\delta_{ij}^{+*} > 0$ implies that $\gamma_i^{+*} > \gamma_i^{+0}$. Thus, $(\boldsymbol{D}^{+0}, \boldsymbol{D}^{-0}, \boldsymbol{\gamma}^{+0}, \boldsymbol{\gamma}^{-0})$ is a solution better than $(\boldsymbol{D}^{+*}, \boldsymbol{D}^{-*}, \boldsymbol{\gamma}^{+*}, \boldsymbol{\gamma}^{-*})$, which leads to a contradiction.

(ii) Assume $\gamma_i^{-*} > 0$. Then, by Lemma 4.2, $\gamma_i^{+*} = 0$. From (i), $\delta_{ij}^{+*} = 0$. Thus,

$$\sum_{j=1}^{n}(a_{ij} - \delta_{ij}^{-*})x_j^0 \le b_i - \gamma_i^{-*}.$$

Choose $\delta_{ij}^{-0} = \delta_{ij}^{-*}$, $\delta_{ij}^{+0} = \gamma_i^{+0} = \gamma_i^{-0} = 0$. Then $(\boldsymbol{D}^{+0}, \boldsymbol{D}^{-0}, \boldsymbol{\gamma}^{+0}, \boldsymbol{\gamma}^{-0})$ is a feasible solution better than $(\boldsymbol{D}^{+*}, \boldsymbol{D}^{-*}, \boldsymbol{\gamma}^{+*}, \boldsymbol{\gamma}^{-*})$, which leads to a contradiction. □

According to Proposition 4.1 we could reduce the number of adjustment variables and obtain the following simplified CSA model.

Program 4.3.

$$\begin{aligned}
\min \quad & \sum_{(i,j)\in\Phi} \left\{\delta_{ij}^{-} / | a_{ij} |\right\} + \sum_{i=1}^{m}\left\{\gamma_i^{+} / h_i\right\} \\
s.t. \quad & \sum_{(i,j)\in\Phi} (a_{ij} - \delta_{ij}^{-})x_j^0 \le b_i + \gamma_i^{+}, i = 1, 2, \ldots, m, \\
& \delta_{ij}^{-} \ge 0, \gamma_i^{+} \ge 0.
\end{aligned} \tag{19}$$

Alternatively, modified goal programming techniques proposed by Li [43] can be applied to reduce the number of deviational variables as follows.

Program 4.4.

$$\begin{aligned}
\min \quad & \sum_{(i,j)\in\Phi} \left\{2\delta_{ij}^{-} / | a_{ij} |\right\} + \sum_{i=1}^{m}\left\{2\gamma_i^{+} / h_i\right\} \\
s.t. \quad & \sum_{(i,j)\in\Phi} (a_{ij} - \delta_{ij}^{-})x_j^0 \le b_i + \gamma_i^{+}, i = 1, 2, \ldots, m, \\
& \delta_{ij}^{-} \ge 0, \gamma_i^{+} \ge 0.
\end{aligned}$$

The interested reader is referred to [43], [44] for more detail. Note that Program 4.3 and Program 4.4 are almost identical except the objective function.

The model we have described considered no adjustment bounds and no costs incurred by adjusting the constraint coefficients. Practically, the degrees of adjustment may be bounded in a certain range as follows.

$$\delta_{ij}^{-} \le l_{ij}, i = 1,2,\ldots,m, j = 1,2,\ldots n, \tag{20}$$

$$\gamma_i^{+} \le u_i, i = 1,2,\ldots,m, \tag{21}$$

where l_{ij} and u_i denote the upper bounds for adjusting a_{ij} and b_i respectively. In addition, the budget constraint could be written as follows.

$$\sum_{i=1}^{m}\left[\left(\sum_{j=1}^{n} w_{ij}\delta_{ij}^{-}\right) + p_i\gamma_i^{+}\right] \le G, \tag{22}$$

where the cost for adjusting a_{ij} and b_i is denoted respectively by w_{ij} and p_i, and G denotes the available budget for adjustment.

By combing (19)-(22), we have a more practical and general CSA model as follows.

Program 4.5.

$$\begin{aligned}
\min \quad & \sum_{(i,j)\in\Phi} \left\{\delta_{ij}^{-} / | a_{ij} |\right\} + \sum_{i=1}^{m}\left\{\gamma_i^{+}/h_i\right\} \\
s.t. \quad & \sum_{(i,j)\in\Phi} (a_{ij} - \delta_{ij}^{-})x_j^0 \le b_i + \gamma_i^{+}, i = 1,2,\ldots,m, \\
& \delta_{ij}^{-} \le l_{ij}, (i,j) \in \Phi, \\
& \gamma_i^{+} \le u_i, i = 1,2,\ldots,m, \\
& \sum_{i=1}^{m}\left[\left(\sum_{j=1}^{n} w_{ij}\delta_{ij}^{-}\right) + p_i\gamma_i^{+}\right] \le G, \\
& \delta_{ij}^{-} \ge 0, \gamma_i^{+} \ge 0.
\end{aligned} \tag{23}$$

Example 4.1. Consider the following LP model.

$$\begin{aligned} \max \quad & 90x_1 + 70x_2 \\ s.t. \quad & 4x_1 + 2x_2 \le 200, \\ & 2x_1 + 3x_2 \le 240, \\ & x_1 \ge 0, x_2 \ge 0, \end{aligned}$$

where the optimal solution $\boldsymbol{x}^*$=(15, 70). Suppose the target solution $\boldsymbol{x}^0$=(20, 90) and the available budget for adjustment G=2,000 are set by the decision maker. $\boldsymbol{L} = \begin{bmatrix} 0.8 & 0.8 \\ 0.6 & 0.75 \end{bmatrix}$ denotes the maximum deviation of adjusting a_{ij}, $\boldsymbol{W} = \begin{bmatrix} 120 & 90 \\ 80 & 100 \end{bmatrix}$ denotes the unit price for adjusting a_{ij}, and $\boldsymbol{p}$=(75,85) denotes the unit price for purchasing extra resources.

According to Program 4.3, the optimal adjustment problem can be formulated as the following mathematical programming.

$$\begin{aligned} \min \quad & (\delta_{11}^-/4) + (\delta_{12}^-/2) + (\delta_{21}^-/2) + (\delta_{22}^-/3) + (\gamma_1^+/200) + (\gamma_2^+/240) \\ \text{s.t.} \quad & 20\delta_{11}^- + 90\delta_{12}^- + \gamma_1^+ \ge 60, \\ & 20\delta_{21}^- + 90\delta_{22}^- + \gamma_2^+ \ge 70, \\ & \delta_{11}^- \le 0.8, \\ & \delta_{12}^- \le 0.8, \\ & \delta_{21}^- \le 0.6, \\ & \delta_{22}^- \le 0.75, \\ & 120\delta_{11}^- + 90\delta_{12}^- + 80\delta_{21}^- + 100\delta_{22}^- + 75\gamma_1^+ + 85\gamma_2^+ \le 2000, \\ & \delta_{11}^- \ge 0, \delta_{12}^- \ge 0, \delta_{21}^- \ge 0, \delta_{22}^- \ge 0, \gamma_1^+ \ge 0, \gamma_2^+ \ge 0. \end{aligned}$$

By using LINGO software, an optimal solution (adjustment) is obtained as follows.

$$\boldsymbol{D}^{-*} = \begin{bmatrix} 0 & 1115/2664 \\ 0 & 3/4 \end{bmatrix}, \quad \boldsymbol{\gamma}^{+*} = (3305/148, 5/2).$$

The total ratio of changes is 0.581344 and the total cost for adjustment is 2,000. The adjusted LP model is presented as follows.

$$\begin{aligned} \max \quad & Z(x) = 90x_1 + 70x_2 \\ s.t. \quad & 4x_1 + 1.58x_2 \le 222.33, \\ & 2x_1 + 2.25x_2 \le 242.5, \\ & x_1 \ge 0, x_2 \ge 0, \end{aligned}$$

where the optimal solution $\boldsymbol{x}^*$=(20, 90). Note that the numerical values of the parameters are rounded off.

The graphical representation of the optimal adjustment of competence set is depicted in Figure 4-1 as follows.

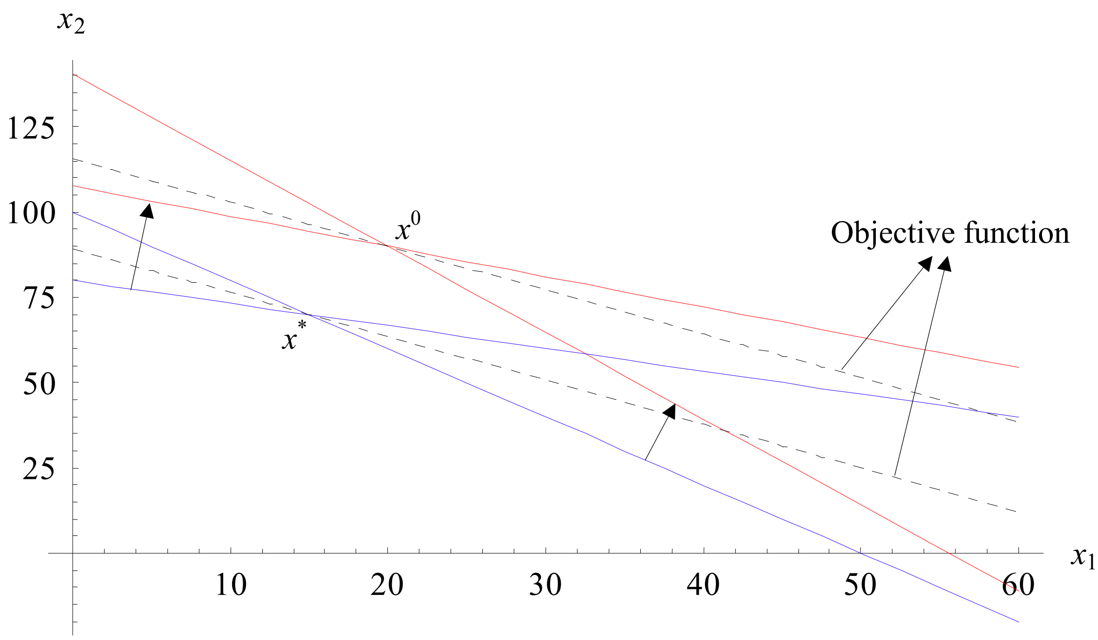

Figure 4-1 Graphical representation of the optimal adjustment of competence set in Example 4.1.

4.4 Bisection Algorithm

We have so far described how CSA models could be used to obtain the optimal adjustment so that x^0 becomes reachable. However, after having solved the CSA model, we may have no solution because of the limitation in the budget level and the bounds of adjustments. This section applies the *bisection algorithm* to find a revised target solution which approximates the original one and obtains its corresponding optimal adjustment.

Utilizing bisection algorithm is motivated in part by the behavior mechanism [11], [12], [13] in *Habitual Domains Theory* (HDs) which characterized two modes of behavior: *active problem solving* or *avoidance justification*. The former attempts to work actively to move the perceived states closer to the ideal states; while the latter tries to rationalize the situations so as to lower the ideal states closer to the perceived states.

Figure 4-2 graphically illustrates the bisection method. When $\boldsymbol{x}^0$ is unlikely to be reached, we bisect the interval $[\boldsymbol{x}^L, \boldsymbol{x}^R]$ and try to obtain the optimal adjustment of competence set with $\boldsymbol{x}^M(1)$ as the target. If $\boldsymbol{x}^M(1)$ is still impossible to be reached, then we bisect the interval $[\boldsymbol{x}^L, \boldsymbol{x}^M(1)]$ and try to obtain the optimal adjustment of competence set with $x^M(3)$ as the target. Otherwise, we bisect the interval $[\boldsymbol{x}^M(1), \boldsymbol{x}^R]$ and try to obtain the optimal adjustment of competence set with $\boldsymbol{x}^M(2)$ as the target. The above procedures continue until the sequence $\{\boldsymbol{x}^M(n)\}$ converges within certain bound.

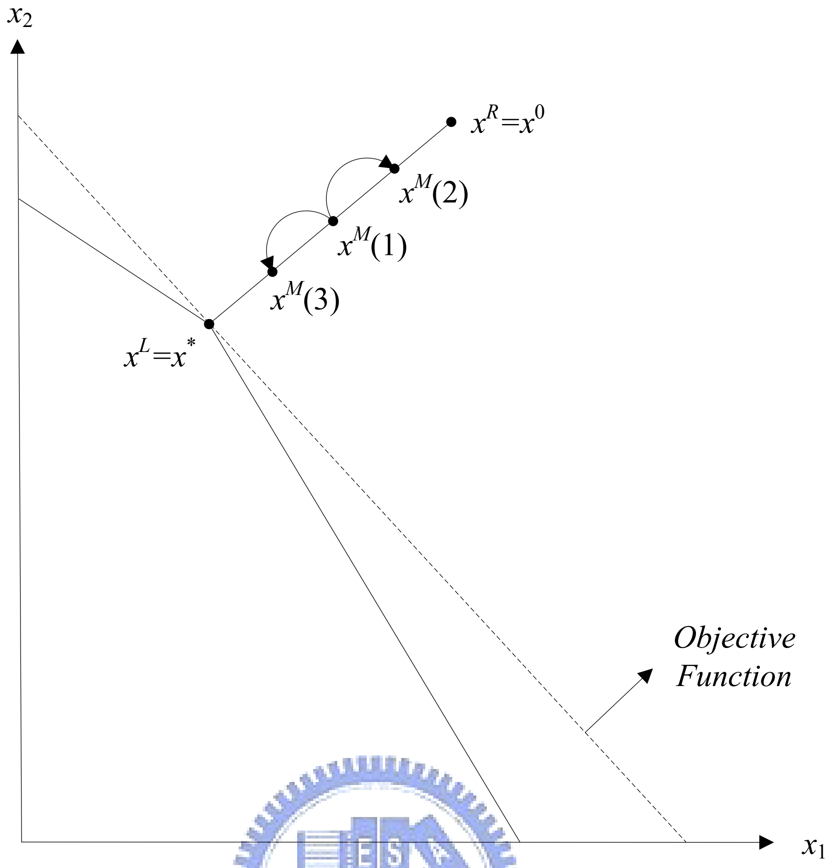

Figure 4-2 Graphical representation of the bisection method.

Based on the two modes of behavior in HDs, we operate the bisection algorithm as Algorithm 4.1 to find a revised target solution and its corresponding optimal adjustment.

Let $\boldsymbol{x}^*$ be the optimal solution in the original system, and $z(\boldsymbol{x}^*)$ be the objective value at $\boldsymbol{x}^*$ (with respect to the original objective function in (5)),

Algorithm 4.1.

Step 1. Set $\boldsymbol{x}^L=\boldsymbol{x}^*$, where $\boldsymbol{x}^L$ denotes the *left end point*, $\boldsymbol{x}^R=\boldsymbol{x}^0$, where $\boldsymbol{x}^R$ denotes the *right end point*, and $\boldsymbol{x}^M=\boldsymbol{x}^R$, where $\boldsymbol{x}^M$ denotes the *middle point* of the interval $[\boldsymbol{x}^L, \boldsymbol{x}^R]$.

Step 2. Choose an $\varepsilon > 0$, where ε denotes the tolerant discrepancy between $z(\boldsymbol{x}^M)$ and $z(\boldsymbol{x}^L)$.

Step 3. Solve (23) with $\boldsymbol{x}^M$ as the target to obtain $\boldsymbol{D}^-$ and γ^+.

Step 4. If (23) has no solution, set $\boldsymbol{x}^R=\boldsymbol{x}^M$ and go to Step 6; otherwise, go to Step 5.

Step 5. If the deviation of the objective value between $\boldsymbol{x}^M$ and $\boldsymbol{x}^L$ is smaller than ε, that is $|z(\boldsymbol{x}^M)\text{-}z(\boldsymbol{x}^L)|<\varepsilon$, stop and $\boldsymbol{x}^M$ is the desired critical target; otherwise, set $\boldsymbol{x}^L=\boldsymbol{x}^M$ and go to Step 6.

Step 6. Set $\boldsymbol{x}^M=(\boldsymbol{x}^L+\boldsymbol{x}^R)/2$ and go back to Step 3.

The flow chart of the Algorithm 4.1 is depicted in Figure 4-3.

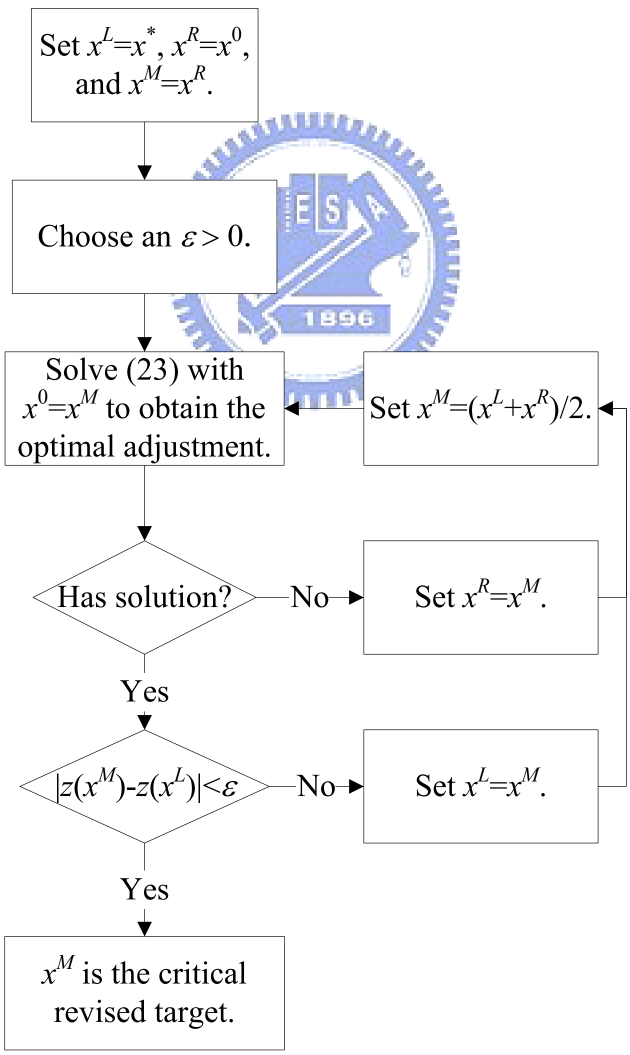

Figure 4-3 Flow chart of the Algorithm 4.1.

Theorem 4.1. Let $\{\boldsymbol{x}^M(n)\}$ be the sequence of middle points generated by the above algorithm. The sequence converges to a critical point $\boldsymbol{x}^c$.

Proof. The proof of this theorem is similar to that of the bisection method. For details, see [45]. □

Remark 4.2. Let $\Delta\boldsymbol{x}=\boldsymbol{x}^0-\boldsymbol{x}^c$. We may interpret Δx as the quantity of products that needs outsourcing if the target must be reached.

Example 4.2. Continue Example 4.1. When the decision maker sets the target to $\boldsymbol{x}^0$ = (70,100), we may obtain no solution by applying the CSA model. In this case, we utilize the bisection algorithm. By choosing ε=10, we obtain the sequence of middle points, $\{\boldsymbol{x}^M(n)\}$, as follows:

$\{\boldsymbol{x}^M(n)\}$={(42.5,85),(28.75,77.5),(35.63,81.25),(39.06,83.13),(37.34,82.19),(38.2,82.66),(38.63, 82.89),(38.42,82.77),(38.53,82.83),(38.47,82.8)}.

The revised target solution is (38.47, 82.80). The optimal adjustment corresponding to the revised target can be derived from the CSA model as follows.

$$\boldsymbol{D}^- = \begin{bmatrix} 0.8 & 0.8 \\ 0.6 & 0.75 \end{bmatrix},\ \boldsymbol{\gamma}^+ = (22.47, 0.28).$$

The total ratio of changes is 1.26206 and the total cost for adjustment is 2,000. The adjusted LP model is listed as follows.

$$\begin{aligned} \max\quad & Z(\boldsymbol{x}) = 90x_1 + 70x_2 \\ s.t.\quad & 3.2x_1 + 1.2x_2 \le 222.47, \\ & 1.4x_1 + 2.25x_2 \le 240.28, \\ & x_1 \ge 0, x_2 \ge 0, \end{aligned}$$

where the optimal solution $\boldsymbol{x}^*$=(38.47, 82.80). Note that the numerical values of the parameters are rounded off.

4.5 Target Revision by the Fuzzy Linear Programming

The use of the *fuzzy linear programming* (FLP) technique in this study is motivated in part by the nature of the optimal adjustment of competence set problems that some constraint coefficients may be adjusted within some tolerant ranges. We may treat these coefficients with the fuzzy sets and then formulate a FLP model with a crisp objective function. In turn, the revised target could be derived by solving the FLP model.

While the bisection method finds a revised target solution which approximates the original optimal solution (a status quo), the FLP techniques allow the decision maker (DM) to interactively select an achievable target. This section demonstrates how the FLP can help the DM to interactively revise the unattainable targets as to get the final target.

FLP problems with a crisp objective function [46], [47] could be represented as follows.

$$\begin{aligned} \max \quad & z = \boldsymbol{cx} \\ s.t. \quad & \tilde{\boldsymbol{A}}x \le \tilde{\boldsymbol{b}}, \\ & \boldsymbol{x} \ge 0, \end{aligned} \tag{24}$$

where $\tilde{\boldsymbol{A}}$ is the $m \times n$ consumption (or productivity) matrix whose elements $\tilde{a}_{ij}$ are fuzzy sets with membership function $\mu_{\tilde{a}_{ij}}$, and $\tilde{\boldsymbol{b}}$ is the $m\times 1$ resource availability vector whose components $\tilde{b}_i$ are fuzzy sets with membership function $\mu_{\tilde{b}_i}$. After having defined the appropriate membership function and α parameter, we could transform (24) into (25) as follows.

$$\begin{aligned} \max \quad & z=\sum_{j=1}^{n} c_j x_j \\ s.t. \quad & \sum_{j=1}^{n} \mu_{\tilde{a}_{ij}}^{-1}(\alpha) x_j \le \mu_{\tilde{b}_i}^{-1}(\alpha), \forall i=1,\ldots,m, \\ & x_j \ge 0, \forall j=1,\ldots,n. \end{aligned} \tag{25}$$

In order to apply (25) to solve FLP problems, membership functions have to be defined for the fuzzy sets of the constraint coefficients first. Assume that $a_{ij} \in [a_{ij}^0, a_{ij}^0 + d_{ij}]$ (interval from a_{ij}^0 to $a_{ij}^0 + d_{ij}$), and $b_i \in [b_i^0 - h_i, b_i^0]$. Note that d_{ij} and h_i are the maximum tolerable deviation from a_{ij}^0 and b_i^0 respectively. Given $\alpha \in [0,1]$, let $a_{ij}(\alpha) = a_{ij}^0 + (1-\alpha) d_{ij}$, and $b_i(\alpha) = b_i^0 - (1-\alpha) h_i$. To illustrate the method, assume that the membership functions of the fuzzy sets $\tilde{a}_{ij}$ and $\tilde{b}_i$ are linear as follows.

$$\mu_{\tilde{a}_{ij}}(a_{ij}(\alpha)) = \begin{cases} 1 \text{ , if } a_{ij}(\alpha) < a_{ij}^0, \\ \alpha, \text{ if } a_{ij}^0 \le a_{ij}(\alpha) \le a_{ij}^0 + d_{ij}, \\ 0 \text{ , if } a_{ij}(\alpha) > a_{ij}^0 + d_{ij}. \end{cases}$$

$$\mu_{\tilde{b}_i}(b_i(\alpha)) = \begin{cases} 1 \text{ , if } b_i(\alpha) > b_i^0, \\ \alpha, \text{ if } b_i^0 - h_i \le b_i(\alpha) \le b_i^0, \\ 0 \text{ , if } b_i(\alpha) < b_i^0 - h_i. \end{cases}$$

The graphical representations of the above two membership functions are depicted in Figure 4-4.

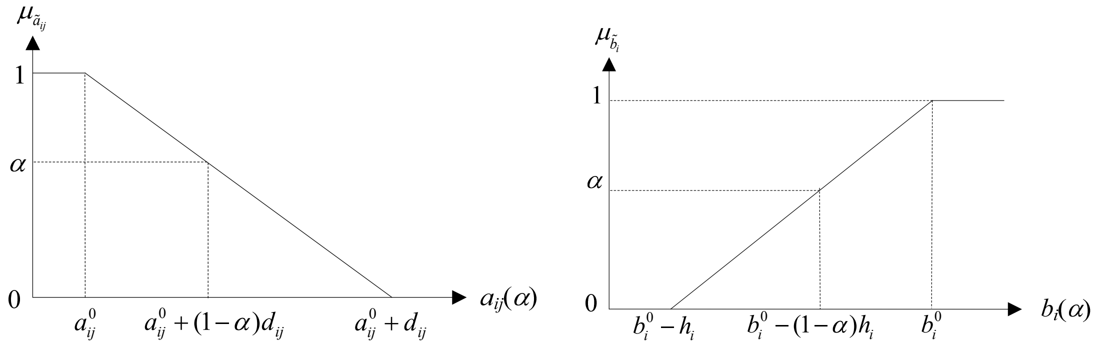

Figure 4-4 The membership functions of the fuzzy sets $\tilde{\boldsymbol{A}}$ and $\tilde{\boldsymbol{b}}$.

Given α, confidence or tolerable level, the following linear programming problem can be set to find the desired target.

$$\begin{aligned} \max \quad & z = \sum_{j=1}^{n} c_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a_{ij}(\alpha) x_j \le b_i(\alpha), \forall i = 1, \ldots, m, \\ & x_j \ge 0, \forall j = 1, \ldots, n. \end{aligned} \tag{26}$$

Therefore, by varying α within 0 to 1 we can derive a set of optimal solutions for the corresponding targets.

Example 4.3. Continue Example 4.1. When the DM set the target to $\boldsymbol{x}^0$ = (70,100), we may obtain no solution by applying the CSA model. In this case, we utilize the fuzzy linear programming techniques as follows.

$$\begin{aligned} \max \quad & 90x_1 + 70x_2 \\ s.t. \quad & [3.2^{\alpha=1}, 4^{\alpha=0}]x_1 + [1.2^{\alpha=1}, 2^{\alpha=0}]x_2 \le [200^{\alpha=0}, 226.6^{\alpha=1}], \\ & [1.4^{\alpha=1}, 2^{\alpha=0}]x_1 + [2.25^{\alpha=1}, 3^{\alpha=0}]x_2 \le [240^{\alpha=0}, 263.5^{\alpha=1}], \\ & x_1 \ge 0, x_2 \ge 0. \end{aligned}$$

Suppose that the value of α has been given by the DM as 0.8, a linear programming problem could be obtained as follows by (26).

$$\begin{aligned} \max \quad & 90x_1 + 70x_2 \\ s.t. \quad & 3.36x_1 + 1.36x_2 \le 221.28, \\ & 1.52x_1 + 2.40x_2 \le 258.8, \\ & x_1 \ge 0, x_2 \ge 0, \end{aligned}$$

where the optimal solution is $\boldsymbol{x}^*$=(29.87,88.92). Then, we may solve Program 4.3 with $\boldsymbol{x}^*$ as the target and obtain the corresponding optimal adjustment of competence set listed as follows.

Table 4-1 shows the optimal adjustments of competence set by varying the value of α within {0, 0.1, 0.2,…, 1}, where $\boldsymbol{x}^*$ presents the optimal solution derived from solving (26) with a given α. The optimal adjustment of competence set is obtained by solving Program 4.3 with $\boldsymbol{x}^*$ as the target. The total adjustment ratio corresponding to the optimal adjustment is also listed. Note that by setting α = 0.9 and α = 1 we obtain the revised target: $\boldsymbol{x}^*$=(32.37,91.99) and $\boldsymbol{x}^*$=(35.08,95.28) respectively and there is no feasible adjustment. This is due to the constraints imposed on the budget and on the tolerant ranges of adjustment. In this case, the DM may decrease the value of α or apply the aforementioned bisection method.

Table 4-1 Optimal adjustment of competence set with different α values.

α	x^*	Optimal adjustment of competence set	Total adjustment ratio
0.0	(15,70)	$D^-=0$, $\gamma^+=0$	0
0.1	(16.47,71.93)	$D^-=0$, $\gamma^+=(9.74, 8.73)$	0.085075
0.2	(18.03,73.97)	$D^-=\begin{bmatrix} 0 & 0 \\ 0 & 0.166781 \end{bmatrix}$, $\gamma^+=(20.06,5.63)$	0.179365
0.3	(19.68,76.12)	$D^-=\begin{bmatrix} 0 & 0.063787 \\ 0 & 0.364162 \end{bmatrix}$, $\gamma^+=(26.1046,0)$	0.283803
0.4	(21.45,78.39)	$D^-=\begin{bmatrix} 0 & 0.214547 \\ 0 & 0.485649 \end{bmatrix}$, $\gamma^+=(25.7617,0)$	0.397965
0.5	(23.35,80.79)	$D^-=\begin{bmatrix} 0 & 0.365915 \\ 0 & 0.607377 \end{bmatrix}$, $\gamma^+=(25.4177,0)$	0.512505
0.6	(25.37,83.33)	$D^-=\begin{bmatrix} 0 & 0.516803 \\ 0 & 0.728789 \end{bmatrix}$, $\gamma^+=(25.0748,0)$	0.626705
0.7	(27.54,86.04)	$D^-=\begin{bmatrix} 0 & 0.78 \\ 0 & 0.75 \end{bmatrix}$, $\gamma^+=(14.90,8.67)$	0.751935
0.8	(29.87,88.92)	$D^-=\begin{bmatrix} 0.8 & 0.8 \\ 0.04 & 0.75 \end{bmatrix}$, $\gamma^+=(2.288,18.614)$	0.959017
0.9	(32.37,91.99)	No feasible adjustment	-
1.0	(35.08,95.28)	No feasible adjustment	-

CHAPTER 5 LINEAR PROGRAMMING MODELS WITH CHANGEABLE PARAMETERS - THEORETICAL ANALYSIS ON "TAKING LOSS AT THE ORDERING TIME AND MAKING PROFIT AT THE DELIVERY TIME"

Corporate competence sets can be expanded through capital investment and be dynamically changed overtime, which can explain the phenomenon of “taking loss at the ordering time and making profit at the time of delivery”. Such phenomenon has existed in practice for a long time, but there are no mathematical model that can explain it adequately.

In this chapter we utilize multiple criteria and multiple constraint levels linear programming (MC^2LP) model and its extended techniques to explore the linear programming models with changeable parameters. The parameters include: unit profit, available resources and input-output coefficients of production function. With those parameters changed with capital investment and/or time, we study how to find dynamic best solutions to make “taking loss at the ordering time and making profit at the time of delivery” feasible. For more general cases we also sketch a generalized mathematical programming model with changeable parameters and control variables.

5.1 Introduction to Linear Programming Models with Changeable Parameters

Why many innovative companies, especially in high-tech industries, are willing to take orders which offer deficits (red figures) at the ordering time? Because in their mind, perhaps, based on their calculation or intuition, they could eventually make profit (black figures) at the delivery time. The time interval from ordering time to delivery time offers the precious window of opportunity for the companies to improve their related technology, market conditions and resource availability.

Similarly, many companies are willing to introduce new products or services which can only offer deficits (red figures) at the time of introduction because their managements believe that eventually their products or services can make profits (black figures) in the due time. The celebrated product Walkman of Sony Inc. is a notable example. It was estimated that Sony would lose $35 for each item sold at the time of first introduction (1979). With changes of parameters, including technology improvement, market conditions and resource availability, Walkman eventually reaps big profit for Sony Inc. (For the details of Walkman transition, see [48])

This research has been motivated by the above observation. For simplicity, the phenomenon described above will be called "*Red in-Black out*" phenomenon. We want to explain such phenomenon by using "programming models in changeable space". Especially we will use multi-criteria and multi-constraint level (MC^2) simplex method to analyze the phenomenon, and show how we can fine tune our computation so that "Red in-Black out" can indeed become a vital business strategy in competition. We will formulate the problems into MC^2-simplex models depending on that the parameter changes are caused either by purposeful investment or by predicted trend of changes. The parameters under consideration includes objective coefficients that reflect changes of market condition, resources availability that could be changed by investment or outsourcing, and productivity coefficients that could be changed by technology and production improvement.

Note that because of changes of relevant parameters, innovative companies are willing to take risk as to have “Red in-Black out” phenomenon. By adapting and/or controlling the changes of the parameters, companies can eventually reap handsome profit. In terms of habitual domain theory [11], [12], this is a proactive attitude toward changes. The vision of strategic planning is over the entire domains within which the parameters could changes. The problems can be studied by using MC^2-simplex method, which consider all possible changes of the parameters. We did not use the formats of sensitivity analysis or parameter variation method such as those studied by Bradley, Hax, and Magnanti [49], Wendell [50], [51], [52] Hiller and Lieberman [53], and Gal [54], because such formats basically are of local properties, not global or entire space of changes. They inherit passive, not proactive attitude, and could not study our problems fully. However, we notice that the format of sensitivity analysis and parameter variation could still provide useful information in other setting of decision making.

5.2 Preliminary: MC^2-simplex Method

A typical linear programming model has the following format.

$$
\begin{aligned}
\max \quad & c_1x_1 + c_2x_2 + \cdots + c_nx_n \\
s.t. \quad & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le d_1, \\
& a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le d_2, \\
& \cdots \\
& a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le d_m, \\
& x_j \ge 0, j = 1, 2, \cdots, n.
\end{aligned} \tag{27}
$$

Let $\boldsymbol{x}=[x_1\ x_2\ \ldots\ x_n]^T$ be the decision vector; $\boldsymbol{c}=[c_1\ c_2\ \ldots\ c_n]^T$, the objective coefficient vector; $\boldsymbol{A}=[a_{ij}]$ (i=1,…,m; j=1,…,n), the resource consumption (or productivity) matrix; and $\boldsymbol{d}=[d_1\ d_2\ \ldots\ d_m]^T$, the resource availability vector.

Then model (27) can be represented by matrix form, as shown in (28).

$$
\begin{aligned}
\max \quad & \boldsymbol{cx} \\
s.t. \quad & \boldsymbol{Ax} \le \boldsymbol{d}, \\
& \boldsymbol{x} \ge 0.
\end{aligned} \tag{28}
$$

For product mix optimization problem, the element of decision vector $\boldsymbol{x}$ represents the production unit; the element of vector $\boldsymbol{c}$, the unit profit for each product; the element in matrix $\boldsymbol{A}$, the consumption unit of different resources by different products; and the element of vector $\boldsymbol{d}$, the available level for each type of resources.

In multiple-criteria and multiple-constraint level (MC^2) simplex method, there are multiple criteria: $\boldsymbol{C}=[\boldsymbol{c}^1\ \boldsymbol{c}^2\ \ldots\ \boldsymbol{c}^q]^T$ is a $q\times n$ matrix, where $\boldsymbol{c}^k$, k=1,…,q, is an n-dimension vector representing the kth criteria; and there are r multiple constraint levels: $\boldsymbol{D}=[\boldsymbol{d}^1\ \boldsymbol{d}^2\ \ldots\ \boldsymbol{d}^r]$ is an $m\times r$ matrix, where $\boldsymbol{d}^k$, k=1,…,r, is the kth constraint levels for the resources. The MC^2-simplex model thus has the following format:

$$
\begin{aligned}
\max \quad & \boldsymbol{Cx} \\
s.t. \quad & \boldsymbol{Ax} \le \boldsymbol{D}, \\
& \boldsymbol{x} \ge 0.
\end{aligned} \tag{29}
$$

In MC^2-simplex literature [14], [55], [56], [57], [58], $\boldsymbol{x}^0$ is a *potential solution* to model (29) if there is a pair of weight vectors ($\boldsymbol{\lambda}$, $\boldsymbol{\sigma}$), $\boldsymbol{\lambda}$>0, $\boldsymbol{\sigma}$>0 such that $\boldsymbol{x}^0$ solves the following model:

$$\begin{aligned} \max \quad & \boldsymbol{\lambda C x} \\ s.t. \quad & \boldsymbol{Ax} \leq \boldsymbol{D\sigma}, \\ & \boldsymbol{x} \geq 0. \end{aligned} \tag{30}$$

The simplex tableau of model (30) can be represented by:

$\boldsymbol{A}$	$\boldsymbol{I}$	$\boldsymbol{D\sigma}$
$-\boldsymbol{\lambda C}$	**0**	**0**

Given a basis B with index set J for the basic variables, let J' be the index set of the corresponding non-basic variables. The simplex tableau of basis J can be represented as:

$\boldsymbol{B}^{-1}\boldsymbol{A}$	$\boldsymbol{B}^{-1}$	$\boldsymbol{B}^{-1}\boldsymbol{D\sigma}$
$\boldsymbol{\lambda C_B B}^{-1}\boldsymbol{A}-\boldsymbol{\lambda C}$	$\boldsymbol{\lambda C_B B}^{-1}$	$\boldsymbol{\lambda C_B B}^{-1}\boldsymbol{D\sigma}$

where, $\boldsymbol{C_B}$ is the submatrix of criteria matrix corresponding to basic variables in J. Dropping the $\boldsymbol{\sigma}$ and $\boldsymbol{\lambda}$ in the simplex tableau, the MC^2 simplex tableau of basis J becomes:

$\boldsymbol{B}^{-1}\boldsymbol{A}$	$\boldsymbol{B}^{-1}$	$\boldsymbol{B}^{-1}\boldsymbol{D}$
$\boldsymbol{C_B B}^{-1}\boldsymbol{A}-\boldsymbol{C}$	$\boldsymbol{C_B B}^{-1}$	$\boldsymbol{C_B B}^{-1}\boldsymbol{D}$

Set $Y(J)=[\boldsymbol{B}^{-1}\boldsymbol{A}, \boldsymbol{B}^{-1}]$, $W(J)=[\boldsymbol{B}^{-1}\boldsymbol{D}]$, $Z(J)=[\boldsymbol{C_B B}^{-1}\boldsymbol{A}-\boldsymbol{C}, \boldsymbol{C_B B}^{-1}]$ and $V(J)=[\boldsymbol{C_B B}^{-1}\boldsymbol{D}]$. The above simplex tableau can be simplified as:

$Y(J)$	$W(J)$

$Z(J)$	$V(J)$

Let $W(J)$ and $Z(J)$ be the sub-matrix of a MC^2 simplex tableau. Define

$$\Gamma(J)=\{\boldsymbol{\sigma}>0 \mid W(J)\boldsymbol{\sigma}\geq 0\}, \tag{31}$$

$$\Lambda(J)=\{\boldsymbol{\lambda}>0 \mid \boldsymbol{\lambda} Z(J)\geq 0\}. \tag{32}$$

The following is well known; see Yu [14] or Shi [56] for instance. (For extensive discussion of the MC^2-simplex method relative to fuzzy programming, the reader is referred to [59], [60], [61], [62].)

Theorem 5.1. The basis with index set J is a potential solution if and only if $\Lambda(J)\times\Gamma(J)\neq\emptyset$. That is, J is a potential solution if and only if there exist some $\boldsymbol{\lambda}>0$ and $\boldsymbol{\sigma}>0$ such that J is the index set of the optimal basic variables for model (29).

We can generate all potential bases or solutions together with their parameter spaces Γ and Λ. An example is provided in the next section.

5.3 Parameter Changes through Investment on Resources and Marketing

It is well known that companies can change the marketing condition through investment in advertisement, service and distribution channels, etc. We shall aggregate the impacts on the changes of marketing conditions by the change of the coefficient, ***c***, of the objective function. Likewise, we aggregate the impact of the investment effort (such as making alliance, outsourcing, extra resource allocation…) on the resource availability by the change of ***d***, the level of resource availability.

In order to facilitate the presentation, we will start with a concrete illustration of a simple example in subsection 5.3.1. Then the concepts are then generalized in subsection 5.3.2.

5.3.1 A Concrete Example

Example 5.3.1. A company produces two types of products, denoted by Type I and Type II, using two kinds of resources said material resource and human resource. The available resource levels of material and human resource are 100 and 120 units, respectively. Unit profits, resource consumption rates and available resource levels are summarized in Table 1. Note that unit profits of Type I and Type II products can make -3 and -5 units of profits, respectively. In other words, producing Type I or/and Type II products will not make profits at current setting. The decision maker wishes to find the optimal products mix for the company by the mathematical programming model.

Table 5-1 Unit profits, resource consumption rates and available resource levels in Example 5.3.1

Resource	Type I	Type II	Available Resource Level
Material Resource	5	3.5	100
Human Resource	2.5	2	120
Unit Profits of Products	-3	-5	

According to Table 5-1, we can set the linear programming model (33) as follows:

$$\begin{aligned} \max \quad & -3x_1 - 5x_2 \\ s.t. \quad & 5x_1 + 3.5x_2 \le 100, \\ & 2.5x_1 + 2x_2 \le 120, \\ & x_1, x_2 \ge 0, \end{aligned} \qquad (33)$$

where x_1 and x_2 are decision variables representing the production units of Type I and Type II products, respectively.

The optimal solution for model (33) is $(x_1, x_2)=(0, 0)$ and the objective value of the model is 0. In other words, since no profit can be made, the optimal decision is not producing any products.

Assume that per unit of investment, the profit rates of Type I and II products can be improved by 0.4 and 0.3 units, respectively, and the available resource levels of material resource and human resource can be improved by 2.5 and 1 units, respectively, as shown in Table 5-2.

Table 5-2 Profit rates and resource levels change for each unit of investment.

Resource	Type I	Type II	Available Resource Level	Change rates for resource level by investment
Material Resource	5	3.5	100	2.5
Human Resource	2.5	2	120	1
Unit Profits of Products	-3	-5		
Change rates for profit improvement by investment	0.4	0.3		

Let y and z be the investment put into improving the profit rate and resource availability respectively. Assume there are upper limit constraints: $y \le 200$, $z \le 300$, and $y+z \le 400$.

With this new information, we can formulate model (34) to solve the model.

$$
\begin{aligned}
\max \quad & (-3x_1 - 5x_2) + y(0.4x_1 + 0.3x_2) \\
s.t. \quad & 5x_1 + 3.5x_2 \le 100 + 2.5z, \\
& 2.5x_1 + 2x_2 \le 120 + z, \\
& 0 \le y \le 200, \\
& 0 \le z \le 300, \\
& y + z \le 400, \\
& x_1, x_2 \ge 0.
\end{aligned} \tag{34}
$$

Model (34) is a mathematical programming model. Since we are more interested in the impact of the solution changes for the changes of the parameter of y and z, we formulate model (34) into model (35) in matrix form, with extra constraint (36). The constraints of (36) are assumed to be imposed on the investment in the market and the resources, including upper limits and the total investment.

$$
\begin{aligned}
\max \quad & \begin{bmatrix} 1 & y \end{bmatrix} \begin{bmatrix} -3 & -5 \\ 0.4 & 0.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\
s.t. \quad & \begin{bmatrix} 5 & 3.5 \\ 2.5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \le \begin{bmatrix} 100 & 2.5 \\ 120 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ z \end{bmatrix}, \\
& \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \ge 0,
\end{aligned} \tag{35}
$$

$$
0 \le \begin{bmatrix} y \\ z \\ y+z \end{bmatrix} \le \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}. \tag{36}
$$

Note that model (35), excluding (36), is a basic MC^2-simplex format. Using the MC^2-simplex method, we can locate all potential solutions (or bases) and their corresponding MC^2-simplex tableaus as listed in Table 5-3.

Table 5-3 MC^2-simplex tableaus for the potential bases of model (35).

Basis of x_3 and x_4; J={3, 4}.

	x_1	x_2	x_3	x_4	RHS	
x_3	5.0000	3.5000	1.0000	0.0000	100.0000	2.5000
x_4	2.5000	2.0000	0.0000	1.0000	120.0000	1.0000
	3.0000	5.0000	0.0000	0.0000	0.0000	0.0000
	-0.4000	-0.3000	0.0000	0.0000	0.0000	0.0000

Basis of x_1 and x_4; J={1, 4}.

	x_1	x_2	x_3	x_4	RHS	
x_1	1.0000	0.7000	0.2000	0.0000	20.0000	0.5000
x_4	0.0000	0.2500	-0.5000	1.0000	70.0000	-0.2500
	0.0000	2.9000	-0.6000	0.0000	-60.0000	-1.5000
	0.0000	-0.0200	0.0800	0.0000	8.0000	0.2000

Basis of x_1 and x_2; J={1, 2}.

	x_1	x_2	x_3	x_4	RHS	
x_1	1.0000	0.0000	1.6000	-2.8000	-176.0000	1.2000
x_2	0.0000	1.0000	-2.0000	4.0000	280.0000	-1.0000
	0.0000	0.0000	5.2000	-11.6000	-872.0000	1.4000
	0.0000	0.0000	0.0400	0.0800	13.6000	0.1800

Basis of x_4 and x_2; J={2, 4}.

	x_1	x_2	x_3	x_4	RHS	
x_4	-0.3571	0.0000	-0.5714	1.0000	62.8571	-0.4286
x_2	1.4286	1.0000	0.2857	0.0000	28.5714	0.7143
	-4.1429	0.0000	-1.4286	0.0000	-142.8571	-3.5714
	0.0286	0.0000	0.0857	0.0000	8.5714	0.2143

Basis of x_1 and x_3; J={1, 3}.

	x_1	x_2	x_3	x_4	RHS	
x_1	1.0000	0.8000	0.0000	0.4000	48.0000	0.4000
x_3	0.0000	-0.5000	1.0000	-2.0000	-140.0000	0.5000
	0.0000	2.6000	0.0000	-1.2000	-144.0000	-1.2000
	0.0000	0.0200	0.0000	0.1600	19.2000	0.1600

The optimal parameter spaces Γ and Λ, defined in (31) and (32) respectively, for each potential basis can be computed systematically using Table 5-3. Note that $\boldsymbol{\lambda}$=(1, y) and $\boldsymbol{\sigma}$=(1, z). Take the basis of x_3 and x_4, i.e. J={3,4}, as an example. By solving:

$$100+2.5z\geq 0,\ 120+z\geq 0,\ z\geq 0,$$

we have the optimal range of z value is: $z\geq 0$. Thus, Γ({3, 4})={z|$z\geq 0$}. Similarly, by solving:

$$3-0.4y\geq 0,\ 5-0.3y\geq 0,\ y\geq 0,$$

we have the optimal range of y is: $0\leq y\leq 7.5$. Thus, Λ({3, 4})={y|$0\leq y\leq 7.5$}.

The optimal parameter spaces in terms of y and z for other potential solutions can be computed similarly. We list the results in Table 5-4.

Table 5-4 The optimal parameter spaces for the potential solutions of model (35).

Basis	y		z	
	Lower bound	Upper bound	Lower bound	Upper bound
J={3, 4}	0	7.5	0	Infinite
J={1, 4}	7.5	145	0	280
J={1, 2}	145	Infinite	146.67	280

J={2, 4}	14.2857	Infinite	0	86.3158
J={1, 3}	6.6667	15.8621	120	Infinite

Table 5-4 offers the information of the potential solution structure for model (35), including each potential solution/basis J together with its $\Lambda(J)\times\Gamma(J)$ in terms of y and z. This information can be depicted as in Figure 5-1. Note in Figure 5-1 by setting $\Theta(J)=\Lambda(J)\times\Gamma(J)$, we see that J is the optimal basis when $(y, z)\in\Theta(J)$.

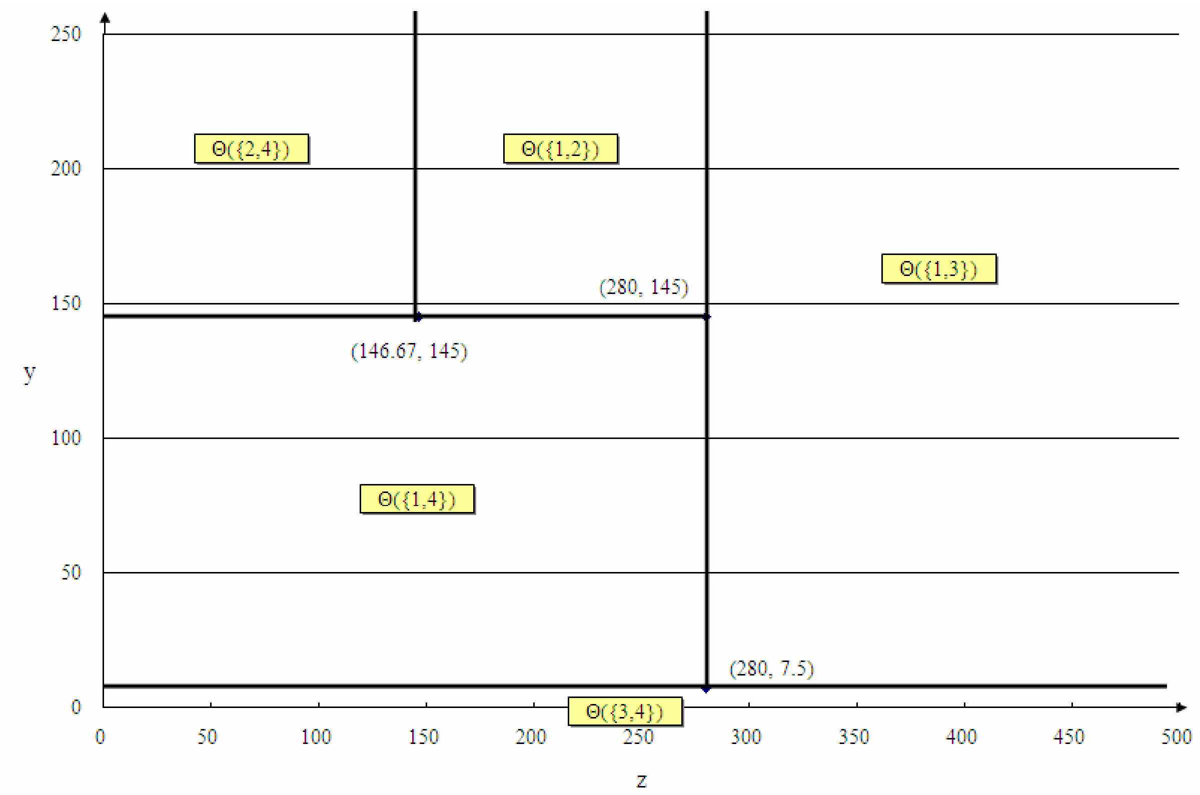

Figure 5-1 The potential solution structure of model (35) when y and z are not limited.

Now, let us consider the investment constraint (36), which can be depicted as shown in Figure 5-2.

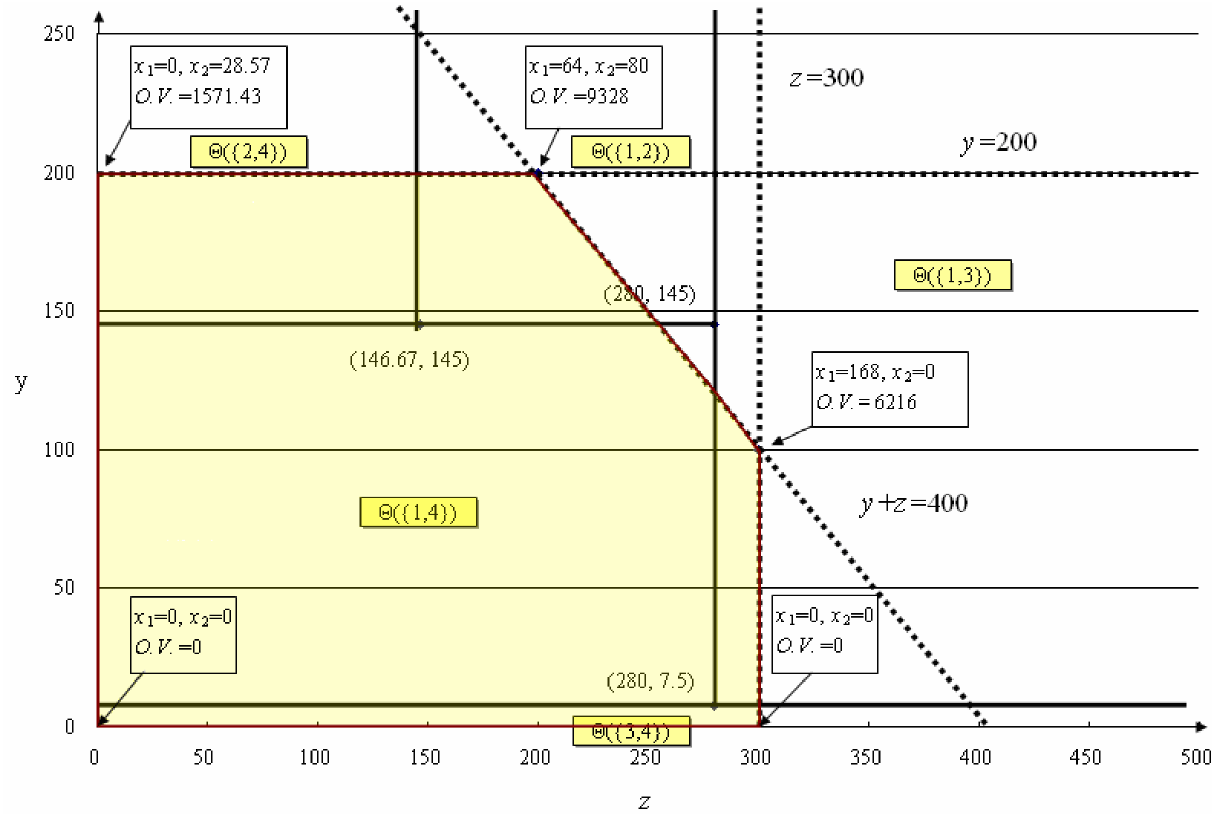

Figure 5-2 The potential solution structure of model (35) with investment constraints (36).

Calculating the corner points of the feasible parameter space in Figure 5-2, we obtain:

1. When (y, z)=(200, 0), optimal solution (x_1, x_2)=(0, 28.57), and the optimal objective value is 1571.43.

2. When (y, z)=(200, 200), optimal solution (x_1, x_2)=(64, 80), and the optimal objective value is 9328.

3. When (y, z)=(100, 300), optimal solution (x_1, x_2)=(168, 0), and the optimal objective value is 6216.

4. When (y, z)=(0, 300), optimal solution (x_1, x_2)=(0, 0), and the optimal objective value is 0.

Figure 5-2 offers useful information. By varying the constraints on (y, z), the decision maker can have a full spectrum of decision outcomes, which can lead to his/her final decision on (y, z) and (x_1, x_2).

We note that in the original model (34), the optimal solution is (x_1, x_2)=(0, 0) with objective value equal to 0. By the change of the parameter (y, z), the optimal solution changes with larger objective value. This change and improvement are due to the change of relevant parameters, which is an important method to expand our habitual domains as to improve our life. For the details of this method and others see Yu [11], [12].

Let us generalize the above concrete illustration in the following subsection.

5.3.2. Parameter Changes by Investment in Resources and Markets

Assume that objective coefficients, namely, elements of vector ***c***, of model (27) are linear functions of the capital investment, which can be represented by equation (37).

$$c_j = f_{c_j}(y) = c_{j,0} + c_{j,1}y, j = 1,\ldots,n, \tag{37}$$

where, $c_{j,0}$ is the original profit rate, $c_{j,1}$ is the increased profit rate for each investment unit, and y is the investment for increasing unit profit rates.

Assume that the available resource levels, namely, elements of array ***d***, are linear functions of the capital investment, which can be represented by equation (38).

$$d_i = f_{d_i}(z) = d_{i,0} + d_{i,1}z, i = 1,\ldots,m, \tag{38}$$

where, $d_{i,0}$ is the original available resource level, $d_{i,1}$ is the increased unit for each investment unit, and z is the investment for increasing resource available levels.

By the definition of c and d in equations (37) and (38), model (27) can be generalized as:

$$
\begin{aligned}
\max \quad & (c_{1,0}+c_{1,1}y)x_1+\cdots+(c_{j,0}+c_{j,1}y)x_2+\cdots+(c_{n,0}+c_{n,1}y)x_n \\
s.t. \quad & a_{11}x_1+\cdots+a_{1j}x_j+\cdots+a_{1n}x_n \le (d_{1,0}+d_{1,1}z), \\
& \cdots \\
& a_{i1}x_1+\cdots+a_{ij}x_j+\cdots+a_{in}x_n \le (d_{i,0}+d_{i,1}z), \\
& \cdots \\
& a_{m1}x_1+\cdots+a_{mj}x_j+\cdots+a_{mn}x_n \le (d_{m,0}+d_{m,1}z), \\
& x_j \ge 0, j=1,2,\cdots,n, \\
& y,z \ge 0.
\end{aligned} \tag{39}
$$

Model (39) in matrix form can be represented as model (40).

$$
\begin{aligned}
\max \quad & \boldsymbol{\lambda C x} \\
s.t. \quad & \boldsymbol{Ax} \le \boldsymbol{D\sigma}, \\
& \boldsymbol{x} \ge 0, \\
& \boldsymbol{\lambda} \ge 0, \\
& \boldsymbol{\sigma} \ge 0.
\end{aligned} \tag{40}
$$

where $\boldsymbol{\lambda}$=(1, y), $\boldsymbol{\sigma}$=(1, z), $\boldsymbol{x}$=[x_1 x_2 … x_n]^T is decision vector, $\boldsymbol{C} = \begin{bmatrix} c_{1,0} & c_{2,0} & \ldots & c_{n,0} \\ c_{1,1} & c_{2,1} & \ldots & c_{n,1} \end{bmatrix}$ is the profit rates matrix (including original objective coefficients and the change rates of profit by investment), $\boldsymbol{A}$=[a_{ij}] (i=1,…,m; j=1,…,n) is the resource consumption matrix, and $\boldsymbol{D}$ = $\begin{bmatrix} d_{1,0} & d_{2,0} & \ldots & d_{m,0} \\ d_{1,1} & d_{2,1} & \ldots & d_{m,1} \end{bmatrix}^T$ is the available resource levels matrix (including original available resource levels and increased units of resource by each unit of investment).

Model (40) can be solved by the MC^2-simplex method. The set of all potential solutions/bases can be obtained systematically as illustrated in subsection 5.3.1.

Observe that in model (40), there are only two parameters, y and z, that are subject to change. Useful information provided by Table 5-4 and Figure 5-1 of Example 5.3.1 can also be constructed for model (40).

Suppose that there are constraints imposed on the investment. We can easily add them to model (40), like inequality (36) adding to model (35). We shall not stop to do so.

Note that the constraints on investment offer useful information for final decision. Nevertheless, the constraints itself can also be subject to change. A bright decision maker certainly would like to keep this option for better decision.

5.4 "Red in-Black out" Phenomenon-Parameter Changes in *c* and *d* due to Time Advancement

In this section, we will focus on the parameter changes of $\boldsymbol{c}$ and $\boldsymbol{d}$ due to time advancement. That is, $\boldsymbol{c}$ and $\boldsymbol{d}$ are both functions of time t, or $\boldsymbol{c} = \boldsymbol{c}(t)$ and $\boldsymbol{d} = \boldsymbol{d}(t)$. Note that we can use day, week or month as the time unit depending on individual cases. We shall start with a concrete simple example in subsection 5.4.1. Then generalize the concept in subsection 5.4.2.

5.4.1. An Illustrative Example

Example 5.4.1. Continue on Example 5.3.1. Assume the original unit profits, resource consumption rate and available resource levels are same as Table 5-1 with linear programming model as model (33). However, for each unit of time advancement, the unit profit of Type I and Type II products will increase by 0.4 and 0.3 unit respectively; and the resource available level for material and human resource will increase by 2.5 and 1 unit respectively. The problem can be summarized as in Table 5-5.

Table 5-5 Example 5.4.1 in a nutshell.

Resource	Type I	Type II	Available Resource Level	Change rates for resource level in time
Material Resource	5	3.5	100	2.5
Human Resource	2.5	2	120	1
Unit Profits of Products	-3	-5		
Change rates for unit profit in time	0.4	0.3		

The problem can be formulated as in model (41).

$$\begin{aligned} \max \quad & (-3x_1 - 5x_2) + t(0.4x_1 + 0.3x_2) \\ s.t. \quad & 5x_1 + 3.5x_2 \le 100 + 2.5t, \\ & 2.5x_1 + 2x_2 \le 120 + t, \\ & x_1, x_2, t \ge 0. \end{aligned} \tag{41}$$

Note that model (41) has only one parameter, t, while model (34) of Example 5.3.1 has two parameters y and z. By setting t at different values, we can obtain the corresponding optimal solutions and the objective values as summarized in Table 5-6.

Table 5-6 The optimal solutions and their objective values for different t values for (41).

t	0	7.5	7.6	50	100	144	146
x_1	0	0	23.8	45	70	92	0
x_2	0	0	0	0	0	0	132.86
Optimal Basis	$J(3,4)$		$J(1,4)$				$J(2,4)$
Objective value	0	0	0.95	765	2590	5023.2	5154.86

t	150	200	250	279	280	300	-
x_1	4	64	124	158	160	168	-
x_2	130	80	30	1	0	0	-
Optimal Basis	$J(1,2)$				$J(1,3)$		
Objective value	5428	9328	14128	17324.38	17440	19656	-

The useful information of Table 5-6 can be further depicted as shown in Figure 5-3.

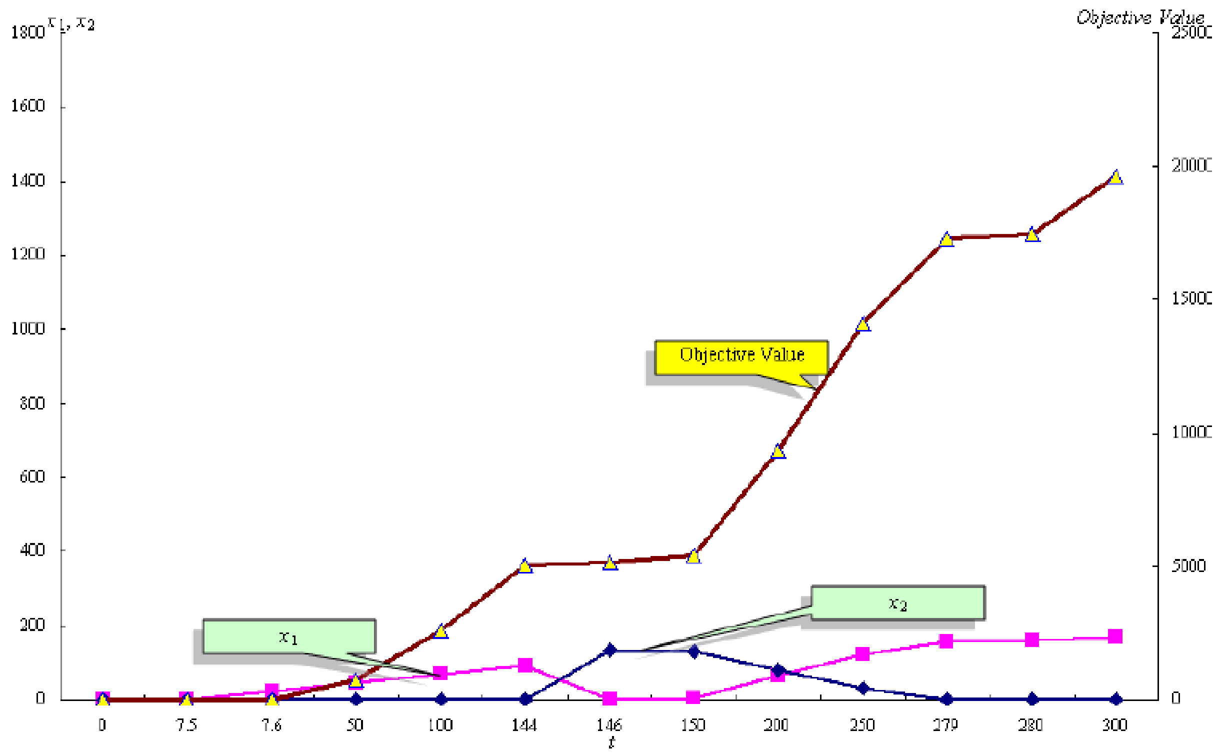

Figure 5-3 Trends of optimal solutions and objective values at different time interval.

In Figure 5-3, when $0 \le t < 7.5$, the optimal basis is J={3, 4} and the optimal objective value is 0, the decision of not producing any product is made due to the fact that no profit can be made. When $7.5 \le t < 145$, the optimal basis is J={1, 4} and the optimal objective value increases by time and only Type I product is produced. When $145 \le t < 146.67$, the optimal basis is J={2, 4} and the optimal objective value increases by time and only Type II product is produced. When 146.67 ≤ t <280, the optimal basis is J={1, 2} and the optimal objective value increases in acceleration by time and both Type I and II products are produced. When t ≥ 280, the optimal basis is J={1, 3} and the optimal objective value increases in time and only Type I product is produced. Note that in Figure 5-3, the critical times of transition are monotonically, not proportionally, deployed.

Note that the changing pace of time for parameters in the objective function, i.e. array ***c***, and in the constraints availability, i.e. array ***d***, is the same. Therefore, the critical time for the changes of optimal bases in Figure 5-3 can be calculated by depicting a line, y=z, in Figure 5-1, which is shown in Figure 5-4. The intersecting points of the line y=z and the range of the different optimal bases are corresponding to the critical time points shown in Table 5-6 and Figure 5-3.

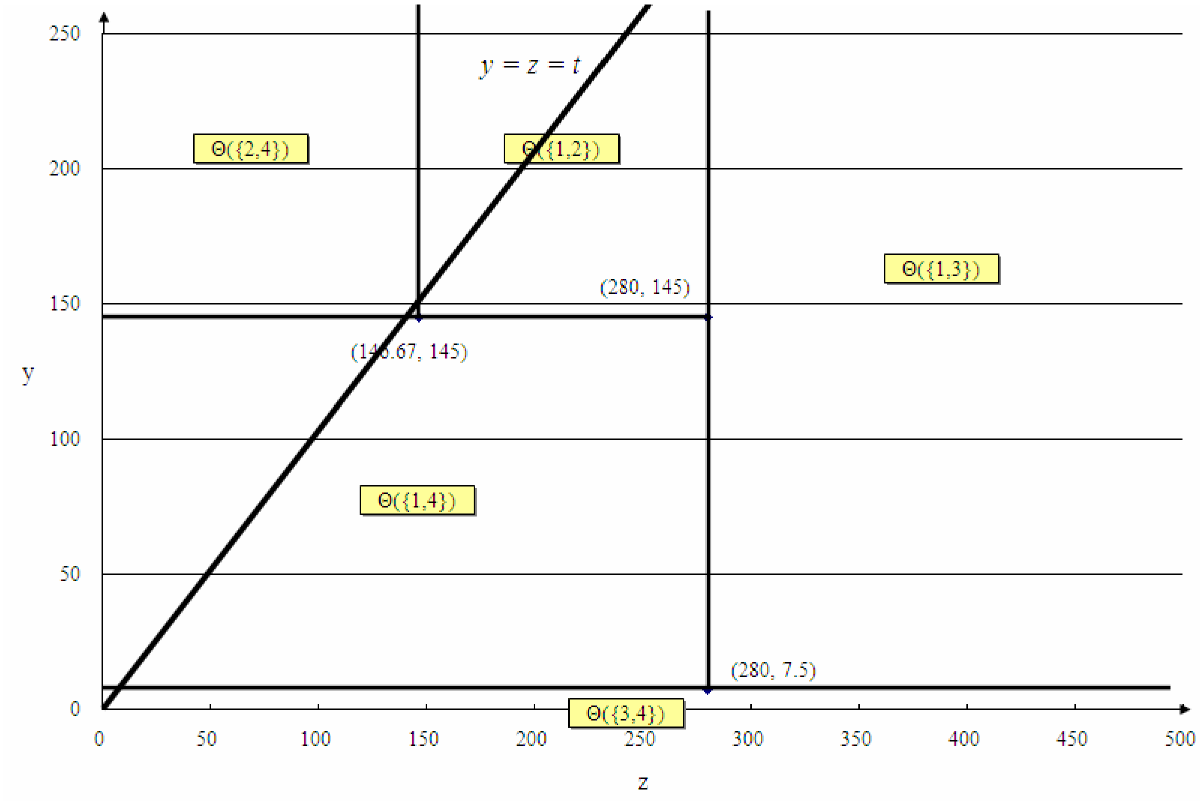

Figure 5-4 Intersecting points of the line of y=z with the potential solution structure in parameter space.

Finally observe, from Table 5-6 and Figure 5-3, that because of optimization formulation, whenever the optimal objective value is zero, the products should not be produced due to the fact that each product produced will bring negative profit or deficit (Red in). However, when $t \geq 7.5$, Type I product began to be able to bring in positive profit (Black out). If the delivery time is set at some time $t > 7.5$, then positive profit can be fulfilled. We shall further discuss this subject in the following subsection.

5.4.2. Generalized Model for Parameter Changes in *c* and *d* due to Time Advancement

With concrete Example 5.4.1 in mind, assume that $\boldsymbol{c}(t)$ and $\boldsymbol{d}(t)$ are linear. More specifically, let

$$c_j(t) = c_{j,0} + c_{j,2}t,\ j = 1,\ldots,n, \tag{42}$$

where, $c_{j,0}$ is the original profit rate, $c_{j,2}$ is the increased profit for each unit of time passed, and

$$d_i(t) = d_{i,0} + d_{i,2}t,\ i = 1,\ldots,m, \tag{43}$$

where, $d_{i,0}$ is the original available resource level, $d_{i,2}$ is change rate of resource availability over time.

Introducing (42)-(43) into model (27), we obtain the following changeable parameter model over c and d due to time advancement.

$$\begin{aligned}
\max\quad & (c_{1,0} + c_{1,2}t)x_1 + (c_{2,0} + c_{2,2}t)x_2 + \cdots + (c_{n,0} + c_{n,2}t)x_n \\
s.t.\quad & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le (d_{1,0} + d_{1,2}t), \\
& a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le (d_{2,0} + d_{2,2}t), \\
& \cdots \\
& a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le (d_{m,0} + d_{m,2}t), \\
& x_j \ge 0,\ j = 1,2,\cdots,n, \\
& t \ge 0.
\end{aligned} \tag{44}$$

Note that model (44) has only one parameter t. It can be formulated as

$$\begin{aligned}
\max\quad & \begin{bmatrix} 1 & t \end{bmatrix} \begin{bmatrix} c_{1,0} & c_{2,0} & \cdots & c_{n,0} \\ c_{1,2} & c_{2,2} & \cdots & c_{n,2} \end{bmatrix} \boldsymbol{x} \\
s.t.\quad & \boldsymbol{Ax} \le \begin{bmatrix} d_{1,0} & d_{1,2} \\ d_{2,0} & d_{2,2} \\ \vdots & \vdots \\ d_{m,0} & d_{m,2} \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix}, \\
& x_j \ge 0,\ j = 1,2,\cdots,n, \\
& t \ge 0.
\end{aligned} \tag{45}$$

By varying t, for $t \geq 0$, one can generate the useful information such as those of Table 5-6 and Figure 5-3 for final decision.

The phenomenon of "Red in-Black out" can be roughly explained as: at the ordering time (t=0), the optimal objective value is less than or equal to 0, and at the delivery time ($t=t_1>0$), the optimal objective value is greater than 0 because the parameters have been changed over time. The following results can help the company to figure out if "Red in-Black out" is a good strategic decision or not.

Given j, define $I(j)=\{i|a_{ij} > 0\}$.

Proposition 5.4.1. Assume there exists $j \in \{1,\ldots,n\}$ such that for all $i \in I(j)$, $c_{j,2} > 0$ and $d_{i,2} > 0$. Then, as time advances, model (44) will eventually make profit.

Proof. Set

$$t_j = \underset{t}{Min}\left\{ t \mid c_{j,0} + tc_{j,2} \geq 0 \text{ and } d_{i,0} + td_{i,2} \geq 0, \text{ for all } i \in I(j) \right\}. \qquad (46)$$

Then, for all $i \in I(j)$, when $t>t_j$, $c_{j,0}+tc_{j,2}>0$ and $d_{i,0}+td_{i,2}>0$, the production solution $\boldsymbol{x}^*$(with $x_j^* = \underset{i}{Min}\{\frac{d_{i,0}+td_{i,2}}{a_{ij}}\} > 0$, and $x_k^* = 0$, for all $k \neq j$) will make a positive profit because of $c_{j,0}+tc_{j,2}>0$, the objective function, $(c_{j,0}+tc_{j,2})x_j^* > 0$, which is smaller than the optimal objective value of model (44). □

Remark 5.4.1. Suppose there exists j, such that $c_{j,0}>0$ and for all $i \in I(j)$, $d_{i,0}>0$. Then according to (46) of the above proof, the production system will make profit at time 0.

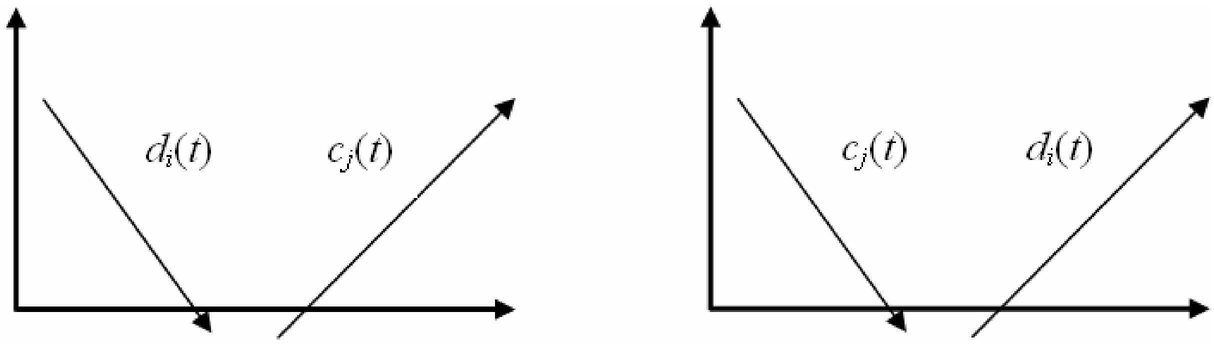

Figure 5-5 Two situations for s_j to be an empty set.

For a given j and small $\varepsilon > 0$, define

$$s_j(\varepsilon) = \underset{s}{Min}\left\{c_{j,0} + sc_{j,2} \ge 0 \ \text{ and } \ d_{i,0} + sd_{i,2} \ge \varepsilon, \ \text{ for all } i \in I(j), s \ge 0\right\}. \qquad (47)$$

Note, s_j can be an empty set, or not defined if $c_{j,0}+sc_{j,2}$ and $d_{i,0}+sd_{i,2}$, $i \in I(j)$ cannot be greater than 0 at the same time such as those shown in Figure 5-5.

Proposition 5.4.2.

(i) Suppose there exists j, such $s_j(\varepsilon)$ is not an empty set, then $s_j(\varepsilon)$ is the time point at which the model (44) will not yield loss. Furthermore, if $c_{j,0}+s_j(\varepsilon)c_{j,2}>0$, then $s_j(\varepsilon)$ is a time point at which the model (44) will yield profit.

(ii) For all j such that $c_{j,0}<0$, let $s^*(\varepsilon) = \underset{j}{Min}\left\{s_j(\varepsilon)\right\}$. Then for time $t>s^*(\varepsilon)$, the system of (44) can make profit.

Proof. For (i). It can be proved similar to that of Proposition 5.4.1.

For (ii). Suppose $s^*(\varepsilon)=s_k(\varepsilon)$. As $c_{k,0}<0$, $c_{k,0}+s_k(\varepsilon)c_{k,2}\ge 0$, $c_{k,0}+tc_{k,2}>0$ for $t>s_k(\varepsilon)$. Therefore, the system can make a profit when $t>s_k(\varepsilon)=s^*(\varepsilon)$. □

Assume the production system will make profit eventually, which can be checked by above Proposition 54.1 and 5.4.2, and that the optimal objective function value $v(t)$ is increasing with time with $v(t=0)\leq 0$. Let t_0 be *the earliest critical time* of making profit in the sense that when $t>t_0$ the system can make profit with $v(t)>0$; and when $t<t_0$ the system will not make profit with $v(t)\leq 0$. The following algorithm, exemplified by the flow chart of Figure 5-4, can be of help to find t_0.

Algorithm 5.4.1.

Step(1): Choose $t_L>0$, where t_L denotes *left end point*, and set $t_R=t_L$, where t_R denotes *right end point*.

Step(2): Solve model (44) with $t=t_L$ to obtain the optimal objective value, $v(t_L)$.

Step(3): If $v(t_L)\leq 0$, go to Step(3-1)-Step(3-3). Otherwise, go to Step(4).

Step(3-1): Set $t_L=t_R$, $t_R=2t_R$.

Step(3-2): Solve model (44) with $t=t_R$ to obtain the optimal objective value, $v(t_R)$.

Step(3-3): If $v(t_R)>0$, go to Step(5). Otherwise, back to Step(3-1).

Step(4): If $v(t_L)>0$, go to Step(4-1)-Step(4-3).

Step(4-1): Set $t_R=t_L$, $t_L=t_L/2$.

Step(4-2): Solve model (44) with $t=t_L$ to obtain the optimal objective value, $v(t_L)$.

Step(4-3): If $v(t_L)\leq 0$, go to Step(5). Otherwise, back to Step(4-1).

Step(5): Set t_M=(t_L+t_R)/2, where t_M denotes the *middle point* of the interval [t_L, t_R].

Step(6): Solve model (44) with $t=t_M$ to find the optimal objective value, $v(t_M)$.

Step(6-1): If $v(t_M)>0$, set $t_R=t_M$ and back to Step(5).

Step(6-2): If $v(t_M)<0$, set $t_L=t_M$ and back to Step(5).

Step(6-3): If $v(t_M)=0$ and $v(t_L)=0$, set $t_L=t_M$ and back to Step(5); if $v(t_M)=0$ and $v(t_L)<0$, then the time point t_M is the earliest critical time of making profit for the system.

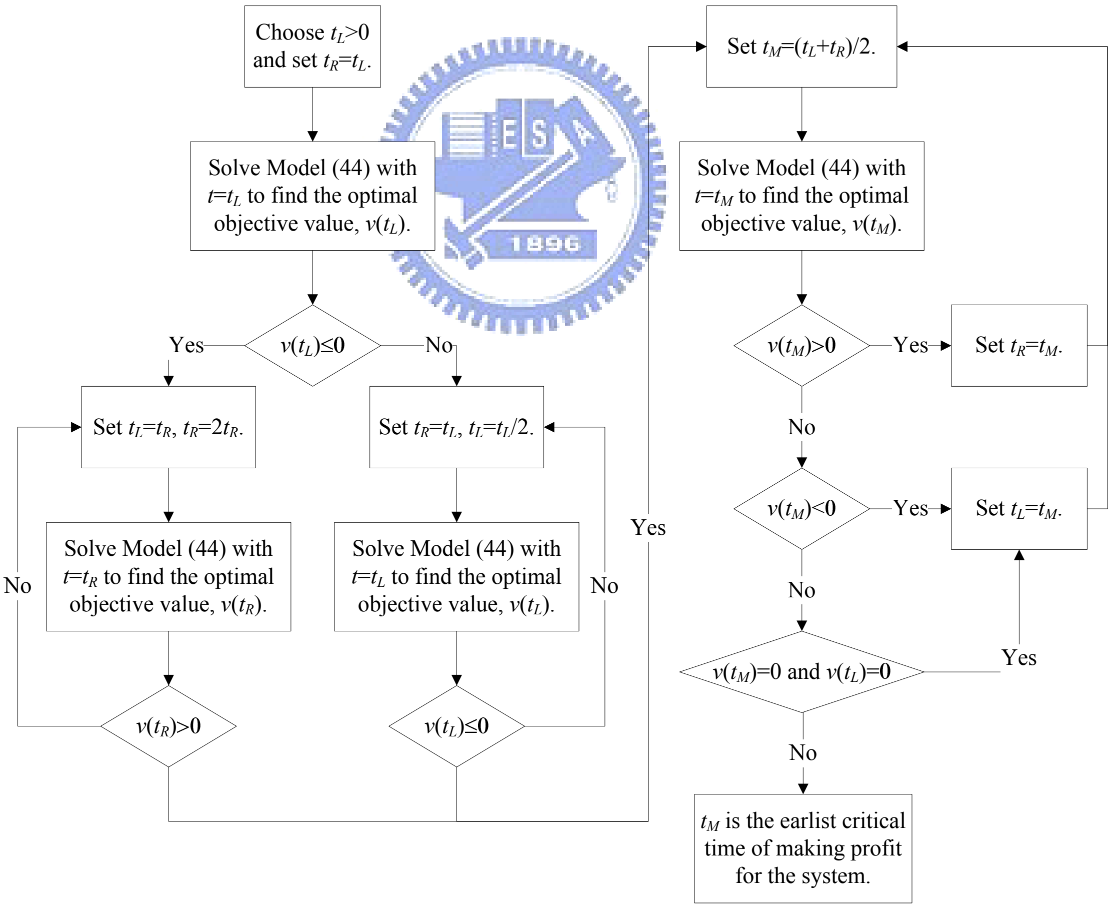

Figure 5-6. Flow chart of Algorithm 5.4.1.

Theorem 4.1. If system (44) will make profit eventually and the objective function value will increase by time, Algorithm 5.4.1 will converge.

Proof. The proof of this theorem is similar to that of bisection method. For details, see [45]. □

5.5 Generalized Model for Parameter Changes Including Elements of *A*

It is well known that parameter changes in elements of ***A*** usually involve nonlinear computation for optimization. However, when the changes follow some specific pattern, the mathematical programming can be reduced to a form of linear inequalities with multi-level resource availability constraints.

Again, we will start with a concrete simple example in subsection 5.5.1. The generalized model for parameter changes including elements of ***A*** and its relation to "Red in-Black out" phenomenon will be given in subsection 5.5.2. Further generalization will be given in subsection 5.5.3.

5.5.1. An Illustrative Example

Example 5.5.1. (Continue on Example 5.4.1.) Assume that the consumption of resources, perhaps due to technological advancement, is reduced at a rate $(1+0.025t)^{-1}$ and $(1+0.00833t)^{-1}$ respectively for material and human resource and the resource availability remains the same. Table 5-7 offers a summary of the problem.

Table 5-7 A summary of the problem of Example 5.5.1.

Resource	Type I	Type II	Available Resource Level	Change rates for resource usage in time
Material Resource	5	3.5	100	$(1+0.025t)^{-1}$
Human Resource	2.5	2	120	$(1+0.00833t)^{-1}$
Unit Profits of Products	-3	-5		
Change rates for unit profit in time	0.4	0.3		

Note that the objective function is the same as in model (41). The constraints of the problem can be rewritten as:

$$\begin{bmatrix} 5*(1+0.025t)^{-1} & 3.5*(1+0.025t)^{-1} \\ 2.5*(1+0.00833t)^{-1} & 2*(1+0.00833t)^{-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \le \begin{bmatrix} 100 \\ 120 \end{bmatrix} \tag{48}$$

or

$$5x_1 + 3.5x_2 \le 100*(1+0.025t) = 100 + 2.5t$$

$$2.5x_1 + 2x_2 \le 120*(1+0.00833t) = 120 + t$$

which reduces to

$$\begin{bmatrix} 5 & 3.5 \\ 2.5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \le \begin{bmatrix} 100 & 2.5 \\ 120 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix}. \tag{49}$$

Note the constraint (49) is identical to that of model (41). Therefore, all the discussion and computation for useful information of model (41) can be carried over to this new problem. We shall not repeat it. Being limited by space, we purposefully choose the change rates for resource usage in time so that we do not have to repeat the computation. Of course, the model can be applied to different rate of change in resource usage.

5.5.2. A Generalization, Including Changes in Elements of *A*

Assume that elements of ***c***, ***d*** and ***A*** can be changed over time and they are all linear functions of time. Specifically,

1. the objective coefficients, ***c***, can be represented by equation (50).

$$c_j = c_{j,0} + c_{j,2}t, j = 1,\ldots,n, \tag{50}$$

where, $c_{j,0}$ is the original profit rate, $c_{j,2}$ is the increased profit for each unit of time, and t represents the time units.

2. the elements of matrix ***A***, will be changed over time and can be represented by equation (51).

$$a_{ij} = a_{ij,0} \big/ \left(1 + a_{ij,1}t\right), i = 1,\ldots,m, j = 1,\ldots,n, \tag{51}$$

where, $a_{ij,0}$ is the original consumption rate for different product j in resource i; $a_{ij,1}$ is the change rate for each unit of time for different product j in resource i. (Note that if for each i, $a_{ij,1}$ is the same for all j, then the constraints reduces to a similar form of (49).)

3. the resource available level, namely, elements of ***d***, can be represented by equation (52).

$$d_i = d_{i,0} + d_{i,2}t, i = 1,\ldots,m, \tag{52}$$

where, $d_{i,0}$ is the original available resource level, and $d_{i,2}$ is change rate of available resource over time.

Introducing (50)-(52) into model (27), we obtain the following changeable parameter model due to time advancement shown in model (53).

$$
\begin{aligned}
\max \quad & (c_{1,0}+c_{1,2}t)x_1+(c_{2,0}+c_{2,2}t)x_2+\cdots+(c_{n,0}+c_{n,2}t)x_n \\
s.t. \quad & [a_{11,0}/(1+a_{11,1}t)]x_1+\cdots+[a_{1n,0}/(1+a_{1n,1}t)]x_n \le (d_{1,0}+d_{1,2}t), \\
& [a_{21,0}/(1+a_{21,1}t)]x_1+\cdots+[a_{2n,0}/(1+a_{2n,1}t)]x_n \le (d_{2,0}+d_{2,2}t), \\
& \cdots \\
& [a_{m1,0}/(1+a_{m1,1}t)]x_1+\cdots+[a_{mn,0}/(1+a_{mn,1}t)]x_n \le (d_{m,0}+d_{m,2}t), \\
& x_j \ge 0,\ j=1,2,\cdots,n, \\
& t \ge 0.
\end{aligned} \tag{53}
$$

Similar to Propositions 5.4.1-5.4.2, Algorithm 5.4.1 and Theorem 5.4.1, we can restate their general cases as follows. Recall that $I(j)=\{i|a_{ij}>0\}$.

Proposition 5.5.1. Assume there exists $j\in\{1,\ldots,n\}$ such that for all $i\in I(j)$, $c_{j,2}>0$ and $d_{i,2}>0$. Then, as time advances, model (53) will eventually make profit.

Recall that $s_j(\varepsilon)=\underset{s}{Min}\left\{c_{j,0}+sc_{j,2}\ge 0 \text{ and } d_{i,0}+sd_{i,2}\ge\varepsilon, \text{ for all } i\in I(j), s\ge 0\right\}.$, as defined in (47).

Proposition 5.5.2.

(i) Suppose there exist j, such $s_j(\varepsilon)$ is not an empty set, then $s_j(\varepsilon)$ is the time point at which the model (53) will not yield loss. Furthermore, if $c_{j,0}+s_j(\varepsilon)c_{j,2}>0$, then $s_j(\varepsilon)$ is a time point at which the model (53) will yield profit.

(ii) For all j such that $c_{j,0}<0$, let $s^*(\varepsilon)=\underset{j}{Min}\left\{s_j(\varepsilon)\right\}$. Then for time $t>s^*(\varepsilon)$, the production system can make profit.

Algorithm 5.5.1.

Step(1): Choose t_L>0, where t_L denotes *left end point*, and set t_R=t_L, where t_R denotes *right end point*.

Step(2): Solve model (53) with t=t_L to obtain the optimal objective value, $v(t_L)$.

Step(3): If $v(t_L)\leq 0$, go to Step(3-1)-Step(3-3). Otherwise, go to Step(4).

Step(3-1): Set t_L=t_R, t_R=2t_R.

Step(3-2): Solve model (53) with t=t_R to obtain the optimal objective value, $v(t_R)$.

Step(3-3): If $v(t_R)$>0, go to Step(5). Otherwise, back to Step(3-1).

Step(4): If $v(t_L)$>0, go to Step(4-1)-Step(4-3).

Step(4-1): Set t_R=t_L, t_L=t_L/2.

Step(4-2): Solve model (53) with t=t_L to obtain the optimal objective value, $v(t_L)$.

Step(4-3): If $v(t_L)\leq 0$, go to Step(5). Otherwise, back to Step(4-1).

Step(5): Set t_M=(t_L+t_R)/2, where t_M denotes the *middle point* of the interval [t_L, t_R].

Step(6): Solve model (53) with t=t_M to find the optimal objective value, $v(t_M)$.

Step(6-1): If $v(t_M)$>0, set t_R=t_M and back to Step(5).

Step(6-2): If $v(t_M)<0$, set $t_L=t_M$ and back to Step(5).

Step(6-3): If $v(t_M)=0$ and $v(t_L)=0$, set $t_L=t_M$ and back to Step(5); if $v(t_M)=0$ and $v(t_L)<0$, then the time point t_M is the earliest critical time of making profit for the system.

Theorem 5.5.1. If the production system will make profit eventually and the objective function value is increasing with time, Algorithm 5.5.1 will converge.

5.5.3. Further Generalization with Parameters as Control Variables

Let k be the investment units for changing the efficiency of resource usage. Model (39) of subsection 5.3.2 can be further expanded as follows.

Let

$$a_{ij} = f_{a_{ij}}(k) = a_{ij,0} \big/ \left(1 + a_{ij,1} k\right), i = 1, \ldots, m, j = 1, \ldots, n, \tag{54}$$

where $a_{ij,0}$ is the original consumption rate for different product j in resource i; $a_{ij,1}$ is the change rate for each unit of investment for different product j in resource i.

Introducing (54) into model (39), we obtain the following model with parameters as control variables.

$$
\begin{aligned}
\max \quad & (c_{1,0}+c_{1,1}y)x_1+(c_{2,0}+c_{2,1}y)x_2+\cdots+(c_{n,0}+c_{n,1}y)x_n \\
s.t. \quad & [a_{11,0}/(1+a_{11,1}k)]x_1+\cdots+[a_{1n,0}/(1+a_{1n,1}k)]x_n \le (d_{1,0}+d_{1,1}z), \\
& [a_{21,0}/(1+a_{21,1}k)]x_1+\cdots+[a_{2n,0}/(1+a_{2n,1}k)]x_n \le (d_{2,0}+d_{2,1}z), \\
& \cdots \\
& [a_{m1,0}/(1+a_{m1,1}k)]x_1+\cdots+[a_{mn,0}/(1+a_{mn,1}k)]x_n \le (d_{m,0}+d_{m,1}z), \\
& y \le y_M, \\
& z \le z_M, \\
& k \le k_M, \\
& y+z+k \le I_M, \\
& x_j \ge 0, j=1,2,\cdots,n, \\
& y,z,k \ge 0.
\end{aligned} \tag{55}
$$

Note, in the above formulation, y, z, k are changeable parameters as well as control variables. When there are other constraints imposing on y, z, k, they can be easily added on. Model (55) is usually nonlinear. As demonstrated before (subsection 5.5.1-5.5.2), with special structures, it can be reduced to the MC^2-simplex format, and can be solved systematically. We shall not repeat it.

CHAPTER 6 CONCLUSIONS AND REMARKS

Human beings encounter many decision problems every day. For each problem we need to gather and analyze lots of information related to solving the problem in order to make a qualified final decision. If our habitual domains have been trapped into certain domain, we may most likely not to avoid blind spots.

Competence set, as an extension of habitual domains, is a projection of our habitual domains with respect to a decision problem. It implicitly embraces actual domain, reachable domain, and activation probability, like habitual domain. Obviously, the competence set cannot be expanded or transformed if our habitual domains get trapped. A firm or supply chain is therefore not able to provide right quality products or services, which satisfy the target customers' needs or desires, or release their charge, pain or frustration ahead of its competitors. In this dissertation, we explored the following two kinds of problems:

1. Develop the insight (a competence) to design and produce a new product or service (a composition of attributes of competence set) to satisfy the customer newly emergent needs.

2. Given a product or service, how to motivate customers so that they are willing to buy our product or service.

This dissertation has discussed four categories of decision problems: routine, mixed-routine, fuzzy, and challenging problems. Many routine problems can be solved by assistance of information technologies (ITs). For mixed-routine and fuzzy problems, we may decompose it into a number of solvable routine sub-problems. As to challenging problems, one must expand his/her habitual domain or think deeper into reachable domain even potential domain, to find effective solution and avoid decision traps.

Though knowledge management (KM)/ITs can clarify what are the needed competence set, and may speed up the process of expansion of competence set. KM/IT may also lead us into traps as to make wrong decisions or transactions. This is most likely when we are confronted with challenging problems and we are in a state of high level of charge. This research has addressed *Innovation Dynamics* for a systematic view of innovation. Many research problems are open for exploration. For instance, in the innovation dynamics, each link of Figure 3-1 and Figure 3-2 involves a number of routine, fuzzy and challenging problems. How do use KM/IT, HD, CS to help the decision maker to make good (optimal) decisions easily and quickly, so that we could relieve their pains and frustration, and create value?

By focusing on the stage of transformation of competence set in the innovation dynamics, we have investigated a class of *optimal adjustment of competence set problems* with a given x^0 as a target to be reached. A *competence set adjustment model* (CSA model) has been formulated to provide useful information for the optimal adjustment of the competence set. The *bisection algorithm* (BA) and the *fuzzy linear programming* (FLP) techniques have been utilized to search for a good target, when the original target is not attainable. The former is to find a solution which is as close as possible to the target from a status quo, and the latter is to help the DM to identify an achievable target depending on fuzzy tolerance. The optimal adjustment could then be derived from the aforementioned CSA model with the new target obtained. The following problems need to further explore: (i) What is the relationship between the optimal adjustment of competence set problem and the ordinary goal programming? (ii) How to effectively determine the optimal adjustment if a set of targets, instead of a single target, is given?

Finally, motivated by the “Red in-Black out” phenomenon (taking loss at the ordering time and making profit at the time of delivery), we have studied linear programming models with changeable parameters using multi-criteria and multi-constraint level linear programming (MC^2LP) models. We have provided formulations, computation methods and analysis as to gain useful insight into the “Red in-Black out” phenomenon. We have also proposed an algorithm to locate the first critical time of making profit for a given system, which is an important information to those decision makers who consider adopting the “Red in-Black out” as a business strategy. At the end, we also sketch a generalized mathematical programming model with changeable parameters and control variables to study more general cases. Many research problems are open. For instances, how to interpret the meaning of the dual problem of the model proposed in Chapter 5? How to deal with the uncertainty and fuzziness of parameter changes due to investment or time advancement? We invite interested readers to explore these interesting and meaningful problems.

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Appendix 1 Behavior Mechanism – 8 hypotheses

1. Circuit pattern hypothesis

Thought, concepts or ideas are represented by circuit patterns of the brain. The circuit patterns will be reinforced when the corresponding ideas are repeated. Furthermore, the stronger the circuit patterns, the more easily the corresponding thoughts are retrieved in our thinking and decision making processes.

2. Unlimited capacity hypothesis

Each normal brain has practically unlimited capacity for encoding and storing more thoughts and concepts that its owner ever intends to.

3. Efficient restructuring hypothesis

The encoded thoughts, concepts and messages are organized and stored systematically as databases for efficient retrieving. Further more, according to the dictation of attention they are continuously restructured so that relevant ones can be efficiently retrieved to release charges.

4. Analogy/association hypothesis

The perception of new events, subjects, or ideas can be learned primarily by analogy and/or association with what is already known. When faced with a new event, subject, or idea, the brain first investigates its features and attributes in order to establish a relationship with what is already known by analogy and/or association.

Once the right relationship has been established, the whole of the past knowledge (preexisting memory structure) is automatically brought to bear on the interpretation and understanding of the new event, subject or idea.

5. Goal setting and state evaluation hypothesis

Each one of us has a set of goal functions and for each goal function we have an ideal state or equilibrium point to reach and maintain. We continuously monitoring, consciously or subconsciously, where we are relative to the ideal state or equilibrium point. Goal setting and state evaluation are dynamic, interactive and subject to physiological forces, self-suggestion, external information forces, current data bank (memory) and information processing capacity.

6. Charge structures and attention allocation hypothesis

Each event is related to a set of goal functions. When there is an unfavorable deviation of the perceived value from the ideal, each goal function will produce various level of charge. The totality of the charges by all goal functions is called the charge structure and it can change dynamically. At any point in time, our attention will be paid to the event which has the most influence on our charge structure.

7. Discharge hypothesis

To release charges, we tend to select the action which yields the lower remaining charge (the remaining charge is the resistance to the total discharge, and thus is called the least resistance principle).

8. Information input hypothesis

Human has innate needs to gather external information. Unless attention is paid, external information inputs may not be proceeded.

Appendix 2 Eight Methods for Expansion of Habitual Domains

1. Learning actively

By active learning we mean obtaining those concepts, ideas and thoughts from various channels including consultations with experts, reading relevant books and following the radio, television, journals, etc.

2. Take the higher position

There is a tendency in all of us to view the world from a very limited, even selfish perspective. By taking the higher position we are, in fact, expanding our habitual domains.

3. Active association

There are many different events, subjects, objects and problems in our daily lives. They all have different features, but common properties. By actively associating them, we may be able to discover the unique features of problems, events, subjects and objects. Once the unique features are discovered, our habitual domains may be expanded.

4. Changing the relative parameter

Make a habit of looking for connections between seemingly disparate objects and events.

5. Changing the environment

Every event or problem has a number of parameters of characteristic elements. By tinkering with these parameters, changing their values, we can produce new concepts and ideas.

6. Brainstorming

Brainstorming is nothing more than effective group thinking. Presented with a particular problem, each member of the group is asked to freely report what comes to mind regarding various aspects of the situation. It can be an enormously creative process, not only to meet a challenge the group faces, but also to encourage individual growth.

7. Retreat in order to advance

Sometimes taking a time-out from the matter can be the most effective mind-expanding technique you can use. By retreating, we change the actual domain and, consequently, the reachable domains.

8. Praying or meditation

Some of the most effective ways to lower our overall charge are through prayer, meditation, relaxation exercises or through a conscious effort to put aside our unfulfilled wishes. The practice can let the ideas of low activation probability to catch our attention and change our actual domain and reachable domain.

Appendix 3 Nine Principles of Deep Knowledge

1. The deep and down principle

This can also be remembered as the ocean principle. The idea is to empty your mind of desires and to insulate oneself from external bombardment of ideas. By doing so, you create an atmosphere conductive to deep thinking. When one is relaxing, his overall charge level is lower, and "hidden" thoughts with much lower charges come bubbling up. Relaxation can also make one more sensitive to emerging problems, allowing one to solve them when they are at a fairly simple stage.

2. The alternative principle

This can be remembered as the door principle. An assumption which is always imposed or always left out will lose its value as an assumption. Sometimes we have to omit or change our combined assumptions so that we can create new ideas from different sets of assumptions.

3. The contrast and complementing principle

This can be remembered as the house principle. A house offers barriers – a roof and walls against the weather – and also, contrasting with and complementing this quality, it has open space within. Even what we see as existing can be contrasted with that which doesn't exist, and these two things complement each other in their functions.

4. The revolving and cycling principle

This principle can be remembered as falling flower seeds. When a flower fades in the autumn and falls to the ground, it carries with it the seeds for renewal in the following spring. Just as each success contains the seeds of failure, so each failure contains the seeds of success.

5. The inner connection principle

This is the blood is thicker than water principle. It means, simply, that a close connection will be honored over simple acquaintance. The idea is to build as many strong channels as possible connecting us to the inner core of another individual's habitual domain. Making inner connections is the real goal of what is sometimes called "networking".

6. The changing and transforming principle

This is the ice and stream principle. The world is constantly changing, and so are the habitual domains of the individuals and organizations that inhabit it. They change when circumstances (or parameters) make them. People who are willing to change are people with a better chance for happiness and success. We must all be on the alert for changes and their implications and be willing to change ourselves. If we do not, we will never tap our potential.

7. Contradiction principle

This is the stand on your head principle. Sometimes it is worth seeing the world upside down, or at least from a different angle, it can clear your mind. If there is an event or information that contradicts our conclusions, then we must revise our assumptions or change our conclusions. Applying this principle to our daily thinking can sharpen our thought processes.

8. The cracking and ripping principle

This is the broken teacup principle. Cracks are the weak point of any structure. If you want to destroy a mighty fortress, we can do so by working on its crack lines and ripping them open.

9. The void principle

This might be called the empty space principle. This principle simply states that the outside of our habitual domains is not empty. Just because we don't perceive it or recognize it doesn't mean it isn't there. And whether we acknowledge it or not, those other HDs can have a profound effect on us.

Appendix 4 Seven Self-perpetuation Operators

1. Everyone is a priceless living entity. We all are unique creations who carry the spark of the divine.

2. Clear, Specific and challenging goals produce energy for our lives. I am totally

committed to doing and learning with confidence.

3. There are reasons for everything that occurs. One major reason is to help us grow and develop.

4. Every task is part of my life's mission. I have the enthusiasm and confidence to accomplish this mission.

5. I am the owner of my living domain. I take responsibility for everything that happens in it.

6. Be appreciative and grateful, and do not forget to give back to society.

7. Our remaining lifetime is our most valuable asset. I want to enjoy it 100 percent and make a 100 percent contribution to society in each moment of my remaining life.