國立交通大學

科技管理研究所

博士論文

由兵力耗損理論探討近代重大戰役之研究

Extending the Lanchester's Square Law to Better Fit the Attrition in the Ardennes Campaign

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摘　要

戰爭是人類社會普遍存在的一種現象，戰爭的特質是交戰雙方意志的衝撞。戰爭是猛烈艱難的工作，危險是其基本的特性。戰爭行為顯而易見的印象是危險，而人類對此危險的反應是恐懼。因其改變國家之命運與國家間的秩序，對人類社會的影響既深且鉅，故交戰雙方都希望藉由瞭解敵軍的戰術、戰略層次及作戰目標，而獲取預想的利益。並試圖為下次作戰找出有利的戰爭條件和方法。

因此中國的孫子兵法始計篇開宗明義就闡述：兵者，國之大事，死生之地，存亡之道，不可不察也。又云：夫未戰而廟算勝者，得算多也；未戰而廟算不勝者，得算少也。多算勝少算不勝，而況於無算乎。謀攻篇提及知勝者有五：知可以戰與不可以戰者勝，識眾寡之用者勝，上下同欲者勝，以虞待不虞者勝，將能而君不御者勝。此五者，知勝之道也。故曰：知己知彼，百戰不殆；不知彼而知，己一勝一負；不知彼不知己，每戰必敗。

克勞塞維茨在戰爭論中曾說：任何理論的主要目的乃在澄清已然困惑不清與糾葛難解的構想及理念；除非已對一些名詞與構想的意義加以界定，否則無人能在此方面獲得任何進展。如有人認為上述說明不具任何意義，則其不是全然無法接受理論上的分析，就是從未接觸到有關戰爭遂行的各種令人困惑而又相互排斥的理念。事實上，理論固然無法提供解決問題的公式，也不能作為據以找出唯一解決方案的原則，但卻能使人深入了解各種紛亂的現象與關係，俾將之提升為更高層次的行動範疇。所以對理念加以釐清、探討與分析，自有其必要性。克勞塞維茨在戰爭論中又說：攻擊和防

禦在戰爭是相互作用的狀態和反應。在進攻和防禦之間轉換將有一段時間的間距很難定義。

人類長久以來一直透過各種技術發展或科學計算，確切解決對戰爭結果的期盼。不論是實兵對抗操演、賽局理論、傳統沙盤推演、新興科技電腦兵棋模擬與蘭徹斯特方程式之解析…等均屬之。近代軍事科技最大的成就不是建造出多麼新穎的武器裝備，而是藉由軟體與硬體的結合，綿密的管理機制，瞭解戰爭與制止戰爭的發生，此乃科技管理運用於軍事層面最佳管理意涵寫照。

於是本研究根據以上的需求，透過第二次世界大戰著名的阿登戰役為事例，藉由著名軍事戰略理論引證、相關戰史討論與文獻蒐整後。進一步著手修改蘭徹斯特平方定律模式。所獲致的成果，除了探討出該模式較符合現代戰爭的型態與精確的交戰雙方兵力損耗外，更進一步透過實證分析，對於兩軍攻守交替的時間問題，作出較合理與適切的說明。

關鍵字：孫子兵法、戰爭論、 蘭徹斯特方程式、阿登戰役

Extending the Lanchester's Square Law to Better Fit the Attrition in the Ardennes Campaign

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Abstract

The war is the human society universal existence one kind of phenomenon; the war special characteristic is joins battle bilateral will dashing. No matter whether history repeats itself or not, we still can learn a lot of useful lessons from it. People have been interested in studying and analyzing historical warfare for thousands of years.

China's Sun Tzu first chapter said: military action is important to the nation, it is the ground of death and life, the path of survival and destruction, so it is imperative to examine it. The one who figures on inability to prevail at headquarters before doing battle is the one who has the least strategic factors in his side. The one with many strategic factors in his favor wins, the one with few strategic factors in his favor. Observing the matter in this way, I can see who will win and who will lose. The third chapter also said: so it is said that if you know others and know yourself, you will not be imperiled in a hundred battles; if you do not know others but know yourself, you win one and lose one; if you do not know others and do not know yourself, you will be imperiled in every single battle.

The war is the violent difficult work; the danger is its basic characteristic. The average person to the war impression, is similar to the humanity regarding the dangerous response is a fear. Joins battle both sides both to hope the affiliation by the attack enemy troop each kind of different social stratum goal, but gains the expectation the benefit. Since

the humanity has been long-time continuously penetrates each kind of technological development or the science computation, hopes to accurate solution to war result.

General Karl Von Clausewitz says in his book On War that attack and defend are a pair of concepts for mutual action and reaction. He considered that defense is more than passive waiting and resistance. The best defense must include the swift and vigorous assumption of the offense. In the shift between offense and defense is a period of deadlock during which both sides seek to seize a key strongpoint, collect intelligence and set up logistics and draw up the next operation plan. Previous researchers did not consider the deadlock of the shift between attack and defense.

General Karl Von Clausewitz believes that offense and defense in warfare are a state of interaction and response. The transition between offense and defense will have a short span of time difficult to define.

No matter is the real troops maneuvers, the game theory, the tradition war game drill, the hi-tech computer war game simulation and so on is it with Lanchaster equation. In this research, we try to improve Bracken's and Chen's work to significantly better fit our extended Lanchester model into the Ardennes Campaign live data. And after revising Lanchester equation, joins battle the bilateral military strength loss besides the discussion, further penetrates the real diagnosis analysis discussion both armies offense and defense in transation question. Realizing that the rapid increase in hi-technology is going to affect warfare, the armed forces are transforming themselves for the digital age. They need analytic tools to help make the best choices possible, and chief among these are good measures of effectiveness that can demonstrate the value of information in terms of military outcomes.

The contemporary greatest military achievement is not to construct the greatest weaponry, but is to affiliate the unification of software and hardware machinery with the

thoughtful management mechanism to prevent the war from happening.This is precisely our diligently goal.

In this research, we try to improve Bracken's and Chen's work to significantly better fit our extended Lanchester model into the Ardennes Campaign live data. According to our numerical experimental result, we improved Bracken's work by 39.26%, and Chen's work by 19.51%.The contribution of this research is that we propose a much better qualitative analysis model for the explanation of modern combat.

Keywords: Lanchaster equation, War game, Sun Tzu , On War, Ardennes Campaign

誌　　謝

從碩士學位的完成到博士學位的獲得，對我而言，是一段辛苦而漫長的路程，對於已習慣軍旅生涯的我而言，能夠躋身國內一流學府交通大學科技管理研究所取得博士學位，甚感幸運。這一路走來雖然艱辛，幸得各方貴人相助，讓我能夠順利畢業，在此表達我的敬意與謝意。

首先要感謝指導教授洪志洋博士，在這四年博士班進修期間的指導與教誨。同時也非常感謝所上袁建中老師、曾國雄老師、虞孝成老師、徐作聖老師、劉尚志老師在企業評價、財務管理、產業分析、科技政策、策略管理、技術預測、創投管理、研究方法、科技與法律等相關領域之指導，讓我得以窺見科技管理的殿堂。對於我未來在返回軍事單位，肯定是受益無窮。此外，論文口試階段也勞煩袁建中老師、曾國雄老師、虞孝成老師給我諸多的引導，使得論文更臻完備。此外雲林科技大學鄭景俗博士，中央警察大學朱錫琛博士在論文模型架構的建立提供許多指導與建議，使我茅塞頓開，在此致上最深的謝意。

要從軍中暫時卸下身邊的職務到民間大學全時進修是相當不容易的事，我能夠順利來到交通大學，非常感謝各級長官的支持讓我得此千載難逢之機。首先，是已卸任的前陸軍總司令暨國防大學校長陳鎮湘上將以及前陸軍總部武計署署長曾祥穎將軍的推薦與鼓勵，中科院副院長金壽豐將軍策勵與提攜。國防部所提供之學雜費與學分費之補助，也讓我在學習時能夠不虞匱乏，免除後顧之憂。

在交通大學科管所求學的四個寒暑裏，暫時脫去了軍服與軍人傳統的價值觀包袱，重溫學生生活，這是彌足珍貴的經驗。尤其是和許多年輕優秀的同學共同學習使我能與時俱進，觀念更新。生活與學習中得到眾人之協助與啟發讓我不至於學業進度落後，諸多先進學長、同學之協助如基生兄、宗耀兄、元惠兄、秋江兄、貴英姊、士其兄、鴻裕兄、華凱兄、辭修兄、以及才華、仁帥、宗偉、筱琪、雅雯、芃婷、昕翰、禾友、宜華等人，目前尚在學之博碩班同學如駕人兄、華鼎兄、有恆、楨屏、燕妮、立翰、家立等人。對於眾人的協助，我由衷表示感謝。

在攻讀博士班這四年間，甘苦參半，雖然獲得全時間進修博士，並晉升上校，但對內人身罹重病，奔波求醫與住院診治，其所受身心疲痛相當不捨。最要感謝內人菡如的成全及父母親和岳父母的關懷與文璇、文穎兩位妹妹協力關心照料支持，一雙兒女德愛與德望的陪伴，讓我精神奕奕、全力以赴，專心於學業，並得以順利畢業。今學有所成，我將此成果分享與我的家人和朋友及眾多關心我的人。學業已告一段落，新的職務與考驗即將開始，我將一本初衷，自我惕勵、自強不息，將所學貢獻於社會國家。謹以此論文獻給所有關心我、幫助我的至親家人、師長、與諸位貴人。

唐文漢　謹誌

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95 年 5 月

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1. Introduction

The greatest military achievement is not to construct the greatest weaponry, but to unify software and hardware machinery with thoughtful management mechanism to prevent the war from happening. This is precisely our diligently goal.

Research background, purposes and methodology are described in this chapter. Additionally, the research process and structure of this dissertation are introduced as followed.

1.1 Research background and problems

China's Sun Tzu ([1]; [35]) first chapter said: military action is important to the nation, it is the ground of death and life, the path of survival and destruction, so it is imperative to examine it. The one who figures on inability to prevail at headquarters before doing battle is the one who has the least strategic factors in his side. The one with many strategic factors in his favor wins, the one with few strategic factors in his favor. Observing the matter in this way, I can see who will win and who will lose.

No matter whether history repeats itself or not, we still can learn a lot of useful lessons from it. People have been interested in studying and analyzing historical warfare for thousands of years. Modern warfare analysis has developed many useful models and systematic methods to make more thorough and appropriate explanations of historical combats. However, since those analytic methods were applied from different viewpoints of quantitative versus qualitative, general versus specific, rough versus detailed etc, it becomes quite complicated and difficult to explain well the causes and effects of a historical combat [2] .

Realizing that the rapid increase in information technology is going to affect warfare, the Army is transforming itself for the digital age. It needs analytic tools to help make the best choices possible, and chief among these are good measures of effectiveness (MOEs) that can demonstrate the value of information in terms of military outcomes [3]. Combat models provide information that assists decision-makers in making and justifying decisions that involve the expenditure of billions of dollars and impact many lives. For example, the simulation Concepts Evaluation Model (CEM) was used to give senior Army leadership insight into potential courses of action in the planning of Desert Storm [4].

In December 1944 Adolph Hitler directed an ambitious counteroffensive with the object of regaining the initiative in the west and compelling the Allies to settle for a negotiated peace. Hitler's generals were opposed to the plan, but the Fuhrer's will prevailed and the counteroffensive was launched on 16 December by some 30 German divisions against Allied lines in the Ardennes region. Allied defenses there had been thinned to provide troops for the autumn defensive. Hitler's intention was to drive through Antwerp and cut off and annihilate the British 21st Army Group and the U.S. First and Ninth Armies north of the Ardennes ([5]; [6]; [7]; [8]).

Aided by stormy weather which grounded Allied planes and restricted observation, the German achieved surprise and made rapid gains at first, but firm resistance by various isolated units provided time for the U.S. First and Ninth Armies to shift against the northern flank of the penetration, for the British to send reserves to secure the line to the Meuse, and for Patton's Third Army to hit the salient from the south. Denied vital roads and hampered by air attack when the weather cleared, the German attack resulted only in a large bulge in the Allied lines which did not even extend to the Meuse River, the Germans' first objective. The Americans suffered some 75,000 casualties in the Battle of the Bulge, but the Germans lost 80,000 to 100, 000. German strength had been irredeemably impaired. By the end of January 1945, American units had retaken all ground they had lost, and the defeat of Germany was clearly only a matter of time. In the east the Red Army had opened a winter offensive that was to carry, eventually, to and beyond Berlin.[1]

"The Ardennes: Battle of the Bulge", is a typical warfare of offense/defense transition, a number of official histories provide carefully documented accounts of operations during the Ardennes-Alsace Campaign. Allied Army operations are covered in Hugh M. Cole, The Ardennes: Battle of the Bulge [9]; Charles B. MacDonald, The Last Offensive [10]; and Jeffrey J. Clarke and Robert Ross Smith, Riviera to the Rhine [11], three volumes in the United States Army in World War II series. Air operations are detailed in Wesley F. Craven and James L. Cate, eds., Europe: Argument to V－E Day, January 1944 to May 1945 [12], the third volume in the Army Air Forces in World War II series, and the British perspective and operations are covered in L. F. Ellis, Victory in the West: The Defeat of Germany [13]. Among the large number of books that describe the fighting in the Ardennes are Gerald Astor, A Blood-Dimmed Tide [14], John S. D. Eisenhower, The Bitter Woods [15], Charles B. MacDonald, A Time for Trumpets ([16]; [17]), S. L. A. Marshall, The Eight Days of Bastogne [18], Jean Paul Pallud, Battle of the Bulge Then and Now [19], Danny S. Parker, Battle of the Bulge [20], and Robert F. Phillips, To Save Bastogne [21].

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1 http://www.army.mil/cmh-pg/reference/eacmp.htm

Data Memory Systems, Inc. provided a daily record of the Ardennes Campaign of World War II from December 15, 1944 to January 16, 1945. A day by day history of forces and casualties on both sides can then be derived from the database. Moreover, the data contains the daily records for tanks, armored personnel carriers, artillery and personnel ([22]; [23]). Using these useful historical data, Bracken was the first researcher who successfully fitted Lanchester's models into the daily record of the Ardennes Campaign [24]. He designed an integrated equation which incorporated both the Lanchester square law model and the Lanchester linear law model. Moreover, he uses the sum of squared errors as the performance measurement of fitness when applying the extended Lanchester model to the Ardennes Campaign data. And he takes the exponents in the extended Lanchester equations as parameters to be fitted to the data. Finally, he utilizes a numerical analysis method to generate the minimum sum of squared errors of the extended model. However, Bracken used too many variables in his extended model in solving the problem of which extended Lanchester model is the best to explain the case of the Ardennes Campaign. Since Bracken's extended model is a generalization of the original Lanchester's models and is too complicated to get an accurate solution, there is still room left for improvement. Bracken's work motivated a series of related researches to improve it ([2]; [25]; [26]; [27]; [28]; [29]; [30]).

Recently, Chen and Chu proposed a much more accurate solution by combining the original Lanchester linear law model with Bracken's tactical factor [29]. Moreover, in that model, they also incorporate a new shift time variable to take account of the situation between attack and defense. Based on this modification, they significantly improved the fitness of the original Lanchester model to the Ardennes Campaign more than Bracken did. And after revising Lanchester equation, joins battle the bilateral military strength loss besides the discussion, further penetrates the real diagnosis analysis discussion both armies offense and defense in turn question.

1.2 Research purposes

According to background and motivation, the multidimensional nature of the concept is not easy to discuss by each part of historian, military personal and mathematician. The purpose of this research is to improve Bracken's [24] and Chen's [29] work to get an even more accurate solution in terms of the sum of squared errors.

Basically, we adopt concepts of the tactical factor variable and the shift time variable to improve the original Lanchester model. Moreover, we use the Lanchester square law model instead of the Lanchester linear law model to reflect the fact that the Ardennes Campaign was not an indirect-fire but a direct-fire combat. More accurately speaking, we assume that in the battle, the cross-firings of each side were aimed at the enemy hiding under bunkers or ditches.

Hence intuitively the Lanchester square law model should be better for the explanation of modern warfare. Verified by our numerical experimental result, demonstrated the Allied armies and the German armed force attack with defend in turn time. We hope that through this study can provide some implications and recommendations the Lanchester square law model to improve the aggression and disaggression strategies [2].

1.3 Framework and methods

The framework in this research is shown in Figure 1. For evaluation of sustainable development issues using by the Lanchester square law model, The first step is to define the notation of the Blue (i.e., Allied) and the Red (i.e., German) combat forces, the actual loss of Blue (Allied) and Red (German) combat forces, the Allied (Blue) and the German (Red) attrition rate without Bracken's tactical factor, Bracken's tactical factor, the last day on which the Germans attack, sum of squared errors.

Mathematical formulation is the second step, we want to select the appropriate analyze approach meeting the relation among criteria and nature of problem and our goal is to find the best fit *a, b, d* and *k,* to minimize the sum of the squared errors between the actual and theoretical attrition.

The third step is Proposition and proof of Proposition. The fourth step is numerical example, we may conclude that our result seems close to the real situation in December 1944 and we have a different point of view from Bracken [24] for the military decision makers as follows. The concept of concentration of force to penetrate, seizure of the initiative of attack and the Lanchester square law are still present in the ArdennesCampaign [2].

1.4 Organization of this dissertation

The structure of this dissertation is showed in Figure 1. The research motivation, background, purposes, framework and methods are described in Chapter 1. Chapter 2 describes the history of The Ardennes Campaign and articles of military warfare analysis tools. We introduce the stream of its development, planning and some modeling. We summarize some comprehensive generalized version of Lanchester equations model in Chapter 3. Furthermore, following the main stream of evolutionary computation, we introduce mathematical formulation of Lanchester equations and evolutionary algorithms .And one empirical study for seeking the a much better qualitative analysis model for the explanation of modern combat in Chapter 4. Finally, concluding remarks, recommendations and future research are given in Chapter 5.

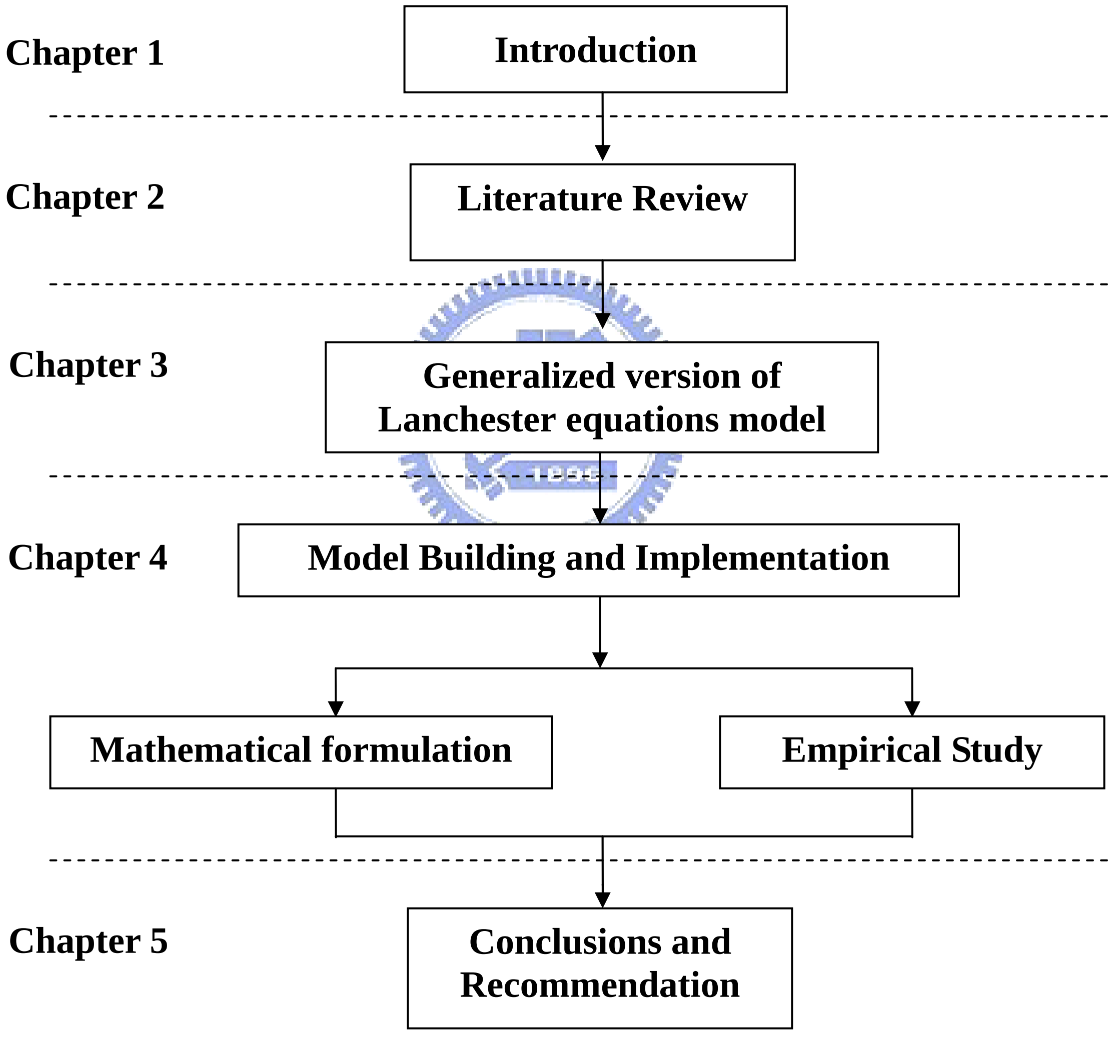

Figure 1. **The research process and organization of the dissertation**

2. Literature review

In this section we summarize related methodology and about development issues of military warfare analysis tools. We also want to describe the history of Ardennes Campaign, a partial article from Thomas D. Morgan (LT. Col.USA Ret.) Army magazine Nov. 2004[2] quoted by us. And cite Sun Tzu Art of War, On War Carl von Clausewitz [31] to discuss the doctrines of military affairs.

2.1 Lanchester equations[3]

The Lanchester laws are perhaps the best-known models of combat. They were developed by F. W. Lanchester [36] just prior to U.S. involvement in World War I and were first published in his now famous book, *Aircraft in Warfare: The Dawn of the Fourth Arm*. In this section, we discuss the Square and Linear law as following:

2.1.1 Lanchester Square Law

The effect of concentrating the force is reflected by the fact that the casualty rate is assumed to depend only on the size of the shooting force. This is due to the firepower delivery available with modern weapons. If we let *R* and *B* represent the initial size of the Red and Blue forces (number of units) respectively, and *N* and *M* ($0 \leq N, M \leq 1$) be the effectiveness of each Red and Blue unit respectively, the rate at which each of the two forces is depleted is given by the relations

$$\frac{dr(t)}{dt} = -Mb(t)$$

$$\frac{db(t)}{dt} = -Nr(t),$$

where $r(t)$ and $b(t)$ represent the Red and Blue force sizes at time t and $r(0) = R$ and $b(0) = B$. The attrition to each side depends on the effectiveness of the shooting side's units and the remaining size of the shooting force. Dividing the two equations, we get

$$\frac{\frac{dr(t)}{dt}}{\frac{db(t)}{dt}} = \frac{dr(t)}{db(t)} = \frac{Mb(t)}{Nr(t)}.$$

Rearranging, we get

$$b(t)db(t) = \frac{N}{M} r(t)dr(t).$$

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[2] http://www.ausa.org/pdfdocs/Morgan.pdf

[3] http://www.rand.org/publications/MR/MR1155/MR1155.ch4.pdf

Integrating from time 0 to time *t*, we get

$$b(t)^2 - B^2 = \frac{N}{M}\left(r(t)^2 - R^2\right)$$

This formulation allows us to examine the requirements for Blue (or Red) to win. For Blue to win, we must have that at time *T*, *r(t)* = 0 and *b*(*t*) > 0. Rewriting the above equation with *t* = *T* and solving for *b(T)*, we get

$$b(T)^2 = B^2 - \frac{N}{M}R^2 \succ 0.$$

Solving the inequality, we get

$$\frac{M}{N} \succ \left(\frac{R}{B}\right)^2.$$

For Blue to win, the relative effectiveness of the two forces must exceed the square of the initial force ratio.

One type of battle described by a Lanchester square law occurs when both sides can employ constant fractions of their forces and have target-rich environments. The size of the force the friendly commander commits to the battle determines the amount of enemy attrition attained rather than the size of the enemy force committed [3].

2.1.2 Lanchester Linear Law

The linear law reflects the inability, or more accurately the futility, of either side to mass its forces effectively. Lanchester referred to this as a characteristic of ancient warfare: In olden times, when weapon directly answered weapon, the act of defence was positive and direct, the blow of sword or battleaxe was parried by sword and shield. . . . Under [these] conditions, it was not possible by any strategic plan or tactical maneuver to bring other than equal numbers of men into the actual fighting line; one man would ordinarily find himself opposed to one man. Under these conditions, attrition depends solely upon the effectiveness of the individual combatant. Another, more modern interpretation of the linear law is that it represents area fires. That is, we assume that the attacker knows the enemy is located within an area, but that he is unable to target each combatant individually. The best he can do is launch indirect fires into the area. In this case, the effectiveness of the attacker depends not only on the effectiveness of the weapon, but also on the number of attackers (number of weapons), the effectiveness of each attacker, and the number of targets in the area fired upon. Both of these cases result in a linear law. As above, we let *M* and *N* be the effectiveness of each combatant, with *r(0)* = *R* and *b(0)* = *B*, the original size of the Red and Blue forces. The number of firing opportunities for Blue is proportional to $b(t)r(t)$, and the number of Red firing opportunities is proportional to $r(t)b(t)$:

$$\frac{dr(t)}{dt} = -M(b(t)r(t))$$
$$\frac{db(t)}{dt} = -N(r(t)b(t)).$$

The effectiveness scores refer to the effectiveness of the individual combatant. Dividing the two equations as above, we get

$$\frac{\frac{dr(t)}{dt}}{\frac{db(t)}{dt}} = \frac{dr(t)}{db(t)} = \frac{M}{N}.$$

Rearranging, we get

$$db(t) = \frac{N}{M} dr(t).$$

Integrating from time 0 to time t, we get

$$b(t) - B = \frac{N}{M}(r(t) - R).$$

For Blue to win, we again must have that at time T, $r(T) = 0$ and $b(T) > 0$. Rewriting the above equation with $t = T$ and solving for $b(T)$, we get

$$b(T) = B - \frac{N}{M} R \succ 0.$$

Solving the inequality, we get

$$\frac{M}{N} \succ \left(\frac{R}{B}\right).$$

In this case, to win, the effectiveness ratio need only exceed the initial force ratio. In the linear case, the impact of the force size on combat outcome is significantly less than in the square case. The area-fires interpretation results in the following attrition rates:

$$\frac{dr(t)}{dt} = -(b(t)M)r(t)$$
$$\frac{db(t)}{dt} = -(r(t)N)b(t),$$

reflecting the effects of force size, weapon effectiveness, and targets available. Here ($b(t)M$) can be interpreted as the firing effectiveness of Blue and ($r(t)N$) can be interpreted as the firing effectiveness of Red. Dividing the two equations as above, we get exactly the same results as above.

2.2 Sun Tzu (Art of War)

Sun Tzu [1] one of the earliest great military thinkers who realized that war, a matter of vital importance to the State, demanded study and analysis. His works are the first known attempt to formulate a rational basis for the planning and conduct of military operations. His purpose, according to Samuel B. Griffith [35], was "to develop a systematic treatise to guide rulers and generals in the intelligent prosecution of successful war". Sun Tzu was also convinced that careful planning based on sound information would contribute to speedy victory.[4] His partial work was quoted by us to discuss in this article.

2.2.1 Initial estimations (Laying plans[5])

Sun Tzu said: The art of war is of vital importance to the State. It is a matter of life and death, a road either to safety or to ruin. Hence it is a subject of inquiry which can on no account be neglected. The art of war, then, is governed by five constant factors, to be taken into account in one's deliberations, when seeking to determine the conditions obtaining in the field. These are: (1) The Moral Law; (2) Heaven; (3) Earth; (4) The Commander; (5) Method and discipline. These five heads should be familiar to every general: he who knows them will be victorious; he who knows them not will fail. … Therefore, in your deliberations, when seeking to determine the military conditions, let them be made the basis of a comparison, in this wise: --

(a) Which of the two sovereigns is imbued with the Moral law?

(b) Which of the two generals has most ability?

(c) With whom lie the advantages derived from Heaven and Earth?

(d) On which side is discipline most rigorously enforced?

The general who loses a battle makes but few calculations beforehand. Thus do many calculations lead to victory, and few calculations to defeat: how much more no calculation at all! It is by attention to this point that I can foresee who is likely to w in or lose.

2.2.2 Planning offensives (Attack by stratagem[6])

Sun Tzu said: In the practical art of war, the best thing of all is to take the enemy's country whole and intact; to shatter and destroy it is not so good. So, too, it is better to

4 http://www.ndu.edu/inss/siws/intro.html
5 http://www.kimsoft.com/polwar1.htm
6 http://www.kimsoft.com/polwar3.htm

recapture an army entire than to destroy it, to capture a regiment, a detachment or a company entire than to destroy them.

It is the rule in war:

(a) If our forces are ten to the enemy's one, to surround him;

(b) If five to one, to attack him;

(c) If twice as numerous, to divide our army into two.

(d) If equally matched, we can offer battle;

(e) If slightly inferior in numbers, we can avoid the enemy;

(f) If quite unequal in every way, we can flee from him.

Thus we may know that there are five essentials for victory:

(a) He will win who knows when to fight and when not to fight;

(b) He will win who knows how to handle both superior and inferior forces;

(c) He will win whose army is animated by the same spirit throughout all its ranks;

(d) He will win who, prepared himself, waits to take the enemy unprepared;

(e) He will win who has military capacity and is not interfered with by the sovereign.

Hence the saying: If you know the enemy and know yourself, you need not fear the result of a hundred battles. If you know yourself but not the enemy, for every victory gained you will also suffer a defeat. If you know neither the enemy nor yourself, you will succumb in every battle.

2.2.3 Military disposition (Tactical dispositions[7])

Sun Tzu said: The good fighters of old first put themselves beyond the possibility of defeat, and then waited for an opportunity of defeating the enemy. To secure ourselves against defeat lies in our own hands, but the opportunity of defeating the enemy is provided by the enemy himself. Thus the good fighter is able to secure himself against defeat, but cannot make certain of defeating the enemy. Hence the saying: One may know how to conquer without being able to **DO it.** The consummate leader cultivates the moral law, and strictly adheres to method and discipline; thus it is in his power to control success. In respect of military method, we have, firstly, Measurement; secondly, Estimation of quantity; thirdly, Calculation; fourthly, Balancing of chances; fifthly, Victory. Measurement owes its existence to Earth; Estimation of quantity to Measurement; Calculation to Estimation of quantity; Balancing of chances to Calculation; and Victory to

[7] http://www.kimsoft.com/polwar4.htm

Balancing of chances. A victorious army opposed to a routed one, is as a pound's weight placed in the scale against a single grain. The onrush of a conquering force is like the bursting of pent-up waters into a chasm a thousand fathoms deep.

2.3 On War

Carl von Clausewitz [31], a Prussian military thinker, is widely acknowledged as the most important of the major strategic theorists. Even though he's been dead for over 170 year, he remains the most frequently cited, the most controversial, and in many respects the most modern.

2.3.1 What is war[8]

We shall not enter into any of the abstruse definitions of War used by publicists. We shall keep to the element of the thing itself, to a duel. War is nothing but a duel on an extensive scale. If we would conceive as a unit the countless number of duels which make up a War, we shall do so best by supposing to ourselves two wrestlers. Each strives by physical force to compel the other to submit to his will: each endeavours to throw his adversary, and thus render him incapable of further resistance. **War therefore is an act of violence intended to compel our opponent to fulfill our will.**

If there was only one form of War, to wit, the attack of the enemy, therefore no defence; or, in other words, if the attack was distinguished from the defence merely by the positive motive, which the one has and the other has not, but the methods of each were precisely one and the same: then in this sort of fight every advantage gained on the one side would be a corresponding disadvantage on the other, and true polarity would exist. But action in War is divided into two forms, attack and defence, which, as we shall hereafter explain more particularly, are very different and of unequal strength. Polarity therefore lies in that to which both bear a relation, in the decision, but not in the attack or defence itself.

If the one Commander wishes the solution put off, the other must wish to hasten it, but only by the same form of action. If it is A’s interest not to attack his enemy at present, but four weeks hence, then it is B’s interest to be attacked, not four weeks hence, but at the

8 http://www.marxists.org/reference/archive/clausewitz/works/on-war/book1/ch01.htm

present moment. This is the direct antagonism of interests, but it by no means follows that it would be for B's interest to attack A at once. That is plainly something totally different.

2.3.2 On the signification of the combat[9]

As war is nothing else but a mutual process of destruction, then the most natural answer in conception, and perhaps also in reality, appears to be that all the powers of each party unite in one great volume, and all results in one great shock of these masses. There is certainly much truth in this idea, and it seems upon the whole to be very advisable that we should adhere to it, and that we should on that account look upon small combats at first only as necessary loss, like the shavings from a carpenter's plane. Still however, the thing is never to be settled so easily.

That a multiplication of combats should arise from a fractioning of forces is a matter of course, and the more immediate objects of separate combats will therefore come before us in the subject of a fractioning of forces; but these objects, and together with them, the whole mass of combats may in a general way be brought under certain classes, and the knowledge of these classes will contribute to make our observations more intelligible.

Destruction of the enemy's military forces is in reality the object of all combats; but other objects maybe joined to that, and these other objects may be at the same time predominant; we must therefore draw a distinction between those in which the destruction of the enemy's forces is the principal object, and those in which it is more the means. Besides the destruction of the enemy's force, the possession of a place or the possession of some object may be the general motive for a combat, and it may be either one of these alone or several together, in which case still usually one is the principal motive.

Now the two principal forms of War, the offensive and defensive, of which we shall shortly speak, do not modify the first of these motives, but they certainly do modify the other two, and therefore if we arrange them in a scheme(see Table 1) they would appear thus:

Table 1. The difference between Offensive and Defensive

Offensive.	Defensive.

[9] http://www.marxists.org/reference/archive/clausewitz/works/on-war/book4/ch05.htm

1. Destruction of enemy's force.	1. Destruction of enemy's force.
2. Conquest of a place.	2. Defence of a place.
3. Conquest of some object.	3. Defence of some object.

These motives, however, do not seem to embrace completely the whole of the subject, if we recollect that there are reconnaissance and demonstrations, in which plainly none of these three points is the object of the combat. In reality we must, therefore, on this account be allowed a fourth class. Strictly speaking, in reconnaissance in which we wish the enemy to show himself, in alarms by which we wish to wear him out, in demonstrations by which we wish to prevent his leaving some point or to draw him off to another, the objects are all such as can only be attained indirectly and under the pretext of one of the three objects specified in the table, usually of the second; for the enemy whose aim is to reconnoitre must draw up his force as if he really intended to attack and defeat us, or drive us off, etc., etc. But this pretended object is not the real one, and our present question is only as to the latter; therefore, we must to the above three objects of the offensive further add a fourth, which is to lead the enemy to make a false move, or, in other words, engage him in a sham fight. That offensive means only are conceivable in connection with this object, lies in the nature of the thing.

On the other hand we must observe that the defence of a place may be of two kinds, either absolute, if as a general question the point is not to be given up, or relative if it is only required for a certain time. The latter happens perpetually in the combats of advanced posts and rear guards.

That the nature of these different intentions of a combat must have an essential influence on the dispositions which are its preliminaries, is a thing clear in itself. We act differently if our object is merely to drive an enemy's post out of its place from what we should if our object was to beat him completely; differently, if we mean to defend a place to the last extremity from what we should do if our design is only to detain the enemy for a certain time. In the first case we trouble ourselves little about the line of retreat, in the latter it is the principal point, &c.

But these reflections belong properly to tactics, and are only introduced here by way of example for the sake of greater clearness. What strategy has to say on the different objects of the combat will appear in the chapters which touch upon these objects. Here we have only a few general observations to make, first, that the importance of the object decreases nearly in the order as they stand above, therefore then, that the first of these objects must always predominate in the great battle; lastly, that the two last in a defensive battle are in reality such as yield no fruit, they are, that is to say, purely negative, and can, therefore, only be serviceable, indirectly, by facilitating something else which is positive.

2.4 Game theory

The concepts and tools of game theory is a branch of microeconomics. Game theory has been widely used not only in business but also to analyze the effects of selecting alternative strategies to achieve a military objective.

2.4.1 The concept of game theory

For games of opposed interests, the basic concepts of maxmin and equilibrium strategies are defined and illustrated. Moving to general noncooperative games, the concepts of Stackelberg equilibrium and disequilibrium are presented in a duopoly game, and two logically consistent foundations for the competitive solution are given. The credibility of threats is discussed, and perfect equilibrium defined. Gaming is used by researchers interested in how people learn and play games and by other analysts interested in exploring strategies and policies, as a vehicle for helping understand complex issues [33].

People learn from gaming by designing games, playing them, or analyzing game results. Unlike many other techniques of analysis, gaming is not a solution method. The output of a good game is increased understanding. Gaming can be used along with other methods in conducting a study. Regardless of whether gaming achieves the rigor early proponents sought, it appears to have continuing value [34].

Game theory has been widely used to analyze the effects of selecting alternative strategies to achieve a military objective. In two-person zero-sum games, i.e., a payoff to player 1 is a loss to player 2, both players have several alternative strategies they may pursue and, although each is aware of the strategies available to his opponent, neither is aware of the strategy his opponent will select. Therefore each player may select a strategy that will maximize his minimum payoff. Such a player will hedge against the likelihood that his opponent will select the strategy that results in the worst payoff. The effects of knowing about an opponent's strategy makes game theory an excellent place to start a discussion of the effects of information on combat outcomes (payoffs). We do this by allowing each of the players (actually, "sides" in a battle) to possess varying amounts of relevant information about the strategy his opponent will select, and then we measure the effect this has on the outcome of the game [3]. In essence, we are postulating varying levels of K_B and K_R. We have designed four games in which the amount of information possessed by each side (K_B and K_R) is allowed to vary. Side 1's information might be thought of, by analogy, as comparable to that available to the U.S. Army in

- The current force, the Army of Excellence (AOE) (Game 1);
- Army XXI (Game 2); and
- Army After Next3 (AAN) (Games 3 and 4).

In addition to four different assumptions about the information available to both sides, we considered three cases of dimensionality with respect to the number of strategies or choices available to both sides. We allow each side three, five, or ten choices. (This feature of the game has some intuitive relationship with warfare, where the value of intelligence relates to the degrees of freedom available to opposing sides, which are usually rather limited.) All the games have the structure depicted in Figure 2 have choices *i* = 1, 2, . . . , *m* and *j* = 1, 2, . . . , *n*, respectively. For each pair of choices there is a payoff $a_{i,j}$ Side 1 receives $a_{i,j}$

$$Side1\ strategies(i)\begin{matrix}1\\2\\\vdots\\m\end{matrix}\begin{bmatrix}a_{1.1} & a_{1.2} & \cdots & a_{1.n}\\ a_{2.1} & a_{2.2} & \cdots & a_{2.n}\\ \vdots & \vdots & \vdots & \vdots\\ a_{m.1} & a_{m.2} & \cdots & a_{m.n}\end{bmatrix}$$
$$Side2\ strategies(j)$$

Figure2. Game Matrix

and Side 2 loses $a_{i,j}$. Side 1 therefore wishes to maximize the payoff and Side 2 wishes to minimize the payoff. This leads Side 1 to pursue what is referred to as a "maximin" strategy and Side 2 to pursue a "minimax" strategy.

2.4.2 Selecting the Optimal Strategy

Side 1's optimal strategy, *i**, is found by first computing, for each of his possible choices *i*, the worst outcome (the outcome that would come about if Side 2 made the best choice consistent with Side 1's having chosen *i*). We call that worst outcome *ai,* min, which is given by

$$a_{i,\min} = \min(a_{i,j}).$$

Side 1's most conservative choice, i^*, is the one that maximizes $a_{i,\min}$. That is, he chooses the row for which $a_{i,\min}$ is largest:

$$a_{\max,\min} = \max_i (a_{i,\min}).$$

His payoff will then be at least as good as $a_{\max,\min}$. For Side 2, we reverse the process. Side 2's optimal strategy, j^*, is found by first computing, for each of his possible choices *j,* the best outcome (the outcome that would come about if Side 1 made the best choice consistent with Side 2's having chosen *j*). We call that the worst outcome, $a_{\max j.}$. It is given by

$$a_{\max.j} = \max_i (a_{i.j}).$$

Now Side 2's most conservative choice, j^*, is the one that minimizes $a_{\max j.}$ That is, he chooses the column for which $a_{\max j.}$ is smallest:

$$a_{\min,\max} = \min_j (a_{\max,j}).$$

His payoff will then be at least as good as $a_{\max,\min}$.

2.4.3 The variable knowledge cases

We might think about war abstractly as follows. In any given battle, Side 1's choice of strategies will have some effect on the outcome, as will Side 2's. Depending on the circumstances of battle (force ratios, terrain, etc.), the strategies may make more or less difference. How, then, do we think about the value of information? As an abstraction, we can consider a vast array of battles in which strategies have very different consequences for the outcomes. We can then ask how much value information would have, on average, over that vast array of battles. This is indeed what we have calculated. For each of 1,000 different battles we generated a payoff matrix as in Figure 2, using random numbers between 0 and 100. We then made various assumptions about how much knowledge each side had about the payoff matrix. Each side then selected strategies based on that knowledge. We did this first assuming that the sides had three strategies each; we repeated the work with five and ten strategies. In the discussions below, we refer to the payoff matrix depicted in Figure 2 as **A**.

• **Game 1: current force (AOE) (both sides have correct information).**

Side 1 and Side 2 have common and correct knowledge of all the values of the payoff matrix **A**. Both sides have the same information about payoffs but are ignorant about each other’s choices. Neither has superior knowledge. This can be thought of as the case in which $K_B = K_R \ and \ \Gamma = 1$.

• Game 2: Army XXI (Side 1 has correct information and Side 2has incorrect information).

Side 1 has correct knowledge of all the values of **A** = **A**1, and Side 2 has a completely *incorrect* understanding of the payoff matrix. We simulate this by providing Side2 with a payoff matrix, **A** = **A**2, composed of a second set of random numbers between 0 and 100. Therefore Side 2 will make decisions based on erroneous information. Although purely an abstraction, this could describe a situation in which Army XXI with superb information fights an enemy who not only lacks valid information but is thoroughly confused. This can be thought of as the case in which Blue (Side 1) has information superiority, i.e., $K_B \succ K_R \ and \ \Gamma \succ 1$.

• Game 3: AAN (Side 1 has correct information, Side 2 has correct information, and Side 1 knows Side 2’s choice).

Side 1 and Side 2 have correct knowledge of the values of **A**, as in Game 1. Side 2 chooses his minimax strategy *j** from the correct matrix **A.** Side 1, however, knows the choice Side 2 makes, and rather than choose his maximin strategy (*i**), he focuses only on the payoffs corresponding to the minimax choice of Side 2 and maximizes his payoff. This simulates the case in which Side 1 has perfect intelligence and, as a result, another kind or higher level of information superiority. Although Side 2’s basic information in this case (as opposed to Game 2) is not bad, it is clearly inferior to Side 1’s.In this case, we have again that $K_B \succ K_R \ and \ \Gamma \succ 1$. , but now Γ is significantly greater than 1.

• Game 4: AAN (Side 1 has correct information, Side 2 has incorrect information, and Side 1 knows Side 2’s choice).

In the fourth game Side 1 has correct knowledge of all the values of **A** = **A**1 and Side 2 has a completely incorrect payoff matrix **A** = **A**2 composed of a second set of random numbers between 0 and 100, as in Game 2. Side 2 chooses his minimax strategy, *j**, from the incorrect information in **A**2. Side 1 knows the choice of Side2.

Rather than using his maximin strategy, he focuses only on the payoffs corresponding to the minimax choice of Side 2 from the incorrect information and makes his choice from the correct matrix, **A**1. Side 1 has perfect information (maximum knowledge). He may even have established this position by actively ensuring (through offensive information operations) that Side 2 has bad information. Thus, Side 1 enjoys not only information superiority but also information dominance, i.e., $K_B \succ \delta_B$ *and* $K_B \succ K_R$.

2.4.4 Results

Table 2 summarizes the results of the four games. In each case, three different sets of strategies, or game sizes, were involved. The entries in the table can be thought of as percentages reflecting the likelihood that Side 1 will be successful given the relative knowledge between the two sides.

Table 2. The effect of knowledge on Game Outcomes

Game Size	Game 1	Game 2	Game 3	Game 4
3*3	50	63	58	75
5*5	50	61	65	83
10*10	49	59	75	91

It is important to note that the table entries do not reflect the likelihood that Side 1 will experience a successful combat outcome, but rather the degree to which relative knowledge contributes to Side 1's successful outcome: relative force ratios, weapon system effectiveness, and other measures discussed later contribute as well. A score of 90, for example, means that relative knowledge contributed 90 percent to Side 1's successful outcome, whereas it contributed only 10 percent to Side 2's successful outcome. The actual outcome is not of interest here, just the contribution of knowledge. The games reflect the effect of knowledge on the likelihood of a successful outcome. Beginning with Game 1, we see that, as predicted, when neither side enjoys information superiority, the likelihood of winning is even—that is, the contribution of knowledge to winning is even. This seems to hold regardless of the number of strategies available to each side. This also applies to Game 2, with Side 2 possessing erroneous information about the outcomes. The pattern appears to change, however, for Games 3 and 4. There appears to be a greater advantage to Side 1 when the number of strategies increases. This phenomenon is easy to explain based on the structure of the game. Side 2's selections in both games approach random choices, where the probability of selecting any of the s strategies is $1/s$. Therefore, the likelihood of succeeding is greater for smaller strategy sets. What is not clear from all this is whether the seeming advantages associated with information superiority and large strategy sets is applicable to real-world engagements. What is missing is some understanding of the relative importance of the choices being made.

2.5 The Ardennes: Battle of the Bulge

Battle of the Bulge was the story of how the high command, American and British, reacted to defeat the German plan once the reality of a German offensive was accepted. But most of all it is the story of the American fighting man and the manner in which he fought a myriad of small defensive battles until the torrent of the German attack was slowed and diverted, its force dissipated and finally spent. It is the story of squads, platoons, companies, and even conglomerate scratch groups that fought with courage, with fortitude, with sheer obstinacy, often without information or communications or the knowledge of the whereabouts of friends. In less than a fortnight the enemy was stopped and the Americans were preparing to resume the offensive. The battle ground of Ardennes we may see in Figure 3.[10] ([9]; [32]).

2.5.1 Weather and terrain analysis of Ardennes[11]

Also spelled Ardennes, wooded plateau covering part of the ancient Forest of Ardennes, occupying most of the Belgian provinces of Luxembourg, Namur, and Liège; part of the Grand Duchy of Luxembourg; and the French department of Ardennes. It is an old plateau comprising the western extension of the Middle Rhine Highlands, stretching in a northeast-southwest direction and covering more than 3,860 sq mi (10,000 sq km). Its geological history is complex; as a result of intense folding, faulting, uplifts, and denudations, some older strata of rock have been thrust over younger strata.

The name Ardennes used in a strict sense refers to the southern half of the area, where the elevations range from 1,150 to 1,640 ft [350 to 500 m], though the high point at Botrange, south of Liège, is 2,277 ft. This part consists of sandstone, quartzite, and some slate and limestone. Its rounded summits are separated by shallow depressions containing

[10] http://www.army.mil/cmh-pg/brochures/ardennes/p04(map).jpg
[11] http://ww2fighters.org/forums/index.php?showtopic=1112

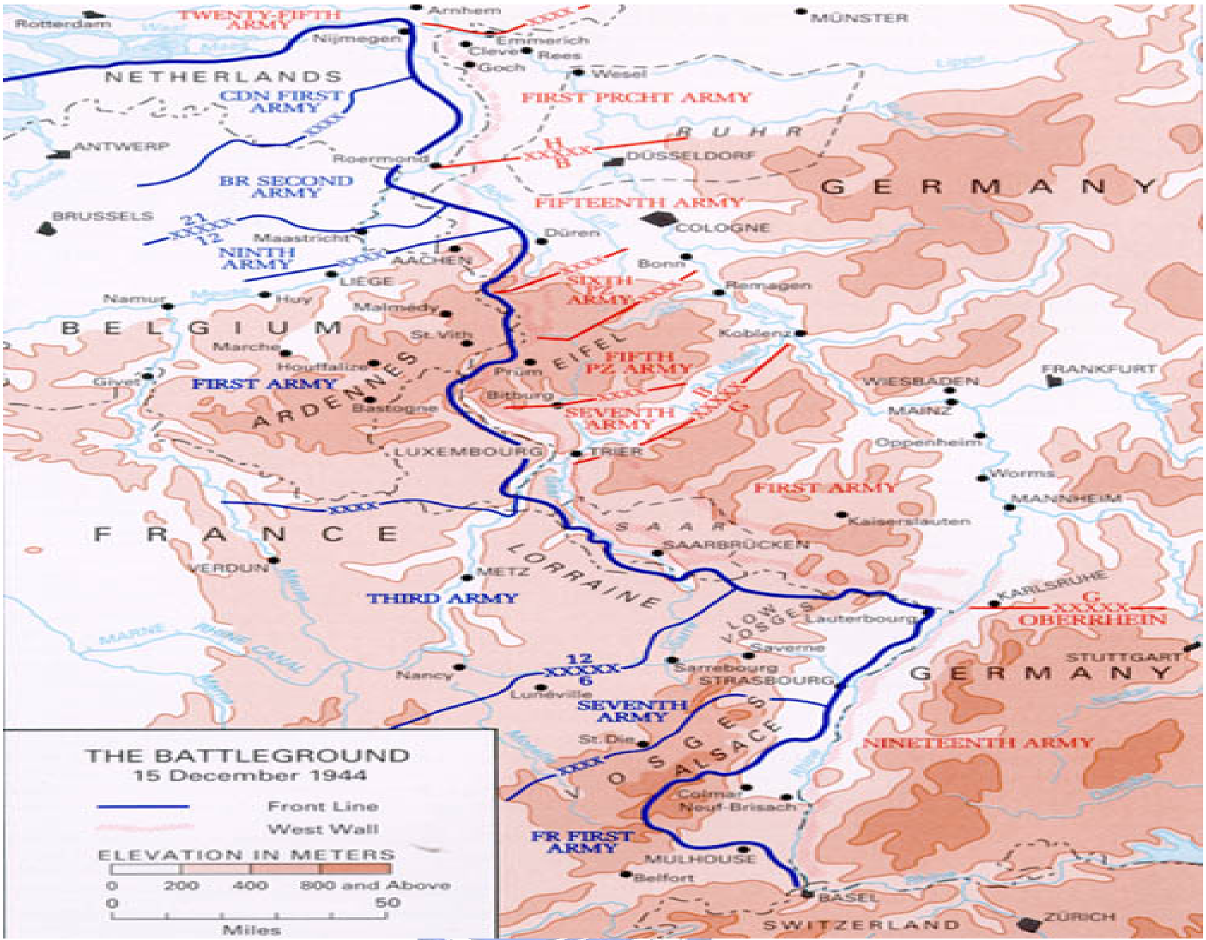

Figure 3. The battle ground of Ardennes

peat bogs, from which rise many rivers that cut narrow and sinuous valleys. This High Ardennes form the watershed between rivers flowing north and west to the Meuse River and south and east to the Moselle River. Heavy precipitation, combined with low clouds, fog, and frost, make the uplands distinctly bleak. Although one-half of the area is covered by forest, the thin, acid, and waterlogged soil is generally infertile, supporting only heath.

The northern part is much lower, between 655 and 985 ft. Most of the small farmland is under permanent grass for pasture, but there is some cultivation of oats, rye, potatoes, and clover in the valleys. Cattle are raised mainly for dairy production, pigs for the ham that has long been a local specialty of the Ardennes, and sheep for a small wool industry. Cattle hides are processed with the abundant local supplies of tannin from the oak trees. Stone quarrying is widespread, but mining and manufacturing are limited.

Despite a certain raw inhospitality of the area, its economy increasingly depends upon the development of tourism. The Ardennes has one of the lowest population densities of Europe, but it is located in the middle of the heavily populated triangle of

Paris-Brussels-Cologne. Mineral springs at Spa, Belg. (whence the English word spa), have made it a favorite health resort since the 16th century. The lonely forests offer respites for central Europeans from the pressures of the surrounding urbanization. During World Wars I and II, the Ardennes became a battleground, the scene of bitter fighting in 1914, 1918, and 1944.

Terrain

Belgium generally is a low-lying country, with a broad coastal plain extending from the North Sea and The Netherlands and rising gradually into the Ardennes hills and forests of the southeast, where a maximum height of 2,277 feet (694 meters) is reached at Botrange. The main physical regions are the Ardennes and Ardennes foothills; the Anglo-Belgian Basin to the north comprising the Central (Bas) Plateaus, the plain of Flanders (Vlaanderen), and the Kempenland (Campine); and the intrusion of the Paris Basin on the south known as the Côtes Lorraines (Belgian Lorraine). The Ardennes region is part of the Hercynian orogenic belt, which reaches from western Ireland into Germany and was formed during the second half of the Paleozoic Era (roughly 300 to 400 million years ago). It is a plateau cut deeply by the Meuse River and its tributaries. Its higher points have poor drainage and are more favourable for peat bogs and upland mossy ground than for crops. A large depression, known east of the Meuse as the Famenne and west of it as the Fagne, separates the Ardennes from the geologically and topographically complex foothills to the north. The principal feature of the area is the Condroz, a plateau more than 1,100 feet in elevation comprising a succession of valleys hollowed out of the limestone between sandstone crests. Its northern boundary is the Sambre-Meuse valley, which transverses Belgium from south-southwest to northeast. Situated south of the Ardennes and cut off from the rest of the country, Côtes Lorraines is a series of hills with north-facing scarps. About half of it remains wooded; in the south lies a small region of iron ore deposits. A region of sand and clay soils lying between 150 and 650 feet in elevation, the Central Plateaus cover northern Hainaut, Walloon Brabant, southern Flemish Brabant, and the Hesbaye plateau region of Liège. The area is dissected by the Dender, Senne, Dijle, and other rivers that enter the Schelde (Escaut) River; it is bounded on the east by the Herve Plateau. The Brussels region lies within the Central Plateaus. Bordering the North Sea from France to the Schelde, the low-lying plain of Flanders has two main sections. Maritime Flanders, extending inland for 5 to 10 miles (8 to 16 kilometers), is a region of newly formed and reclaimed land (polders) protected by a line of dunes and dikes and having largely clay soils. Interior Flanders comprises most of East and West Flanders and has sand-silt or sand soils. At an elevation of 80 to 300 feet, it is drained by the Leie, Schelde, and Dender rivers flowing northeastward to the Schelde estuary. Several shipping canals interlace the landscape connecting the river systems. Covered by pasturelands and industry and lying between 160 and 330 feet in elevation, the Kempenland forms an irregular watershed of plateau and plain between the extensive Schelde and Meuse drainage systems.

Weather

Belgium has a temperate, maritime climate predominantly influenced by air masses from the Atlantic. Rapid and frequent alternation of different air masses separated by fronts gives Belgium considerable variability in weather. Frontal conditions moving from the west produce rainy weather, with rainfall heavy and frequent, averaging 30 to 40 inches (750 to 1,000 millimeters) a year. Winters are damp and cool with frequent fogs; summers are rather mild. The annual mean temperature is around 50° F (10° C). Brussels, which is roughly in the middle of the country, has a mean minimum temperature of 31° F (-0.3° C) in January and a mean maximum of 71° F (21.6° C) in July. Regional climatic differences are determined by elevation and distance inland. Farther inland, maritime influences become weaker, and the climate becomes more continental, characterized by greater seasonal extremes of temperature. The Ardennes region, the highest and farthest inland, is the coldest. In winter, frost occurs on about 120 days, snow falls on 30 to 35 days, and January mean minimum temperatures are lower than elsewhere. In summer, the elevation counteracts the effect of distance inland, and July mean maximum temperatures are the lowest in the country. Because of the topography, the region has the highest rainfall in Belgium. In contrast, the Flanders region enjoys generally higher temperatures throughout the year. There are fewer than 60 days of frost and fewer than 15 of snow. On the seacoast these figures are reduced to below 50 and 10, respectively. There are a few hot days, especially on the coast, where the annual rainfall is the lowest in the country. All of Belgium except the Ardennes lies within the zone of broad-leaved deciduous forestation. The dominant tree is the oak; others include beech, birch, and elm. Little remains of the forest that covered this area 2,000 years ago. Most of lowland Belgium is now used for agriculture or human settlement; small clumps of deciduous trees and grasses dominate the remaining open spaces. In the Kempenland, however, significant areas are devoted to planted forests of silver birch and Corsican pine. The Ardennes lies within the zone of mixed deciduous and coniferous forestation. The area has been heavily logged for centuries.

Hence, little old-growth forest remains. The Ardennes is dominated now by coniferous forests in the higher elevations and by zones of mixed coniferous and deciduous trees, especially beech and oak, in the foothills. Hautes Fagnes, which is located at the northeastern edge of the Ardennes, is covered with peat bogs. Drainage has improved, however, and the area, forested with spruce, is part of a nature reserve.

2.5.2 The term Battle of the Ardennes

The term Ardennes Offensive (or Battle of the Ardennes) [12] refers to multiple battles throughout history, all of which took part in or around the Ardennes Forest in France and Belgium shown in Table 3.

Table3. The Ardennes Offensive (The Battle of the Bulge)

Conflict	World War I	World War II	World War II
Date	August 21-23, 1914	May 10-12, 1940	December 16, 1944 – January 15, 1945
Place	The Ardennes	The Ardennes	The Ardennes
Result	German victory	German victory	Allied victory

The Ardennes is a region of extensive forests and rolling hill country, primarily in Belgium and Luxembourg, but stretching into France (lending its name to the Ardennes department and the Champagne-Ardennes region).

The Battle of the Ardennes was one of the opening battles of World War I. It took place from August 21-23, 1914, part of the Battle of the Frontiers. French commander-in-chief Joseph Joffre ordered an attack through the Ardennes forest in support of the French invasion of Lorraine. The French forces consisting of the Third and Fourth Armies, expecting only light resistance ran into a German advance consisting of the German Fourth and Fifth Armies. The initial engagement took place in a heavy fog and the Germans built defensive positions before heavy fighting commenced the second day, The French forces were badly routed by entrenched German machine guns, falling back to Verdun and Sedan.

[12] http://encyclopedia.thefreedictionary.com/Ardennes%20Offensive

In World War II, the Battle of France was the German invasion of France and the Low Countries, executed 10 May, 1940 which ended the Phony War. German armored units punched through the Ardennes, outflanking the Maginot Line and unhinging the Allied defenders. Paris was occupied and the French government fled to Bordeaux on 14 June.

Ardennes Offensive, which was actually known to the Germans as Operation Wacht Am Rhein, was also known as Second Battle of the Ardennes and popularly known as the Battle of the Bulge, started in late December 1944 and was the last major German offensive on the Western Front during World War II.

2.5.3 The battle of the Bulge remembered

At 5:30 A.M. Sunday, December16, 1944, all hell broke loose along the lightly defended U.S. sector of the Ardennes Forest on the German-Belgian-Luxembourg border as German *Panzers* attacked after a short artillery preparation. German tanks, attacking with searchlights glaring, pressed through antitank obstacles along the Siegfried Line while rockets fired from Nebelwerfers screamed overhead.

The American defenses crumbled and two great Panzer armies broke through and headed for the Meuse River and the vital English Channel ports beyond the Meuse. The German attack was a complete surprise to the Allies. Field Marshal Bernard L. Montgomery only the day before had said, "The enemy is at present fighting a defensive campaign ...he cannot stage any major offensive operations." In December 1944, the victorious Allies were advancing on a wide front toward Hitler's Third Reich. Except for a temporary setback in Holland in September 1944 (the ill-fated "Bridge Too Far"), Allied forces had pushed Hitler's once invincible legions behind the fabled West Wall defense of the Siegfried Line. In the north, Field Marshal Montgomery's armies were advancing on the Upper Rhine and the Ruhr, the industrial heart of Germany. In the center, Gen. Omar Bradley's 12th Army Group was advancing against the Siegfried Line. Operating as half of Bradley's army group was the First Army under Lt. Gen. Courtney Hodges, preparing to attack the Roer dams while defending the Ardennes front. Maj. Gen.Troy Middelton's VIII Corps defended the 80-mile Ardennes front, stretching from Monschau in Germany to Echternach in Luxembourg, with the equivalent of four divisions. Gen. Dwight D. Eisenhower advocated an offensive attitude across his wide front of advance, but he held some sectors with comparatively weak forces to gain strength at his points of attack. The Ardennes sector was known as the"Ghost front." It was a cold, quiet place where only occasional artillery rounds were fired and patrols probed enemy lines only to keep in practice. It was known as a rest area for each side. The order of battle of both sides we may see as follow in Table 4[13].

Table 4. The order of battle for period of 16 December 1944 to 2 January 1945

The Allied Army order of battle	The German Army order of battle
Armored division: 2nd , 3rd , 4th , 6th , 7th , 9th , 10th ,11th Infantry division: 1st , 2nd , 4th , 5th , 9th , 26th , 28th , 30th , 35th , 75th , 80th , 83rd , 84th , 87th , 99th , 106th Airborne division: 101st , 82nd	SS Panzer Division: 1st , 2nd , 9th , 12th Panzer Division Panzer Lehr, 2nd , 9th , 116th Parachute Division 3rd, 5th Grenadier Division 9th , 12th , 18th , 26th , 62nd , 79th , 167th , 212th , 246th , 272nd , 276th , 277th , 326th , 340th , 352nd , 560 Volks Fuehrer-Grenadier Fuehrer Begleit Brigades

The key to the sector was the seven-mile wide Losheim Gap defended by the 14th Cavalry Group attached to the 106th Infantry Division. The 106th had just arrived from the States and had never been in combat. In choosing the Ardennes for a rest area, the Allies had forgotten that the Ardennes and the Losheim Gap were classic east-to-west invasion routes, used successfully by the Germans in 1914 and 1940. Hitler conceived a last, decisive offensive in the West in September and October 1944. This was after the Normandy landings, the destruction of the German armor in the Falaise Gap, and the Allied landings in southern France. German armament production had increased in 1944, but armament minister Albert Speer warned Hitler that it would be impossible to continue unless a decisive solution in the West was found. New units were being formed for the Wehrmacht. Luftwaffe and naval units were pressed into service, and units were transferred secretly from the Russian front. Also, new Volksgrenadier (people's infantry) were formed from very young and middle-aged men freed from factory work by slave laborers from conquered countries. In all, about 30 divisions (250,000 men, 2,000 guns, 1,000 armored vehicles and 1,500 aircraft) were assembled along the Western front opposite the Ardennes for the great offensive. Secrecy and surprise were key to the success of the offensive. Only German army commanders were told of the plan, code-named

[13] http://www.army.mil/cmh-pg/books/wwii/7-8/7-8_25.htm

Wacht am Rhein (Watch on the Rhine).Operational security was enhanced further by making all preparatory moves at night, driving tracked vehicles over straw to muffle the sound and prevent telltale tracks, moving artillery into position with horses and using aircraft overflights to mask noises not subdued by other means. The objective of the offensive was to break through the Allied front at its weakest point, separate the American and British forces, and occupy Antwerp before it could be put into full operation. If that could be done, perhaps disaster in the West could be averted and a full effort made against the Russians. Field Marshal Karl Rudolf Gerd von Rundstedt, nearing 70 and Germany's senior field commander, was brought back from retirement to command the Western front. Although only brought in a few weeks before the offensive, his name has been given to the offensive because of his prestige and lukewarm attitude to the national socialism of the Nazis. Under von Rundstedt was Field Marshal Walther Model's Army Group B, consisting of four armies. The main attack would be made by the 6th *SS Panzer* Army and the 5th *Panzer* Army with a total of 16 divisions, eight of them armored. Unhappy with such an ambitious plan and limited resources, both von Rundstedt and Model tried to get Hitler to accept a modified, limited offensive. Hitler refused and the die was cast for the last great gamble in the West. On December 16, bad weather grounded the Allied air forces. The Germans attacked in a thick fog, achieved com-plete surprise and made deep penetrations along the front[14](Figure 4).

American units were overrun, surrounded and routed. The 14th Cavalry Group guarding the critical Losheim Gap withdrew prematurely and it took the relief of several commanders to bring it under control. The 106th Infantry Division was destroyed. Two regiments of the 106th Division, with supporting troops, were cut off and surrounded in the Schnee Eifel. On December 19, both regimental commanders surrendered their commands. Next to Bataan in the Philippines, it was the largest mass surrender of American soldiers in history. It was also the most severe defeat for U.S. forces during the European campaign. From his Supreme Headquarters Allied Expeditionary Force (SHAEF) in Versailles, Gen. Eisenhower ordered his reserves, the 82nd and 101st Airborne Divisions, to the Ardennes, and diverted armored divisions from the north and south to the Ardennes sector. Brig. Gen. Bruce Clarke brought the vanguard of the 7th Armored Division to St. Vith. Here, Clarke took over the shattered remains of the 106th Division and held the Germans at bay until December 23 when the last American units pulled out. Clarke's defense of St. Vith had

[14] http://www.army.mil/cmh-pg/brochures/ardennes/p23(map).jpg

delayed the German 5th *Panzer* Army for three days, long enough to upset Hitler's timetable and to allow Allied reinforcements to arrive.

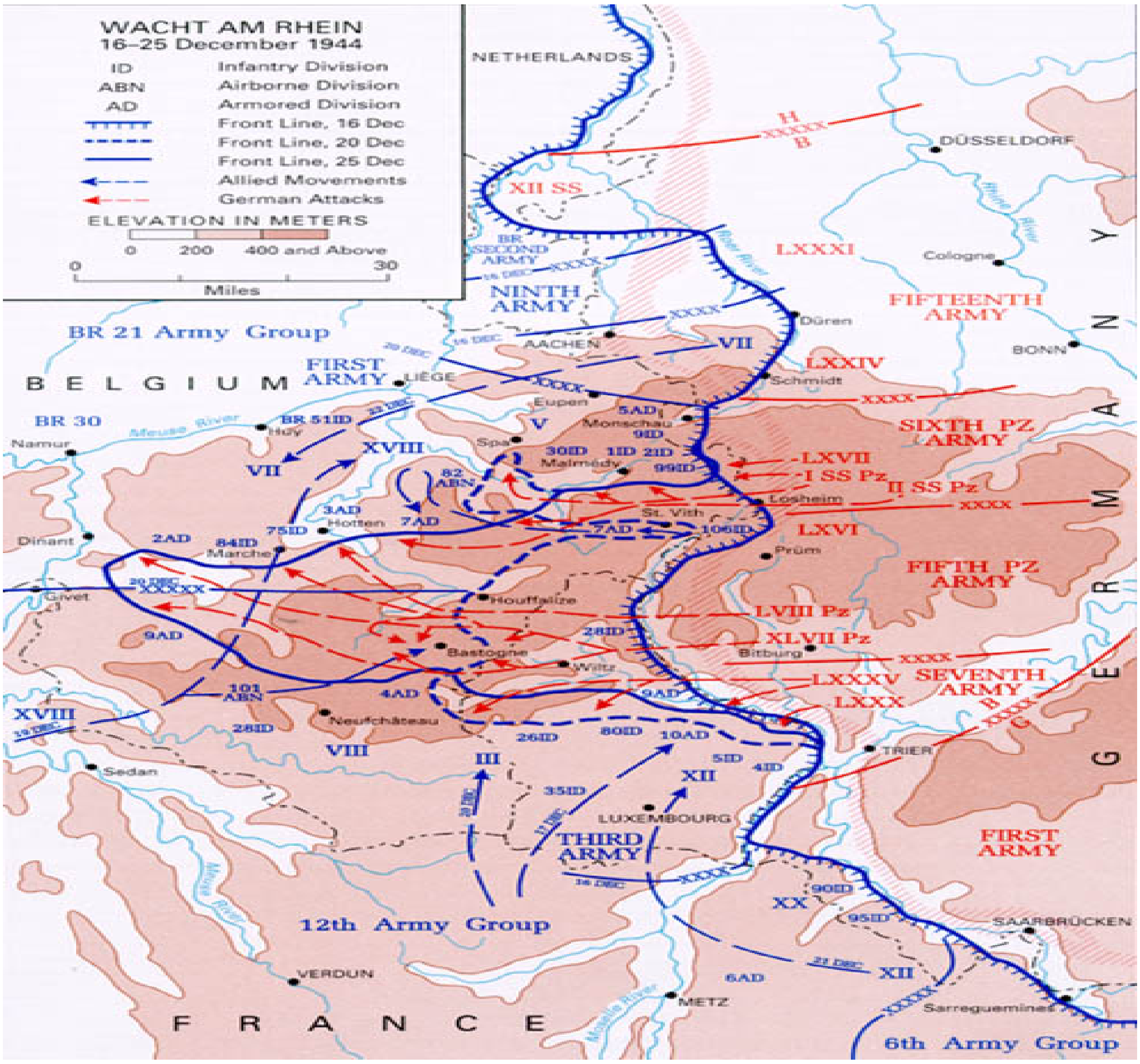

Figure 4. The German offensive and Allies defense

While Clarke was holding out at St. Vith, another drama took place to the south. Ordered to the Ardennes from SHAEF reserves, the 101st Airborne Division was bivouacked near Reims, France. The division immediately left France in the late afternoon of December 18, drove all night in 3,000 hurriedly commandeered trucks and arrived at Bastogne on the morning of December 19 just ahead of the 2nd *Panzer*, *Panzer-Lehr* and 26th *Volksgrenadier* Divisions, all closing in from the east. The 101st would become famous for its stubborn defense of that vital town and the defiance of its acting commander, Brig. Gen. Anthony McAuliffe. By December 20, Bastogne, the logistics center of the

Ardennes, was surrounded and the 101st was trapped inside without badly needed combat service support. During the motor march to Bastogne, the division's major medical unit, the 326th Medical Company, had been cut off and captured, and the 426th Quartermaster Company ambushed and diverted to VIII Corps control. The 101st was without surgeons, medical supplies and normal supply services. The weather was bad and little resupply got in by air. Things did not look good for the "battered bastards of Bastogne." In spite of the Germans' crushing attacks, the north and south shoulders of the Bulge salient held. In the north, the 2nd and 99th Divisions' defense at Monschau and along the Elsenborn Ridge, reinforced by the 47th and 39th Infantry regiments of the 9th Division, stopped the 6th *SS Panzer* Army. In the south, the heroic delaying action of the 28th Infantry Division had slowed the Germans long enough for the 101st Airborne to get to Bastogne. Also, the 4th Infantry Division's defense of the area around Echternach was equally effective against the German 7th Army. In the center, elements of the 5th *Panzer* Army had bypassed Bastogne and penetrated almost to the Meuse River. At Celles, just short of the Meuse, the 2nd *Panzer* Division was crushed by an aggressive Maj. Gen. Ernest Harmon and his 2nd (Hell on Wheels) Armored Division during a two-day battle that began on Christmas. Good weather heartened Bastogne's defenders on December 23. "Air Force Day" was the result of good flying weather that made possible an aerial resupply of badly needed food, ammunition and medicine, not to mention a good aerial pounding of the Germans by B-17 bombers and fighter-bombers. When the weather socked in again, the infantrymen and artillerymen were ready for the German main attack that took place on Christmas Eve. Hitler had been promised Bastogne for Christmas. At a council of war in Verdun on December 19, Gen.George Patton flamboyantly promised Eisenhower that he would shift his attack on the Saar Basin 90 degrees and attack north to relieve Bastogne ithin 48 hours. None of the generals at the conference believed that he could do that so fast. On the morning of December 22, the III Corps of Patton's Third Army launched its attack northward. The vanguard of III Corps was the 4th Armored Division. **L**ate in the afternoon of December 26, Combat Command R, with Lt. Col. Creighton W. Abrams' 37th Tank Battalion leading, was just a few miles south of Bastogne. Abrams suggested a direct thrust into Bastogne. Thirteen artillery battalions fired a 90-second preparation and the lead elements of Abram's tank battalion blasted its way into the 101st Airborne's perimeter. This lifted the siege of Bastogne that had occupied as many as nine German divisions. It was at Malmedy that one of the greatest tragedies of the campaign occurred. On December 17, a 125-man convoy of B Battery, 285th Field Artillery Observation Battalion had just passed a crossroads near Malmedy when it collided with SS Col. Joachim Peiper's Kampfgruppe (battle group) that was the spearhead of the 1st SS Panzer Division of Gen. Josef (Sepp) Dietrich's 6th SS Panzer Army. Peiper was a fanatic *Waffen-SS* officer known for his ruthlessness on the Russian front. Dietrich was an ex-sergeant major and former bodyguard of Hitler from the street-brawling days in Munich. Dietrich had directed, "No time is to be wasted in the matter of prisoners." The U.S. unit was captured, lined up in a snow-covered field and machine-gunned. Eighty-six soldiers were killed. When the news of the atrocity got out, the incident became known as the "Malmedy Massacre." Word went out to U.S. units that no *SS* prisoners were to be taken. That word was countermanded later, but the damage was done.

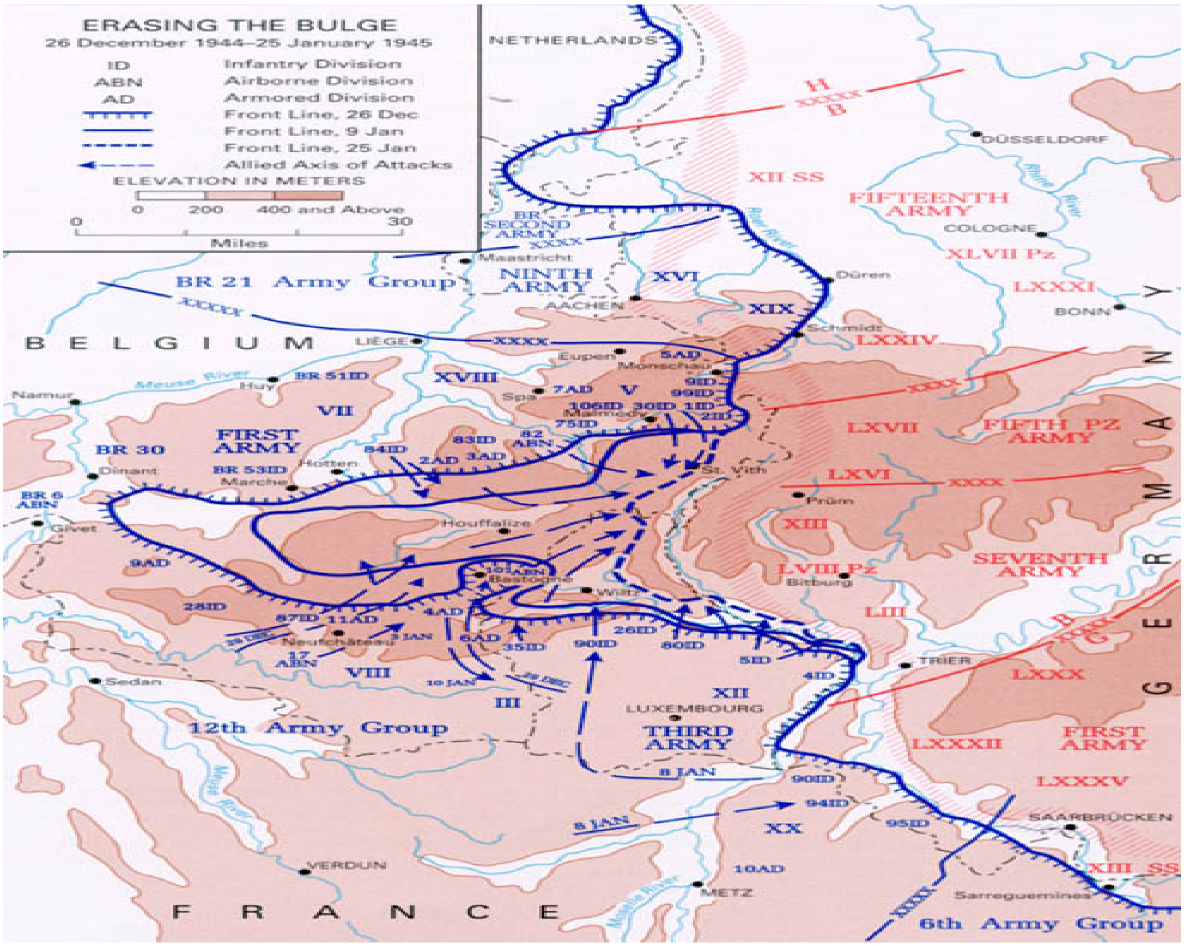

Figure 5. The Allies offensive and the German defense

After the war, Peiper and Dietrich were tried as war criminals, but they got off with only prison sentences because their sentences were influenced by the revenge taken by U.S. troops after the Malmedy Massacre was publicized. Gen. McAuliffe objected strongly when they were released in the 1950s. With the Third U.S. Army attacking from the south and the First U.S. Army attacking from the north, the Germans in the Bulge made one last desperate effort after Christmas to regain the initiative. They failed, and Hitler refused to allow a timely withdrawal behind the Siegfried Line. By January 20, 1945, the Bulge had been eliminated and the Germans were back at their starting points[15](Figure 5). In 34 days, the Germans had lost about 70,000 killed, wounded and captured, and about 800 armored vehicles and 1,000 aircraft were destroyed. American losses were equally severe. About 80,000 were killed, wounded and captured, and 700 armored vehicles and 500 aircraft were destroyed[16](Table 5) ; but, the Americans were able to replace their losses within 15 days. The Germans could not. Hitler lost the last of his mobile reserves in the Ardennes, and he was ill-prepared for Stalin's January 1945 offensive in the east. Churchill called the Battle of the Bulge "the greatest American battle of the war ... an everfamous American victory."

Table 5. The Ardennes Campaign (The Battle of the Bulge)

Conflict	World War II
Date	December 16, 1944 – January 15, 1945
Place	The Ardennes
Result	Allied victory
Combatants	
Allies	Germany
Commanders	
Dwight Eisenhower	Gerd von Rundstedt
Strength	
80,000 men, 400 tanks, 400 guns (Dec 16 - start of the Battle)	200,000 men, 600 tanks, 1,900 guns (Dec 16 - start of the Battle)
Casualties	
78,000 casualties (8,607 dead, 21,144 captured/missing, 47,139 wounded), 733 tanks lost	68,000 casualties (17,236 dead, 16,000 captured/missing, 34,439 wounded), 700 tanks lost

2.6 Concluding remarks

No matter whether history repeats itself or not, we still can learn a lot of useful lessons from it. People have been interested in studying and analyzing historical warfare

15 http://www.army.mil/cmh-pg/brochures/ardennes/p41(map).jpg
16 http://encyclopedia.laborlawtalk.com/Battle_of_the_Bulge

for thousands of years ([2]; [37]; [53]).

Since the humanity has been long-time continuously penetrates each kind of technological development or the science computation, hopes to accurate solution to war result.

However, since those analytic methods were applied from different viewpoints of quantitative versus qualitative, general versus specific, rough versus detailed etc, it becomes quite complicated and difficult to explain well the causes and effects of a historical combat. We try to summarize some comprehensive generalized version of Lanchester equations model in Chapter 3and to seek a much better qualitative analysis model for the explanation of modern combat in Chapter 4.

3. Generalized version of Lanchester equations model

Since Lanchester [36] proposed his models for attrition in combat much has been written on the subject. One strand of the literature has been concerned with making the original differential equations more realistic [38]. Another strand has studied stochastic versions of the original equations we may see in Clark [39]; Goldie [40] and Kress and Talmor [41]. Other authors have tried to fit the equations to historical battles and campaigns; e.g. the Battle of Gilead[42], the Land Battles of the Years 1618–1905[43], the American Civil War ([44]; [45]) or the Korean War [46]. For a recent discussion of data from the Ardennes campaign in the Second World War ([24]; [29]; [47]; [48]). Looking at Lanchester's original formulation it is perhaps not surprising that such attempts have produced no clear cut evidence that the models hold [27].

3.1 Original formulation of Lanchester's Square and Linear Law

F W Lanchester (1868–1946) was a man of many talents. This British engineer not only built the first car in Britain in 1895, but made significant contributions to aeronautics (e.g. the Lanchester–Prandtlgeneral equations of flight), operations research and military strategy described by Borges [49] . In fact, there is an annual prize offered in his name by the Operations Research Society of America. But perhaps, more importantly, Lanchester is remembered for his insights into military tactics. His book Aircraft in Warfare: the Dawn of the Fourth Arm written in 1916 just before World War I laid down laws of combat which involved insights using force strengths in terms of numbers, fighting capabilities of individual soldiers or weapons, and the concentrations of forces. Lanchester considered aircraft as a fourth type of military force after cavalry, infantry and artillery and hence the title of his book [36]. Lanchester divided warfare into two basic types: ancient and modern. Of the former type he says: In olden times, when weapon directly answered weapon, the act of defense was positive and direct, the blow of sword or battle-ax was parried by sword and shield. Under the old conditions, it was not possible by any strategic plan or tactical maneuver to bring other than approximately equal numbers of combatants into the actual firing line; one man would ordinarily end himself opposed by one man. Even were a general to concentrate twice the number of men on any given portion of the field to that of the enemy, the number of men actually wielding their weapons at any given time, was, roughly speaking, the same on both sides.

3.1.1 Lanchester Linear Law

Based on this line of reasoning, Lanchester came up with a Linear Law of combat which says that the chances of winning depend therefore not so much on numbers of attacking units but more importantly on the effectiveness of each attacking unit especially if the battle is actually composed of a series of duels. This is how the Lanchester Linear Law may be derived. Let *m* and *n* be the numbers of opposing forces *M* and *N* at time *t*. Let β and α be the fighting ability of one unit of force *M* and *N*. The rate of attrition of the two sides is calculated as:

$$\frac{dm}{dt} = -\alpha m \text{ and } \frac{dn}{dt} = -\beta n \left(\text{where } m \prec n \right).$$

Therefore

$$\frac{dm}{dn} = \frac{\alpha}{\beta}, dm = \left(\alpha / \beta \right) dn$$

$$\text{and} \quad \beta dm = \alpha dn.$$

Integrating from time 0 to time *t*, $\beta(m - m_0) = \alpha(n - n_0)$.

When side *M* wins, *n* = *0*. Therefore, $m = m_0 - \frac{\alpha}{\beta} n_0$.

Consequently, side *M* will win if $m_0 \succ \frac{\alpha}{\beta} n_0$.

This means that if $m_0 = n_0$ to begin with, then side *M* can only win if the fighting ability of each unit of *M* is greater than that of each unit of *N*. This is a straightforward commonsensical result if the battle is set up as a set of duels.

3.1.2 Lanchester Square Law

Lanchester then dealt with combat situations that he likened to modern warfare. In this case every unit of side *M* can inflict damage on every unit of side *N* at the same time, which means that a concentration of forces will be a much better strategy. For example, in the earlier linear situation of duels, arraying 100 attacking units against 50 units would not have any particular advantage. However, in a modern warfare context, concentrating 100 units against 50 would be a decided advantage as each unit of the enemy force would be attacked by two units of the other side. In situations such as these, could a smaller force win over a larger force? This could be achieved if the smaller force divided up the larger force by tactical maneuvers into two parts and then concentrated its attack on each part separately. To derive the rate of attrition according to the conditions of modern warfare, similar equations apply.

Here, $\frac{dm}{dt} = -\alpha n, and \frac{dn}{dt} = -\beta m.$

Therefore, $\frac{dm}{dn} = (\alpha / \beta) n^2 - n_0^2$, which leads to $mdm = (\alpha / \beta) ndn.$

Integrating this from time 0 to time *t*, $m^2 - m_0^2 = (\alpha / \beta) n^2 - n_0^2.$

When side *M* wins, *n* = 0. Rearranging, $m^2 = m_0^2 - (\alpha / \beta) n_0^2.$

Therefore, $m = \sqrt{(m_0^2 - \alpha / \beta n_0^2)}.$ This is Lanchester's Square Law.

On examination of this law it can seen that for side *M* to win, the condition $m_0 \succ (\alpha / \beta) n_0^2$ must apply. This means that even if each unit of side *N* has 25 times or *a*/*b* times the fighting value of each unit of side *M*, side *M* has merely to have 5 times or α / β times the number of attacking units of side *N* to match the fighting strength of side *N*. This is an important result with predictive value if α / β is known.

3.1.3 Numerical example

What about the strategy in which the smaller side divides up the larger side and then concentrates on smaller enemy subsets sequentially? A famous example is Nelson's strategy at Trafalgar. In the Battle of Trafalgar (21 October 1805), Lord Nelson won a famous victory over the combined French and Spanish fleets. Nelson split the enemy fleet into two using a small subset of his fleet; then the larger subset engaged the two portions of the enemy fleet separately. This ended in the capture of 20 enemy ships by the English. Was this a lucky accident or the outcome of a carefully planned strategy?

Apparently this was a premeditated ploy [50]. In a memo Nelson wrote on 9 October 1805, he assumed that his entire fleet of 40 ships would encounter the larger French and Spanish fleet consisting of 46 ships. He decided that he would use 8 ships to split the enemy fleet into two sections; he would then use his remaining 32 ships to destroy the first half of the fleet, and engage the second half later. He thus felt that he would be able to increase his odds of destroying a greater proportion of the enemy fleet. In retrospect, it appears as if Nelson was using Lanchester's Square Law of combat. If the fighting ability of each combat unit of the two warring sides was equal, then applying Lanchester's Square Law, the French and Spanish fleet could be expected to win with 23 survivors $\left[\sqrt{\left(46^2-40^2\right)}\right]$. However if Nelson employed 8 ships to split the enemy fleet into two equal parts, the English would have been left with 22 surviving ships $\left[\sqrt{\left(32^2-23^2\right)}\right]$. This figure could also be augmented by any survivors of the initial 8 ships; Nelson would thus increase his odds of winning the overall battle over those calculated by simultaneously pitting the two entire fleets against each other. History proved him right. Lanchester's Square Law can further be used to show that if the number of combat units or combatants is doubled by splitting each into a unit half the size, so that each combat unit now has half its original fighting ability, then the number of casualties suffered by the now more numerous side is proportionately lowered. Thus a disproportionately greater benefit will be obtained by increasing numbers of combat units than by simply increasing the fighting ability of each combat unit, if all combat units have equal probability of being engaged with all enemy combatants at the same time.

The essence of the Lanchester's two laws of combat appears to be that small combat units can be highly successful if they are greatly in the majority and if they encounter the larger units all at once. However, if large combat units are greatly outnumbered by smaller units, they can be successful only if they engage the smaller ones in a series of one-to-one fights. Lanchester's strategies are used in planning several real war games and tactics for deploying forces as they lend themselves to precise predictions. They are apparently also being used successfully in Japanese business and marketing strategies, an effort that has been pioneered by Nobuo Taoka [51]. These marketing strategies employ terms such as "local battles", "close combat", and "one point concentration" in their lexicon [52], and focus on offensive and defensive strategies of the weak and the strong.

For example, advice given to the weak is to fight in only one market, i.e. to concentrate forces, and to differentiate to produce better products. The corresponding defensive strategies of the strong would be to take the combat to large markets to swamp the offensive efforts of the weak, to the extent of even flooding the market with copycat products as soon as possible. Although these are qualitative descriptions, they have been refined into quantitative prescriptions [51]. This view of business as war has apparently contributed to the success of Japanese business and marketing strategies in the post-World

War II scenario. Curiously, Taoka was apparently inspired by the use of Lanchester's laws by the US Navy against the Japanese in the Pacific war arena.

3.2 Models of ground combat

No matter were the ancient times 3000 ago the battle of Gibeah[53], regular combat , 1991 Persian Gulf War, even guerrilla warfare [47] the ground combat [54] has the decisive significance on gain victory.

3.2.1 Lanchester-type aggregated-force model of conventional ground combat

Taylor [38] develops a Lanchester-type model of large-scale conventional ground combat between two opposing forces in a left double quote sector. right double quote It is shown that nonlinear R. L. Helmbold-type equations of warfare with operational losses may be used to represent the loss-rate curves that have been used in many aggregated-force models. These nonlinear differential equations are used to model the attrition of combat capability in conjunction with a rate-of-advance equation that relates motion of the contact zone or forward edge of the battle area (FEBA) between the opposing forces to the force ratio and tactical decisions of the combatants. This simplified auxiliary model is then used to develop some important insights into the dynamics of FEBA movement used in large-scale aggregated-force models.

3.2.2 Lanchester's equations and the structure of the operational campaign: between-campaign effects

Speight [55] was to reconcile common combat modelling practice with the evidence stemming from the analysis of historical battle outcomes. In many aggregated models variants of Lanchester's 'Square Law', or a 'Square-Linear Law', are used to represent the direct fire attrition process. These place a heavy premium on the concentration of force with, other things being equal, the balance of attrition strongly favouring the side with the greater number of combatants. However, almost without exception, the relationships actually observed in collected samples of historical battles are in line with a 'Log-Linear' version of Lanchester's equations. This would suggest that, in attrition terms, concentration of force should positively be avoided. The greater the number of combatants assembled on the battlefield, the greater will be the likely number of casualties accruing to that side. In this paper it is pointed out that the battles which feature in historical samples are self-selecting. By definition, these samples do not feature those instances where a would-be assailant chose not to launch an attack because he calculated that his chances of success were negligible. Nor do they include those occasions where the defender chose to abandon his position because he saw that defeat was almost inevitable. For those cases that remain a key task for the attacking commander would have been to assemble the resources he deemed necessary to ensure a reasonable chance of mission success. In practical terms this means that he would be prepared to enter battle with a smaller force ratio if he perceived the opposing forces to be militarily ineffective; if he had confidence in the prowess of his own troops; if he felt that he had the edge in terms of weapon effectiveness; and/or if he judged that the chances of local concentration and other terrain features were in his favour. These sorts of considerations are in line with the between-campaign relationships actually observed in the main historical data base assembled by UK analysts.

This paper describes a process of theoretical modelling and simulation, based on evidence from live trials and from battle. The results suggest that, even though the affrays within a campaign may obey a version of Lanchester's 'Square-Linear Law', the mechanisms outlined above will ensure that their outcomes will appear to obey a 'Log-Linear' relationship when they are aggregated over a collected sample of campaigns. This effect will be enhanced if casualties from the direct fire battle are simply combined with those from other quasi-independent sources, such as those due to air power or the taking of prisoners.

3.2.3 Modelling the mobile land battle: combat degradation and criteria for defeat

Speight and Rowland [56] reviews some of the evidence concerning what is known about the degradation of combat skills in battle. It puts forward a scheme for the representation of this effect in battle models, and then links this to the odds of 'victory' or 'defeat' in mobile land warfare. The historical evidence suggests that, in mortal combat, only a modest proportion of weapon crews can be relied on to make a fully active contribution to the battle. Of the remainder some will make only an intermittent contribution, and some no contribution at all. It appears that there are relatively stable differences in these proportions from one army to another. The evidence also suggests that the contribution of the less effective is likely to be somewhat more in the attack than in the defence.

The article shows how this phenomenon could affect the form of mathematical models and predictions commonly used to represent combat attrition. Historical analysis also suggests that, at the tactical level, successful resistance to attack depends less on attrition than it does on maintaining the spatial integrity of the defence. Clearly, this integrity is more likely to be compromised as the proportion of non-contributing defenders increases. A simple modeling scheme is therefore proposed. That sector in which the attacker intends to break through is designated as the 'critical point'. If, when the attacker reaches this 'critical point', the number of his survivors equals or exceeds a pre-determined multiple of the active surviving defenders in this sector, then the attack will be deemed to have 'succeeded'. Although simplistic, and obviously in need of refinement, this scheme does provide a plausible explanation for some observed operational relationships: that armies which characteristically impose low casualty rates on their attackers tend to surrender when their own casualty rates are low, and also tend to retreat at a faster rate as a function of local force ratio.

3.3 Attrition models

Attrition is a reduction in the number of personnel, weapons, and equipment in a military unit, organization or force [5]. Combat attrition is one of the most important aspects of combat modeling. It is the most studied combat process, there is no agreement on the best way to model it. The following are from different view of attrition models.

3.3.1 Attrition models of the Ardennes campaign

Past empirical validation studies include the work of Engel [57] on the Iwo Jima campaign of World War II, Busse [58] on the Incheon–Seoul campaign of the Korean War, and Bracken [24] on the Ardennes Campaign of World War II. Various reanalysis of the Engel and Busse work have been conducted by Samz [43], Hartley [59], and Hartley and Helmbold [46]. Bracken [24] introduced an additional parameter d to the standard Lanchester equation which he called the *tactical parameter,* to account for a battle in which it is known that defense and offense switch during the course of the campaign. Engel [57] concluded that the square law might fit the Iwo Jima data, but the data were incomplete on the Japanese side. Hartley [59] concluded that, depending on the assumptions made for the Japanese data, other laws could be made to fit. With complete daily data available for the Incheon–Seoul campaign, work by Busse [58], Hartley [60], and Hartley [61] proved inconclusive.

The Ardennes data has complete daily tallies, but some of the German data were estimated. The estimation was based on extrapolation from existing records and was done by World War II historical experts ([22]; [23]). For the Ardennes Campaign, Bracken [24] concluded that the Lanchester linear law fit the data. Thus, the empirical evidence needed to validate Lanchester theory is sparse, and the results to date are somewhat inconclusive and conflicting. In spite of this, the Lanchester equations are commonly employed to explain observed or reported phenomena, such as the work of David [42], which models the biblical battles of Gibeah or the work of Franks and Partridge [62] modeling ant warfare.

Fricker [30] revisits the modeling by Bracken of the Ardennes campaign of World War II using the Lanchester equations. It revises and extends that analysis in a number of ways:

(1) It more accurately fits the model parameters using linear regression;

(2) it considers the data from the entire campaign;

(3) it adds in air sortie data. In contrast to previous results, it concludes by showing that neither the Lanchester linear nor Lanchester square laws fit the data.

A new form of the Lanchester equations emerges with a physical interpretation.

3.3.2 New look at the 3:1 rule of combat through Markov Stochastic Lanchester models

The question of how wars are fought and what factors constitute victory in them has drawn much attention throughout history. In particular, the issue of force ratio and its impact on the outcome of battles was addressed as early as the 5th century BC by Sun Tzu and later by Clausewitz, Lanchester, Liddell Hart and many others. The question of 'necessary' or 'optimal' force ratio that is required to achieve a victory has been analysed extensively ([31]; [35]; [63]; [64]; [65]; [66]). Mearsheimer JJ [67] echoes the rule of thumb shared by many that 'An attack requires more than a 3:1 advantage on each main axis to succeed'. In more recent papers this general statement is challenged by Epstein [68]. The arguments that are used to support or to oppose this rule range from quotations of combat commanders, and reference to historical data, to the utilization of formal macroscopic models.

Kress and Talmor [41] describes the 3:1 rule of combat states that in order that for the attacker to win the battle, his forces should be at least three times the force of the defender.

This somewhat vague statement has resulted in numerous interpretations and discussions from historical and military science points of view. In this paper the authors attempt to examine this rule by utilizing a number of Markov Stochastic Lanchester models that correspond to various basic combat situations and to draw some conclusions from their implementations. They identify general combat situations where the 3:1 rule is reasonable as well as situations where the force ratio should be either smaller or larger. Since the analysis is performed in the formal and somewhat 'sterile' setting of (pure) mathematical modeling, the results should be appropriately interpreted as reasoning of a certain abstraction of the battlefield.

3.3.3 Lanchester models of the Ardennes Campaign

A detailed data base of the Ardennes campaign of World War II (December 15, 1944 through January 16, 1945) has recently been developed ([22];[23]). Bracken [24] formulates four Lanchester models of the campaign there are

Model 1— Combat Forces;

Model 2— Total Forces;

Model 3— Combat Forces, No Tactical Parameter;

Model 4— Total Forces, No Tactical Parameter, and estimates their parameters for these data. Two-sided time histories of warfare on battles and campaigns are very rare, so Lanchester models have seldom been validated with historical data. The models are homogeneous in that tanks, armored personal carriers, artillery, and manpower are weighted to yield a measure of strength of the Allied and German forces. This weighting is utilized for combat power and for losses. The models treat combat forces in the campaign (including infantry, armor, and artillery manpower) and total forces in the campaign (including both combat manpower and support manpower.) Four models are presented. Two models have five parameters (Allied individual effectiveness, German individual effectiveness, exponent of shooting force, exponent of target force, and a tactical parameter reflecting which side is defending and attacking.) The other two models remove the tactical parameter, which is not generally known prior to warfare, and estimate the other parameters without the tactical parameter.

The objective of Bracken's research is to fit Lanchester equations to Ardennes campaign data. The data cover 33 days for the campaign, from December 15, 1944 through January 16, 1945. however, the data for the first day seem to be incomplete on the German side. The Germans attacked during days 1-6 and the Allies attacked during days 7-33. Preliminary research has shown that a form of the model that includes specification of which side is attacking and defending fits the data significantly better than if this factor is left out of the model. Also, the heaviest attrition takes place at the beginning of the campaign. On the basis of these considerations, the analysis treats data for days 2-11, or five days during which each side is attacking— 2-6 for the Germans and 7-11 for the Allies.

The main results of this research are

(a) the lanchester Linear model fits the Ardennes campaign data in all four cases, and (b)when combat forces are considered Allied individual effectiveness is greater than German individual effectiveness, whereas when total forces are considered Allied and German individual effectiveness is the same. The interpretation of the latter result is that the two sides had essentially the same individual capabilities but were organized differently- the Allies chose to have more manpower in the support forces, which yield greater individual capabilities in the combat forces. The overall superiority of the Allies in the campaign led to the attrition to the Allies being a smaller portion of their forces.

Lanchester equations fit to with-war two- sided data have been very rare due to the nonavailability of such data. Thus the present investigation is of interest in that regard. In the future, two-sided data may be more available. Methodology for the analysis of such data will become an important topic; this analysis should be thought of as a prototype.

3.4 Applications in bi-tech and business

As we know Lanchester Laws are widely used in calculating military force, but here we particularly introduce two issues in business and bio-tech.

3.4.1 Special experiments in bio-tech

McGlynn [73] attempted to test whether ants use Lanchester's laws in interspecific competition at food sources. He designed two types of bait platforms, and conducted these experiments in natural settings at the La Selva Biological Station in Costa Rica. In one bait platform (modified Petri dishes), he had large semicircular openings which provided access to the bait, while the other had a narrow entrance which provided bait access. McGlynn found that, although ants of all sizes were attracted to the baits, smaller ant species significantly dominated the bait platforms that has large entry access holes, while there was no clear pattern of domination by ants of any particular size at the platforms with the small entry holes. McGlynn [73] suggests that these results may be interpretable from Lanchester's laws, at least for the results from the smaller ants. However, he does caution that other factors such as territoriality, nutritional needs of the colony, nest locations, and the suite of competing species near particular bait platform sites, could have contributed to the findings. Considering the very significant result obtained with the bait platforms provided with the large openings, McGlynn [73] suggests that this could be used to design baits platforms to administer insecticides for ant control, especially since the problem ants worldwide seem to be small non-native invading species [74]. In a comparative study, McGlynn [74] found that in all of the ant genera with monomorphic worker castes that fight during competition, the non-native invasive species were smaller than the native species. He believes that the fact that fighting nonnative ants are smaller than their closest native relatives may provide powerful insights into the mode of success of the non-native invading species. While other factors including life history attributes such as polygyny and colony budding, could be responsible for their competitiveness, it is certainly intriguing to consider whether some of this success could be attributed to the fact that the ants are following Lanchester's laws of combat.

Lanchester's Linear Law appears to be followed in slave-making ants which steal brood from heterospecific colonies to augment the worker force in their own nests. The slave-makers are usually outnumbered by workers in the colonies that they wish to raid. Do the slave-makers try to organize one-to-one duels? According to Franks and Partridge [62] the slave-making ants *Formica pergandei* and *F. subintegra* which produce "propaganda" or confusing substances (esters such as decyl, dodecyl and tetradecyl acetates) that cause alarm amongst the defending workers (Regnier and Wilson [75]), are actually trying to split the enemy ranks to force limited engagements or one-to-one duel situations. In these duels, victory is ensured by the superior individual fighting ability of the slavemaking ants, many of which are equipped with powerful mandibles and stings. Another possible example of the use of the Linear Law is the slave-making ant *Harpagoxenus sublaevis* which produces the "propaganda" alkanes *n*-heptadecene and *n*-heptadecadiene (Ollet *et al* [76]) that cause the defenders to attack each other [77].

3.4.2 The Lanchester strategy on sales and marketing[17]

The Lanchester Strategy is based on the Lanchester Laws, Yano [52] describes the strategy of the weak.The New Lanchester Strategy includes strategies for both the strong (market leader) and the weak (second and lower market share). The starting point of stategy is, after all, the desire on the part of the weak to find a way to defeat the strong. Recently, diversification has led to increasing competition among companies engaged in disparate industries. Companies entering the fray for the first time need to implement the strategy of the weak. Even the strong would be well advised to familiarize themselves with the strategy of the weak in order to protect their positions of strength. This volume is a continuation of the story presented in Volume One. Supervisor Sakamoto remains the chief protagonist, and the story revolves mainly around him and his colleagues at Company W, a manufacturer. Other characters from manufacturing companies, retailers, and service industries are introduced in this story.

Differentiation here means having something that the competition doesn't. This is the most basic strategy for the weak. Weak companies would be seriously mistaken to adopt the same strategies used by their stronger rivals. Since differentiation strategy involves having something that the competition does not, companies must be prepared to change their differentiation strategy when a competitor's reaction warrants such a change. For that reason, differentiation strategy must never be rigid. Constant alertness and flexibility are called for. In this chapter, we discuss the importance of differentiation strategy and describe the various forms it may take.

Local battles are waged on a limited front. In terms of sales strategy, this has two meanings:

1. Fighting local battles, and
2. Creating local battle conditions.

Fighting local battles means competing in specific regions. Creating local battle conditions in specific regions. Creating local battle conditions means segmenting the market. Not only must regions be segmented, but also merchandise and customer bases. This step is necessary when a company is attempting to set priorities.

We consider single (man-to-man) combat. In terms of sales strategy, this means penetrating a market dominated by one company and going after that company's customers. This strategy is needed when a company enters a new industry or region, or is attempting

[17] http://www.lanchester.com/YANO2.html

to cultivate a new customer base, and is in the process of setting its priorities.

In relation to sales strategy, close combat embraces the following four concepts:

1. Adopting a direct sales system,
2. Launching a "downstream" campaign,
3. Reinforcing one's home base, and
4. Using personal approach as a weapon.

Close combat is vital to distribution strategy and in developing tactics.

In a battle, the weaker army has no hope of winning unless it focuses its small forces on one area. In sales strategy, as well, the weaker company must launch a concentrated offense. There are several types of one-point concentration (involving regions, products, and customer bases), but the most problematic aspect is where to focus the effort. The best strategy for the weaker company is to select a relatively easy target or to focus on an area where it already has strength.

Diversionary operations confuse the enemy. It is important to keep such operations secret, since once an army's battle plan is discovered by the enemy, defeat is inevitable. Decoy maneuvers are an important aspect of this particular strategy. In sales strategy, the purpose of diversionary operations are:

1. To demoralize the enemy, and
2. To scatter the enemy's forces.

3.5 Concluding remarks

General Karl Von Clausewitz says in his book On War[18] that attack and defend are a pair of concepts for mutual action and reaction. He considered that defense is more than passive waiting and resistance. The best defense must include the swift and vigorous assumption of the offense. In the shift between offense and defense is a period of deadlock during which both sides seek to seize a key strongpoint, collect intelligence and set up logistics and draw up the next operation plan.

Previous researchers did not consider the deadlock of the transition between offense and defense ([24]; [26]; [37]; [63]; [78]). General Karl Von Clausewitz believes that offense and defense in warfare are a state of interaction and response. The transition

[18] K.V. Clausewitz, *On war*, Harmondsworth, Penguin Books. New York, 1968. Extracts selected from the English translation by Vom Kriege published in London 1908.

between offense and defense will have a short span of time difficult to define.

In chapter 4 we will seek a much better qualitative analysis model not only for the explanation of modern combat but also discuss the transition between offense and defense by empirical study.

4. Model building and implementation: empirical study

The purpose of this research is to improve Bracken's [24], Chen and Chu's [29] work to get an even more accurate solution in terms of the sum of squared errors. Basically, we adopt concepts of the tactical factor variable and the shift time variable to improve the original Lanchester model. The contribution of this research is that we want to propose a much better qualitative analysis model for the explanation of modern combat.

4.1 The problem of the original Lanchester model

Many qualitative combat analysis researches ([12]; [29]; [30]; [38]; [57]; [69]; [70]; [71]; [72]) have been proposed using the Lanchester's models. They introduced their text with an illuminating discussion of the evolution of OR methods in the Department of Defense (DOD) as a tool for guiding commanders in troop disposition and logistics requirements in specific tactical situations.

The Lanchester equations is applied to exploration of the force ratios required to win in guerrilla-counterguerrilla warfare. It is shown that an attacking guerrilla force can, by using tactics that compensate for its weaknesses, defeat a force of defending regulars that has over-all superiority in numbers and weapons. The defenders can win by appropriate selection of weapons, countertactics, and group sizes for individual engagements. In general, however, the high over-all ratios of defending regulars to attacking guerillas that have characterized recent warfare of this kind in which the defense has been successful are extremely difficult to reduce [69]. Protopopescu et al [71] introduced an analytic approach for describing combat that is based on non-linear differential equations. This method introduces an analytic approach that yields both the spatial and temporal distribution of forces. The approach was motivated by the work of Lanchester in particular, the limitations of the ordinary differential equations that he used to describe force-on-force conflict.

Taylor [38] has even published a two-volume book on the mathematical analysis of Lanchester's equations. Recently, Fricker [30] gave a new interpretation of the Lanchester linear equation such that multiplication of attrition parameter and enemy force is the probability of being killed. Since the principal purpose of a weapon system is to destroy enemy targets, we begin with the development by considering attrition characteristics of a weapon. Certain assumptions about firing disciplines are contained in most discussions of Lanchester's linear equations. Some particular firing disciplines do occur in reality and a particular set of attrition equations does reflect the actual situation. Engel [57] verified the Lanchester law using battlefield data. Samz [72] presented related results with little adjustment. However, it is very hard to get daily records of both sides in a battle.

4.2 Bracken's and Chen's work can be improved to better fit the attrition in the Ardennes Campaign

Chen and Chu [29] investigates the famous Ardennes Campaign in the Second World War. The daily history of the battle, from December 15 (day 1), 1944 to January 16 (day 33), 1945, was given by the Data Memory Systems, Inc.. Records of troops and casualties on both sides were contained in that database. The information includes the daily history for artillery, tanks, armored personnel carriers, and manpower. In the real world, damaged manned-weapon systems usually lead to personnel casualties. And applied the Lanchester linear law to model the Ardennes data with the Bracken's tactical factor. Fricker [30] analyzed the Ardennes data following Bracken [24]. His paper fit logarithmic linear models and found significantly different results from those of Bracken's. Fricker showed how to use a linear regression approach to solve model parameters in a statistical setting. He concluded that the Lanchester linear (and square) model fitting the Ardennes Campaign does not hold. In modern warfare, an important tactical factor is to seize the opportunity for attack, which is usually based on the concentration of available force to penetrate the enemy's defense line. From historic data they know that on day 1 the Germans suddenly attacked the Allies, then penetrated the Allies front line and tried to arrive the port city of Antwerp to split the Allies. On day 33, the Allies restored the front line in Ardennes to its positions before December 15.

Hence, during these days, the attack initiative shifted from the Germans to the Allies. Owing to the lack of historic evidence, it is allowed to assume that there is no exact record when the Allies changed their position from defense to attack because there were more than 700,000 soldiers from both the Allies and the Germans involved in the Ardennes Campaign. We believe that the commanders of both sides tried to take control over the battle by seizing the opportunity to attack. Consequently, the exact date when the mode shifted from defense to attack allows discussion. Their result supported by the minimum sum of squared residuals may indicate a reasonable date of the mode shift. Bracken [24] evenly partitioned the 10-day period such that from day 2 to day 6, the Germans attacked and from day 7 to day 11, the Allies attacked. They believe that the commanders of both sides tried to control over the battle by seizing the opportunity to attack, so we treat the shift date, *k*, as a new parameter such that from day 2 to day *k*, the Germans attacked and from day $k + 1$ to day 11, the Allies attacked. They tried to find the optimal solution for $(a; b; d; k)$ to minimize the sum of squared residuals and evaluate partial derivatives of the sum of squared residuals between the real and estimated attrition.

Therefore, they can find a minimum for each different transformation. Comparing those minimums, they can partition the data of those 10 days in two subsets, and attain the minimum for the sum of squared residuals **(** Table 6 **).** We learn from the historical data that the Germans were unable to concentrate their forces to keep on attacking after the first days. Consequently, they changed their role to defenders and led to failure. Through this way they find a better correspondence between the model's results and the real historical record.

Table 6. Results of SSR_k, a, b and d

Dates	**Day k**	**SSR_k**	$a(\times 10^{-9})$	$b(\times 10^{-9})$	**d**
Dec.16	2	20616429	8.5428	10.4690	1.0971
Dec.17	3	20637065	8.3661	10.7131	1.0774
Dec.18	4	19766895	8.2991	10.8260	1.1031
Dec.19	5	17532651	8.3316	10.8146	1.1514
Dec.20	6	15325785	8.2197	10.7436	1.1756
Dec.21	7	16628244	7.8311	11.1351	1.1458
Dec.22	8	13066407	7.3999	11.7239	1.2043
Dec.23	9	12319246	7.3703	12.1632	1.2360
Dec.24	10	19001170	6.7087	12.9562	1.1438

Chen and Chu [29] proposed a much more accurate solution by combining the original Lanchester linear law model with Bracken's tactical factor. Moreover, in that model, they also incorporate a new shift time variable to take account of the situation between attack and defense. Based on this modification, they significantly improved the fitness of the original Lanchester model to the Ardennes Campaign more than Bracken did. Hung et al [2] want to improve Bracken's and Chen's work to get an even more accurate solution in terms of the sum of squared errors.

4.3 Building a much better qualitative analysis model for the explanation of modern combat

We try to improve Bracken's [24], Chen and Chu's [29] work to significantly better fit our extended Lanchester model into the Ardennes Campaign live data. Essentially, we adopt the concepts of the tactical factor variable and the shift time variable to improve the original Lanchester's model. Moreover, we use the Lanchester square law model instead of Lanchester linear law model to reflect the fact that the Ardennes Campaign was not an indirect-fire but a direct-fire combat. According to our numerical experimental result, we improved Bracken's work by 39.26%, and Chen's work by 19.51%.The contribution of this research is that we want to propose a much better qualitative analysis model for the explanation of modern combat.

Mathematical formulation

We use the Lanchester square law model instead of the Lanchester linear law model to reflect the fact that the Ardennes Campaign was not an indirect-fire but a direct-fire combat. More accurately speaking, we assume that in the battle, the cross-firings of each side were aimed at the enemy hiding under bunkers or ditches. Hence intuitively the Lanchester square law model should be better for the explanation of modern warfare.

Notation

We define the following notation:

1. B : the Blue (i.e., Allied) combat forces, including tanks, armored personnel carriers, artillery and personnel. The Allies include a British Corps as well as the US forces,
2. $\dot{B}$: the actual loss of Blue (Allied) combat forces,
3. R : the Red (i.e., German) combat forces, including tanks, armored personnel carriers, artillery and personnel,
4. $\dot{R}$: the actual loss of Red (German) combat forces,
5. a : the Allied (Blue) attrition rate without Bracken's tactical factor,
6. b : the German (Red) attrition rate without Bracken's tactical factor,
7. d or $1/d$: Bracken's tactical factor,
8. k : the last day on which the Germans attack,
9. SSE : sum of squared errors.

Recall the differential equations for Lanchester's square law without reinforcements

$$\frac{dB}{dt} = -\alpha R$$

$$\frac{dR}{dt} = -\beta B$$

We want to find the attrition rate coefficients α and β. The attrition rate is equal to the reciprocal of the expected time to destroy enemy personnel or equipment system. No matter what the tactical decision may be, the original square law implies that the full force of each side operates with unchanged destroyed effect on its enemy. However, in some circumstances this seems unbelievable.

However, for a more realistic modelling, tactical factor inherent in these models should be incorporated. In accordance with the above reasoning, a modification of the familiar Lanchester formulation is suggested by Bracken[24]. From Bracken[24], if we introduce his tactical factor, *d* or *1/d*, then the estimation will be more accurate, so we suppose that

(1) When the Blue force defends and the Red force attacks, we take

$$\alpha = ad,\ \beta = b\frac{1}{d}$$

(2) When the Blue force attacks and the Red force defends, we take

$$\alpha = a\frac{1}{d},\ \beta = bd$$

The constant terms (i.e. independent of the battle time and the relative strengths of the opposing force) *a* and *b* are usually named the Lanchester attrition-rate coefficients to represent the individual effectiveness of each side's forces.

According to Hartley [49], during a battle, most commanders prefer to attack so their enemy unwillingly must defend. From the historical record, we know that for the first 10 days of the Ardennes Campaign, the Germans attacked and at the end the Allies attacked. Hence, on some day *k*, the attack initiative shifted from the Germans to the Allies. Our goal is to find the best fit *a, b, d* and *k*, to minimize the sum of the squared errors between the actual and theoretical attrition. For $2 \le k \le 10$, define the objective function SSE_k (*a, b, d*) as

$$SSE_k(a,b,d) = \sum_{n=2}^{k}\left(\dot{B}_n - adR_n\right)^2 + \sum_{n=2}^{k}\left(\dot{R}_n - b\frac{1}{d}B_n\right)^2$$

$$+\sum_{n=k+1}^{11}\left(\dot{B}_n - a\frac{1}{d}R_n\right)^2 + \sum_{n=k+1}^{11}\left(\dot{R}_n - bdB_n\right)^2$$

Given k, our procedure is to obtain the local critical points of $SSE_k(a, b, d)$ under the restrictions $0 \prec a$, b and d. To simplify the notation, put

$$g(1,k) = \sum_{n=2}^{k}\dot{B}_n R_n,\ g(2,k) = \sum_{n=k+1}^{11}\dot{B}_n R_n,\ g(3,k) = \sum_{n=2}^{k}\dot{R}_n B_n,\ g(4,k) = \sum_{n=k+1}^{11}\dot{R}_n B_n,$$

$$g(5,k) = \sum_{n=2}^{k}R_n^2,\quad g(6,k) = \sum_{n=k+1}^{11}R_n^2,\ g(7,k) = \sum_{n=2}^{k}B_n^2,\quad g(8,k) = \sum_{n=k+1}^{11}B_n^2.$$

$$h(1,k) = g(1,k)g(2,k)g(8,k) + g(3,k)g(4,k)g(5,k),$$

$$h(2,k) = g(1,k)g(2,k)g(7,k) + g(3,k)g(4,k)g(6,k),$$

$$h(3,k) = g(2,k)g(2,k)g(5,k) - g(1,k)g(1,k)g(6,k),$$

$$h(4,k) = g(3,k)g(3,k)g(8,k) - g(4,k)g(4,k)g(7,k).$$

$$\theta_6 = h(1,k)g(5,k)g(8,k),$$

$$\theta_5 = h(3,k)\left[g(8,k)\right]^2 + h(4,k)\left[g(5,k)\right]^2,$$

$$\theta_4 = 3h(2,k)g(5,k)g(8,k) - h(1,k)\left[g(5,k)g(7,k) + g(6,k)g(8,k)\right],$$

$$\theta_3 = 2h(4,k)g(5,k)g(6,k) + 2h(3,k)g(7,k)g(8,k),$$

$$\theta_2 = h(2,k)\left[g(5,k)g(7,k) + g(6,k)g(8,k)\right] - 3h(1,k)g(6,k)g(7,k),$$

$$\theta_1 = h(3,k)\left[g(7,k)\right]^2 + h(4,k)\left[g(6,k)\right]^2,$$

$$\theta_0 = -h(2,k)g(6,k)g(7,k).$$

We now consider the partial derivatives of $SSE_k\,(a, b, d)$ to find the critical points of $SSE_k\,(a, b, d)$.

Proposition 1：

Given k, there exist local critical points for $SSE_k\,(a, b, d)$ such that a, b and d satisfy the following equations:

$$a = \frac{d^3 g(1,k) + dg(2,k)}{d^4 g(5,k) + g(6,k)},\ b = \frac{d^3 g(4,k) + dg(3,k)}{d^4 g(8,k) + g(7,k)}\ and\ \sum_{j=0}^{6} \theta_j d^{2j} = 0$$

Proof:

See Appendix A.

4.4 Empirical study

To induce a homogeneous system for this research, we make the same assumptions that Bracken did and weigh artillery, tanks, armored personnel carriers, and personnel by 40, 20, 5 and 1, respectively. Based on the data shown in Table 7, which was adopted directly from Table 5 of Bracken [24], it is apparent that the record of the first day is incomplete on the German side. After Christmas Day, the sky was clear over the Ardennes area when the superior Allied air forces began striking the Germans. Hence in this paper, we concentrate ourselves on the records from December 16 to 25, 1944.

Table7. Data on combat forces and losses

Dates	Day *k*	Blue forces	Blue losses	Red forces	Red losses
Dec. 15	1	558820	478	144	0
Dec. 16	2	555482	2594	577446	2656
Dec. 17	3	553625	3833	571923	4303
Dec. 18	4	562661	3615	567134	3415
Dec. 19	5	576795	4200	563255	3263
Dec. 20	6	644252	3424	570018	3275
Dec. 21	7	665746	1804	566877	3799
Dec. 22	8	681412	2350	578629	2866
Dec. 23	9	683076	2698	576223	4518
Dec. 24	10	698910	2858	580074	6985
Dec. 25	11	715759	2177	570005	5638

According to different times of the shift from attack to defense, we partition the 10 days data (from December 16 to 25, 1944) into two phases. With the help of Mathcad PLUS 6.0, for each k, the equation $\sum_{j=0}^{6}\theta_j d^{2j}=0$ has just one positive solution. The uniqueness of the positive root is not guaranteed to be the case for other data sets. Hence, it must be checked a new for each data set. We thus obtain the results shown in Table 8. Recall the method of Jerome Bracken. He used a more complicated model with five parameters: p, q, a, b and d such that p and q are the exponents of the attacking and defending forces. He only considered the case k=6. Without taking partial derivatives, he directly chose 1875 combinations of parameters and compared their sum of squared error values. His minimum occurs at $p=q=1$ and the sum of squared errors is expressed as 16.3×10^6. From Chen and Chu [29], we know that the exact results for the sum of squared errors with respect to Lanchester's linear law are 15.3×10^6 for k= 6 and 12.3×10^6 for k= 9. From Table 8, for k= 6 we get a smaller sum of squared errors equal to 13.8×10^6. Moreover, for k= 9 we attain an even better fit such that the sum of squared errors is 9.94×10^6. The Ardennes Campaign included many large engagements in which combats on both sides were visible to each other (especially including the city fighting). So the direct-fire combat principle is more suitable than the area or indirect-fire combat principle.

Table 8. Results of *SSE, a, b* and *d*

Dates	Day k	$SSE(\times10^6)$	$a(\times10^{-3})$	$b(\times10^{-3})$	d
Dec. 16	2	17.635930	5.348	6.222	1.049557
Dec. 17	3	17.7047778	5.261	6.329	1.032958
Dec. 18	4	17.292666	5.278	6.274	1.056622
Dec. 19	5	15.799438	5.277	6.193	1.103013
Dec. 20	6	13.839646	5.167	6.255	1.138036
Dec. 21	7	14.430477	5.022	6.463	1.124027
Dec. 22	8	10.933808	4.717	6.726	1.090527
Dec. 23	9	9.943100	4.466	7.095	1.227492
Dec. 24	10	16.085690	4.673	7.068	1.130073

Hence, it is predictable that Lanchester's square law will provide a better fit than Lanchester's linear law (Table 9). This coincides with our results. From $a=4.5\times10^{-3}$

and $b = 7.1\times10^{-3}$, the individual effectiveness of the Blue force is better than the individual effectiveness of the Red force. We may conclude that our result seems close to the real situation in December 1944. At that time, the Blue force has the advantage in the amount of armed equipment and technological invention. It coincides with the results in Chen and Chu [29]. They also imply that the individual effectiveness of the Blue force was superior to that of the Red force. During the first phase, the Red force depended on concentration of forces and having the initiative of attack to penetrate the Blue defensive line. With the help of the tactical factor, the Red force suffered acceptable losses. After the Blue force seized the initiative of attack, the tactical factor became favourable to them, so the Red force suffered heavier losses.

Therefore, we conclude a different point of view from Bracken [24] for the military decision makers as follows. The concept of concentration of force to penetrate, seizure of the initiative of attack and the Lanchester square law are still present in the Ardennes Campaign.

Table 9.The sum of squared errors of Bracken's, Chen and Chu's, and Hung et al's

	Dates	Day k	$SSE(\times10^6)$	a	b	d
Bracken	Dec.20	6	16.3	8×10^{-9}	10×10^{-9}	1.25
Chen and Chu	Dec.23	9	12.319246	7.3703×10^{-9}	12.1632×10^{-9}	1.2360
Hung et al	Dec. 23	9	9.943100	4.466×10^{-3}	7.095×10^{-3}	1.227492

4.5 Discussion

In 1914, Lanchester [36] proposed two interesting qualitative analysis models and related processing methods for better explanation of the attrition in warfare. Based on different assumptions of firing disciplines, Lanchester's square law model is designed to explain the direct-fire combat, and Lanchester's linear law model is used to explain the area of indirect-fire combat. From then on, many qualitative combat analysis researches have been proposed using the Lanchester's models ([37]; [79]; [80]; [81]; [82]; [83]). For example, Engel [57] has fitted warfare data into various Lanchester models. With small modifications, Samz [48] and Helmbold [46] also get similar results. Recently Data Memory Systems, Inc. ([22]; [23]) provides a daily record of the Ardennes Campaign of

World War II from December 15, 1944 to January 16, 1945. A day by day history of forces and casualties on both sides can then be derived from the database. Moreover, the data contains the daily records for tanks, armored personnel carriers, artillery and personnel. Using these useful historical data, Bracken [24] was the first researcher who successfully fitted Lanchester's models into the daily record of the Ardennes Campaign. He designed an integrated equation which incorporated both the Lanchester square law model and the Lanchester linear law model. Moreover, he uses the sum of squared errors as the performance measurement of fitness when applying the extended Lanchester model to the Ardennes Campaign data. And he takes the exponents in the extended Lanchester equations as parameters to be fitted to the data. Finally, he utilizes a numerical analysis method to generate the minimum sum of squared errors of the extended model. However, Bracken[24] used too many variables in his extended model in solving the problem of which extended Lanchester model is the best to explain the case of the Ardennes Campaign. Since Bracken's extended model is a generalization of the original Lanchester's models and is too complicated to get an accurate solution, there is still room left for improvement. Bracken's work motivated a series of related researches to improve it. Recently, Chen and Chu [29] proposed a much more accurate solution by combining the original Lanchester linear law model with Bracken's tactical factor. Moreover, in that model, they also incorporate a new shift time variable to take account of the situation between attack and defense. Bracken's[24] is from day 2 to day 6, the Germans attacked and from day 7 to day 11, the Allies attacked.

Based on this modification, they significantly improved the fitness of the original Lanchester model to the Ardennes Campaign more than Bracken did. More precisely, the sum of squared errors for Bracken [24] is 16.3×10^6 , for Chen and Chu [29] is 12.3×10^6, and for us is 9.9×10^6. Chen and Chu [29] and Hung et al [2] has the same new shift time variable to take account of the situation between attack and defense. The result supported by the minimum sum of squared errors may indicate a reasonable date of the mode shift, from day 2 to day9, the Germans attacked and from day 10 to day 11, the Allies attacked.

That is to say, we improved the Bracken's work by 39.26%, and Chen's [29] work by 19.51%. The contribution of this research is that we propose a much better qualitative analysis model for the explanation of modern combat.

5. Conclusions and recommendations

In this section, we can conclude that based on the Ardennes Campaign live data, we have successfully demonstrated that our proposed solution dramatically outperforms the previous related works.

5.1 Research finding and concluding remarks

No matter whether history repeats itself or not, we still can learn a lot of useful lessons from it. People have been interested in studying and analyzing historical warfare for thousands of years. Modern warfare analysis has developed many useful models and systematic methods to make more thorough and appropriate explanations of historical combats. However, since those analytic methods were applied from different viewpoints of quantitative versus qualitative, general versus specific, rough versus detailed etc., it becomes quite complicated and difficult to explain well the causes and effects of a historical combat.

Recall the method of Jerome Bracken [24], the sum of squared errors is expressed as 16.3×10^{6} for *k*= 6. Chen and Chu [29], we know that the exact results for the sum of squared errors with respect to Lanchester's linear law are 15.3×10^{6} for *k*= 6 and 12.3×10^{6} for *k*= 9. From Table 8, for *k*= 6 we get a smaller sum of squared errors equal to 13.8×10^{6}. Moreover, for *k*= 9 we attain an even better fit such that the sum of squared errors is 9.94×10^{6}.

Bracken [24] evenly partitioned the 10-day period such that from day 2 to day 6, the Germans attacked and from day 7 to day 11, the Allies attacked. The result of Hung et al's [2]supported by the minimum sum of squared errors may indicate a reasonable date of the mode shift, from day 2 to day9, the Germans attacked and from day 10 to day 11, the Allies attacked. It coincides with the result in Chen and Chu [29].

From Table 9, for $a = 4.5\times10^{-3}$ and $b = 7.1\times10^{-3}$, the individual effectiveness of the Blue force is better than the individual effectiveness of the Red force. We may conclude that our result seems close to the real situation in December 1944. At that time, the Blue force has the advantage in the amount of armed equipment and technological invention.

According to the results of empirical study intuitively the Lanchester square law model should be better for the explanation of modern warfare. Thus, we use the Lanchester square law model instead of the Lanchester linear law model to reflect the fact that the Ardennes Campaign was not an indirect-fire but a direct-fire combat.

Before talking about our future work, we would like to quote a veteran scholar of warfare, Noel Falconer, as follows:

The Ardennes Campaign—the WW2‘ Battle of the Bulge’—is used as a source of test data for Lanchester’s Laws because daily overall force levels and casualties have been extracted. It was timed, brilliantly, to exploit bad weather and thereby avert Allied air attacks. In direct consequence, visibility was poor and combats were mostly between small groups. Losses there from aggregate exactly only under linear assumptions to those calculated from total-force strengths. Then, the power of armor in particular cannot be meaned. The German tanks included the extremely formidable King Tiger, others were Panthers, the best all-rounder of the day, but many were war-weary Panzer IVs, and there were assault guns without turrets, powerful where their cannon could bear but weak and incapable of counter-fire to the sides and rear; their effectiveness varied by at least an order. None—on the Nazi side—was adequately resupplied. A key Gruppe ran completely out of petrol and had to abandon its immobile, and hence suicidally vulnerable, tanks. Ammunition needed to be conserved ever more rigorously. The quality, and state, of the forces engaged also varied. Initially, an 80-mile Allied front was held but thinly, with one unblooded and three exhausted divisions; whereas the reinforcements were first-class troops, accustomed to winning and confident of victory, the Americans under Patton moreover, their best fighting general. More, in diametric contrast to all German forces, these were comprehensively re-supplied. In net, the unit effectiveness, far from being constant (even after separating attack and defense), varied between elements and increased— erratically!—with time on one side as it decreased on the other. The Lanchester equations are algebraically correct; which means that what is assessed in validating them is the correctness of their assumptions. Altogether cleaner data is essential for this. It exists—deep buried in dusty, distant archives, surely, the very devil to extract, inconvenient and expensive (and hard to fund!) to research. And still OR, like science itself, depends upon accurate, applicable measurement.

5.2 Recommendations

Sun Tzu said: In the practical art of war, the best thing of all is to take the enemy's country whole and intact; to shatter and destroy it is not so good. So, too, it is better to recapture an army entire than to destroy it, to capture a regiment, a detachment or a company entire than to destroy them.

He also said: the general who loses a battle makes but few calculations beforehand. Thus do many calculations lead to victory, and few calculations to defeat: how much more no calculation at all! It is by attention to this point that I can foresee who is likely to w in or lose [1].

Hence no matter what kind of methods are used, the overall objective for modern combat analysis is that we can explain the historical data even better, which enables us to learn more from the history. So for the future work, we may try to build a hybrid method which incorporates different analytic methods such as critical success factor analysis and Lanchester models, etc, and more practical, detailed and influential variables such as weather, terrain, morale, material resource re-supply, commander quality and/or characteristics, etc, to better fit the historical data and also extract more valuable information from the model.

The contemporary greatest military achievement is not to construct the greatest weaponry, but is to affiliate the unification of software and hardware machinery with the thoughtful management mechanism to prevent the war from happening. This is precisely our diligently goal

Appendix A [2]

Proof of Proposition 1

First we compute $\partial SEE_k / \partial a, \partial SEE_k / \partial b \text{ and } \partial SEE_k / \partial d$

$$\frac{\partial SEE_k}{\partial a} = 2\sum_{n=2}^{k}\left(B_n - adR_n\right)\left(-dR_n\right) + 2\sum_{n=k+1}^{11}\left(B - \frac{a}{d}R_n\right)\left(\frac{-1}{d}R_n\right) \quad \text{(A. 1)}$$

$$\frac{\partial SEE_k}{\partial b} = 2\sum_{n=2}^{k}\left(R - \frac{b}{d}B\right)\left(\frac{-1}{d}B_n\right) + 2\sum_{n=k+1}^{11}\left(R - bdB_n\right)\left(-dB_n\right) \quad \text{(A. 2)}$$

$$\begin{aligned}\frac{\partial SEE_k}{\partial d} &= 2\sum_{n=2}^{k}\left(B - adR_n\right)\left(-aR_n\right) + 2\sum_{n=2}^{k}\left(R - \frac{b}{d}B_n\right)\left(\frac{b}{d^2}B_n\right) \\ &\quad + 2\sum_{n=k+1}^{11}\left(B_n - \frac{a}{d}R_n\right)\left(\frac{a}{d^2}R_n\right) + 2\sum_{n=k+1}^{11}\left(R_n - bdB_n\right)\left(-bB_n\right)\end{aligned} \quad \text{(A. 3)}$$

Second we solve the critical points of $SEE_k(a,b,d)$. By Equations (A.1)–(A.3), with notation of $g(i, k)$, we know that

$$\frac{\partial SEE_k}{\partial a} = 0 \Leftrightarrow a = \frac{d^3 g(1,k) + dg(2,k)}{d^4 g(5,k) + g(6,k)} \quad \text{(A. 4)}$$

$$\frac{\partial SEE_k}{\partial b} = 0 \Leftrightarrow b = \frac{d^3 g(4,k) + dg(3,k)}{d^4 g(8,k) + g(7,k)} \quad \text{(A. 5)}$$

$$\begin{aligned}\frac{\partial SEE_k}{\partial d} = 0 \Leftrightarrow\ & ag(1,k) + \frac{b^2}{d^3}g(7,k) + \frac{a^2}{d^3}g(6,k) + bg(4,k) \\ &= a^2 dg(5,k) + \frac{b}{d^2}g(3,k) + \frac{a}{d^2}g(2,k) + b^2 dg(8,k)\end{aligned} \quad \text{(A. 6)}$$

If we substitute the expressions for a and b from Equations (A.4) and (A.5) into Equation (A.6), then we get an equation in d only. That equation can be written as

$$\sum_{j=0}^{6}\theta_j d^{2j} = 0 \quad \text{(A. 7)}$$

We now consider the positive roots of Equation (A.7).

Let $f(d) = \sum_{j=0}^{6}\theta_j d^{2j}$. From $\theta_6 \succ 0$, we have that $\lim_{d\to\infty} f(d) = \infty$. On the other hand, we know that $f(0) = \theta_0 \prec 0$. Since $f(d)$ is a continuous function, we deduce that $f(d)$ has positive roots.

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Education

- Bachelor Course in Management Science, Chinese Military Academy (August 1982- November 1986).
- Artillery Regular Course in Army Artillery and Missile School (April 1988-October 1988).
- English Course in National Defense Language Center (August 1989 - March 1991).
- Master Course in Institute of Resources Management, National Defense Management College (August 1994 - June 1996).
- Military Course in Army Command and Staff School, National Defense University (August 1999 - February 2001).

Work Experience

- Platoon leader, Chinese Military Academy (November 1986 – November 1987).
- Headquarters battery commander, artillery brigade, 319^{th} infantry division (June 1991 - February 1993).

- Staff, artillery brigade, 333rd infantry division (May 1993 - July 1994).
- 608th Artillery battalion commander, artillery headquarter, 8th field army (August 1996 - August 1999).
- Staff, operations division, Army General Headquarters (February 2001 - September 2001).
- Military instructor, Army Command and Staff School, National Defense University (since September 2002).

Publications

A. Referred Papers

A-1. International Journal

1. Daisy, M. H. Huang, Chih-Yang Hung, Titus Tang,Gino young,Hsiao-Jung Chen, Peter Chu, 2004, "The battle of Gibeah: a further analysis", *Journal of Information & Optimization Science*, Vol. 25, No. 3, pp. 441-451.
2. C-Y Hung, GK Yang, PS Deng, T Tang, S-P Lan and P Chu, 2005, "Fitting Lanchester's square law to the Ardennes Campaign", *Journal of the Operational Research Society*, pp.1–5. **(SCI)**.
3. Chih-Yang Hung, Titus Tang, Daisy, M. H. Huang,Peter Shaohua Deng, Rober Huang-Jing Lin, Peter Chu, 2005, "Analystic inventory model with a mixture of back orders and lost sales", *Journal of Information & Optimization Science*, Vol. 26, No. 2, pp. 247-259.
4. Wen-Han Tang, Chih-Young Hung, Gwo-Hshiung Tzeng, 2004, "Evaluating the Strategies of Enhancing Educational Effects Of the Department of Command & General Staff of Army of Defense University－A Case Study of the Application of Fuzzy Integral MCDM", *Journal of Multi-Criteria Decision Analysis*,. (Paper in review)

A-2. Domestic Journal

1. 唐文漢、洪志洋、曾國雄，「運用模糊積分多評準決策法提升軍事學院教學效益方案之評選」，國防大學國防管理學報，第 26 卷第 1 期，31-48 頁，2005 年 5 月。(已刊登發表)
2. 唐文漢、洪志洋、曾國雄，「運用 DEA 與模糊迴歸時間數列對研究機構科技研發績效之評估與預測」，淡江大學管理研究學報，第 6 卷第 1 期，75-110

頁，2006 年 1 月。 (已刊登發表)

B. Conference Papers

B-1. International Conference

1. Hua-Kai Chiou, Gwo-Hshiung Tzeng and Wen-Han Tang. "Imprecise DEA with FMOP method for Evaluating the Performance of R&D Projects in Taiwan", *The 5th World Automation Congress*, Seville, Spain, June 28 – July 1, 2004, ISSCI076: pp.1-6.
2. Wen-Han Tang, Chih-Young Hung and Gwo-Hshiung Tzeng, "Evaluating the Strategies of Enhancing Educational Effects Of the Department of Command & General Staff of Army of Defense University－A Case Study of the Application of Fuzzy Integral MCDM", *The XVII-th International Conference on Multiple Criteria Decision Making*, Whistler, British Columbia,Canada, August 6- 11, 2004
3. Wen-Han Tang Chih-Young Hung and Gwo-Hshiung Tzeng,"DEA & Fuzzy Regression Time Series Approach for Measuring and Forecasting the Performance Efficiency of R&D in Taiwan's Science and Technology Research Institutes", *PICMET '05 Conference*, Portland, Oregon, USA, July 31 – August 4, 2005,

B-2. Domestic Conference

1. 洪志洋、黃元惠、唐文漢、許銘祥：以模糊多目標決策法和財務績效衡量進行台灣生物技術產業發展項目之選擇 中華民國科技管理學會論文，2003 年 12 月。
2. 洪志洋、黃元惠、唐文漢、許銘祥：我國生物技術產業發展現況與主要生物技術國家產業發展模式研究 中華民國科技管理學會論文，2003 年 12 月。
3. 唐文漢、洪志洋、曾國雄，「運用模糊多評準決策法評估教學效益提昇方案一以國防大學軍事學院陸軍學部為例」，第 11 屆國防管理實務暨學述研討會，2003 年 7 月。
4. 唐文漢、洪志洋、曾國雄，「國家級研究機構科技研發績效之 DEA 評估與模糊迴歸時間數列之預測」，2004 年第十二屆國防管理學術暨實務研討會。

5. 唐文漢、洪志洋，「比較經濟附加價值(EVA)與傳統財務指標探討臺灣IC製造業營運績效」，中華企業評價學會2004年論文研討會論文集，2004年11月。