Chapter 3 Research Methodology

3.1 Fundamentals of Catastrophe Theory

Catastrophe theory (Thom, 1975) is a mathematical theory that describes the relation between two sets of variables, control variables and behavioural variables (state variables), is so-called gradient system. In the gradient system, with fixed value of the control variables, the system always seeks an equilibrium state which means that the value of the behavioural variable changes until the minimum or maximum of a certain quantity is obtained. Given state variable $X = (x_1, x_2, ...x_n)$ and control variable $C = (c_1, c_2, ...c_n)$, the potential function V(x,c) and the equilibrium surface can be defined as $\nabla_x V = 0$. Figure 3.1 shows the relationship between state variable and control variable.

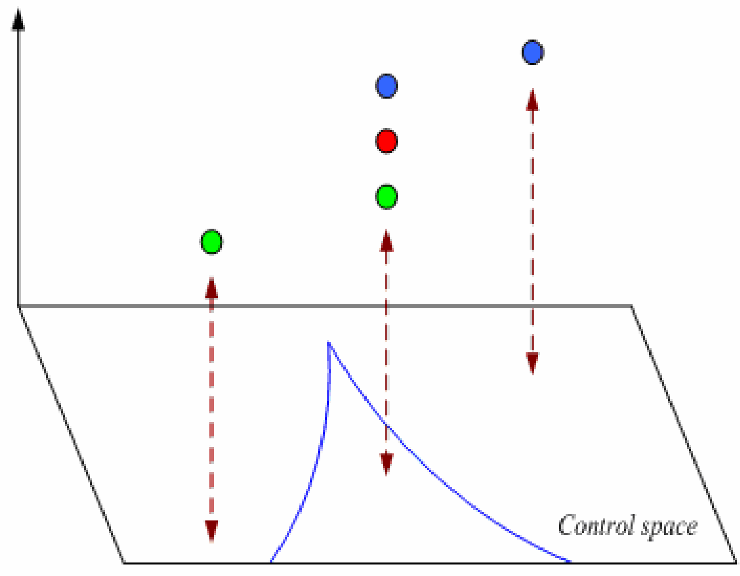

Figure 3.1 Relationships between State Variables and Control Variables

Tom has demonstrated through his classification theorem that all discontinuous phenomena that can be expressed in terms of four or fewer independent variables (also called control dimensions) which exit in many branches of science can be modeled accurately using one of only seven elementary catastrophes; Table 3.1 gives prototypical examples for equations showing each type of catastrophe. More specifically, for any system with fewer than five control factors and fewer than three behavior axes, these are the only seven catastrophes possible. Only the cusp catastrophe model is considered in the thesis, so the cusp model will be defined in greater detail.

Table 3.1 Rene Thom's Seven Elementary Catastrophes

	control dimension	behavior dimension	Potential function
Fold	1	1	$\frac{1}{3}z^3 - xz$
Cusp	2	1	$\frac{1}{4}z^4 - xz - \frac{1}{2}yz^2$
Swallowtail	3	1	$\frac{1}{5}z^5 - xz - \frac{1}{2}yz^2 - \frac{1}{2}vz^3$
Butterfly	4	1	$\frac{1}{6}z^6 - xz - \frac{1}{2}yz^2 - \frac{1}{3}vz^3 - \frac{1}{4}uz^4$
Hyperbolic	3	2	$z^3 + w^3 + xz + yw + vzw$
Elliptic	3	2	$z^3 - zw^2 + xz + yw + vz^2 + vw^2$
Parabolic	4	2	$z^2 w + w^4 + xz + yw + vz^2 + uw^2$

3.2 Cusp Catastrophe Model

The catastrophe structure most commonly has been applied the cusp model (Gresov, Haveman, & Oliva 1993). Figure 3.2 shows the basic form of the deterministic cusp model generated. Each catastrophe model can be formalized by potential or gradient structures, a potential function $F(x, c)$ is a function of both the system state x and the control parameter(s) c. The Cusp Catastrophe Model (CCM; see Thom, 1975) consists of one behavior variable and only two control variables. The potential is represented by Eq. (3.1), the equilibria of Eq. (3.1) is three-dimensional.

$$F(u,v,x) = -\frac{1}{4}x^4 + \frac{1}{2}ux^2 + vx \tag{3.1}$$

Where the state variable x is controllability, and u and v are environmental control parameters. As a stable equilibrium state x for this potential gives relative value x of a function F (u, v, x), a set of point (u, v, x) is defined as Eq. (3.2),

$$\begin{aligned} &\frac{\partial F}{\partial x} = -x^3 + ux + v = 0 \\ &M_F : \{(u,v,x) \,|\, -x^3 + ux + v = 0\} \end{aligned} \tag{3.2}$$

Where M_F is said to be cusp catastrophe manifolded. The values of x in correspondence to which attains a local maximum or minimum satisfies the condition as Eq. (3.3),

$$3x^2 + u = 0 \tag{3.3}$$

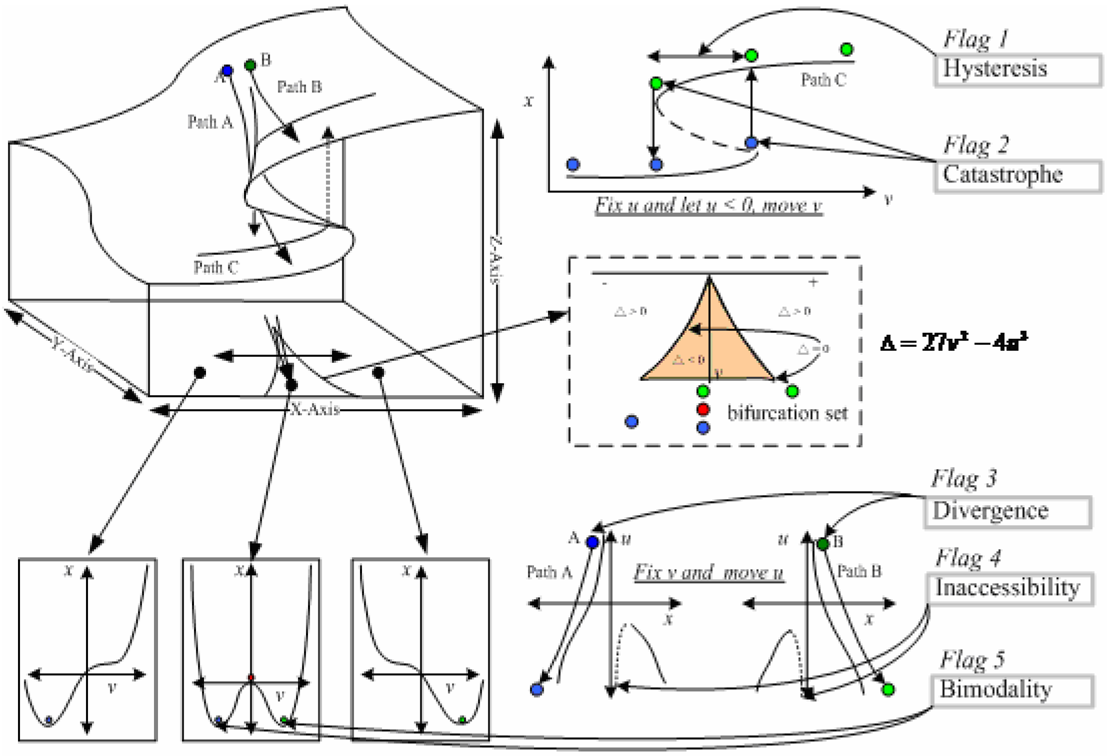

Figure 3.2 A Cusp Catastrophe Model and its Five Different Flags

Eliminating x from Eq. (3.2) and Eq. (3.3), the bifurcation set is express by Eq. (3.4). In Zeeman's terminology u is a splitting factor and v is normal factor.

$$4u^3 = 27v^2 \tag{3.4}$$

A switch in topology takes place at the values of u and v satisfying Eq. (3.4), which constitute the catastrophe set. In the equation Eq. (3.3) x is the state variable, and u, v are control parameters. The parameter u determines whether the system has one or can have two stable equilibria. When $u > 0$ only one stable equilibrium can exit whatever the value of v. When $u < 0$ it depends upon the value of v whether the system has a single low level of stable equilibrium, or a low level and a high level equilibria, or a single high level of equilibrium.

According to the different variable sets, three different cases can be defined. Case 1: There is one stable equilibrium point; Case 2: There are two stable and one unstable equilibrium point; or Case 3: There is one stable equilibrium point, and one at which an instantaneous jump in the state variable occurrence.

Changes in the control or independent variables (v-right/left movement, and u-back/front movement) cause the changes in the behavior or dependent variable (x-vertical movement). If u is low, smooth changes in v occur in proportion to change in x as shown by examining the travel of point A and B in Figure When u is high (past the singularity) changes in v producing relatively small changes in x until a threshold is reached when there is a sudden discontinuous shift in x. This is depicted by the path from point C to D in Fig3.2. Note, that a reversal in v back to the point of the shift in x, will not cause x to return back to its original position, since v will have to move well past to cause x to shift back. This is shown as the movement from point C to E.

The various moves on the surface are characterized by five qualities that Thom (1975) described as: bimodality, divergence, catastrophe, hysteresis, and inaccessibility for more details of these catastrophe flags are discussed as the following:

1. Bimodality

Over some parts of phenomenon, the behavior is ambiguous; that is, bimodality indicates that either two stables or distinctly different behaviors can be occurred.

2. Divergence

The divergence indicates small differences in the starting position can result in vastly different and opposite ending positions. In short, small initial differences can bring about totally different behavior.

3. Catastrophe (sudden transitions)

If changes in the normal and splitting factor produce a path which crosses the bifurcation set, an abrupt, catastrophe changes, in the value of the dependent variable will be occurred. At that point, an abrupt transition is made from the lower to the upper surface.

4. Hysteresis

After the sudden transitions, although the path is returned, the hysteresis phenomena show that the abrupt change from one mode of behavior to another takes place at different values of the control factors depending on the direction of change.

5. Inaccessibility

Over part of phenomenon, there is a middle region between the two types of behavior that are inaccessible.[1]

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[1] For further discussion of catastrophe theory, the following websites are proposed readable presentations:

http://www.sbm.temple.edu/~oliva/cat-theory.htm

3.3 Approaches for Estimating Catastrophe Models

Estimation of chaos models, in general, and catastrophe models, in particular, is difficult because of nonlinear dynamic characteristics. Several cusp-fitting procedures have been proposed, but none is completely satisfactory. On the following literature, there are three techniques for fitting the cusp catastrophe models. These methods are GEMCAT of Oliva *et al.* (1987), the maximum likelihood method of Cobb (1978) and the regression method of Guastello (1982). GEMCAT and the method of Cobb for instance can be applied to cross sectional data; the method of Guastello can only be applied to time series of data. Figure 3.3 shows that the major researchers on catastrophe theory and its estimating approach.

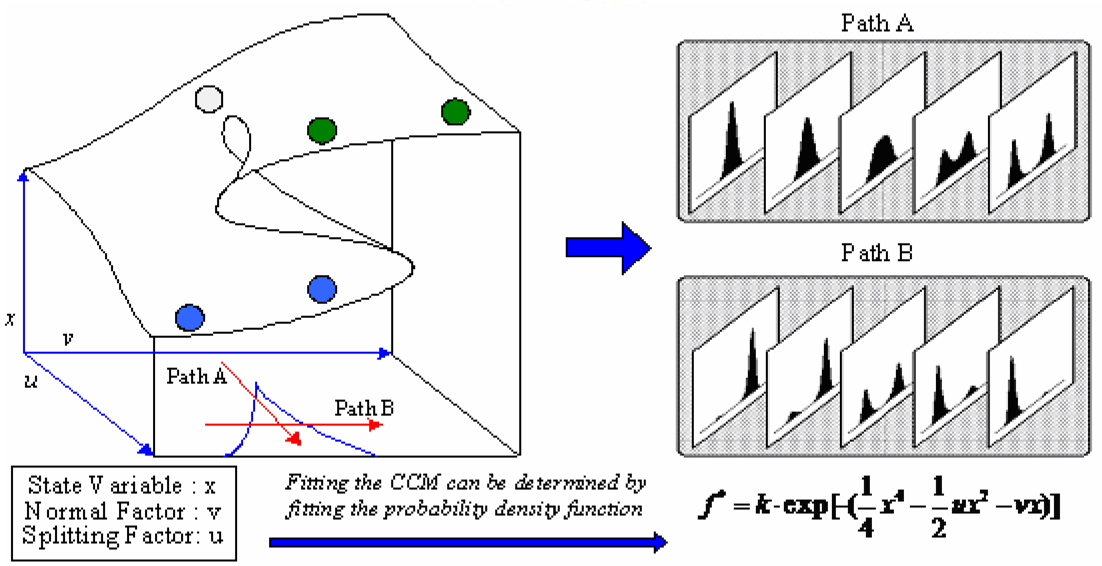

Figure 3.3 Major Researchers on Catastrophe Theory and its Estimating Approach

A limitation of Cobb's and Guastello's methods does not allow researchers to specify models in terms of specific combinations of multiple indicator variables. Rather the technique finding catastrophe if it exists and identifies which independent variables are associated with the control factor and which independent variables are associated with the splitting factor. Cleary, this is a problem when the researchers are trying to develop a confirmatory that estimates a specific catastrophe model. Additionally, the dependent variable is required to be univariate. Consequently, its usefulness is limited when the catastrophe model uses or requires a multivariate dependent construct.

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http://www.marquette.edu/psyc/guastell.html

http://www.aetheling.com/models/cusp/Intro.htm

http://perso.wanadoo.fr/l.d.v.dujardin/ct/eng_index.html

GEMCAT approaches have been successfully applied in a number of different organizational research contexts (e.g. Oliva, 1992; Gresov *et al*., 1993; Kalph, Kauffman and Oliva, 1994). Next, the GEMCAT approach is described in some details.

Oliva *et al*.'s (1987) GEMCAT approach allows all variables in a catastrophe to be latent composites. To accomplish this, the variable *X*, *Y*, and *Z* in the canonical cusp is presented by Equation (3.5),

$$f(x, y, z) = \frac{1}{4}z^4 - \frac{1}{2}yz^2 - xz \tag{3.5}$$

Let:

i = 1… I dependent variables;

j = 1…J "splitting" independent variables;

k = 1…K "normal" independent vatiables;

t = 1…T observations;

Z_{it} = the value of the i-th dependent variable on observation t;

Y_{it} = the value of the j-th splitting independent variable on observation t;

X_{kt} = the value of the k-th normal independent variable on observation t;

Now, define three "latent" unobservable variables:

$$Z_t^* = \sum_{i=1}^{I} \alpha_i Z_{it} \tag{3.6}$$

$$Y_t^* = \sum_{j=1}^{J} \beta_j Y_{jt} \tag{3.7}$$

$$X_t^* = \sum_{k=1}^{K} \gamma_k X_{kt} \tag{3.8}$$

where:

α_i = the estimated coefficient for the i-th dependent variable in $Z = Z_{it}$;

β_j = the estimated coefficient for the j-th splitting independent variable in

$\mathrm{Y} = \mathrm{Y}_{jt}$

γ_k = the estimated coefficient for the k-th normal independent variable in $X = X_{kt}$

$Z_t^* = Z\alpha = \sum_{i=1}^{I} \alpha_i Z_{it}$ = the value of the latent performance variable on observation t, where $\alpha = \alpha_i$

$Y_t^* = Y\beta = \sum_{j=1}^{J} \beta_j Y_{jt}$ = the value of the latent splitting variable on observation t, where $\beta = \beta_j$

$X_t^* = X\gamma = \sum_{k=1}^{K} \gamma_k X_{kt}$ = the value of the latent normal variable on observation t, where $\gamma = \gamma_k$

Thus, the equation (3.5) can be redefined as these three "latent" unobservable constructs which can thus accommodate univariate or multivariate measurements for each type of variable. This allows the cusp catastrophe model to be rewritten as shown in Eq. (3.9):

$$f(X_t^*, Y_t^*, Z_t^*) = \frac{1}{4} Z_t^{*4} - X_t^* Z_t^* - \frac{1}{2} Y_t^* Z_t^{*2} \tag{3.9}$$

In these terms, the estimation problem, given $X = X_{kt}$, $\mathrm{Y} = \mathrm{Y}_{jt}$ and $Z = Z_{it}$, and its derivative set equal to zero can be stated as:

$$\begin{aligned} \frac{\partial f(X_t^*, Y_t^*, Z_t^*)}{\partial Z_t^*} &= 0 \\ &= Z_t^{*3} - X_t^* - Y_t^* Z_t^* \end{aligned} \tag{3.10}$$

From equation Eq. (3.10) the estimating goal is to minimize Eq. (3.11):

$$\mathrm{Min}_{\alpha i, \beta j, \gamma k} \Phi = \left\| e_t^{\,2} \right\| = \sum_{t=1}^{T} \left[Z_t^{*3} - X_t^* - Y_t^* Z_t^* \right]^2 \tag{3.11}$$

where the e_t = error. That is, for a given empirical data on various specified dependent, splitting, and normal variables, one wishes to estimate the impact coefficients that define their respective latent variables, which make Φ as close to zero as possible. Minimizing Φ is equivalent to find the best fitting cusp catastrophe surface to the empirical data.

More recently, Lange et al. (2000) developed an improved version of the algorithm called GEMCAT Ⅱ (the GEMCAT Ⅱ software is developed in Delphi V3.0), which provides greater speed, efficiency, utility and flexibility in terms of analysis and testing. GEMCAT Ⅱ uses a combination of the Downhill Simplex method and Powell's Conjugate Gradient approach. GEMCAT estimates the various indicator weights by minimizing the total squared residual (Φ) across observations; the default procedure is to run the Downhill Simplex.

3.4 Conclusion Remarks

Interest in developing more parsimonious approaches to the modeling of complex behavior has been stimulated by catastrophe theory (Thom, 1975; Zeeman, 1976). These approaches have become intriguing to researchers in behaviorally based disciplines such as economics (Zeeman, 1977; Lange *et al*., 2001), (Byrne, 2001), and management (Oliva, et al., 1988; Guastello, 1988). The models' strengths can capture complex behavior by using significantly fewer nonlinear equations than the number of linear equations needs to describe the same phenomena.

Catastrophe theory is a mathematical theory that describes the relation between two sets of variables, such as control variables and behavioural variables. The first important step in experimental research concerned with catastrophe systems is to get some strong indications that the system under survey indeed shows catastrophic transitional behavior (Zeeman, 1976; Glimore, 1981). When one has obtained strong indications by the use of the flags mentioned in the previous paragraph it seems reasonable to try determining the appropriate catastrophe model. Therefore, the second step in experimental research, catastrophe modeling, is concerned with fitting catastrophe models to experimental observations.

Currently, three different approaches are considered the most appropriate for estimating catastrophe models. The first is by Cobb (1978), the second is by Guastelo (1982) and the third is GEMCAT, developed by Oliva *et al*, (1987). GEMCAT, in contrast, is a confirmatory multivariate analytical procedure. Theses three method are different, having advantage and disadvantages. Cobb's procedure is classified as an exploratory statistical method in which researchers cannot indicate a priori which measured variables relate to which independent variable (Oliva *et al*., 1992).

To deal with the Cobb's and Guastello's approach, researchers using their estimating techniques have typically averaged or otherwise scaled the measures to get a single dependent measure. Unfortunately, such averaging techniques can cause the loss of value information when a true catastrophe models is present as demonstrated in Oliva *et al*. (1987). More recently, Alexander, Herbert, Deshon and Hanges (1992) noted a problem with Guastello's difference regression approach, arguing that this technique will yield catastrophe results in the face of linear data. In short, the procedure can indicate that catastrophe data are present when they are not. Alexander and colleagues went on to suggest that researchers interested in estimating catastrophe model should use either Cobb's (1978) technique or the GEMCAT developed by Oliva *et a*l. (1987). The choice of technique depends on the nature of the research context. For exploratory, situations in which theory construction is the focus, or when the existence of catastrophe data is the issue and univariate dependent measures are sufficient, the Cobb's approach is best; alternately, GEMCATE is more appropriate for theory-testing or confirmatory contexts, and those requiring multivariate indicators.

Alexander *et al*. (1992) comparison of Cobb (1981) and Guastello (1995) techniques versus the GEMCAT approach note that for exploratory situations in which theory construction is the focus, or when the existence of catastrophe data is the issue and univariate dependent measures are sufficient, Cobb related approaches are the best choice. However, Alexander *et al*. (1992) argue the GEMCAT is the best choice for theory testing or confirmatory contexts, and those requiring multivariate indicators in the dependent variables. Given the use of multivariate dependent construct and confirmatory mature of this work, the GEMCATII procedure is the appropriate estimation technique to be used. According to literature review, the analysis framework of catastrophe model is shown as the Figure 3.4.

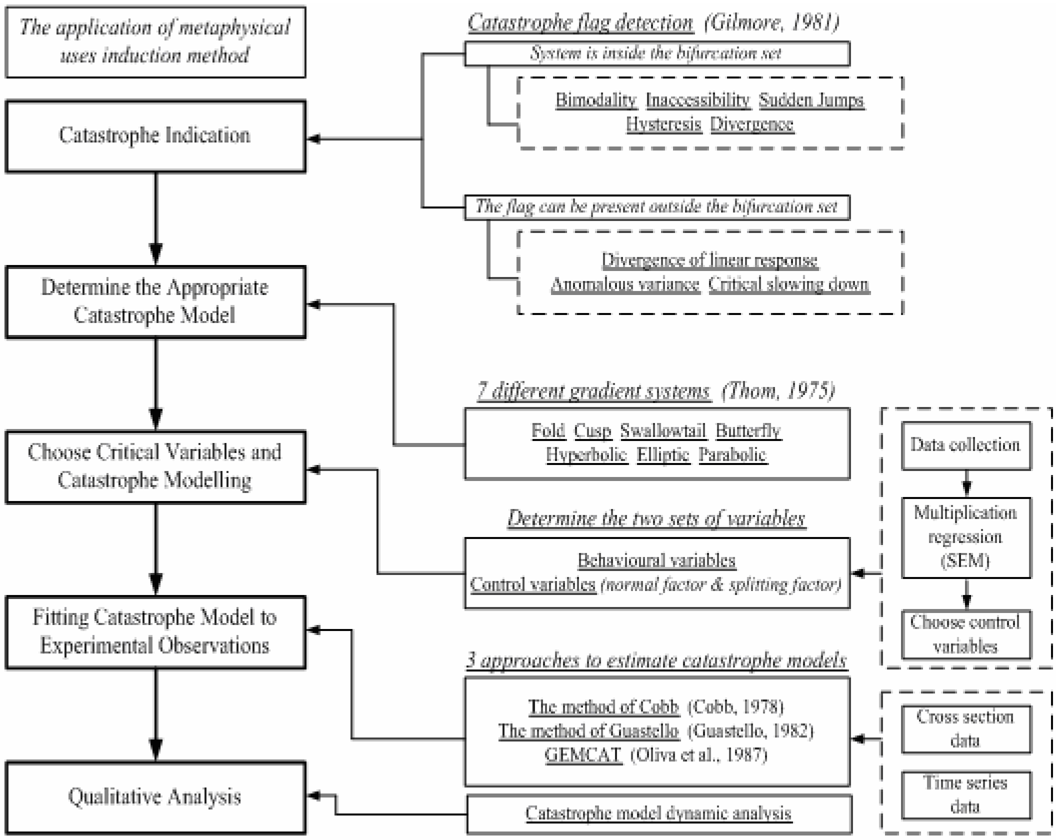

Figure 3.4 Analysis Framework of Catastrophe Model