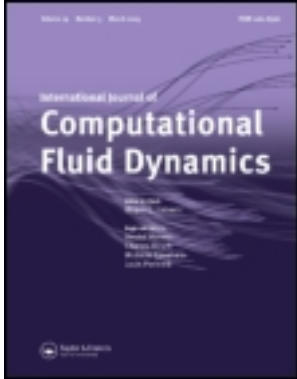


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The lowest stability and bifurcation in supercritical Taylor vortices

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The lowest stability and bifurcation in supercritical Taylor vortices

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Here, we numerically investigate the lowest stability and bifurcation boundary of supercritical Taylor vortices in flows with different wavenumbers and for various radius ratios; the radius ratios range from those corresponding to axisymmetrical Taylor vortex flow (TVF) to those corresponding to wavy vortex flow (WVF). The variation in the wavenumber of a supercritical TVF is found to affect the stability of the flow, because the wavenumber of the Taylor vortices remains constant only when the flow is quasi-static. The variation in the wavenumber is examined and found to be significant when the radius ratio is less than 0.7842. The results for TVF are compared with those for the flow during the quasi-static transition from TVF to WVF.

Keywords: hydrodynamic stability; wavy vortex flow; bifurcation; wavenumber; Taylor-Couette flow

1. Introduction

Fluid flows between two concentric rotating cylinders are often encountered in fluid dynamics. These flows are interesting and complex. Thus, they attract the attention of many scholars and researchers. Here, we numerically analyse and simulate the flow patterns and flow characteristics between two concentric rotating cylinders.

The Taylor vortex problem, for many years, was on the verge of being classified as a nonlinear problem until Coles (1965) first reported on the nonuniqueness of the wavy flow in the Taylor-Couette flow. The entire pattern of wavy vortices moves with a uniform velocity in the azimuthal direction. Since the term ‘wavy’ is typically associated with motion that includes periodic vertical oscillations, this study emphasises that the wavy Taylor vortices move as rings in the azimuthal direction. The rings have k_1 fixed sinusoidal upward and downward deformations, where k_1 is an integer number of azimuthal waves. Wavy Taylor vortices were observed by Taylor (1923), Lewis (1928), Schultz-Grunow and Hein (1956) and Coles (1965). However, they were not recognised as a characteristic feature of the flow. After Coles’ preliminary results were published, wavy vortices were also observed by Nissan *et al.* (1963).

Burkhalter and Koschmieder (1974) found that in the case of large radius ratios, the wavelength of

axisymmetrical vortices was independent of the Reynolds number in fluid columns of infinite length if the Reynolds number in such fluid columns increases quasi-statically. Jones (1985) presented the stability boundary for an axial wavenumber of 3.13, the critical value for a quasi-static transition, for a wide range of radius ratios. Jones (1985) considered the problem of calculation of nonlinear axisymmetrical Taylor vortices. A spectral method together with Newton–Raphson iterations was used to solve the nonlinear algebraic equations.

While Taylor’s study analysed such flow under supercritical conditions, Stuart (1958) observed that the shape, i.e. the size of the vortices remains unchanged above the critical Reynolds number. Numerous studies (see Coles 1965, Burkhalter and Koschmieder 1973, 1974, Park *et al.* 1981, Ahlers *et al.* 1983, Andereck *et al.* 1986, Antonijoan and Sanchez 2002) have demonstrated the importance of considering the acceleration/deceleration of the flow in determining the final state of the flow. These vortices have axial wavelengths that are different from those of vortices observed after a quasi-static transition. The present study demonstrates that the stability boundary occurs at a critical wavelength corresponding to the quasi-static transition in addition to another wavelength. These solutions are related to the standard Taylor vortices and can be obtained quasi-statically for

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certain radius ratios when a mechanism is used for modifying the axial wavelength (see Hall and Blennerhasset 1979).

2. Formulation

Figure 1 shows the system geometry specified by the inner and outer radii, R_1 and R_2 , of cylinders with an infinite aspect ratio and the dimensionless parameter in the problem is the radius ratio $\eta = R_1/R_2$. The inner cylinder rotates with the Reynolds number Re , $Re = R_1\Omega_1 d/\nu$ where ν is the dynamic viscosity, Ω_1 is the angular velocity of the inner cylinder rotation, and $d = R_2 - R_1$ is the radial gap between cylinders. The outer cylinder is considered to be at rest under all conditions. First, the Taylor vortex flow (TVF) is solved numerically and the details have been published by Yang and Lin (2009). The flow is described by the incompressible, three-dimensional, and dimensionless Navier–Stokes equations with the cylindrical

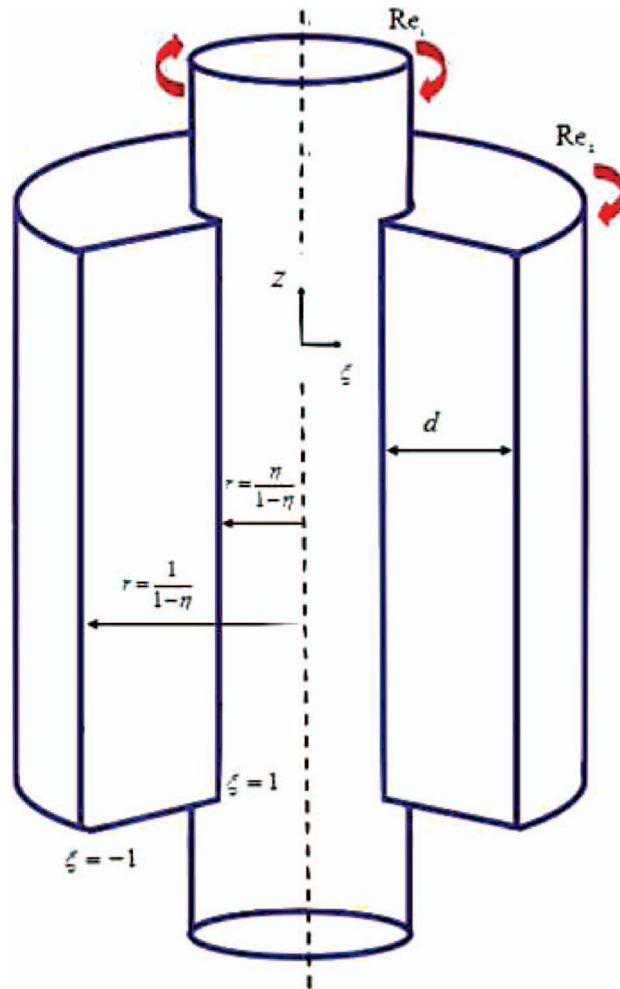


Figure 1. The computational model of flow field.

coordinates (r, θ, z) in an absolute frame of reference according to the velocity–pressure formulation.

The stabilities of the supercritical TVF are studied by introducing disturbances in the TVF. This flow type is expressed as follows:

$$f(V_r, V_\theta, V_z, p) = \bar{f}(\bar{V}_r, \bar{V}_\theta, \bar{V}_z, \bar{p}) + f'(V'_r, V'_\theta, V'_z, p') \quad (1)$$

where \bar{f} denotes the flow velocity and pressure profile of the supercritical TVF, and f' represents the perturbations. The equations employed for the analysis – only out-of-phase wavy modes are investigated – of perturbations in normal modes are as follows:

$$V'_r = \sum_{q=1}^Q \sum_{s=2}^{S+1} a_{qs} \phi_s(\xi) \sin q\alpha z \cdot \exp[\sigma\tau + i(k_1\theta + k_2z)] \quad (2)$$

$$V'_\theta = \sum_{q=1}^Q \sum_{s=2}^{S+1} b_{qs} \phi_s(\xi) \sin q\alpha z \cdot \exp[\sigma\tau + i(k_1\theta + k_2z)] \quad (3)$$

$$V'_z = \sum_{q=0}^{Q-1} \sum_{s=2}^{S+1} c_{qs} \phi_s(\xi) \cos q\alpha z \cdot \exp[\sigma\tau + i(k_1\theta + k_2z)] \quad (4)$$

$$p' = \sum_{q=1}^Q \sum_{s=0}^{S-1} d_{qs} T_s(\xi) \sin q\alpha z \cdot \exp[\sigma\tau + i(k_1\theta + k_2z)] \quad (5)$$

Here, τ is the dimensionless time and α is the axial wavenumber of the TVF. Q and S are the number of terms in the Fourier series expansion and Chebyshev polynomial expansion, respectively. k_1 (an integer) and k_2 (a real number) are the wavenumbers of the perturbations in the azimuthal and axial directions, respectively; a_{qs} , b_{qs} , c_{qs} and d_{qs} are the amplitude coefficients. The space can be defined as: $T_s(\xi) = \cos(s \cdot \cos^{-1} \xi)$; ϕ_s is a basis function that satisfies the boundary conditions. ϕ_s is expressed as follows:

$$\phi_s(\xi) = T_s - [1 - (-1)^s] \frac{T_1}{2} - [1 + (-1)^s] \frac{T_0}{2}, \quad s = 2, 3, 4, \dots \quad (6)$$

where $\xi \in [-1, 1]$. The domain of r in the governing equation is transformed from $\eta/(1-\eta) \leq r \leq 1/(1-\eta)$ to $-1 \leq \xi \leq 1$ through the relational equation $\xi = 2r - (1 + \eta)/(1 - \eta)$.

The dimensionless Navier–Stokes and continuity equations are as follows:

$$\partial_i \vec{f} + \vec{f} \cdot \nabla \vec{f} = -\nabla p + \frac{1}{Re} \Delta \vec{f}, \quad \nabla \cdot \vec{f} = 0 \quad (7)$$

The boundary conditions are as follows:

$$f' = 0 \text{ at } r = \frac{\eta}{1-\eta} \quad \text{and} \quad r = \frac{1}{1-\eta} \quad (8)$$

Substituting Equation (1) into Equation (7) and linearising the dimensionless Navier-Stokes equation, we can obtain the characteristic perturbation equations, which constitute a generalised eigenvalue problem as follows:

$$AX = \sigma BX, \quad A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & 0 & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} B_{11} & 0 & 0 & 0 \\ 0 & B_{22} & 0 & 0 \\ 0 & 0 & B_{33} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} a_{rs} \\ b_{rs} \\ c_{rs} \\ d_{rs} \end{bmatrix} \quad (9)$$

In the above equations, A and B are complex matrices that depend on the values of k_1 and k_2 , and the eigenvector X contains the amplitudes of the eigenfunctions. The stability of the flow can be determined by the real part of the growth rate of a complex disturbance σ_r . When $\sigma_r < 0$, the entire flow is stable. The disturbance decreases with an increase in time. When $\sigma_r > 0$, the disturbance increases with time and the flow becomes unstable. When $\sigma_r = 0$, the flow has neutral stability.

The eigenvalue of the generalised eigensystem is obtained by using the subroutine DGVLCG in the IMSL library, which determines all eigenvalues with a high level of accuracy. R_c is searched on the neutral stable curve, i.e. the curve on which the real part of the most unstable eigenvalue vanishes, using the secant method that requires two initial guesses. The iteration is not terminated until the real part of the most unstable eigenvalue is less than 10^{-6} . The R_c values for different wavenumbers can be obtained for neutral stable states. The minimum Reynolds number is called the critical Reynolds number and corresponds to the critical wavenumber.

3. Results and discussion

Prior to the computation of the flow field, we analysed the degree of accuracy, which served as the basis for the post computation. In theory, the greater the number of terms expanded, the higher is the accuracy. However, the limit to the increase of the number of terms can be determined from the round-off error and the computation process becomes time consuming.

Therefore, the best option was to use the expansion with the least number of terms for which a certain degree of accuracy can be guaranteed. The results have been published by Yang and Lin (2009). For the computation in this study, both Q and S, which are the number of terms in the Fourier series expansion and Chebyshev polynomial expansion, respectively, were maintained at 10 terms.

Figure 2 shows the numerical result (with $k_2 = 0$) together with the experimental data obtained by Ahlers *et al.* (1983). Each symbol (solid circle) corresponds to a solution in their study (onset of the wavy vortex flow (WVF) at $k_1 = 3$). The range of wavenumbers considered is 2.6–4.0, and the range of R_c/R_{ec} is 0.8–2.0. R_{ec} is the critical value of R_c , i.e. the value at which the TVF occurs. The model used in the present study assumes that wavy Taylor vortices are perfectly periodic in the axial direction and thus ignore the end effects. This model is similar to that developed by Ahlers *et al.* (1983).

A comparison of the model used in the present study with the models developed by Park (1984) and Jones (1981) indicated a good agreement between the experimental and theoretical values for $k_1 = 2$ (see Table 1).

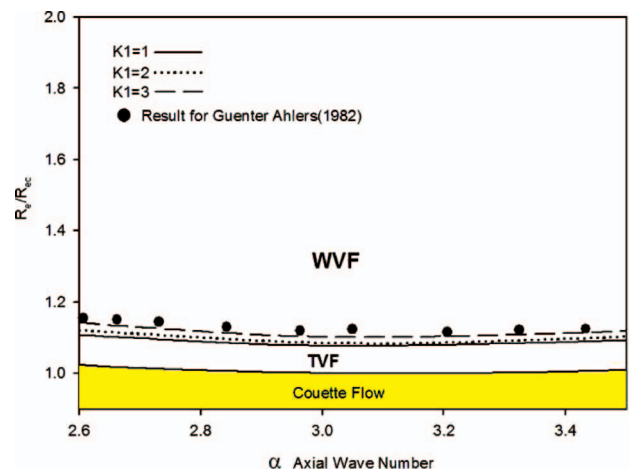


Figure 2. Combines both numerical and experimental results for the onset of WVF for $\eta = 0.893$.

Table 1. Comparison of experimental and theoretical R_c values for $\eta = 0.782$.

	Experiment of Park (1984)	Study of Jones (1981)	This study
Onset of $k_1 = 1$	Not observed	110	109.5
Onset of $k_1 = 2$	137.3	120	119.5
$k_1 = 2$ gone	161.3	163	167.5
$k_1 = 1$ gone	Not observed	169	167.8
Onset of $k_1 = 3$	322	None	None

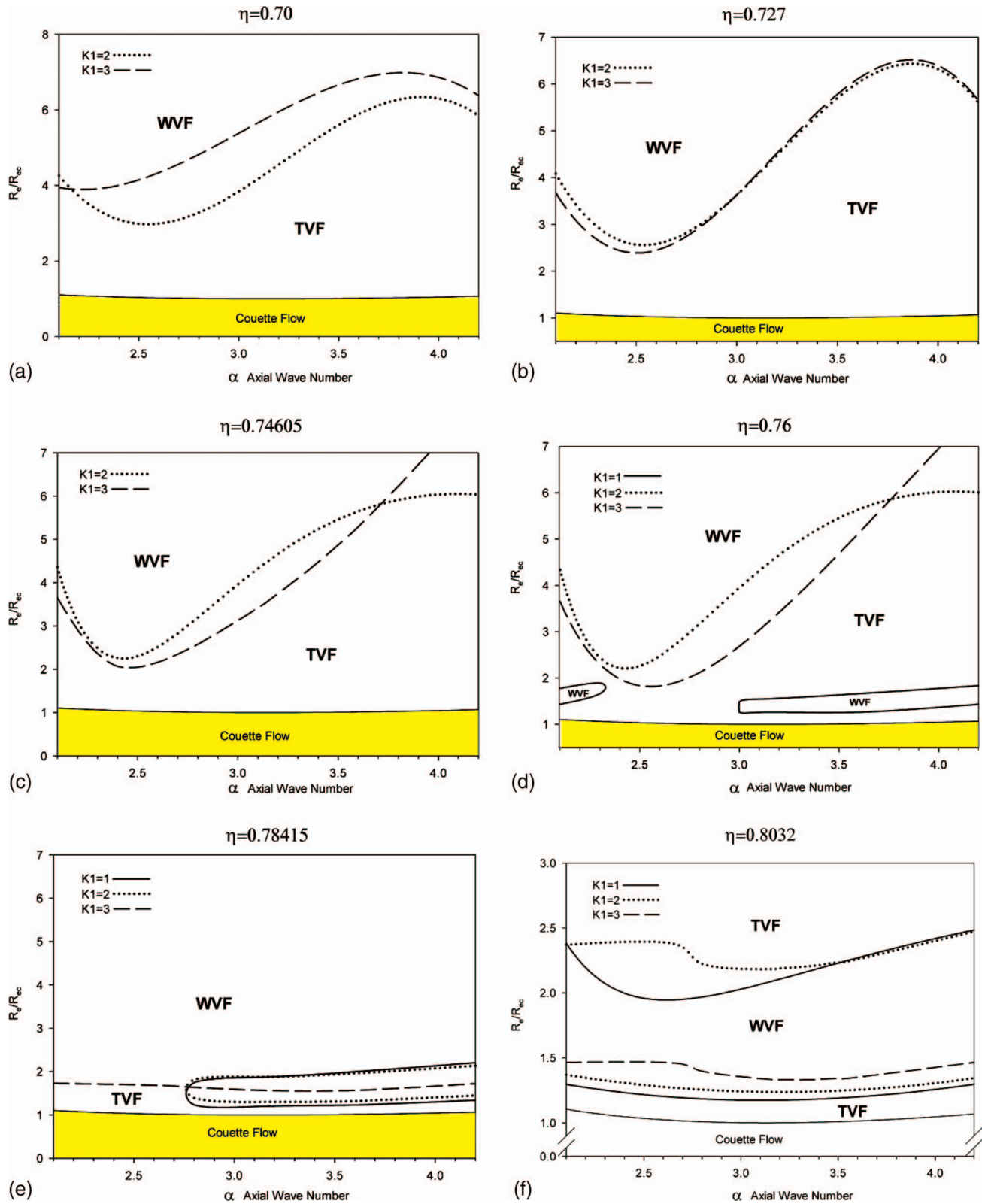


Figure 3. Neutral stability curves of the transition of Taylor vortex flow to wavy vortex flow for various (a) $\eta = 0.7$, (b) $\eta = 0.727$, (c) $\eta = 0.74605$, (d) $\eta = 0.76$, (e) $\eta = 0.78415$, (f) $\eta = 0.8032$, (g) $\eta = 0.82$, (h) $\eta = 0.88$, (i) $\eta = 0.93$.

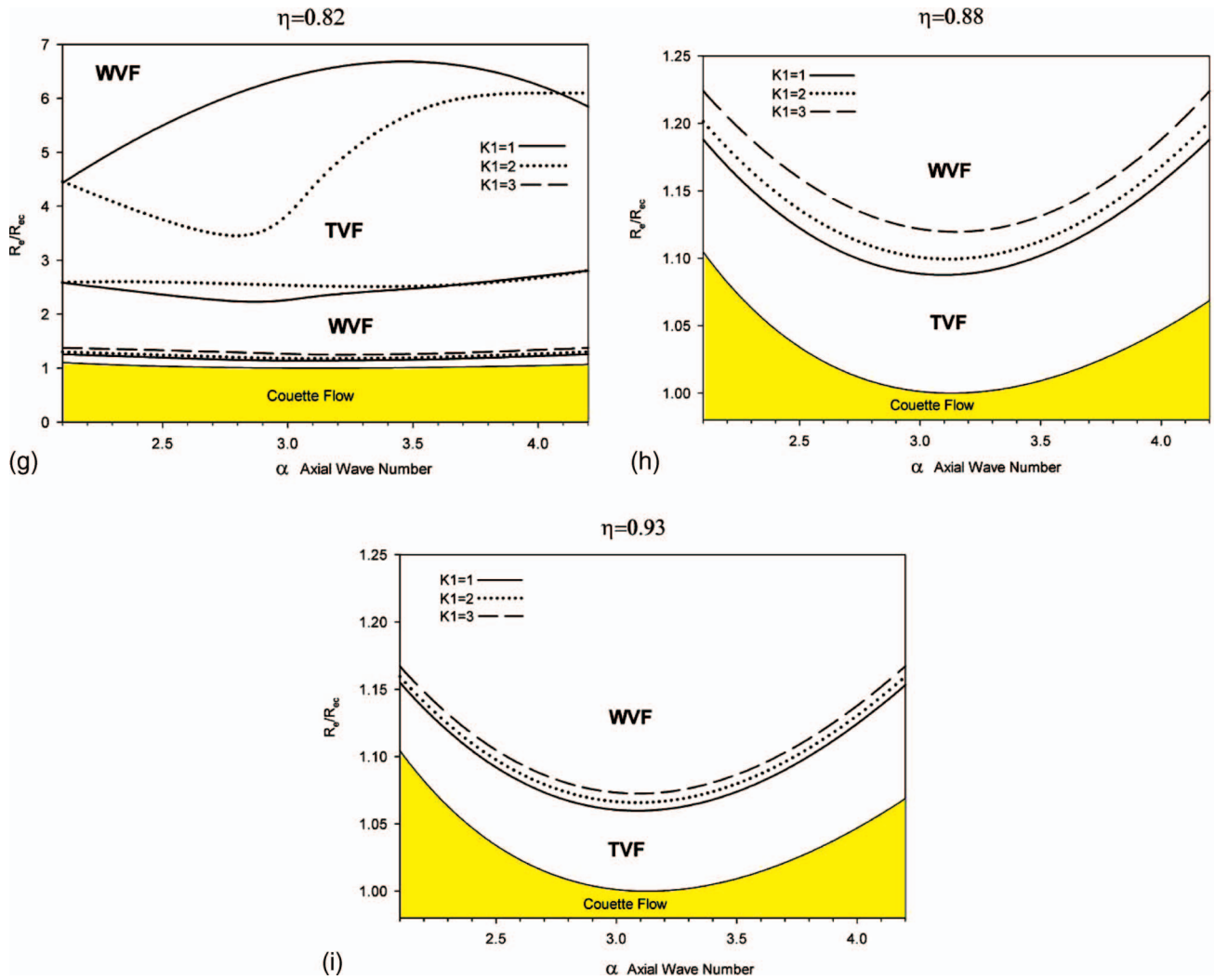


Figure 3. (Continued).

The plots in Figure 3a-i show the stability boundaries at the onset of wavy Taylor vortices with k_1 in the range 1-3 in the parameter plane ($\alpha, R_c/R_{ec}$). A good agreement has been observed on comparing every plot with the plots obtained by Jones (1985).

For η in the range 0.7-0.74605, the lowest stability of the transition from TVF to WVF depended heavily on the value of α , which was less than 3.13. For $\eta = 0.76$ and $\eta = 0.78415$, the dominant transition occurred at $k_1 = 3$, which was below a certain value for α . When η was increased above 0.78415, a new transition curve with $k_1 = 1$, for which α was equal to approximately 3.13, represented the lowest stability boundary.

Figure 4 presents the wavenumbers of the lowest stability boundary for various values of η . For the transition from TVF to WVF, α is less than 3.13 for

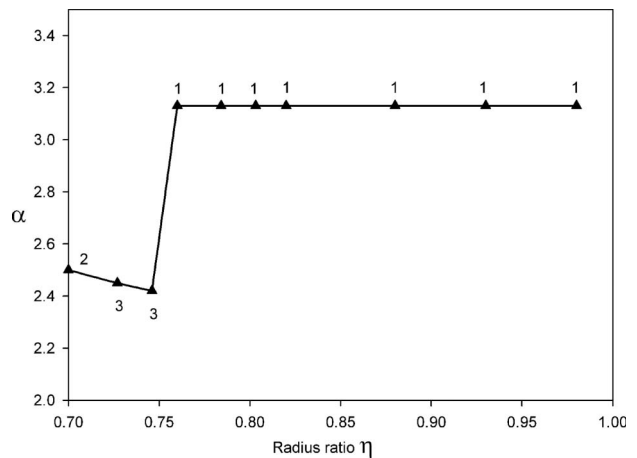


Figure 4. The wave number of the lowest stability boundary for various. The numbers denote k_1 .

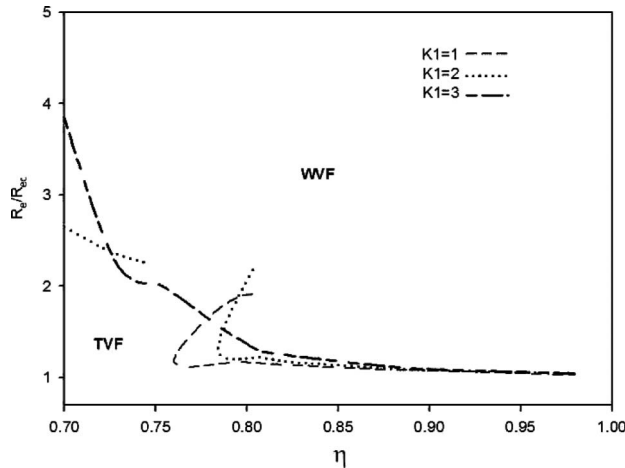


Figure 5. The lowest stability boundary for the bifurcation from Taylor vortices (in this study).

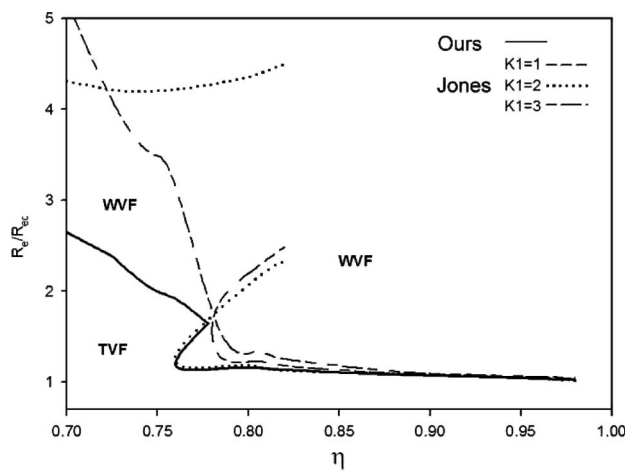


Figure 6. The lowest stability boundary for the bifurcation from Taylor vortices (with the simulated result obtained by Jones (1985)).

three sections: the first section is $0.76 \leq \eta \leq 0.7842$ with an upper branch of $k_1 = 1$, the second section is $0.7 \leq \eta \leq 0.7248$ with $k_1 = 2$ and the third section is $0.7248 \leq \eta \leq 0.7842$ with $k_1 = 3$ (see Figure 5). The critical Reynolds number shifted to values of α that were substantially less than 3.13; that is, the azimuthal waves were easily generated. Figure 6 also shows the new stability boundary curves for the transition from TVF to WVF and presents different stability boundary curves for axisymmetrical TVF when η was lower than approximately 0.7842.

4. Conclusion

The effect of a variation in the axial wavenumber of a TVF on the stability of the flow can be studied by the infinite cylinder approximation. The wavenumber was

considered as an external parameter and is not determined theoretically, but is measured experimentally. In some apparatuses, such as those used by King and Swinney (1983), fluid could be added or removed even when the cylinders were rotating, thereby allowing direct control over the wavelength. The present study determined a new curve for the lowest stability boundary for the transition from a supercritical TVF to a WVF. This curve differed from that obtained by Jones (1985), who assumed that the Reynolds number of the inner cylinder increased quasi-statically.

This study also investigated the lowest stability boundary for different axial wavenumbers and various radius ratios ranging from the ratio corresponding to a supercritical TVF to that corresponding to a WVF. The variation in the axial wavenumber was found to affect the stability of the flow for radius ratios less than 0.7842.

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