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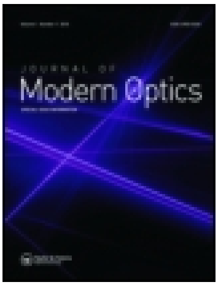

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Method of solving cemented triplets with given primary aberrations

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Method of solving cemented triplets with given primary aberrations

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Abstract. An effective algorithm is proposed for solving the structure of thin-lens cemented triplets which consist of three different glass types and have the given target values of power, primary spherical aberration, coma and longitudinal chromatic aberration. The formulae for the aberrations are combined with the power formula into a fifth-order polynomial equation in one variable, whose roots can all be solved by a combination of numerical and algebraic methods.

1. Introduction

Optical design problems in which the system will be designed from the primary aberration theory normally begin with two stages. First, the optical thin-lens layout and the aberration targets of each optical component are decided. Thus the focal length and aberration targets of each component, the separation between adjoining components, and the paraxial, marginal and chief ray heights at each component are determinate. The aberration targets are normally three in number: the Seidel coefficients S_1, S_{2C} and C_L, representing spherical aberration, central coma and longitudinal chromatic aberration. A technique to solve primary aberration targets was proposed by Hopkins [1]. Second, there follows a thin-lens design stage for each component. During this stage, a suitable type of lens, which may be a doublet or a triplet, etc., is chosen for each component and then the glass materials and curvatures are found to meet the power K and aberration targets. Hence, systematic thin-lens design methods for various lens types are useful tools and have been the subject of research by several workers.

Cemented doublets can meet two of the three aberration targets. Khan and MacDonald [2] used a set of precomputed graphs to find the solutions near these three target values. They also discussed the advantages and limitations of several different methods provided by earlier workers. If cemented doublets cannot provide suitable solutions, lenses with more free parameters must be adopted. Contact doublets have four surfaces and can meet the three aberration targets. One of the design methods for the contact doublets is described by Smith [3].

On the other hand, cemented triplets can also meet three aberrations and provide more chosen solutions than cemented doublets. Conrady [4, 5] has given a study of cemented triplets which are produced by dividing cemented doublets and thus have the same glass type as the front and rear elements. The method has three

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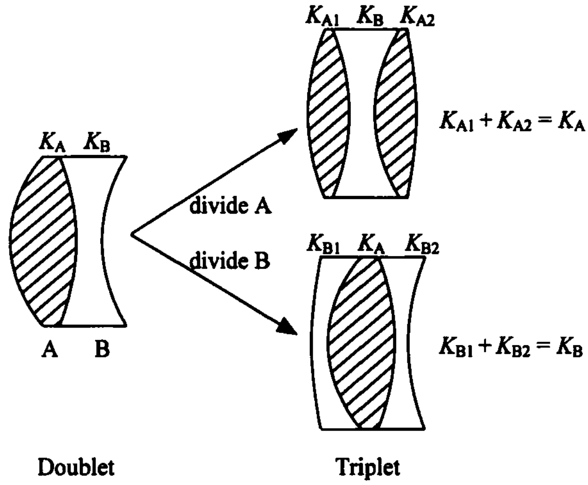

Figure 1. Cemented triplets with two glass types are obtained by dividing either the flint lens or the crown lens of cemented doublets into two parts. The triplets have the same values of K and C_L as the doublet; they also produce the same kind of secondary spectrum.

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steps. First, the powers of the two elements of a cemented doublet are determined to meet the total power and longitudinal chromatic aberration targets. Second, the flint element of the cemented doublet is divided into two parts, whose total power is equal to the power of the original element, and one part is placed in front of and the other part behind the crown to make a cemented triplet (figure 1). The new triplet has the same K and C_L as the original doublet. Of course, the crown element could be divided in this way. Third, in performing Conrady's G sum analysis, the spherical aberration formula is a quadratic, while the central coma formula is linear. Hence there may be two solutions to the problem. Since the triplet is obtained from dividing a doublet, it produces the same kind of secondary spectrum.

In this paper, we propose an algorithm to solve the kind of cemented triplets which consist of three different glass types. In general, these triplets are more effective in reducing the secondary spectrum compared with triplets using only two glass types. From a mathematical point of view, the problem is to solve a set of nonlinear simultaneous equations including four equations with four variables. To solve the triplet, we combine the formulae for the three aberrations with the power formula into a fifth-order polynomial equation in one variable. Hence, there exists one solution and at most five solutions for each set of three chosen glasses. All the roots can be solved effectively by a combination of the Newton-Raphson method [6] and the algebraic method of solving quartic equations [7, 8]. The procedure is described in section 2.2.

2. Method

The notation used here is in most respects that of Welford [9]. The scheme of the Gaussian optics is shown in figure 2. K is the power of the lens which is the inverse of the focal length. The paraxial angles of the marginal and principal rays in the object and image spaces are denoted by u, $\bar{u}$, u', and $\bar{u}'$, respectively. A ray

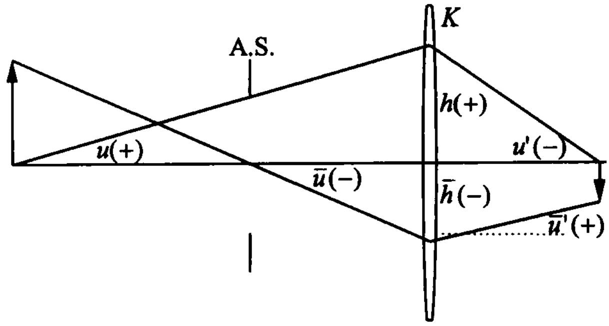

Figure 2. The parameters used to define the Gaussian optics of the thin lens. $u' = u - hK$ and $\bar{u}' = \bar{u} - \bar{h}K$. A.S. denotes the aperture stop.

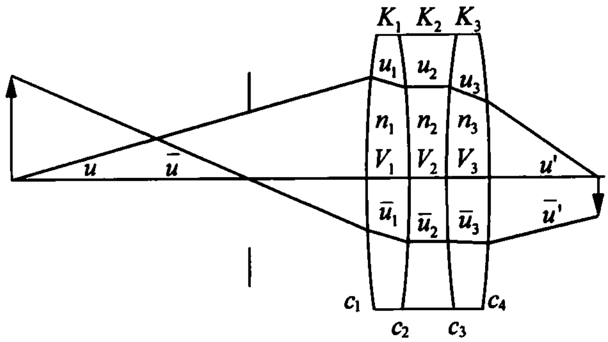

Figure 3. Scheme of the thin lens cemented triplet.

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angle is regarded as positive if a clockwise rotation of the ray brings it parallel to the optical axis. h and $\bar{h}$ are the heights of the marginal and principal rays at the lens, being positive if they are above the axis. The values of those parameters shown in figure 2 are given to define the Gaussian optics of the lens.

The scheme of the triplet is shown in figure 3. n and V denote the refractive index and Abbe number of the glass, respectively. c denotes the surface curvature. The formulae of the power and the aberrations are [9]

$$K_1 + K_2 + K_3 = K, \tag{1}$$

$$\begin{aligned} S_1 &= -\sum AAh\Delta(u/n) \\ &= -h[A_1^2\Delta(u/n)_1 + A_2^2\Delta(u/n)_2 + A_3^2\Delta(u/n)_3 + A_4^2\Delta(u/n)_4], \end{aligned} \tag{2}$$

$$\begin{aligned} S_{2C} &= H\sum A\Delta(u/n) \\ &= H[A_1\Delta(u/n)_1 + A_2\Delta(u/n)_2 + A_3\Delta(u/n)_3 + A_4\Delta(u/n)_4], \end{aligned} \tag{3}$$

$$C_L = h^2\left(\frac{K_1}{V_1} + \frac{K_2}{V_2} + \frac{K_3}{V_3}\right), \tag{4}$$

where $H = \bar{h}u - h\bar{u}$ is the optical invariant; $A_i = n_{i-1}(hc_i + u_{i-1}) = n_i(hc_i + u_i)$ is the refraction invariant of surface i; and $\Delta(u/n)$ denotes the difference of (u/n) on refraction.

2.1. *The normalized forms of the aberrations and the power formulae*

It is simpler to use the dimensionless normalized forms of the Seidel coefficients and some parameters instead of the conventional forms [2, 10]. The symbol '$\sim$' over a parameter denotes the normalized form of that parameter. The normalized parameter is equal to the scaling factor times the original parameter. For example,

$$\tilde{S}_1 = \frac{1}{h^4 K^3} S_1, \tag{5}$$

where $1/h^4K^3$ is the scaling factor. Table 1 shows those normalized parameters. By using these normalized parameters, equations (1)–(4) can be rewritten as

$$\tilde{K}_1 + \tilde{K}_2 + \tilde{K}_3 = 1, \tag{6}$$

$$\tilde{S}_1 = -[\tilde{A}_1^2\Delta(\tilde{u}/n)_1 + \tilde{A}_2^2\Delta(\tilde{u}/n)_2 + \tilde{A}_3^2\Delta(\tilde{u}/n)_3 + \tilde{A}_4^2\Delta(\tilde{u}/n)_4], \tag{7}$$

$$\tilde{S}_{2c} = \tilde{A}_1\Delta(\tilde{u}/n)_1 + \tilde{A}_2\Delta(\tilde{u}/n)_2 + \tilde{A}_3\Delta(\tilde{u}/n)_3 + \tilde{A}_4\Delta(\tilde{u}/n)_4, \tag{8}$$

$$C_L = \frac{\tilde{K}_1}{V_1} + \frac{\tilde{K}_2}{V_2} + \frac{\tilde{K}_3}{V_3}. \tag{9}$$

These dimensionless normalized aberrations are independent of the aperture, field, and power of the triplet. We choose $\tilde{K}_{1,2,3}$ and $\tilde{A}_2$ as the four independent variables of the above equations. It is then necessary to express $\tilde{A}_{1,3,4}$ and $\Delta(\tilde{u}/n)_{1\text{-}4}$

Table 1. The normalized parameters used for the cemented triplet.

$\tilde{H} = \frac{1}{h^2K} H$
$(\tilde{c}_{1,2,3,4}, \tilde{K}_{1,2,3}) = \frac{1}{K}(c_{1,2,3,4}, K_{1,2,3})$
$(\tilde{u}, \tilde{u}_{1,2,3}, \tilde{u}') = \frac{1}{hK}(u, u_{1,2,3}, u')$ $\tilde{u}' \equiv \tilde{u} - 1$
$\tilde{A}_{1,2,3,4} = \frac{1}{hK} A_{1,2,3,4}$
$\Delta(\tilde{u}/n)_{1,2,3,4} = \frac{1}{hK}\Delta(u/n)_{1,2,3,4}$
$\tilde{S}_1 = \frac{1}{h^4K^3} S_1$
$(\tilde{S}_2, \tilde{S}_{2c}) = \frac{1}{Hh^2K^2}(S_2, S_{2c})$
$\tilde{C}_L = \frac{1}{h^2K} C_L$

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as functions of $\tilde{K}_{1,2,3}$ and $\tilde{A}_2$. The normalized form of A_i is

$$\tilde{A}_i = n_{i-1}(\tilde{c}_i + \tilde{u}_{i-1}) = n_i(\tilde{c}_i + \tilde{u}_i). \tag{10}$$

Hence, it is easy to get the following relationships

$$\begin{aligned} \tilde{A}_1 &= \tilde{A}_2 + n_1(\tilde{c}_1 - \tilde{c}_2) = \tilde{A}_2 + \frac{n_1\tilde{K}_1}{n_1 - 1}, \\ \tilde{A}_3 &= \tilde{A}_2 - \frac{n_2\tilde{K}_2}{n_2 - 1}, \\ \tilde{A}_4 &= \left(\tilde{A}_2 - \frac{n_2\tilde{K}_2}{n_2 - 1}\right) - \frac{n_3\tilde{K}_3}{n_3 - 1}. \end{aligned} \tag{11}$$

Using equation (10) for surface 4, we have $\tilde{A}_4 = n_3(\tilde{c}_4 + \tilde{u}_3) = \tilde{c}_4 + \tilde{u}'$, and then

$$\tilde{u}_3 = -\frac{n_3 - 1}{n_3}\tilde{A}_4 + \tilde{u}'. \tag{12}$$

$\Delta(\tilde{u}/n)_4$ can be expressed as

$$\Delta(\tilde{u}/n)_4 = \frac{n_3 - 1}{n_3}\left(\frac{\tilde{A}_4}{n_3} + \tilde{u}'\right). \tag{13}$$

Similar expressions are used for surfaces 1 and 3:

$$\begin{aligned} \Delta(\tilde{u}/n)_1 &= -\frac{n_1 - 1}{n_1}\left(\frac{\tilde{A}_1}{n_1} + \tilde{u}\right), \\ \Delta(\tilde{u}/n)_3 &= \left(\frac{1}{n_3} - \frac{1}{n_2}\right)\left(\frac{\tilde{A}_3}{n_2} + \tilde{u}_3\right). \end{aligned} \tag{14}$$

Since

$$\Delta(u/n)_1 + \Delta(u/n)_2 + \Delta(u/n)_3 + \Delta(u/n)_4 = -hK, \tag{15}$$

then

$$\Delta(\tilde{u}/n)_2 = -1 - \Delta(\tilde{u}/n)_1 - \Delta(\tilde{u}/n)_3 - \Delta(\tilde{u}/n)_4. \tag{16}$$

Thus, $\tilde{A}_{1,2,3}$ and $\Delta(\tilde{u}/n)_{1\text{-}4}$ are all expressed as functions of $\tilde{K}_{1,2,3}$ and $\tilde{A}_2$.

2.2. *Combining the four equations into one fifth-order polynomial equation in $\tilde{K}_2$*

From equations (6) and (9), $\tilde{K}_1$ and $\tilde{K}_3$ can be expressed as functions of $\tilde{K}_2$:

$$\begin{aligned} \tilde{K}_1 &= q_1\tilde{K}_2 + q_2, \\ \tilde{K}_3 &= q_3\tilde{K}_2 + q_4, \end{aligned} \tag{17}$$

where

$$\begin{aligned} q_1 &= \frac{V_1}{V_3 - V_1}\frac{V_2 - V_3}{V_2}, \\ q_2 &= \frac{V_1}{V_3 - V_1}(\tilde{C}_\mathrm{L}V_3 - 1), \\ q_3 &= -1 - q_1, \\ q_4 &= 1 - q_2. \end{aligned} \tag{18}$$

Substituting equations (11), (12), (14), (16) and (17) into equations (7) and (8), after some reduction, we can get the following two equations in $\tilde{A}_2$ and $\tilde{K}_2$:

$$\begin{aligned} t_1\tilde{K}_2^2+t_2\tilde{A}_2\tilde{K}_2+t_3\tilde{A}_2+t_4\tilde{K}_2+t_5 &= 0, \\ -2t_2\tilde{A}_2^2\tilde{K}_2+t_6\tilde{A}_2\tilde{K}_2^2+t_7\tilde{K}_2^3+t_8\tilde{A}_2^2+t_9\tilde{A}_2\tilde{K}_2+t_{10}\tilde{K}_2^2+t_{11}\tilde{A}_2+t_{12}\tilde{K}_2+t_{13} &= 0, \end{aligned} \tag{20}$$

where the coefficients t_1 through t_{13} are as follows:

$$\begin{aligned} t_1 &= -G_3(n_1)q_1^2+[G_3(n_3)q_3+G_4(n_2)G_1(n_3)-1]q_3+G_3(n_2), \\ t_2 &= -\left(\frac{1}{n_1}q_1+\frac{1}{n_3}q_3+\frac{1}{n_2}\right), \\ t_3 &= -\left(\frac{1}{n_1}q_2+\frac{1}{n_3}q_4+1\right), \\ t_4 &= -[2G_3(n_1)q_2+1]q_1+\left[\frac{G_4(n_2)}{n_3}+G_3(n_2)+2G_3(n_3)q_3\right]q_4, \\ t_5 &= -[G_3(n_1)q_2+1]q_2+G_3(n_3)q_4^2-\tilde{u}'-\tilde{S}_{2c}, \\ t_6 &= 3[G_3(n_1)q_1^2-G_3(n_3)q_3^2-G_3(n_2)]+2q_3[1-G_4(n_2)G_2(n_3)], \\ t_7 &= \{G_5(n_3)q_3^2+G_4(n_2)[3G_3(n_3)q_3-1]+G_6(n_2)G_2(n_3)\}q_3+G_5(n_1)q_1^3+G_5(n_2), \\ t_8 &= 2\left(\frac{q_2}{n_1}+\frac{q_4}{n_3}\right)+1, \\ t_9 &= 2\{[3G_3(n_1)q_2+1]q_1+[-3G_3(n_3)q_3-G_4(n_2)G_2(n_3)+1]q_4\}, \\ t_{10} &= [3G_5(n_1)q_2+G_4(n_1)\tilde{u}]q_1^2-G_4(n_2)[q_4+2q_3\tilde{u}'+\tilde{u}'] \\ &\quad +[3G_5(n_3)q_4-G_4(n_3)\tilde{u}']q_3^2+[G_6(n_2)G_2(n_3)+6G_4(n_2)G_3(n_3)q_3]q_4, \\ t_{11} &= 3[G_3(n_1)q_2^2-G_3(n_3)q_4^2]+2(q_2\tilde{u}+q_4\tilde{u}'), \\ t_{12} &= [3G_5(n_1)q_2+2G_4(n_1)\tilde{u}]q_1q_2-2G_4(n_2)q_4\tilde{u}' \\ &\quad +G_4(n_3)[3G_3(n_3)q_4-2\tilde{u}']q_3q_4+3G_4(n_2)G_3(n_3)q_4^2, \\ t_{13} &= [G_5(n_1)q_2+G_4(n_1)\tilde{u}]q_2^2+[G_5(n_3)q_4-G_4(n_3)\tilde{u}']q_4^2-\tilde{S}_1, \end{aligned} \tag{21}$$

with functions $G_1(n)$ through $G_6(n)$ defined as

$$\begin{aligned} G_1(n) &= \frac{n+1}{n}, \quad G_2(n) = \frac{n+2}{n}, \quad G_3(n) = \frac{1}{n-1}, \\ G_4(n) &= \frac{n}{n-1}, \quad G_5(n) = \frac{n}{(n-1)^2}, \quad G_6(n) = \frac{n^2}{(n-1)^2}. \end{aligned} \tag{22}$$

From equation (19)

$$\tilde{A}_2 = \frac{-t_1\tilde{K}_2^2-t_4\tilde{K}_2-t_5}{t_2\tilde{K}_2+t_3}, \tag{23}$$

and substituting $\tilde{A}_2$ into equation (20), we get a fifth-order polynomial equation

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in $\tilde{K}_2$

$$w_5\tilde{K}_2^5 + w_4\tilde{K}_2^4 + w_3\tilde{K}_2^3 + w_2\tilde{K}_2^2 + w_1\tilde{K}_2 + w_0 = 0, \tag{24}$$

where the coefficients w_5 through w_0 are as follows

$$\begin{aligned}
w_5 &= t_2(-2t_1t_1 - t_1t_6 + t_2t_7),\\
w_4 &= t_1(t_1t_8 - 4t_2t_4 - t_2t_9 - t_3t_6) + t_2(t_2t_{10} + 2t_3t_7 - t_4t_6),\\
w_3 &= t_1(-4t_2t_5 - t_2t_{11} - t_3t_9 + 2t_4t_8)\\
&\quad + t_2(t_2t_{12} + 2t_3t_{10} - 2t_4t_4 - t_4t_9 - t_5t_6) + t_3(t_3t_7 - t_4t_6),\\
w_2 &= t_1(-t_3t_{11} + 2t_5t_8) + t_2(t_2t_{13} + 2t_3t_{12} - 4t_4t_5 - t_4t_{11} - t_5t_9)\\
&\quad + t_3(t_3t_{10} - t_4t_9 - t_5t_6) + t_4t_4t_8,\\
w_1 &= t_2(2t_3t_{13} - 2t_5t_5 - t_5t_{11}) + t_3(t_3t_{12} - t_4t_{11} - t_5t_9) + 2t_4t_5t_8,\\
w_0 &= t_3(t_3t_{13} - t_5t_{11}) + t_5t_5t_8.
\end{aligned} \tag{25}$$

Since all the coefficients are real, the polynomial has one, three, or five real roots. Abel showed in 1824 that a polynomial equation of degree higher than four cannot be solved by purely algebraic methods. On the other hand, the roots (real or complex) of a quartic equation can always be solved by purely algebraic methods [7, 8]. Let

$$\begin{aligned}
F(\tilde{K}_2) &= w_5\tilde{K}_2^5 + w_4\tilde{K}_2^4 + w_3\tilde{K}_2^3 + w_2\tilde{K}_2^2 + w_1\tilde{K}_2 + w_0,\\
F'(\tilde{K}_2) &= \frac{\mathrm{d}}{\mathrm{d}\tilde{K}_2}F(\tilde{K}_2) = 5w_5\tilde{K}_2^4 + 4w_4\tilde{K}_2^3 + 3w_3\tilde{K}_2^2 + 2w_2\tilde{K}_2 + w_1.
\end{aligned} \tag{26}$$

$F'(\tilde{K}_2)$ is an algebraic quartic equation whose roots can be solved by purely algebraic methods and these roots indicate the turning points of $F(\tilde{K}_2)$. These points are helpful for finding the roots of $F(\tilde{K}_2)$. For example, consider a function $F(\tilde{K}_2)$ with the curve as shown in figure 4. $P_{1\text{-}4}$ are the horizontal coordinates of the turning points. There is no root in the interval $[P_1, P_2]$ because $F(P_1)$ and $F(P_2)$ have the same sign. Since $F(P_3)$ and $F(P_4)$ are of opposite sign, one root exists between P_3 and P_4 and this root can quickly be found by the numerical Newton–Raphson method [6]. Since $F(P_4)$ is negative and the slope of the curve on the right-hand side of P_4 is positive, one root exists in this region. By the same

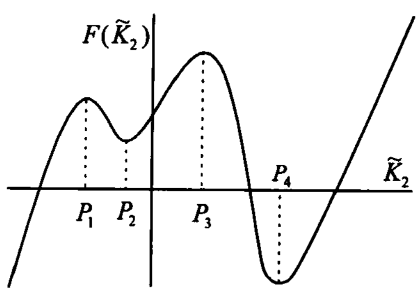

Figure 4. A fifth-order equation with four turning points and three roots.

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reasoning, one root is located on the left-hand side of P_1. These roots can also be solved by the Newton–Raphson method. In this way, all the roots can be found.

After the value of $\tilde{K}_2$ has been obtained, we can find $\tilde{A}_2$ by means of equation (23), $\tilde{K}_1$ and $\tilde{K}_3$ by means of equation (17), and $\tilde{A}_1$ by means of the first term in equation (11). Finally, the four normalized curvatures $\tilde{c}_{1\text{-}4}$ can be calculated sequentially from the following equations:

$$\begin{aligned} \tilde{A}_1 &= \tilde{c}_1 + \tilde{u}, \\ \tilde{K}_1 &= (n_1 - 1)(\tilde{c}_1 - \tilde{c}_2), \\ \tilde{K}_2 &= (n_2 - 1)(\tilde{c}_2 - \tilde{c}_3), \\ \tilde{K}_3 &= (n_3 - 1)(\tilde{c}_3 - \tilde{c}_4). \end{aligned} \tag{27}$$

3. Example

As an example to demonstrate the calculating process of the method in detail, we solve the thin-lens structures of the triplet which has the first order layout as shown in figure 5. The optical invariant $H = -h\bar{u} = -0{\cdot}1246$. Let F (0·000 486 1 mm), d (0·000 587 6 mm), and C (0·000 656 3 mm) lines be the short, middle, and long wavelengths over the band of interest. The given target values of S_1, $S_{2\mathrm{C}}$, and C_L represented in wavefront, Seidel and normalized Seidel forms are listed in table 2. Table 3 shows the explicit values of the refractive indices, Abbe numbers and G_is of the three glasses (KF9, KZFS4, FK52) which have been chosen from the Schott catalogue. The values of the q_i and t_i coefficients are listed in table 4. Substituting these values into equation (25), we have the following fifth-order polynomial equation in $\tilde{K}_2$:

$$0{\cdot}002\,526\,86\,\tilde{K}_2^5 - 0{\cdot}309\,78\,\tilde{K}_2^4 + 2{\cdot}953\,09\,\tilde{K}_2^3 + 17{\cdot}7303\,\tilde{K}_2^2 + 17{\cdot}4582\,\tilde{K}_2 - 7{\cdot}123\,99 = 0 \tag{28}$$

The equation has five real roots which are −2·703 61, −1·973 05, 0·307 355, 111·553 and 15·4154. Table 5 lists the normalized solutions and the corresponding un-normalized solutions of the system. The surface radii of solutions 4 and 5 are too small to satisfy the aperture requirement and thus these solutions should be neglected. As a general rule, weak lens surfaces are preferred to strong surfaces because weak surfaces induce less high order aberrations and are cheaper to make. Hence, solution 3 is not a good one because it has two strong inner surfaces. The thin lens design is now complete.

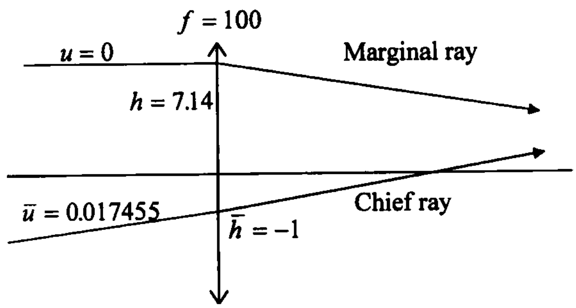

Figure 5. The first order layout of the lens in the example.

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Table 2. The aberration target values presented in three forms (*a*). The corresponding values of coma are shown in (*b*).

Aberration name	Wavefront[a] (in units of λ_d)	Seidel coefficient	Normalized Seidel coefficient
Spherical	$\frac{S_1}{8\lambda_d} = 1$	$S_1 = 0{\cdot}004\,700\,8$	$\tilde{S}_1 = \frac{S_1}{h^4K^3} = 1{\cdot}808\,75$
Central coma	$\frac{S_{2C}}{2\lambda_d} = -0{\cdot}2$	$S_{2C} = -0{\cdot}000\,235\,04$	$\tilde{S}_{2C} = \frac{S_{2C}}{Hh^2K^2} = 0{\cdot}369\,937$
Longitudinal chromatic	$\frac{C_L}{2\lambda_d} = -0{\cdot}25$	$C_L = -0{\cdot}000\,293\,8$	$\tilde{C}_L = \frac{C_L}{h^2K} = -0{\cdot}000\,576\,309$

[a] $\lambda_d = 0{\cdot}000\,587\,6$ nm (*a*)

Aberration name	Wavefront	Seidel coefficient	Normalized Seidel coefficient
Coma[b]	$\frac{S_2}{2\lambda_d} = -0{\cdot}7602$	$S_2 = -0{\cdot}000\,893\,42$	$\tilde{S}_2 = \frac{S_2}{Hh^2K^2} = 1{\cdot}406\,17$

[b] $S_2 = S_{2c} + \frac{\bar{h}}{h} S_1$ (*b*)

Table 3. The glass data.

	Glass 1	Glass 2	Glass 3
Glass name	KF9	KZFS4	FK52
refractive index	$n_1 = 1{\cdot}523\,41$	$n_2 = 1{\cdot}6134$	$n_3 = 1{\cdot}486\,05$
Abbe number V	$V_1 = 51{\cdot}4676$	$V_2 = 44{\cdot}2773$	$V_3 = 81{\cdot}7818$
$G_1(n)$–$G_6(n)$ (See equation (22))	$G_1(n_1) = 1{\cdot}656\,42$	$G_1(n_2) = 1{\cdot}619\,81$	$G_1(n_3) = 1{\cdot}672\,92$
	$G_2(n_1) = 2{\cdot}312\,84$	$G_2(n_2) = 2{\cdot}239\,62$	$G_2(n_3) = 2{\cdot}345\,85$
	$G_3(n_1) = 1{\cdot}910\,55$	$G_3(n_2) = 1{\cdot}630\,26$	$G_3(n_3) = 2{\cdot}057\,40$
	$G_4(n_1) = 2{\cdot}910\,55$	$G_4(n_2) = 2{\cdot}630\,26$	$G_4(n_3) = 3{\cdot}057\,40$
	$G_5(n_1) = 5{\cdot}560\,74$	$G_5(n_2) = 4{\cdot}288\,00$	$G_5(n_3) = 6{\cdot}290\,30$
	$G_6(n_1) = 8{\cdot}471\,29$	$G_6(n_2) = 6{\cdot}918\,25$	$G_6(n_3) = 9{\cdot}347\,70$

4. Conclusion

We have proposed a general algorithm as a computational tool for designing the cemented triplets which consist of three different glasses with given aberration targets of S_1, S_{2c}, and C_L. Combination of the formulae for lens power and aberrations leads to a 5th order polynomial equation whose roots can all be solved by the method described in section 2.2.

Damped least squares (DLS) is an optimization method which is widely used in optical design and can also be used to solve the triplets. It is a method for solving simultaneous equations with the restriction that the initial guesses of the variables are close to the real roots. A single number called the merit function, which is the sum of weighted squares of the deviations from targets, is used in the DLS

Table 4. The values of q_1–q_4 (*a*) and t_1–t_{13} (*b*).

Coefficients q_1 through q_4 (See equation (18))	
$q_1 = -1\cdot4381$	$q_3 = 0\cdot438\,103$
$q_2 = -1\cdot777\,83$	$q_4 = 2\cdot777\,83$

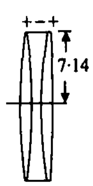
(*a*)

Coefficients t_1 through t_{13} (See equation (21))	
$t_1 = -0\cdot436\,49$	$t_8 = 2\cdot404\,53$
$t_2 = 0\cdot029\,383\,1$	$t_9 = -17\cdot3146$
$t_3 = -1\cdot702\,26$	$t_{10} = 31\cdot5352$
$t_4 = 6\cdot121\,56$	$t_{11} = -35\cdot0665$
$t_5 = 12\cdot2448$	$t_{12} = 135\cdot292$
$t_6 = 1\cdot248\,26$	$t_{13} = 125\cdot367$
$t_7 = -2\cdot648\,14$	

(*b*)

Table 5. Solutions of the triplet. f_{1-3} and r_{1-4} are the un-normalized focal lengths and surface radii, respectively. The surface radii of solutions 4 and 5 are too small to satisfy the aperture requirement.

	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5	Note
$\tilde{K}_1$	2·1103	1·0596	−2·2198	−162·197	−23·946	See equation (17)
$\tilde{K}_2$	−2·7036	−1·9731	0·3074	111·549	15·415	
$\tilde{K}_3$	1·5934	1·9134	2·9125	51·648	9·531	See equation (17)
$\tilde{A}_2$	−4·2073	−0·8706	8·3185	3006·38	2·312	See equation (23)
$\tilde{A}_1$	1·9348	2·2135	1·8575	2534·30	−67·384	See equation (11)
$\tilde{c}_1$	1·9348	2·2135	1·8575	2534·30	−67·384	See equation (27)
$\tilde{c}_2$	−2·0970	0·1890	6·0987	2844·19	−21·634	
$\tilde{c}_3$	2·3106	3·4056	5·5976	2662·33	−46·764	
$\tilde{c}_4$	−0·9676	−0·5311	−0·3946	2556·07	−66·374	

	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5
f_1	47·388	94·374	−45·048	−0·6165	−4·1760
f_2	−36·987	−50·683	325·349	0·8965	6·4872
f_3	62·761	52·262	34·335	1·9362	10·4919
r_1	51·686	45·178	53·835	0·03946	−1·4840
r_2	−47·687	529·052	16·397	0·03516	−4·6224
r_3	43·279	29·364	17·865	0·03756	−2·1384
r_4	−103·351	−188·286	−253·438	0·03912	−1·5066
				unreasonable solution	unreasonable solution

method. DLS attempts to reduce the merit function to zero by an iterative procedure in which the aberrations and lens power are calculated at each step. Some problems are usually encountered while using the DLS method. If there is no solution or if the initial guesses of surface curvatures are not close to the real roots, the solving process will lead to a local minimum of the merit function. If the merit function has more than one zero, different initial guesses are necessary to find different solutions. How to make the initial guesses is usually a problem with the DLS method. DLS is also usually used to find the glass material but the result is often unsatisfactory because of the discontinuous distribution of the glass material map.

As compared with the DLS method, the proposed algorithm has the following advantages: (1) it avoids the need and difficulty of making an initial guess, (2) all the solutions can be found, (3) the procedure is more effective in treating more interesting glasses in the same time.

We have written a computer program and set constraints on it to restrict the acceptable surface curvatures so that those solutions with harder surfaces are not adopted. The algorithm is of particular use in setting up a preliminary system for subsequent optimization.

Algorithms for solving the airspaced triplets which consist of one singlet and one cemented doublet will give freedom to find more desirable solutions. This will be considered in the future.

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