Pergamon

Int. J. Heat Mass Transfer. Vol. 40, No. 6, pp. 1401–1410, 1997

0017-9310/97 $17.00+0.00

PII: S0017-9310(96)00194-9

Stability of viscoelastic fluids in a modulated gravitational field

WEN-MEI YANG

Department of Mechanical Engineering, National Chiao Tung University, Hsinchu, Taiwan 30010, Republic of China

(*Received* 22 *February* 1996 *and in final form* 17 *May* 1996)

Abstract—The instability of a viscoelastic fluid layer heated from below in a modulated gravitational field is studied numerically. Fluids satisfying the Maxwellian model and the Boussinesq approximation are considered. A system of linear equations with periodic coefficients describing the behavior of disturbances, is obtained by linear stability theory. The disturbances are expanded by double series of mixed Fourier and Chebyshev form. An algorithm combining Galerkin and collocation methods is employed to trace the stability boundary between stable and unstable states. For the case of viscoelastic fluids acted on by a constant gravity, a transition Deborah number is found for each Prandtl number. Below and above this transition value stationary and oscillatory convections, respectively, will develop at the onset of instability. For the case of Newtonian fluids acted on by a modulated gravity, modulation has a destabilization effect at low frequencies and a slight stabilization effect at high frequencies, which increases with increasing the amplitude of modulation. The critical Rayleigh number approaches the quasi-steady limit as the frequency tends to zero. For the case of a viscoelastic fluid acted on by a modulated gravity, modulation has the same effects at both very low and very high frequencies, as those of Newtonian fluids. While in the range of intermediate frequency, subharmonic disturbances are found to enhance the stabilization effect at small Deborah numbers and the destabilization effect at large Deborah numbers. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

Viscoelastic fluids are noted because of their frequent appearance in manufacturing processes, such as crystal growth, injection molding, transport of chemical substances and in the petroleum industry. Some investigators treated viscoelastic fluids as Newtonian fluids and accordingly, ignored the elastic behavior of fluids which may be important under some circumstances. The stability of viscoelastic fluids was noted but limited in shear and extensional flows [1]. Among thermally-induced instability of viscoelastic fluids, Herbert [2] considered the stability of viscoelastic liquids in heated plane Couette flow and found that the presence of elasticity has a destabilizing effect on the flow. Green III [3] found an oscillating convective motion was possible at the onset of instability. Vest and Arpaci [4] found that overstability will occur at the lowest possible adverse temperature gradient at which the rate of change of kinetic energy can balance, in a synchronous manner, the periodically varying rates of energy dissipation by the shear stresses and energy release by the buoyancy force, assuming that stationary convection has not been initiated. Later, Hamabata [5] further considered the effect of internal heat generation on the overstability of a viscoelastic liquid layer.

The effect of modulation on the dynamical system has long been of interest because stabilization or destabilization may occur in the presence of modulation which thus enhances the mass, momentum and heat transport [6]. The effects of temperature modulation on the thermal instability were studied by: Venzian [7], Rosenblat and Herbert [8], Yih and Li [9] and Finucane and Kelly [10] and of gravity modulation by Gresho and Sani [11]. Recently, Yang [12] further studied the effect of modulation on radiation-induced instability of a fluid layer and confirmed that destabilization and stabilization, respectively, occur at low and high modulation frequencies. Meanwhile, the quasi-steady limit is valid as the frequency tends to zero.

In the present study the effect of gravity modulation on the thermal instability of viscoelastic fluids is investigated. The limiting cases, viscoelastic fluids in a constant gravitational field and Newtonian fluids in a modulated gravitational field, are examined and compared with the available data. Finally the viscoelastic fluids in a modulated gravitational field are considered.

FORMULATION

We consider a viscoelastic fluid layer confined between two infinite plates of distance L apart that are oscillating vertically. If the coordinate system, is attached to the lower plates, then a modulated gravitational field which consists of a constant part plus a sinusoidally varying part can be considered to be acting on the system. Let X_1 and X_2 be the Cartesian

NOMENCLATURE

A, B	coefficient matrices	Θ	magnitude of the disturbance of temperature
G	elastic modulus	θ	temperature
g	gravity	λ	relaxation time
k	wavenumber	κ	thermal diffusivity
L	thickness of a fluid layer	μ	dynamic viscosity
M, N	resolutions of double series in time and spatial coordinates	ν	kinematic viscosity
P	dimensional pressure	ρ	density
p	pressure	σ	growth rate of disturbances
Pr	Prandtl number	τ	dimensional time
Ra	Rayleigh number	τ_{ij}	stresses
T	dimensional temperature	Φ	magnitude of the disturbance of ϕ
T_{ij}	dimensional stresses	Φ_n	trial functions for velocity
T_n	nth degree Chebyshev polynomial	ϕ	$\partial v_3/\partial t$
t	time	Ψ_n	trial functions for temperature
V_i	dimensional velocities	Ω	dimensional frequency of gravity modulation
v_i	velocities	ω	frequency of gravity modulation.
W	magnitude of the disturbance of velocity		
X_i	dimensional coordinates	Superscripts	
x_i	coordinates.	$\bar{}$	basic state
		$'$	disturbances.
Greek symbols			
β	thermal expansion coefficient	Subscripts	
Δ	$(Ra_c - Ra_0)/Ra_0$	c	critical value
Γ	Deborah number	0	critical value of unmodulated case
ε	amplitude of gravity modulation	1	upper plate
ζ	coordinate for Chebyshev polynomials	2	lower plate.

coordinates parallel to the plates and X_3 perpendicular to the plates. Suppose the bottom plate is at temperature T_2 which is higher than the temperature of the upper plate T_1.

For an incompressible fluid satisfying the Boussinesq approximation, if the energy dissipation is negligible, the equation governing the motion of the fluid can be written as follows:

$$\frac{\partial V_j}{\partial X_j} = 0 \tag{1}$$

$$\frac{\partial V_i}{\partial \tau} + V_j \frac{\partial V_i}{\partial X_j} = g\beta(1+\varepsilon\cos\Omega\tau)(T-T_0)e_i - \frac{1}{\rho}\frac{\partial P}{\partial X_i} + \frac{\partial T_{ij}}{\partial X_j} \tag{2}$$

$$\frac{\partial T}{\partial \tau} + V_j \frac{\partial T}{\partial X_j} = \kappa \frac{\partial^2 T}{\partial X_j \partial X_j} \tag{3}$$

where T_0 is a reference temperature, P the pressure relative to its hydrostatic value, T_{ij} the deviatoric stress tensor, β the coefficient of thermal expansion, ρ the density at the reference temperature, κ the thermal diffusivity, ε and Ω the amplitude and angular frequency of gravity modulation, and $e_i = (0, 0, 1)$.

There are many different models proposed for viscoelastic fluids [13, 14]. The first attempt to obtain a viscoelastic constitutive equation, over a century ago, appears to have been that of Maxwell. He proposed that fluids with both viscosity and elasticity could be described by:

$$T_{ij} + \lambda \frac{\partial}{\partial \tau} T_{ij} = \mu\left(\frac{\partial V_i}{\partial X_j} + \frac{\partial V_j}{\partial X_i}\right) \tag{4}$$

where $\lambda = \mu/G$ is called the relaxation time, μ is the dynamic viscosity and G is the elastic modulus. When $G \to \infty$ or $\lambda = 0$, it becomes a Newtonian fluid. Although the Maxwellian model is empirical and its range of validity is somewhat limited, because of simplicity, it is employed in this study to explore the effect of elastic modulus in the onset of convection.

Introducing the following dimensionless variables

$$x_i = X_i/L \qquad t = \tau\kappa/L^2 \qquad v_i = V_i L/\kappa$$

$$\theta = T/(T_2 - T_1) \qquad p = P\mu\kappa/L^2 \qquad \tau_{ij} = T_{ij}\mu\kappa/L^2$$

equations (1)–(4) become

$$\frac{\partial v_j}{\partial x_j} = 0 \tag{5}$$

$$\frac{1}{Pr}\left(\frac{\partial v_i}{\partial t}+v_j\frac{\partial v_i}{\partial x_j}\right) = Ra(1+\varepsilon\cos\omega t)(\theta-\theta_0)e_i-\frac{\partial p}{\partial x_i}+\frac{\partial \tau_{ij}}{\partial x_j} \quad (6)$$

$$\frac{\partial \theta}{\partial t}+v_j\frac{\partial \theta}{\partial x_j}=\frac{\partial^2\theta}{\partial x_j\partial x_j} \quad (7)$$

$$\tau_{ij}+\Gamma\frac{\partial}{\partial t}\tau_{ij}=\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i} \quad (8)$$

where $Pr = \nu/\kappa$ is the Prandtl number, $Ra = g\beta(T_2-T_1)L^3/\nu\kappa$ is the Rayleigh number, $\omega = \Omega L^2/\kappa$ is the dimensionless angular frequency of modulation, $\Gamma = \lambda\kappa/L^2$ the ratio of stress relaxation tome to the characteristic process time, is called the Deborah number.

When the temperature difference is small, the fluid is in a stationary state. This basic state is described by

$$\bar{v}_i = 0, \quad \bar{\tau}_{ij} = 0 \quad \bar{\theta} = \theta_2 - x_3,$$

$$\bar{p} = Ra(1+\varepsilon\cos\omega t)(\theta_2 x_3 - x_3^2/2)+p_2$$

where p_2 is the pressure on the bottom plate.

Adding small disturbances on the basic state

$$\tau'_{ij} \quad p' \quad v'_1 \quad \text{and} \quad v'_2$$

then substituting into equations (5)–(8) and neglecting the nonlinear terms we can obtain the linear disturbance equations. Successively eliminating τ'_{ij}, p', v'_1 and v'_2 then the disturbance equations can be reduced to

$$\frac{1}{Pr}\left(1+\Gamma\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}(\nabla^2 v'_3) = Ra\left\{(\nabla_1^2\theta')(1+\varepsilon\cos\omega t)+\Gamma\left[\frac{\partial(\nabla_1^2\theta')}{\partial t}(1+\varepsilon\cos\omega t) - (\nabla_1^2\theta')\varepsilon\omega\sin\omega t\right]\right\}+\nabla^2(\nabla^2 v'_3) \quad (9)$$

$$\frac{\partial \theta'}{\partial t}=\frac{\partial^2\theta'}{\partial x_j\partial x_j}+v'_3 \quad (10)$$

where $\nabla_1^2 = \partial^2/\partial x_1^2+\partial^2/\partial x_2^2$, and the boundary conditions to

$$v'_3=\frac{\partial v'_3}{\partial x_3}=\theta'=0 \quad \text{at } x_3=0 \quad \text{and} \quad 1. \quad (11)$$

Equation (9) is a second-order time-derivative differential equation. To avoid the appearance of a nonlinear algebraic eigensystem we may let $\phi' = \partial v'_3/\partial t$, then the second-order equation (9) can be converted to two first-order equations. The disturbances in the form of normal modes can be expressed as

$$\begin{bmatrix} v'_3 \\ \phi' \\ \theta' \end{bmatrix} = \begin{bmatrix} W(x_3,t) \\ \Phi(x_3,t) \\ \Theta(x_3,t) \end{bmatrix} e^{i(k_1x_1+k_2x_2)+\sigma t} \quad (12)$$

where: W, Φ and Θ are, according to the Floquet theory [15, 16], periodic functions of time with the same period as the gravity modulation, k_1 and k_2 are the wavenumbers of the disturbance in the x_1 and x_2 directions, respectively, and $\sigma = \sigma_r + i\sigma_i$ is the growth rate of the disturbances. Let σ_1 be the eigenvalue with the greatest real part. The basic state, with respect to the infinitesimal disturbances, is unstable if σ_{r1} is greater than zero or stable if σ_{r1} is less than zero. Here, unstable means that a disturbance experiences net growth over each modulation cycle, or grows during part of the cycle, but ultimately decays; while stable means that every disturbance decays at every instant. At the neutral stable state σ_{r1} is equal to zero. If σ_{i1} is simultaneously equal to zero, the disturbance is synchronous with the periodic basic state. If $\sigma_{i1}/\omega = m/n$ is rationally dependent and irreducible, where m and n are two integers, harmonic motion of period $2\pi/\omega$ will develop for $n = 1$, or subharmonic motion of period $2n\pi/\omega$ will develop for $n > 1$. If σ_{i1} and ω are two incommensurate frequencies, i.e. rationally independent, a quasi-periodic motion will develop.

Substituting the normal modes into the disturbance equations, we obtain

$$-\frac{\partial W}{\partial t}+\Phi=\sigma W \quad (13)$$

$$Pr(D^2-k^2)^2W-\left(1+\Gamma\frac{\partial}{\partial t}\right)(D^2-k^2)\Phi - Pr\,Ra\,k^2\left[(1+\varepsilon\cos\omega t-\Gamma\varepsilon\omega\sin\omega t)\Theta + \Gamma(1+\varepsilon\cos\omega t)\frac{\partial\Theta}{\partial t}\right]$$
$$= \sigma[\Gamma(D^2-k^2)\Phi + Pr\,Ra\,\Gamma k^2(1+\varepsilon\cos\omega t)\Theta] \quad (14)$$

$$-\frac{\partial\Theta}{\partial t}+W+(D^2-k^2)\Theta=\sigma\Theta \quad (15)$$

where $D = \mathrm{d}/\mathrm{d}x_3$ and $k^2 = k_1^2+k_2^2$. The associated boundary conditions are

$$W = DW = \Phi = D\Phi = \Theta = 0 \quad \text{at } x_3 = 0 \quad \text{and} \quad 1. \quad (16)$$

Equations (13)–(15) with the boundary conditions (16), are all homogeneous and thus constitute an eigenvalue problem. For the existence of nontrivial solutions, the eigenvalues σ are dependent on the parameters as

$$\sigma = f(Ra, Pr, \Gamma, \varepsilon, \omega, k). \quad (17)$$

The neutral state at which the real part of the most

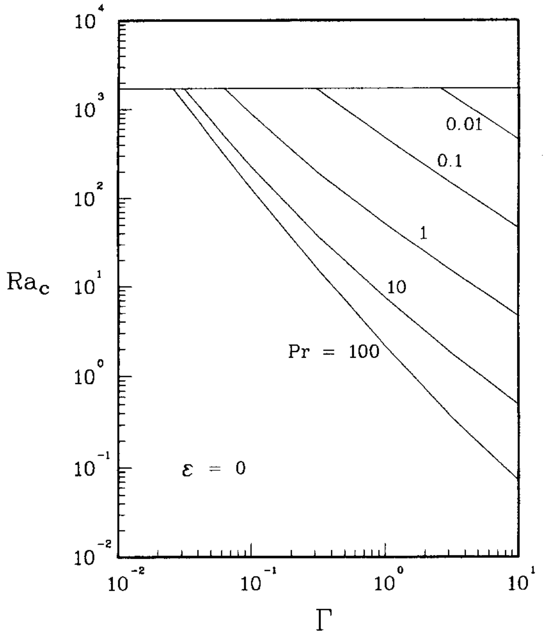

Fig. 1. The critical Rayleigh number vs Deborah number at different Prandtl numbers for the unmodulated case.

unstable eigenvalue vanishes can be found by the method of linear interpolation. The critical Rayleigh number, denoted by Ra_c, occurs at a neutral state which has a minimum Rayleigh number at the corresponding critical wavenumber k_c.

NUMERICAL METHOD

We will expand the unknowns in equations (13)–(15) into double series, i.e. Fourier series with respect to time and Chebyshev polynomials with respect to space. Because the nth-degree Chebyshev polynomial of the first kind is defined by

$$T_n(z) = \cos(n\cos^{-1}\zeta)$$

in the interval $\zeta \in [-1, 1]$, to fit the domain of definition of Chebyshev polynomials, the domain of the present problem is transformed from $0 \leqslant x_3 \leqslant 1$ to $-1 \leqslant \zeta \leqslant 1$ by $\zeta = 2x_3 - 1$.

Now

$$W = \sum_{m=-\infty}^{\infty}\sum_{n=4}^{\infty} a_{mn}\Phi_n e^{im\omega t} \tag{18}$$

$$\Phi = \sum_{m=-\infty}^{\infty}\sum_{n=4}^{\infty} b_{mn}\Phi_n e^{im\omega t} \tag{19}$$

$$\Theta = \sum_{m=-\infty}^{\infty}\sum_{n=2}^{\infty} c_{mn}\Psi_n e^{im\omega t} \tag{20}$$

where a_{mn}, b_{mn} and c_{mn} are unknown coefficients, and Φ_n and Ψ_n are the trial functions defined by

$$\Phi_n = \begin{cases} T_n - \dfrac{n^2}{4}T_2 + \left(\dfrac{n^2}{4} - 1\right)T_0 & \text{even } n \\ T_n - \dfrac{n^2-1}{8}T_3 + \left(\dfrac{n^2-1}{4} - 1\right)T_1 & \text{odd } n \end{cases} \tag{21}$$

$$\Psi_n = \begin{cases} T_n - T_0 & \text{even } n \\ T_n - T_1 & \text{odd } n \end{cases} \tag{22}$$

which satisfy the boundary conditions (16).

In view of the structure of equations (13)–(15), the even mode and odd mode disturbances are separable. Thus we expand the unknowns each time by either even or odd trial functions which may reduce a large amount of computation time. By taking the lowest $M \times N$ terms of the expansions and substituting them into equations (13)–(15), we may obtain three equations of functions of t and x_3. To eliminate t and x_3, we multiply each of the equations by $e^{-im\omega t}$, $m = -M, \ldots, 0, \ldots, M$, and integrate over one period, then substitute the collocation points

$$\zeta_n = \cos[n\pi/(N+1)] \quad n = 1, 2, \ldots, N$$

into the equations. An algebraic eigensystem is obtained:

$$AX = \sigma BX \tag{23}$$

where A and B are two $3(M \times N) \times 3(M \times N)$ coefficient matrices. X is a $3(M \times N)$ vector composed of the unknown coefficients. The eigenvalues of the generalized eigensystem, equation (23) can be solved directly by the QZ algorithm and the procedures are referred to in Yang, ref. [12].

RESULTS AND DISCUSSION

We will present the results for some special cases and compare them with the available data, then we will examine the effect of gravity modulation on the instability of viscoelastic fluids.

(i) Overstability of viscoelastic fluids

When the gravity modulation is not considered, $\varepsilon = 0$, the viscoelastic fluid layer is subjected to a constant gravity and heated from below. Here we set $M = 1$ and include both even and odd mode disturbances to calculate the critical values by increasing N from one to six. For $Pr = 0.1$, $\Gamma = 1$, and $Pr = 1000$, $\Gamma = 0.1$ and 1, the results obtained with a wavenumber accuracy of 0.01 are listed in Table 1. It is seen that the results show a good convergence for $N > 4$.

Vest and Arpaci [4] found that, at the onset of instability, stationary convection occurs for smaller Γ and overstability for larger Γ. The comparison of the critical Rayleigh number, wavenumber and the corresponding σ_i for $Pr = 0.1$–1000 at $\Gamma = 0.1$ and 1 obtained by $N = 6$, with the results of Vest and Arpaci

Table 1. Critical values obtained by different N

N	$Pr = 0.1, \Gamma = 1$ k_c	Ra_c	σ_{ic}	$Pr = 1000, \Gamma = 0.1$ k_c	Ra_c	σ_{ic}	$Pr = 1000, \Gamma = 1$ k_c	Ra_c	σ_{ic}
1	3.48	493.78	1.451	22.32	110.22	2245.57	12.74	1.343	421.87
2	3.06	374.83	2.052	16.67	105.15	1674.04	10.76	1.210	356.40
3	3.60	479.34	1.644	18.48	106.31	1848.77	11.46	1.246	378.72
4	3.46	474.10	1.647	19.38	107.31	1943.31	11.60	1.269	384.45
5	3.48	476.43	1.646	19.30	107.35	1935.68	11.58	1.269	383.81
6	3.48	476.40	1.646	19.30	107.37	1935.74	11.56	1.269	383.21

Table 2. Comparison of present results by $N = 6$ with ref. [4]

Γ	Pr	Present results k_c	Ra_c	σ_{ic}	Ref. [4] k_c	Ra_c	σ_{ic}
0.1	0.1	3.116	1707.73	0	—	—	—
	1	4.915	870.48	15.08	4.917	877.8	15.07
	10	7.256	226.68	76.22	7.309	230.0	76.68
	100	11.59	127.89	371.78	11.96	130.1	385.8
	1000	19.30	107.37	1935.74	20.46	108.0	2052.0
1	0.1	3.484	476.39	1.647	3.484	478.9	1.647
	1	3.700	51.19	6.062	3.696	51.58	6.061
	10	4.719	7.418	20.78	4.724	7.496	20.77
	100	7.199	2.160	82.66	7.297	2.203	83.45
	1000	11.57	1.269	383.50	12.76	1.289	418.8

[4] are shown in Table 2. Both results are in good agreement, with a difference less than 2%.

The critical Rayleigh number, wavenumber and σ_{ic} vs Deborah number for various Prandtl numbers are shown in Figs. 1–3, respectively. In Fig. 1 the critical Rayleigh number is seen to be 1707.76, as the Deborah number Γ is small. Stationary convection with $k_c = 3.117$ develops because $\sigma_{ic} = 0$, at the onset of instability. For each Prandtl number, there can be found a transition Deborah number, as the Deborah number is greater than this value, Ra_c is smaller than 1707.76, which means that the elasticity modulus has a destabilization effect on the onset of instability. The marginal curves for $Pr = 1$, $\Gamma = 0.062$ and 0.063 are

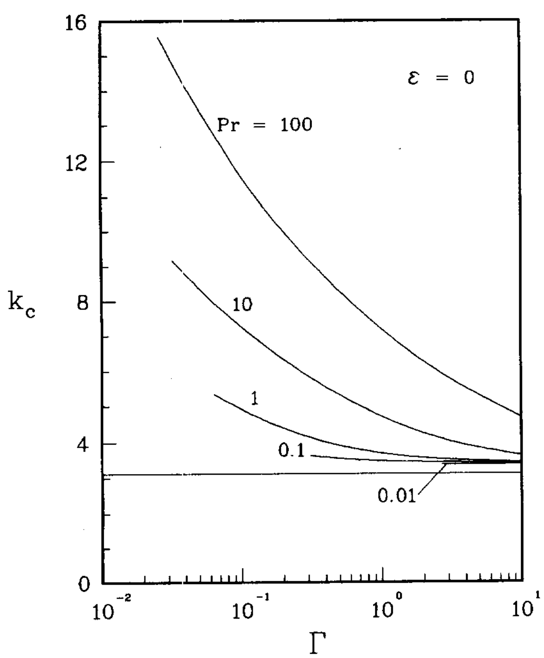

Fig. 2. The critical wavenumber vs Deborah number at different Prandtl numbers for the unmodulated case.

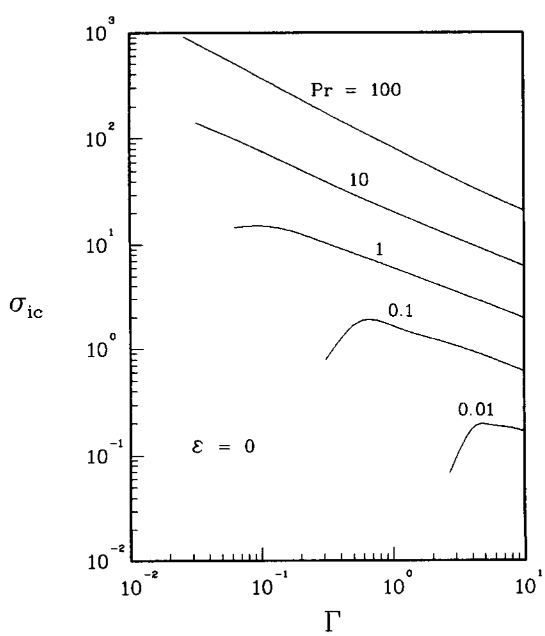

Fig. 3. The imaginary part of the most unstable eigenvalue at critical Rayleigh number vs Deborah number at different Prandtl numbers for the unmodulated case.

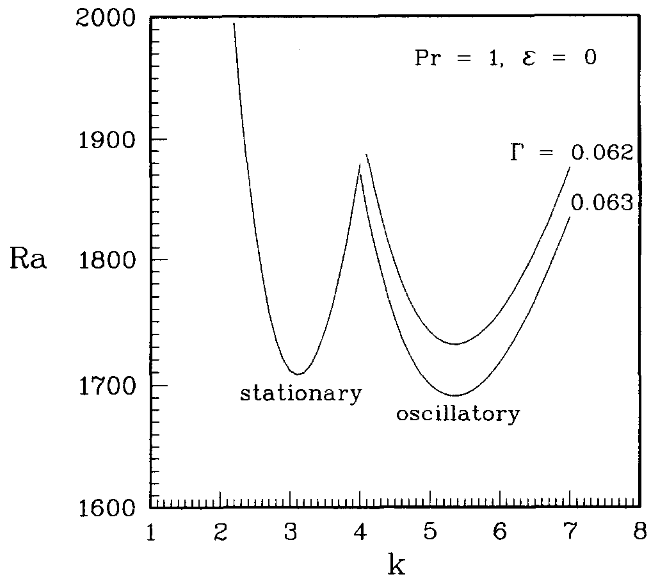

Fig. 4. The marginal curves of stationary and oscillatory modes for $Pr = 1$, $\varepsilon = 0$, $\Gamma = 0.062$ and 0.063.

shown in Fig. 4. The marginal curve for the stationary mode disturbance is the same for all values of Γ. For $\Gamma = 0.062$ the stationary modes have a smaller Rayleigh number than the oscillatory modes. While for $\Gamma = 0.063$ the oscillatory modes have a smaller Rayleigh number than the stationary modes. The transition Deborah number is thus between 0.062 and 0.063 for $Pr = 1$ and decreases with increasing Prandtl number, which can be seen from Fig. 1.

The critical wavenumber is equal to 3.117, when the Deborah number is less than the transition value. As Γ increases over the transition value, the critical wavenumber changes suddenly from 3.117, corresponding to stationary modes, to a greater value corresponding to oscillatory modes. As Γ continues increasing, the critical wavenumber decreases monotonically, as shown in Fig. 2.

The σ_i at the critical condition is equal to zero when Γ is smaller than the transition value for which a stationary convection develops at the onset of instability. As Γ is greater than the transition value, σ_i is no longer equal to zero, which means that overstability occurs and the convection shows periodic travelling waves at the onset of instability. From Fig. 3, it is also seen that σ_i increases with increasing Pr.

(ii) Effect of gravity modulation on Newtonian fluids

When $\Gamma = 0$, a layer of Newtonian fluids in a modulated gravitational field is heated from below. From the definition of the Rayleigh number, basically, the effect of modulation on the thermal instability is similar to that of a fluid layer in a constant gravitational field, but with a fixed temperature at the upper plate and a higher modulated temperature at the lower plate [10, 12]. The cases for $Pr = 7$ and $\varepsilon = 0-1$ are shown in Fig. 5 where the ordinate is defined by the percentage change of the critical Rayleigh number compared with the unmodulated case $Ra_0 = 1707.76$, $\Delta = (Ra_c - Ra_0)/Ra_0$. The modulation has a stabilization effect for a positive Δ and a destabilization effect for a negative Δ, on the onset of instability. At low frequencies, it is seen that modulation has an effect of destabilization. In the quasi-steady limit $\omega \to 0$, Δ is, within a certain accuracy, close to the theoretical result $-\varepsilon/(1+\varepsilon)$, which is expressed by the horizontal lines below each curve. At high frequencies, modulation has a very slight effect of stabilization which was also found in the temperature modulation problem [8] and radiation modulation problem [12]. The destabilization effect at low frequencies, as well as the stabilization effect for $\omega = 3.5$ and $0.8 < \varepsilon < 1$, were experimentally approved for air by Finucane and Kelly [10], for the temperature modulated case.

Gresho and Sani [11] solved the same problem by one term expansion. They found that the instability occurs for synchronous modes at lower frequencies and for subharmonic modes at higher frequencies. Although stabilization was shown for the considered range of parameters in Fig. 4 of their paper, they proposed that destabilization may occur at large ε/ω^2 and high frequency. A recalculation of a wider range of frequency is shown in Fig. 6 which shows a good qualitative agreement with the Fig. 4 of Gresho and Sani [11]. The lower curves represents the instability due to synchronous modes, for which the critical Rayleigh number increases with increasing frequency. While for a frequency higher than about 1720, the instability due to synchronous modes can no longer be found. Instead, the subharmonic modes occur and the critical Rayleigh number decreases as frequency increases, as shown by the upper curve. Actually, from Fig. 5 we have seen that the destabilization occurs at low frequencies and is especially obvious for large ε.

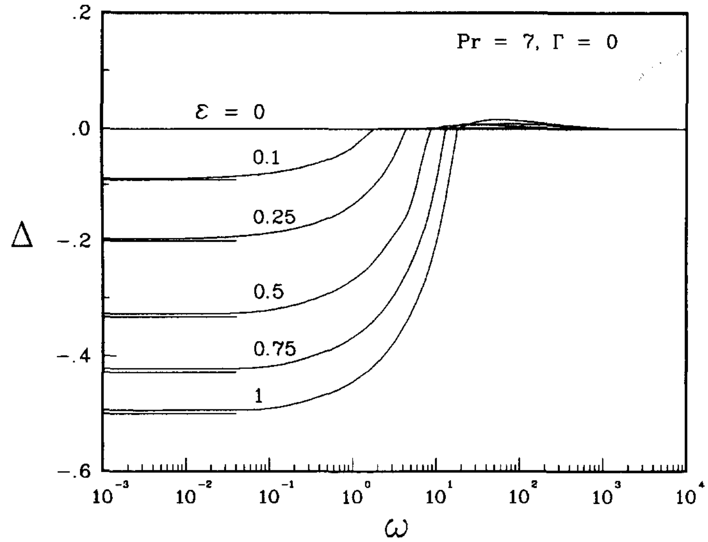

Fig. 5. Δ vs ω at different ε for $Pr = 7$ and $\Gamma = 0$.

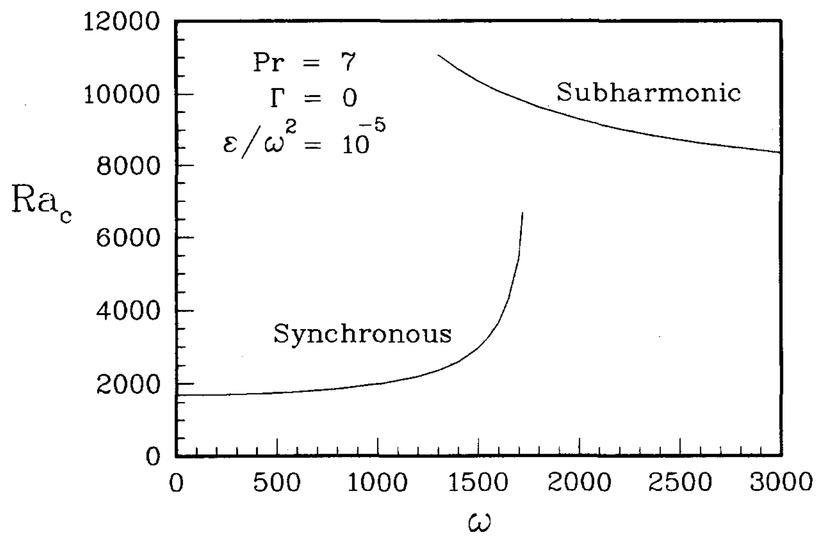

Fig. 6. The critical Rayleigh number vs frequency for $Pr = 7$, $\Gamma = 0$ and $\varepsilon/\omega^2 = 10^{-5}$.

(iii) Effect of gravity modulation on viscoelastic fluids

When Γ is greater than zero, the effect of elasticity on the instability of viscoelastic fluids under modulated gravity can be seen. An accuracy test at the critical condition for the modulated case $Pr = 1$, $\Gamma = 0.1$, $\varepsilon = 1$ and $\omega = 0.01$, by varying the combinations of M and N, is shown in Table 3. Here only the even mode disturbances are considered. It is seen that at a fixed M the critical Rayleigh number converges rapidly, as N is increased from 1 to 2, which shows the exponential convergence of the Chebyshev polynomials, whereas a much larger M for the expansion in time, by a complete set of Fourier series, is needed. In the calculations of modulated cases, the results are obtained from two consecutive resolutions, for which the relative variation is less than 1% and the error for the case of quasi-steady limit, $\omega \to 0$, compared with the theoretical value $\Delta \to -\varepsilon/(1+\varepsilon)$ is within 2%.

From the results in (i) we know that there is a transition value of Γ for each Pr and below and above this transition value the instabilities are respectively due to stationary and oscillatory mode of disturbances. Here we consider the fluids with $Pr = 7$ for which the transition value of Γ is about 0.04. It is also known from (ii) that the destabilization effect is most significant at low frequencies and approaches the quasi-steady limit as $\omega \to 0$ and the stabilization effect is found at high frequencies and disappears at very high frequencies. When the elastic modulus is present in the fluids the same behavior is still present at both very low and very high frequencies, but the phenomenon in the range of intermediate frequency is more complicated. As shown in Fig. 7, the destabilization

Table 3. Ra_c (upper) and σ_{ic} (lower) vs M and N by even modes for $Pr = 1$, $\Gamma = 0.1$, $\varepsilon = 1$, $\omega = 0.01$ and $k_c = 4.92$ ($k_c = 4.74$ for †)

	N			
M	1	2	3	4
3	451.23 15.991†	510.87 15.089	509.92 15.090	509.95 15.090
5	412.81 15.991†	467.37 15.089	446.50 15.090	466.52 15.090
7	400.40 15.991†	453.32 15.089	452.48 15.030	452.50 15.090
9	394.84 15.990†	447.03 15.088	466.20 15.089	446.22 15.089
11	391.89 15.989†	443.69 15.087	442.86 15.088	442.89 15.088
13	390.16 15.988†	441.73 15.086	440.90 15.087	440.93 15.087
15	389.08 15.985†	440.51 15.083	439.69 15.084	439.72 15.084

effect at low frequencies and the slight stabilization effect at high frequencies, are seen for all values of Γ. For $\Gamma = 0$, 0.01 and 0.02 the elastic modulus is small and the stationary convection occurs at $Ra = Ra_0$ as the gravity modulation is absent. The three curves approach the same values at both low and high frequency limits. At $\omega \sim O(10^2)$, stronger stabilization for $\Gamma = 0.01$ and destabilization for $\Gamma = 0.02$ are found because of the different mode of disturbances. This can further be understood from Fig. 8, which shows the critical Rayleigh number vs Γ at $\omega = 200$. The upper horizontal curve represents the instability due to the syncronous modes while the inclined curve due to the subharmonic modes. For $\Gamma = 0.01$ the critical Rayleigh number is due to syncronous modes and is higher than Ra_0. Thus elasticity enhances the stabilization effect of modulated flow. For $\Gamma = 0.02$ the subharmonic modes compete to occur at the onset of instability and decrease the critical Rayleigh number to a value lower than Ra_0. Elasticity strongly destabilizes the modulated flow which would be stabilized if the elasticity were absent.

The marginal curves for $\Gamma = 0.01$ and $\omega = 150$ are shown in Fig. 9. It is seen that the syncronous disturbances occur at smaller wavenumbers and the subharmonic disturbances at larger wavenumbers. The critical Rayleigh number is the minimum and is due to the subharmonic disturbance. At a larger value of elastic modulus $\Gamma = 0.045$ and for $\omega = 450$, an additional local minimum is found at a higher wavenumber by the subharmonic disturbance, as shown in Fig. 10.

CONCLUSION

The instability of viscoelastic fluids heated from below in a modulated gravitational gravity is studied. A numerical method based on the linear theory and Floquet theory is developed to trace the stability boundary.

For the case of viscoelastic fluids acted on by a constant gravity, a transition Γ is found for each Prandtl number. Below this transition value stationary convection occurs and above it oscillatory convection occurs, at the onset of instability. The critical Rayleigh number for Γ less than the transition value is equal to 1707.76, while for Γ greater than the transition value it is always smaller than 1707.76. Therefore, the elastic modulus has destabilization effect on the occurrence of instability.

For the case of Newtonian fluids acted on by a modulated gravity, modulation has a destabilization effect at low frequencies and a slight stabilization effect at high frequencies, which increase with increasing amplitude of modulation. The destabilization approaches the quasi-steady limit as the frequency tends to zero.

When the modulated gravity is acting in the vis-

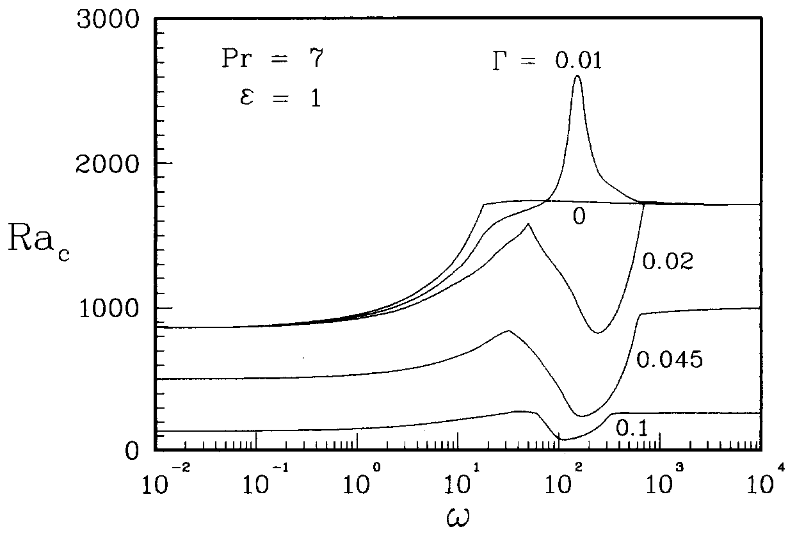

Fig. 7. The critical Rayleigh number vs frequency at different Deborah numbers for $Pr = 7$ and $\varepsilon = 1$.

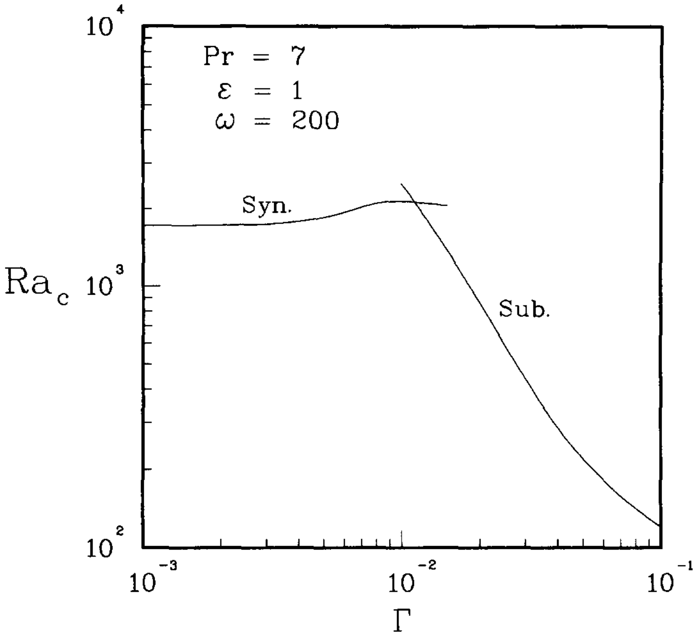

Fig. 8. The critical Rayleigh number for syncronous and subharmonic modes vs Deborah number for $Pr = 7$, $\varepsilon = 1$ and $\omega = 200$.

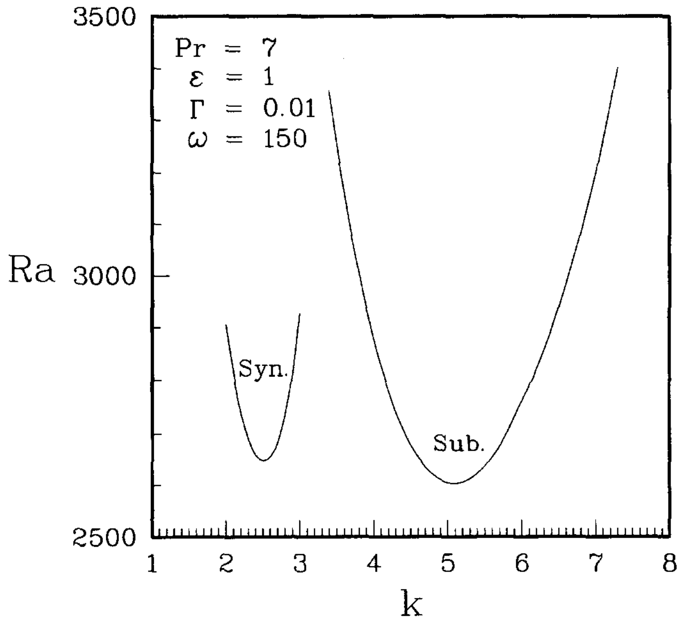

Fig. 9. The marginal curves for syncronous and subharmonic modes with $Pr = 7$, $\varepsilon = 1$, $\Gamma = 0.01$ and $\omega = 150$.

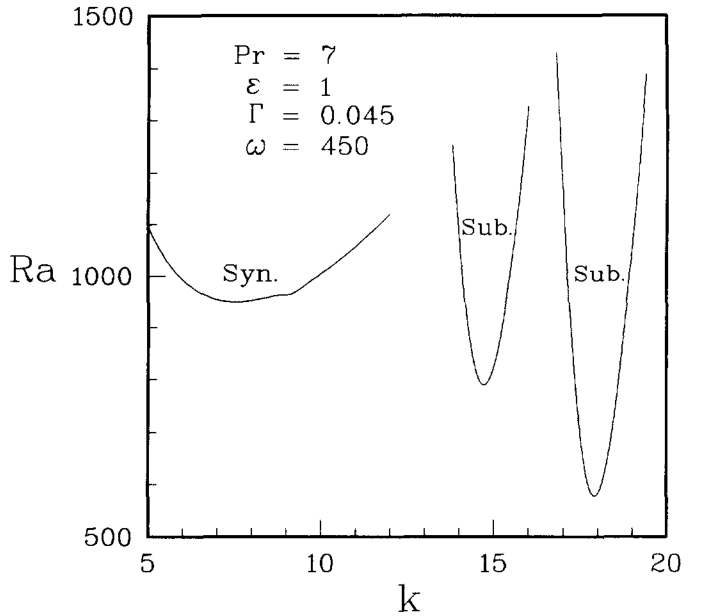

Fig. 10. The marginal curves for syncronous and subharmonic modes with $Pr = 7$, $\varepsilon = 1$, $\Gamma = 0.045$ and $\omega = 450$.

coelastic fluids, modulation has the same effects at both low and high frequency ranges as those of Newtonian fluids. While at the intermediate frequency range, subharmonic disturbances are found to enhance the stabilization effect for smaller Γ and the destabilization effect for larger Γ.

Acknowledgements—The author gratefully acknowledges the support of the National Science Council, Republic of China, through the project NSC 82-0401-E009-307.

REFERENCES

1. Petrie, C. J. S. and Denn, M. M., Instabilities in polymer processing. *AIChE Journal*, 1976, **22,** 209–236.
2. Herbert, D. M., On the stability of visco-elastic liquids in heated plane Couette flow. *Journal of Fluid Mechanics*, 1963, **17,** 353–359.
3. Green, T., III, Oscillating convection in an elasticoviscous liquid. *Physics of Fluids*, 1968, **11,** 1410–1412.
4. Vest, C. M. and Arpaci, V. S., Overstability of a viscoelastic fluid layer heated from below. *Journal of Fluid Mechanics*, 1969, **36,** 613–623.
5. Hamabata, H., Overstability of a viscoelastic liquid layer with internal heat generation, *International Journal of Heat and Mass Transfer*, 1986, **29,** 645–647.
6. Davis, S. H., The stability of time-periodic flows. *Annual Review of Fluid Mechanics*, 1976, **8,** 57–74.
7. Venezian, G., Effect of modulation on the onset of thermal convection. *Journal of Fluid Mechanics*, 1969, **35,** 243–254.
8. Rosenblat, S. and Herbert, D. M., Low-frequency modulation of thermal instability. *Journal of Fluid Mechanics*, 1970, **43,** 385–398.
9. Yih, C.-S. and Li, C.-H., Instability of unsteady flows or configurations—2. Convective instability. *Journal of Fluid Mechanics*, 1972, **54,** 143–152.
10. Finucane, R. G. and Kelly, R. E., Onset of instability in a fluid layer heated sinusoidally from below. *International Journal of Heat and Mass Transfer*, 1976, **19,** 71–85.
11. Gresho, P. M. and Sani, R. L., The effects of gravity modulation on the stability of a heated fluid layer. *Journal of Fluid Mechanics*, 1970, **40,** 783–806.
12. Yang, W.-M., Effect of modulation on radiation-induced instability. *International Journal of Heat and Mass Transfer*, **38,** 1995, 47–53.
13. Findley, W. N., Lai, J. S. and Onaran, K., *Creep and Relaxation of Nonlinear Viscoelastic Materials with an Introduction to Linear Viscoelastic*. North-Holland, Amsterdam, 1976.
14. Bird, R. B., Armstrong, R. C. and Hassager, O., *Dynamics of Polymeric Liquids*, Vol. 1 : *Fluid Mechanics*, 2nd edn. Wiley, New York, 1987.
15. Ince, E. L., *Ordinary Differential Equations*. Dover, New York, 1969.
16. Coddington, E. A. and Levinson, N., *Theory of Ordinary Differential Equations*. McGraw-Hill, New York, 1955.