Performance evaluation of a bigrating as a beam splitter

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The design of a bigrating for use as a beam splitter is presented. It is based on a rigorous formulation of plane-wave scattering by a bigrating that is composed of two individual gratings oriented in different directions. Numerical results are carried out to optimize the design of a bigrating to perform 1×4 beam splitting in two dimensions and to examine its fabrication and operation tolerances. It is found that a bigrating can be designed to perform two functions: beam splitting and polarization purification.

Key words: Bigrating, beam splitter, periodic structure, cascaded gratings.

1. Introduction

We present here an investigation of the scattering of a plane wave by a bigrating that is composed of two single dielectric gratings separated by a uniform dielectric layer. In the limiting case in which the uniform dielectric layer is absent, we have a structure with cascaded gratings. The two gratings may have different physical and structural parameters, and their periodic variations may be oriented in different directions. Specifically, the two gratings generally cross each other at angle φ, which is referred to as the crossing angle. To avoid confusion in terminology, we distinguish from the outset between the usages of two commonly referenced phrases, doubly periodic and two-dimensionally periodic. Namely, a structure is said to be doubly periodic if it is periodic in one direction with two different periods, and a structure is said to be two-dimensionally periodic if it is periodic in two orthogonal directions. Therefore, regardless of the periods of the two constituent gratings, a bigrating as a whole is more than a two-dimensional periodic structure; it is doubly periodic in one direction and singly periodic in the orthogonal direction. Our interest here is in exploiting such a special feature for the design of an optical beam splitter.

For optical interconnections, there has been a continuing search in the literature for a better design for optical beam splitters.[1–4] For example, utilizing the concept of integrated planar micro-optics, Walker *et al.*[3] reported the design of a 1×4 beam splitter that uses nine gratings of the reflection type, and Noponen and Turunen[4] synthesized two-dimensional periodic structures to achieve $1 \times N$ beam splitters of the transmission type. On the other hand, a bigrating consists of only two single gratings; it is easy to design and relatively simple to fabricate. More important, it offers some unique characteristics suitable to the design of 1×4 beam splitters.

Analyses of planar structures consisting of multiple gratings have been presented by many authors. To mention a few, single and cascaded anisotropic dielectric gratings with the same period along the boundary surfaces have been analyzed with the three-dimensional (3D) coupled-wave theory.[5] Multilayered periodic structures were analyzed on the basis of the generalized scattering matrix theory.[6] A rigorous analysis was presented for guided waves in doubly periodic structures.[7] All these analyses have been carried out in certain special conditions: (1) When the grating rulings are parallel and have the same period or (2), when the incidence is on the principal plane, i.e., the direction of the incidence is perpendicular to the grating rulings. On the other hand, the overall diffraction phenomenon associated with a bigrating may be analyzed simply in terms of multiple reflections between the two constituent gratings as will be explained. This approach inevitably involves the scattering of plane waves by a grating at the non-principal-plane incidence, which is by itself an important canonical problem in the sense that it is a 3D boundary-value problem requiring the

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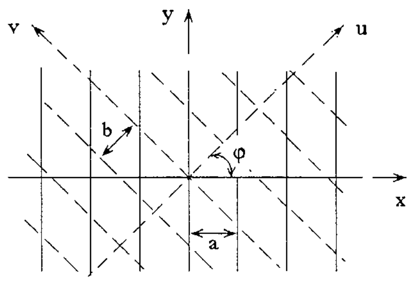
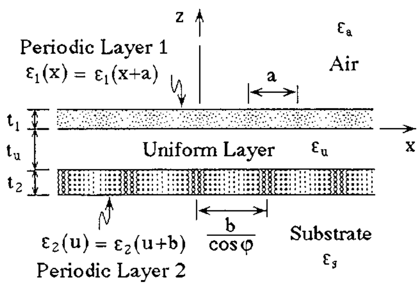


(a) Top view



(b) Front view

Fig. 1. Structure configuration of a bigrating.

coupling of TE- and TM-polarized waves for the boundary conditions to be satisfied. A rigorous treatment of such a canonical problem and the physical consequences associated with it have been reported in the past.[8–10] With the results of such a canonical problem available, the case of bigratings can be handled simply as an extension of the treatment of doubly periodic structures, as in this paper.

To set the stage, in Section 2 we describe first the physical model and all the relevant parameters for the ensuing analysis. A mathematical procedure is outlined in Section 3 for solving the plane-wave scattering by a bigrating as a boundary-value problem. It takes into account the hybrid nature of the electromagnetic fields associated with the type of structure. The design procedure and numerical examples are given in Section 4 with particular emphasis on the Bragg-regime operation so each individual grating splits the beam into two beams with almost equal intensity. Finally, some concluding remarks are in Section 5.

2. Statement of the Problem

A rigorous analysis is presented for the scattering of a plane wave by a bigrating that consists of two parallel single gratings separated by a uniform dielectric layer. In general, the two constituent gratings may be different in physical as well as structural parameters, and also their periodic variations may be oriented in different directions with an angle φ between them, as shown in Fig. 1. With the coordinate systems shown, the upper grating (grating 1) is periodic along the x direction and uniform along the y direction, whereas the lower grating (grating 2) is periodic along the u direction and uniform along the v direction. Specifically, grating 1 has dielectric constants $\varepsilon_1(x)$, with period a and thickness t_1, and grating 2 has dielectric constants $\varepsilon_2(u)$ with period b and thickness t_2. Between the two gratings is a uniform layer of the dielectric constant and thickness, ε_u and t_u, respectively. Finally, the upper half-space is referred to as the air region with the dielectric constants designated as ε_a and that of the lower half-space as the substrate region with the dielectric constants designated as ε_s. Note that, although the dielectric constant of the air is unity, symbol ε_a is intentionally kept in the formulation for generality. In this way, another value may be assigned to it to account for a cover layer, such as a prism over the bigrating. Such a bigrating as a whole is generally doubly periodic in the x direction and singly periodic in the y direction.

In general, each of the two gratings may have any spatial variation of its dielectric constants; in this paper we consider exclusively the canonical case of a sinusoidally modulated medium with the dielectric constant given by

$$\varepsilon_i(s) = \varepsilon_i\left[1 + \delta_i \cos\left(\frac{2\pi}{\Lambda_i} s + p_i\right)\right] \quad \text{for } i = 1 \text{ and } 2, \quad (1)$$

where s stands for x for grating 1 and u for grating 2, and ε_i, δ_i, Λ_i, and p_i are, respectively, the average dielectric constant, the modulation index, the period, and the phase of the ith grating. Such a modulated dielectric medium corresponds to the case of a volume holographic grating, and its wave-propagation characteristics have been extensively analyzed in terms of the Mathieu functions, as reported in the literature.[11] On the other hand, a periodic dielectric constant with an arbitrary variation may be represented by a Fourier series, and the model of a modulated medium may be regarded as its first-order approximation.

A plane wave is incident obliquely from the air region and is scattered by the bigrating structure with the direction of the propagation indicated in Fig. 2. In the spherical coordinated system, the incident plane wave is characterized by the propagation constant k_a, the polar angle θ_{inc}, and the azimuth angle ϕ_{inc}. On the other hand, in the rectangular xyz-coordinate system, the propagation vector of the incident plane wave generally has three components that are related to incident angles θ_{inc} and ϕ_{inc} by

$$k_x = k_a \sin \theta_{inc} \cos \phi_{inc}, \quad (2a)$$

$$k_y = k_a \sin \theta_{inc} \sin \phi_{inc}, \quad (2b)$$

$$k_z = k_a \cos \theta_{inc}. \quad (2c)$$

Because we are interested mostly in the power flow perpendicular to the grating layers, we refer to the z direction as the longitudinal direction and the xy plane as the transverse plane. Thus k_z is called the longitudinal propagation constant, and k_x and k_y are the components' transverse propagation vector.

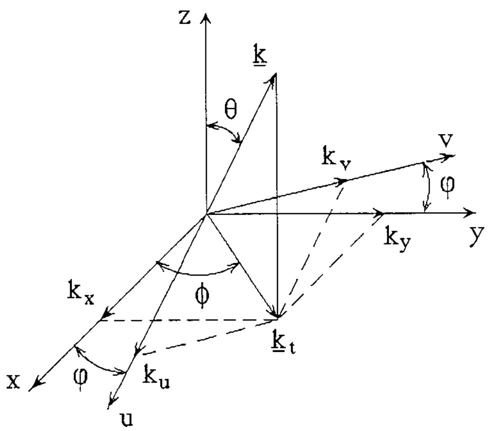


Fig. 2. Propagation vector in spherical and rectangular coordinate systems.

As a result of the multiple reflections between the two gratings, a host of space harmonics is generated. Because grating 2 can be considered as having been rotated about the z axis by angle φ with respect to grating 1, it can be considered as having periods $b/\cos\varphi$ and $b/\sin\varphi$ in the x and y directions, respectively. Therefore the bigrating structure is doubly periodic in the x direction and singly periodic in the y direction; for the mth space harmonic of grating 1 and the nth space harmonic of grating 2, referred to simply as the mnth harmonic, the components of the transverse propagation vector are related to those of the incident plane wave by

$$k_{xmn} = k_x + m\frac{2\pi}{a} + n\frac{2\pi}{b}\cos\varphi, \tag{3a}$$

$$k_{yn} = k_y + n\frac{2\pi}{b}\sin\varphi, \tag{3b}$$

where m and n are the harmonic indices of gratings 1 and 2, respectively, and can be any integer ranging from negative infinity to positive infinity. Furthermore k_x and k_y are related to the direction of plane-wave incidence, as given in Eq. (1). By coordinate rotation, the components of the transverse propagation vector of the mnth space harmonic are given in the uvz-coordinate system as

$$k_{umn} = k_u + m\frac{2\pi}{a}\cos\varphi + n\frac{2\pi}{b}, \tag{4a}$$

$$k_{vm} = k_v - m\frac{2\pi}{a}\sin\varphi, \tag{4b}$$

where k_u and k_v are the transverse components of the incident propagation vector in the uvz-coordinate system; they are related to those in the xyz-coordinate system by the transformation formula:

$$\begin{pmatrix} k_u \\ k_v \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}\begin{pmatrix} k_x \\ k_y \end{pmatrix}. \tag{5}$$

Evidently from Eqs. (3)–(5), when the incident plane wave and the bigrating structure are specified, the transverse propagation vector of each space harmonic can readily be determined in both coordinate systems. This means physically that for every space harmonic in each subregion the direction of propagation can be easily obtained, but the amplitude must be determined by a 3D boundary-value problem, to be explained below.

3. Method of Analysis

The scattering of a plane wave by a dielectric grating layer sandwiched between two uniform media has been rigorously formulated in the most general condition of non-principal-plane incidence. As for a special class of doubly periodic structures,[7] a special case of the crossing angle, $\varphi = 0$, a bigrating structure can be likewise analyzed in terms of multiple scattering between the two constituent gratings. Specifically, we may simply make use of the previous results of the plane-wave scattering by a single grating at a non-principal-plane incidence[8] as a building block, so that we may skip altogether the details of solving the boundary-value problem requiring the Floquet representations for the fields in the periodic regions and the subsequent matching of the boundary conditions on the surfaces of the gratings. Therefore only an outline of the method is given here with a list of expressions for the space harmonic amplitudes that are relevant to the ensuing discussions.

In a uniform dielectric medium, each space harmonic appears as a plane wave of which the tangential field components may be generally represented as a superposition of those of the TE- and TM-polarized plane waves. For the mnth harmonic the tangential-field components can be written as

$$\boldsymbol{z}_0 \times \boldsymbol{E}_{tmn}(\boldsymbol{\rho}, z) = [\boldsymbol{a}_{mn}'V_{mn}'(z) + \boldsymbol{a}_{mn}''V_{mn}''(z)] \times \exp(-j\boldsymbol{k}_{\rho mn}\cdot\boldsymbol{\rho}), \tag{6a}$$

$$\boldsymbol{H}_{tmn}(\rho, z) = [\boldsymbol{a}_{mn}'I_{mn}'(z) + \boldsymbol{a}_{mn}''I_{mn}''(z)] \times \exp(-j\boldsymbol{k}_{\rho mn}\cdot\boldsymbol{\rho}), \tag{6b}$$

where the single and double primes denote the TE and TM polarizations, respectively, $\boldsymbol{k}_{\rho mn}$ is the transverse propagation vector, $\boldsymbol{\rho}$ is the transverse coordinate vector, and a_{mn}' and a_{mn}'' are unit vectors defined as

$$a_{mn}' = \boldsymbol{k}_{\rho mn}/k_{\rho mn}, \tag{7a}$$

$$a_{mn}'' = \boldsymbol{z}_0 \times a_{mn}' = \boldsymbol{z}_0 \times \boldsymbol{k}_{\rho mn}/k_{\rho mn}. \tag{7b}$$

In Eqs. (6) and (7) all the phase quantities are supposed to be known, but not the amplitudes, $V_{mn}(z)$ and the $I_{mn}(z)$, which represent vertical variations of the electric and magnetic fields of the mnth harmonic, respectively, and can be written generally as a superposition of the forward and backward traveling waves as

$$V_{mn}(z) = V_{mn}\exp(-jk_{zmn}z) + \overline{V}_{mn}\exp(jk_{zmn}z), \tag{8a}$$

$$I_{mn}(z) = Y_{mn}[V_{mn} \exp(-jk_{zmn}z) - \bar{V}_{mn} \exp(jk_{zmn}z)], \tag{8b}$$

where V_{mn} and $\bar{V}_{mn}$ are the amplitudes of the forward and backward traveling waves, respectively. Note that the primes over the V and I terms are omitted here for simplicity, and these expressions hold for either the singly or the doubly primed quantities, denoting the TE- or TM-polarized fields. Finally, the longitudinal propagation constant k_{zmn} and the wave admittance Y_{mn} of the mnth harmonic in a uniform medium of the dielectric constant ε are defined as

$$k_{xmn}{}^2 + k_{yn}{}^2 + k_{zmn}{}^2 = k_0{}^2\varepsilon, \tag{9a}$$

$$Y_{mn} = \begin{cases} Y_{mn}' = \dfrac{k_{zmn}}{\omega\mu_0} & \text{for TE polarization,} \\ Y_{mn}'' = \dfrac{\omega\varepsilon_0\varepsilon}{k_{zmn}} & \text{for TM polarization,} \end{cases} \tag{9b}$$

where k_{xmn} and k_{yn} are given in Eq. (3) and k_0 is the free-space propagation constant. With the electromagnetic fields of each space harmonic represented above, the total electromagnetic fields in each uniform medium can then be written as a superposition of all the space harmonics and they are required to satisfy the boundary conditions at every surface of the two gratings. In view of Eqs. (6) and (8) there are four sets of unknowns to be determined for each uniform region, including the forward and backward traveling-wave amplitudes of both TE and TM polarizations. Making use of the previous results on plane-wave scattering by a single grating at an oblique incidence,[8] we skip altogether the details of the Floquet representations for the fields in the periodic regions and the matching of the boundary conditions on the surfaces of the gratings and list only the expressions for the space harmonic amplitudes that are relevant to the ensuing discussions.

To account for the interaction of the two constituent gratings, the key step is to determine the fields in the uniform layer in between. Consider first the plane-wave scattering by grating 2. In Eqs. (6) and (8) the backward traveling-wave amplitudes are related to the forward ones by

$$\bar{V}_{mn}' = \sum_{p=-\infty}^{\infty} G_{mnp}{}^{(1,1)}V_{mp}' + \sum_{p=-\infty}^{\infty} G_{mnp}{}^{(1,2)}V_{mp}'', \tag{10a}$$

$$\bar{V}_{mn}'' = \sum_{p=-\infty}^{\infty} G_{mnp}{}^{(2,1)}V_{mp}' + \sum_{p=-\infty}^{\infty} G_{mnp}{}^{(2,2)}V_{mp}'', \tag{10b}$$

where $G_{mnp}{}^{(i,j)}$ is a reflection coefficient with the superscripts specifying the coupling between the two polarizations and the subscripts denoting the harmonic indices. Explicitly, $G_{mnp}{}^{(1,1)}$ is the reflection coefficient from the pth TE harmonic to the nth TE harmonic of grating 2, when the mth TE harmonic of grating 1 is incident, $G_{mnp}{}^{(1,2)}$ is the reflection coefficient from the pth TM harmonic to the nth TE harmonic of grating 2, when the mth TM harmonic of grating 1 is incident, and similarly for other G terms. Here we have made use of the fact that these reflection coefficients can readily be determined when grating 2 and its surroundings are specified. Substituting the two expressions in Eq. (10) into Eq. (6), we can then define the relationship between the electric and magnetic fields of all the space harmonics, in the form of impedance or admittance matrices, at the bottom surface of grating 1. Thus the problem of plane-wave scattering by the bigrating is reduced to a case of plane-wave scattering by a single grating with a known termination at the output surface; this enables us to determine the scattering characteristics of grating 1 including the effect of grating 2. For an incidence of a single unpolarized plane wave that may consist of TE and TM plane waves of amplitudes A_{00}' *and* A_{00}'', respectively, the amplitudes of the reflected TE and TM mnth harmonic are given by

$$B_{mn}' = \Gamma_{mn}{}^{(1,1)}A_{00}' + \Gamma_{mn}{}^{(1,2)}A_{00}'', \tag{11a}$$

$$B_{mn}'' = \Gamma_{mn}{}^{(2,1)}A_{00}' + \Gamma_{mn}{}^{(2,2)}A_{00}'', \tag{11b}$$

where $\Gamma_{mn}{}^{(1,1)}$ is the reflection coefficient from the TE incident wave to the TE-reflected mnth harmonic and $\Gamma_{mn}{}^{(1,2)}$ is the reflection coefficient from the TM incident wave to the TE-reflected mnth harmonic, and similarly for other Γ terms.

With the reflection coefficients of the overall bigrating structure given above for all the space harmonics, the electromagnetic fields in the air region are considered to be completely determined. In turn, the fields in every other region can be determined consecutively through the requirements of the continuity tangential field across the interface boundaries. In the substrate region there are only the transmitted space harmonics propagating in the forward direction; the amplitudes of TE and TM mnth harmonics are given by

$$C_{mn}' = T_{mn}{}^{(1,1)}A_{00}' + T_{mn}{}^{(1,2)}A_{00}'', \tag{12a}$$

$$C_{mn}'' = T_{mn}{}^{(2,1)}A_{00}' + T_{mn}{}^{(2,2)}A_{00}'', \tag{12b}$$

where $T_{mn}{}^{(1,1)}$ is the transmission coefficient from the TE incident wave to the TE-transmitted mnth harmonic and $T_{mn}{}^{(1,2)}$ is the transmission coefficient from the TM incident wave to the TE transmitted mnth harmonic and similarly for other T terms.

Based on the approach described above, we have developed a computer program to generate reliable numerical data for the design of dielectric bigratings. Some of the results are illustrated below.

4. Design Considerations and Numerical Examples

With the rigorous formulation outlined in Section 3, the scattering characteristics of the bigratings can be systematically investigated. We have carried out considerable numerical data to establish the validity and accuracy of the analysis method employed. However, in this paper, we focus on the design of bigratings for use as beam splitters only. Before embarking on the details of specific examples, it is useful and instructive to determine some general characteristics of a bigrating, based on well-established knowledge of single gratings. This provides not only a line of thought for design consideration but also a good starting point for numerical analysis. For illustration purposes, the two gratings are set to have the same average dielectric constant, $\bar{\varepsilon}_1 = \bar{\varepsilon}_2 = 1.1$, and the same modulation index, $\delta_1 = \delta_2 = 0.011$, but different periods: $a = 0.6$ μm and $b = 1.0$ μm. Note that the average dielectric constants are deliberately set to be relatively small, so that most of the electromagnetic energy may transmit through the bigrating and be diffracted in different directions. These parameters are used exclusively for all the numerical examples below.

A. Design Considerations

Referring to Fig. 2, we consider here the incidence of a light beam with its electric-field vector polarized in the y direction and the propagation vector in the xz plane, and the cross section of the light beam is assumed to be so large that it can be regarded as a uniform plane wave. Such an incident light beam may be diffracted first by grating 1 into many beams, and each of the transmitted beams is further diffracted by grating 2. With respect to grating 1, we have a case of principal-plane incidence; therefore the transmitted light is diffracted only in the x direction into various space harmonics, some propagating and the other decaying along the z direction. Each of the propagating harmonics appears as a light beam with the incident polarization preserved. With respect to grating 2, however, every diffracted beam transmitted through grating 1 is an incident beam at a non-principal-plane incidence because of the crossing angle between the two gratings. Thus the subsequent scattering of each harmonic from grating 1 by grating 2 generally results in the excitation of both polarizations in the further diffracted beams. In other words, polarization conversions may occur in the scattering process, and the diffracted beams from grating 2 generally contain both TE and TM components.

The bigrating structure consists of two separate single gratings, each of which may diffract an incident optical beam into two beams of almost equal intensity in the Bragg condition. Therefore the two gratings can diffract an incident beam into four beams, if the Bragg conditions of both gratings are satisfied simultaneously. In the case of two gratings crossing at an angle, the beams may be diffracted in both the x and the y directions, as indicated in Fig. 2. Such a two-dimensional splitting provides 1 degree of freedom more for the design of beam splitters for various applications, such as optical interconnections. For this study we are particularly interested in evaluating the fabrication and operation tolerance of the bigrating as a 1×4 beam splitter to achieve a desired intensity distribution among the four diffracted beams.

The propagation vector of the incident wave may be related to the incident parameters; for the tangential components we have in the uvz-coordinate system

$$k_u = k_x \cos\varphi + k_y \sin\varphi = k_a \sin\theta_{\rm inc} \cos(\varphi - \phi_{\rm inc}), \tag{13a}$$

$$k_v = -k_x \sin\varphi + k_y \cos\varphi = k_a \sin\theta_{\rm inc} \sin(\varphi - \phi_{\rm inc}), \tag{13b}$$

where k_x and k_y are determined as in Eqs. (2). Because the periodicity of grating 1 is in the x direction, the Mth-order Bragg condition is given by $k_x a = M\pi$, from which we obtain, after invoking Eq. (2a),

$$\sin\theta_{\rm inc} \cos\phi_{\rm inc} = M\lambda/2a. \tag{14a}$$

On the other hand, the periodicity of grating 2 is in the u direction, the Nth-order Bragg condition is given by $k_u b = N\pi$, from which we obtain, after invoking (13a),

$$\sin\theta_{\rm inc} \cos(\varphi - \phi_{\rm inc}) = N\lambda/2b. \tag{14b}$$

Dividing Eq. (14a) by Eq. (14b), we obtain the necessary condition

$$\frac{\cos\phi_{\rm inc}}{\cos(\varphi - \phi_{\rm inc})} = \frac{Mb}{Na} \tag{15}$$

for the two individual gratings forming the bigrating to both be in the Bragg conditions simultaneously. For example, when the bigrating is fixed and the order of the Bragg interaction chosen, i.e., a, b, φ, M, and N are given, the incident angles of $\phi_{\rm inc}$ and $\theta_{\rm inc}$ are determined successively from Eq. (15) and either Eq. (14a) or Eq. (14b).

Returning to Fig. 2, we have an incident wave of the TE polarization with respect to the vertical z direction. Here the azimuth angle of incidence is $\phi_{\rm inc} = 0$. For the grating periods that were chosen, $a = 0.6$ μm, $b = 1.0$ μm and at wavelength $\lambda = 1.0$ μm, we obtain, for both gratings 1 and 2 to be in the first-order Bragg conditions, $M = 1$ and $N = 1$; the crossing angle is obtained from Eq. (15) to be $\varphi = 53.13°$ and the incident angle from Eq. (14a) to be $\theta_{\rm inc} = 56.443°$. In these conditions, the incident beam is diffracted by grating 1 into two beams propagating in different directions, one corresponding to the fundamental harmonic and the other to the $m = -1$ harmonic. Each of these two beams is then diffracted by grating 2 into two beams, corresponding to the fundamental and $n = -1$ harmonics. Thus the incident beam is split into four beams altogether after transmission through the bigrating as a whole.

Note that with respect to the xyz-coordinate system the diffraction of grating 2 causes a shift not only in the x direction but also in the y direction. Thus the diffraction of an incident optical beam by the bigrating designed above results in four-beam splitting in two dimensions. Certainly, multiple reflections occur between the two gratings; they are omitted in the discussion above for simplicity. Although the direction of propagation of the four beams can be easily determined as described above, we must determine the distribution of intensity and polarization among the four beams by solving the overall structure as a boundary-value problem, taking into consideration the multiple-scattering effect, as outlined in Section 3. Some numerical examples are given below.

B. Numerical Results

To achieve a desired intensity distribution among the four beams, it is necessary to design the bigrating structure by adjusting the overall structural parameters other than the periods of the two gratings. To explore the basic concept, we consider here the case in which a prism is put over the bigrating to reduce the reflection taking place at the upper surface of grating 1. We may normalize all the dielectric constants of the structure to that of the prism, so that the dielectric constant of the prism region becomes unity, $\varepsilon_a = 1.0$, as in the case of air. In the numerical analysis to follow, the normalized dielectric constants of the uniform layer and the substrate are chosen to be equal to the average value of that of the gratings, i.e., $\varepsilon_u = \varepsilon_s = 1.1$. This is done to reduce the reflections from the two gratings, so that the multiple reflections between the two gratings are negligible and almost all the energy is transmitted through the bigrating. Finally, the thickness of the uniform layer does not play an important role as long as it is not too small; for computation purposes we set $t_u = 5$ μm, so that the two gratings show good separation and the effect of the higher-order evanescent harmonics can be ignored in the design considerations.

We have carefully examined the beam-splitting effect for a large range of grating thicknesses, and we determined that the thicknesses of the two gratings may be chosen at $t_1 = 6.6$ μm and $t_2 = 10.5$ μm for the four diffracted beams to share almost equal intensities. Note that we have chosen here the smallest thicknesses of the two gratings for the intended purpose. We may also choose larger thicknesses such as $t_1 = 19.8$ μm and $t_2 = 37.5$ μm to obtain similar results. With one grating thickness fixed and the other varied, Fig. 3 shows the intensity variations of the four diffracted beams for the case of TE incidence. From these results, we observe that a 10% change in the grating thickness results in an ~3% change in the diffraction efficiency of every beam. This offers a fabrication tolerance that should be acceptable in practice.

Using the structure designed above, we look at the fabrication and operation tolerance of the beam splitter. First, for the alignment of the two gratings, Fig. 4 shows the effect of the crossing angle on the diffraction efficiencies. We observe that for a variation in the crossing angle within ±0.5° around $\varphi = 53.13°$, the intensities of the four beams remain practically unchanged. The fabrication tolerance of ±0.5° should not pose any practical difficulty in optical applications. To determine the effect of frequency drifting of the source, the diffraction efficiency versus incident wavelength is shown in Fig. 5. We observe that within 1% around the wavelength of 1 μm, the

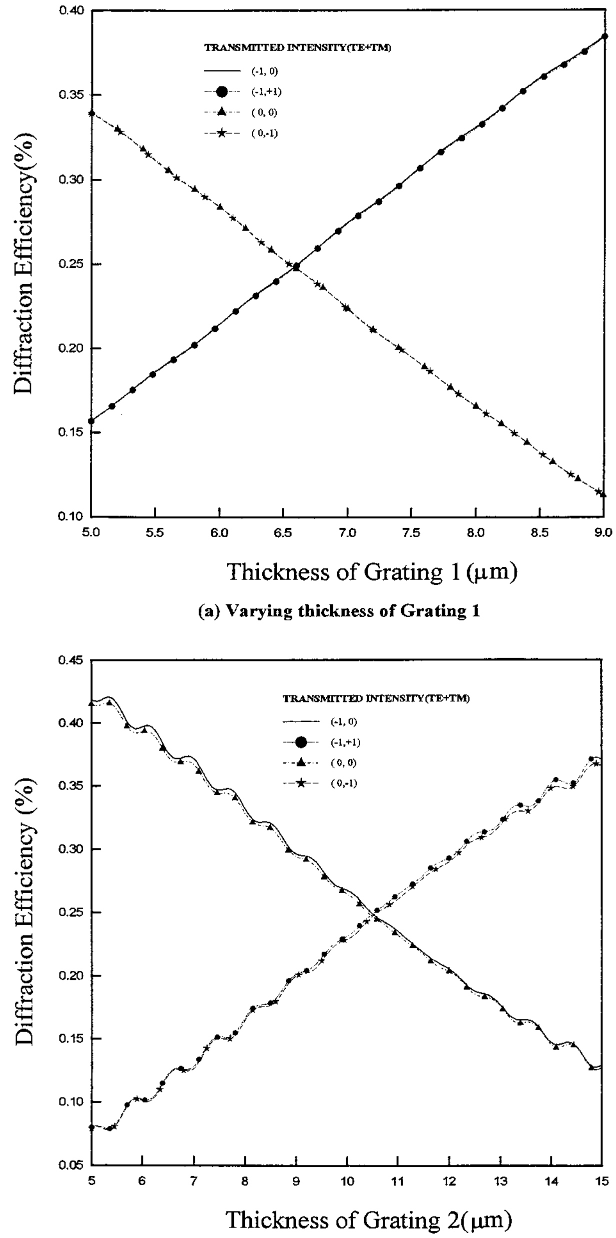


Fig. 3. Effect of grating thickness on the diffraction efficiency.

efficiencies remain more or less unchanged. Therefore the frequency drifting of the source should not pose a threat to the operation of a bigrating as a beam splitter.

As mentioned above, a bigrating is inherently a 3D boundary-value problem that cannot support pure TE or TM electromagnetic fields. In other words, for the incidence of either a TE- or a TM-polarized beam, both TE and TM polarizations are generally excited in the diffracted beams. However, the relative intensities of the two polarizations in a beam may be controlled by adjusting the thickness of the grating. Based on such a concept, we have designed a bigrating to perform the one-to-four beam splitting in two dimensions, with each of the beams having a single

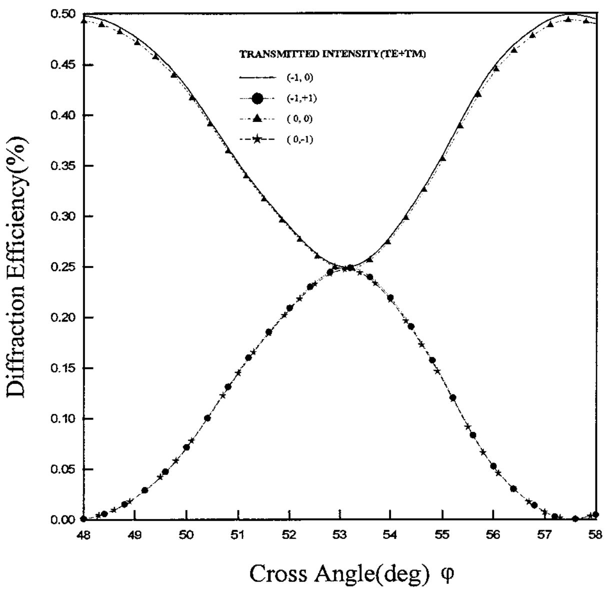


Fig. 4. Effect of crossing angle on the diffraction efficiency.

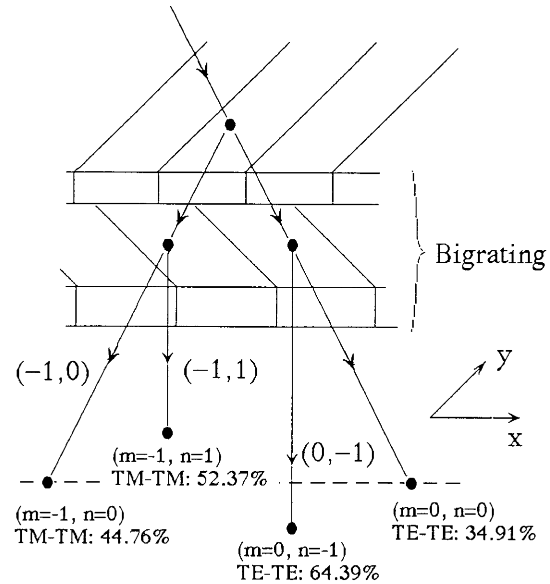


Fig. 6. 1 × 4 beam splitting in two dimensions.

polarization. Specifically, we have designed a bigrating that diffracts an unpolarized incident beam into two pure TE-polarized beams and two pure TM-polarized beams, as depicted in Fig. 6. It is emphasized that in this case the distribution of the beam intensities is somewhat uneven, as indicated numerically therein. Nevertheless the bigrating performs two functions simultaneously: beam splitting and polarization purification.

Although not shown here, we have also performed calculations for the case in which the two gratings remain the same, but all the uniform regions are filled with air instead. Our results show that the performance of the bigrating as a beam splitter deteriorates slightly. Our interpretation is that such a phenomenon is due to the multiple reflections that take place between the gratings. Thus a reduction in the reflection effect is essential to the enhancement of the beam splitter's performance.

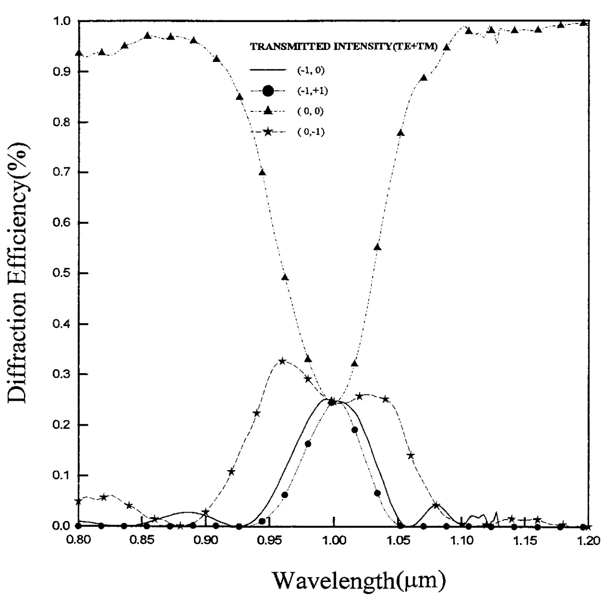


Fig. 5. Effect of the wavelength on the diffraction efficiency.

5. Conclusion

Based on a rigorous treatment of plane-wave scattering by a single grating at a non-principal-plane incidence, the diffraction of a light beam by a bigrating is systematically formulated as a 3D boundary-value problem, including the effect of polarization couplings. Numerical data were carried out with particular emphasis on the design of a bigrating for use as a 1 × 4 optical beam splitter. The fabrication and operation tolerances of the bigrating are examined, and it is found that a bigrating can indeed perform the beam-splitting function. With an appropriate design, it can also realize the polarization purification of the diffracted light beams. Thus the class of bigratings seems to have the potential to provide the free-space optical interconnections.

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