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Quantum kinetic equation for spin relaxation and spin Hall effect in GaAs

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Abstract. We present a general quantum kinetic theory of spin transport based on the Kadanoff- Baym equation (KBE), which we use to study dynamical spin processes in semiconductors right down to femtosecond and nanometer scales. In our application of KBE we describe the evolution of the non-equilibrium 2×2 matrix Green function for carrier spin, averaged over the thermal bath. Spin relaxation effects are treated within the Kadanoff-Baym Ansatz (KBA), while carrier interactions are treated within the random-phase model of screening. We track the detailed oscillation of the spin- polarized carrier state within the coherence time. Our general kinetic approach also allows description of the spin Hall effect when both impurity scattering and the Fröhlich interaction are included in the KBE collision term. We find that the level of spin current is very sensitive to the density of impurities, and that the Fröhlich interaction can generate a considerable spin current. Significantly, the Fröhlich term leads to a unique type of oscillatory behaviour in the spin current that is independent of impurity scattering effects.

PACS. 05.30.-d Quantum statistical mechanics – 05.60.Gg Quantum transport – 72.25.Dc Spin polarized transport in semiconductors – 73.63.Hs Quantum wells

1 Introduction

Manipulation of the spin degree of freedom in semiconductors has attracted considerable attention over the last decade [1–4] due to potential applications such as in quantum computation [1,2], magnetic random access memory [3] and spin transistors [4]. Recently, a very fast spin relaxation in GaAs was reported [5]. The time as short as 110 fs ± 10% for heavy holes was experimentally discovered. Such a time scale is likely too short for industrial applications, but is significant in quantum kinetic theory since energy non-conserving events [6] and memory effects [7,8] are active in this time scale. In a previous report [7], memory effects are shown to be appreciable in the time evolution of non-equilibrium carriers, implying that the memory effect on carrier- carrier scattering (CCS) resembles Rabi oscillation. That result [7] motivates us to study whether the carrier also oscillates between distinct spin-polarized states due to the memory effect prior to spin relaxation. In this work, starting with the Pauli equation, a 2×2 spin-dependent non-equilibrium Green-function matrix was utilized to construct the KBE, which was then applied to spin-dependent non-equilibrium CCS in the presence of a D'yakonov Perel' (DP) magnetic field [9]. The quantum kinetic oscillation between distinct spin-polarized states due to the memory (non-Markovian) CCS is demonstrated. Additionally, another oscillation that is the spin precession caused by spin-orbit coupling (SOC) term, Δ_{ij}, at the frequency of $\hbar^{-1}\,|\mathrm{Im}\Delta_{12}|$ is described.

The spin Hall effect (SHE) [10,11], namely, the appearance of spin transverse transport driven by a longitudinal electric field, was predicted by D'yakonov Perel' over 30 years ago. Recent experimental verification of the SHE [12] has prompted considerable discussion [13–15]. Since reversely spin-polarized particles have distinct directions of spin current (SC), such spin-particle separation without using magnetic fields, magnetic materials or magnetic dopants makes spin-based device fabrication compatible with conventional semiconductor process technology and is likely to be important in spintronic applications [1–4]. Theoretical interests of the SHE involve intrinsic (or extrinsic) scatterings [16,17] and ballistic (or diffusive) transport [18,19]. Another concern is that, once spintronic devices are manufactured, the device model based on Boltzmann theory [20,21] is no longer appropriate as the device shrinks rapidly down to the scale in

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length with momentum uncertainty compared to carrier's momentum. Thus a quantum transport description [22,23] for the SHE is needed. In contrast to Kubo formula [24,25] and Keldysh formalism under quasiclassical approximation [26], we base on an ab initio method to construct the spin-dependent KBE by using the non-equilibrium Green functions, into which the retarded Green functions by solving the spin-dependent Dyson equation are input. This work derives the KBE for the SHE, which incorporates intrinsic, extrinsic effects, impurity and Fröhlich interactions. Within the delta interaction approximation, a concise formula for the first-order SC with respect to an electric field can be obtained, indicating that only a quantum well (QW), not bulk, can have a non-zero SC due to the presence of both bulk (BIA) and surface inversion asymmetries (SIA). Furthermore, our numerical results show that the SC is very sensitive to the impurity density and that Fröhlich interaction can generate a remarkable SC. Significantly, Fröhlich interaction also leads a unique oscillatory behaviour in the SC, probably one kind of quantum kinetic oscillation, which does not occur in the impurity-induced SC.

The remainder of this paper is organized as follows. In Section 2, a general spin-dependent KBE is derived. In Section 3, the general KBE is applied to spin relaxation. Section 4 is devoted to derive and solve numerically a spatially-independent KBE for the SHE. By solving a long standing problem regarding spatially-entangled collision integrals, a spatially-dependent KBE for spin accumulation on lateral sides due to the SHE [27,28] is presented in Section 5 Conclusions are finally drawn in Section 6.

2 Derivation of spin-dependent KBE

The Pauli Hamiltonian is considered first. The field operator $\psi(\mathbf{r})$ is defined as $\sum_{\mathbf{k}} u_{\mathbf{k}}(\mathbf{r})c_{\mathbf{k}}$, where the annihilation operator $c_{\mathbf{k}}$ is for Fermions and $u_{\mathbf{k}}(\mathbf{r})$ is the single-particle state with wave vector $\mathbf{k}$. The field operator is anti-commute. The Heisenberg equation of motion for $\psi_\uparrow$ and $\psi_\downarrow$ [29–32] can then be written as $\partial_t\psi_\uparrow = -i\hbar^{-1}([\psi_\uparrow, H_{11}] + [\psi_\downarrow, H_{12}])$ and $\partial_t\psi_\downarrow = -i\hbar^{-1}([\psi_\uparrow, H_{21}] + [\psi_\downarrow, H_{22}])$, respectively, where the diagonal and off-diagonal elements of Pauli Hamiltonian $\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22}\end{pmatrix}$ are $-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r},t) + \Delta_{ii}$ and $\Delta_{ij(i\neq j)}$, respectively; [,] denotes the commutator; $U(\mathbf{r},t)$ is the potential energy; and $\mathbf{\Delta}$ is the SOC term (see Appendix A). To keep the article concise, a compact form for either definitions or equations is used. This will greatly reduce the length of mathematical expression. Original expressions for these compact forms are separately shown in Appendix B for better understanding.

The contour Green function is defined as $G^c(1,1') \equiv \frac{1}{i\hbar}\langle T_C[\psi_H(1)\psi_H^+(1')]\rangle$, where $1^{(')}$ presents $(\mathbf{r}_{1^{(')}}, t_{1^{(')}})$, $\langle\rangle$ stands for the thermal average operator, and contour $T_c[\ ]$ follows the conventional path [33]. Via the equation of motion for $\psi_{\uparrow(\downarrow)}$ and Green-function definition, the spin-dependent Dyson equation is written as

$$\mathbf{D}(1)\mathbf{G}^c(1,1') = \delta_C(1-1')\mathbf{I} + \int_C d2\mathbf{\Sigma}^c(1,2)\mathbf{G}^c(2,1'), \tag{1a}$$

$$[\mathbf{D}^*(1')\mathbf{G}^c(1,1')]^T = \delta_C(1-1')\mathbf{I} + \int_C d2\mathbf{G}^{c^T}(1,2)\mathbf{\Sigma}^{c^T}(2,1'), \tag{1b}$$

where $\delta_C(t_1 - t_{1'})$ is a contour delta function [33]. $\mathbf{D}^{(*)}(1^{(')})$ is a 2×2 matrix, of which the diagonal element is $(-)i\hbar\partial_{t_{1^{(')}}} - H_{jj}^{(*)}$ ($j = 1$ or 2) and the off-diagonal element is $-H^{(*)}_{jj'(j\neq j')}$. $\mathbf{\Sigma}^c(1,1')$ [$\mathbf{G}^c(1,1')$] is a 2×2 spin-dependent contour self- energy [Green-function] matrix, of which the diagonal element is $\Sigma^c_{ss}(1,1')$ [$G^c_{ss}(1,1')$] and the off-diagonal element is $\Sigma^c_{ss'}(1,1')[G^c_{ss'}(1,1')]$, where $s\neq s'$ in this context.

Via equations (1a), (1b) and the Langreth theorem [34,35], two kinds of KBE are obtained.

$$\Big[\mathbf{D}(1)\mathbf{G}^{\langle}(1,1')\Big]_{ss^{(')}} - \Big[\mathbf{D}^*(1')\mathbf{G}^{\langle}(1,1')\Big]_{ss^{(')}} - \sum_{s''=\uparrow,\downarrow}\Big(\Big[\Sigma_{ss''}, G^{\langle}_{s''s^{(')}}\Big] + \Big[\Sigma^{\langle}_{ss''}, G_{s''s^{(')}}\Big]\Big) = \frac{1}{2}\sum_{s''=\uparrow,\downarrow}\Big(\Big\{\Sigma^{\rangle}_{ss''}, G^{\langle}_{s''s^{(')}}\Big\} - \Big\{G^{\rangle}_{s''s^{(')}}, \Sigma^{\langle}_{ss''}\Big\}\Big) \tag{2a}$$

$$\Big[\mathbf{D}(1)\mathbf{G}^{\langle}(1,1')\Big]_{ss^{(')}} - \Big[\mathbf{D}^*(1')\mathbf{G}^{\langle}(1,1')\Big]_{ss^{(')}} = \sum_{s''=\uparrow,\downarrow}\Big(\Sigma^r_{ss''}G^{\langle}_{s''s^{(')}} + \Sigma^{\langle}_{ss''}G^a_{s''s^{(')}} - G^r_{s''s^{(')}}\Sigma^{\langle}_{ss''} - G^{\langle}_{s''s^{(')}}\Sigma^a_{ss''}\Big), \tag{2b}$$

where the retarded, advanced, lesser and greater Green functions (self energies) follow the definition in references [22,23]. Notably, $\Sigma(1,1') = [\Sigma^r(1,1') + \Sigma^a(1,1')]/2$ while $G(1,1') = [G^r(1,1') + G^a(1,1')]/2$, and ΣG and $G\Sigma$ are abbreviated forms of $\int_C d2\Sigma(1,2)G(2,1')$ and $\int_C d2G(1,2)\Sigma(2,1')$, respectively. Additionally, $\{,\}$ stands for the anti-commutator. Equations (2a) and (2b) will be applied to spin relaxation and the SHE (including the subsequent topic for spin accumulation), respectively.

3 KBE for spin relaxation

This section focuses on spin relaxation. Assume a one-band (conduction band) model and spatial independence. The equation for the spin-dependent carrier's distribution matrix $\mathbf{f}_{\mathbf{k}}(t)$ at the wave vector $\mathbf{k}$ on the band structure can be obtained using equation (2a) at an equal-time limit [22], where $\mathbf{f_k}(t)$ equals $-i\hbar\mathbf{G}^{\langle}_{\mathbf{k}}(t,t'=t)$.

$$\frac{i}{\hbar}\left[\mathbf{D}(t)\mathbf{f_k}(t)-\mathbf{D}^*(t)\mathbf{f_k}(t)\right]_{ss^{(\prime)}}= \frac{1}{2}\int_{-\infty}^{t}dt'\sum_{s''=\uparrow,\downarrow}\Big[\Sigma^{\rangle}_{ss'',\mathbf{k}}(t,t')G^{\langle}_{s''s^{(\prime)},\mathbf{k}}(t',t) +G^{\langle}_{s''s^{(\prime)},\mathbf{k}}(t,t')\Sigma^{\rangle}_{ss'',\mathbf{k}}(t',t)-G^{\rangle}_{s''s^{(\prime)},\mathbf{k}}(t,t')\Sigma^{\langle}_{ss'',\mathbf{k}}(t',t) -\Sigma^{\langle}_{ss'',\mathbf{k}}(t,t')G^{\rangle}_{s''s^{(\prime)},\mathbf{k}}(t',t)\Big]. \quad (3)$$

By applying the Langreth theorem [34,35], the two-time lesser (greater) CCS self energy within the random phase approximation can be expressed as

$$i\hbar\sum_{\mathbf{q}}G^{\langle,\rangle}_{s_1s_2,\mathbf{k-q}}(t,t')\bar{V}^{\langle,\rangle}_{s_1s_2,\mathbf{q}}(t,t'),\ \text{where}\bar{V}^{\langle,\rangle}_{s_1s_2,\mathbf{q}}(t,t')$$

is the mean-field screened potential at the exchanged wave vector $\mathbf{q}$ and can be presented as

$$\int_{-\infty}^{t}dt_1\int_{-\infty}^{t'}dt_{1'}\bar{V}^r_{\mathbf{q}}(t,t_1)L^{\langle,\rangle}_{s_1s_2,\mathbf{q}}(t_1,t_{1'})\bar{V}^a_{\mathbf{q}}(t_{1'},t')$$

according to the Dyson-like equation [22]. Note that the notation of s_1s_2 can be spin-polarized or spin-flip, unlike that of ss'. The polarization function $L^{\langle,\rangle}_{s_1s_2,\mathbf{q}}(t_1,t_{1'})$ equals $-2i\hbar\sum_{\mathbf{k}'}G^{\langle,\rangle}_{s_1s_2,\mathbf{k'+q}}(t_1,t_{1'})G^{\rangle,\langle}_{s_1s_2,\mathbf{k}'}(t_{1'},t_1)$. The retarded (advanced) screened potential $\bar{V}^{r(a)}_{\mathbf{q}}(t,t')$ is assumed as $\bar{V}_{\mathbf{q}}\delta(t-t')$, where $\bar{V}_{\mathbf{q}}$ is the screened Coulomb interaction. Therefore, $\Sigma^{\langle,\rangle}_{s_1s_2,\mathbf{k}}(t,t')$ can be written as

$$2\hbar^2\sum_{\mathbf{q},\mathbf{k}'}\bar{V}_{\mathbf{q}}(t)\bar{V}_{\mathbf{q}}(t')G^{\langle,\rangle}_{s_1s_2,\mathbf{k-q}}(t,t')G^{\langle,\rangle}_{s_1s_2,\mathbf{k'+q}}(t,t') \times G^{\rangle,\langle}_{s_1s_2,\mathbf{k}'}(t',t).$$

Using the KBA [36]

$$G^{\langle,\rangle}_{s_1s_2}(t,t')=i\hbar[G^r_{s_1s_2}(t,t')G^{\langle,\rangle}_{s_1s_2}(t',t') - G^{\langle,\rangle}_{s_1s_2}(t,t)G^a_{s_1s_2}(t,t')]$$

and the plane-wave approximation, $G^{r,a}_{s_1s_2,\mathbf{k}}(t,t')=\mp\frac{i}{\hbar}\theta(\pm t\mp t')\exp[(-ie_{\mathbf{k}}\mp\gamma)(t-t')/\hbar]$, where $e_{\mathbf{k}}$ is kinetic energy and γ is the damping constant; thus equation (3) can be rewritten as [37]

$$\partial_t f_{ss^{(\prime)},\mathbf{k}}(t)-\frac{1}{\hbar}f_{s's^{(\prime)},\mathbf{k}}(t)\begin{cases}\mathrm{Im}\ \Delta_{12},\ s'=\downarrow\\ \mathrm{Im}\ \Delta_{21},\ s'=\uparrow\end{cases}= \frac{1}{\hbar^2}\sum_{\mathbf{q},\mathbf{k}'}\bar{V}_{\mathbf{q}}(t)\int_{-\infty}^{t}dt'\bar{V}_{\mathbf{q}}(t')\exp[-\gamma(t-t')/\hbar]\cos[\delta(t-t')/\hbar] \times\sum_{s''=\uparrow,\downarrow}\Big\{f_{ss'',\mathbf{k-q}}(t')f_{ss'',\mathbf{k'+q}}(t')[1-f_{ss'',\mathbf{k}'}(t')] \times[1-f_{s''s^{(\prime)},\mathbf{k}}(t')]-f_{s''s^{(\prime)},\mathbf{k}}(t')f_{ss'',\mathbf{k}'}(t')[1-f_{ss'',\mathbf{k-q}}(t')] \times[1-f_{ss'',\mathbf{k'+q}}(t')]\Big\},\quad (4)$$

where δ, which is the non-conserving energy of CCS, equals $e_{\mathbf{k-q}}+e_{\mathbf{k'+q}}-e_{\mathbf{k}'}-e_{\mathbf{k}}$. $f_{\mathbf{k-q}}f_{\mathbf{k'+q}}(1-f_{\mathbf{k}'})(1-f_{\mathbf{k}})$ and $f_{\mathbf{k}}f_{\mathbf{k}'}(1-f_{\mathbf{k-q}})(1-f_{\mathbf{k'+q}})$ are two kinds of Pauli factor.

The physical picture of quantum kinetic oscillation can be captured by drawing the memory integral in equation (4) that is parallel to an atomic system. For example, take the last term on the right-hand side (RHS) in equation (4), states $\mathbf{k}$ and $\mathbf{k-q}$ can be regarded as two distinct levels of an atom, while the scattering from $\mathbf{k}'$ to $\mathbf{k'+q}$ can be regarded as an applied field at the strength of $V(\mathbf{q})\exp(-i\omega t)$. With the quantum dynamical derivation [38], a Rabi-oscillation-like equation can be obtained as $i\hbar\dot{f}_{\mathbf{k}}=f_{\mathbf{k-q}}V_{\mathbf{k},\mathbf{k-q}}$ and $i\hbar\dot{f}_{\mathbf{k-q}}=f_{\mathbf{k}}V_{\mathbf{k-q},\mathbf{k}}$. Integrating $\dot{f}_{\mathbf{k-q}}$ to time and inputting it into the other equation yields the memory integral of $\dot{f}_{\mathbf{k}}=-\int_0^t dt'\hbar^{-2}|V_{\mathbf{k},\mathbf{k-q}}|^2f_{\mathbf{k}}$. The derived result demonstrates the equivalence between the memory integral and oscillation. Thus the behaviour of the memory integral in equation (4) is essentially equivalent to the carrier-carrier oscillation; however, equation (4) is somewhat complex due to Pauli factors and the summation over momentum space in band structure. The oscillation results from the quantum coherence between carriers. As time passes, the quantum coherence collapses and the quantum process reduces to a monotonic scattering process, which no longer oscillates between the $\mathbf{k}$ and $\mathbf{k-q}$ states, as if an atom stops oscillating when the quantum coherence between the photon and atom collapses and eventually emits (or absorbs) a photon. Similarly, equation (4) indicates an initially spin-coherent carrier oscillates between distinct spin-polarized states within the quantum coherence time due to the memory integral. As the quantum coherence collapses, the carrier stops oscillating and eventually completes spin relaxation process. This oscillation is generic and thus valid for other interactions. The CCS, not Fröhlich interaction, was considered in this section because the exchange energy of CCS can be zero, which favors the oscillation between two distinct spin-polarized states with the same energy.

In addition to the carrier-carrier oscillation, equation (4) shows another oscillation (spin precession) that can be understood from the corresponding homogeneous solution. By inputting $\partial_t f_{ss',\mathbf{k}}(t)$ in equation (4) into ∂_t equation (4), the following equation is generated, $\ddot{f}_{ss}-\frac{\mathrm{Im}\Delta_{12}\mathrm{Im}\Delta_{21}}{\hbar^2}f_{ss}=CT$, where CT is the collision term. As $\Delta_{21}=\Delta^*_{12}$, $\mathrm{Im}\Delta_{21}$ equals $-\mathrm{Im}\Delta_{12}$. Accordingly, the homogeneous solution of equation (4) indicates the spin-polarized distribution function has an oscillating frequency at $\hbar^{-1}|\mathrm{Im}\Delta_{12}|$.

4 KBE for the spin Hall effect

This section describes how the KBE is applied to the SHE. The driven potential energy, $U(\mathbf{r}_1,t_1)=-q_c\mathbf{r}_1\cdot\mathbf{E}$, is considered, where q_c is the charge and $\mathbf{E}$ is an electric field. By applying Wigner transformation to equation (2b), where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_{1'}$, $\tau=t_1-t_{1'}$, $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_{1'})/2$ and $T=(t_1+t_{1'})/2$, the left-hand side (LHS) and RHS of equation (2b) in Fourier domain after transformation under the scalar potential gauge ($\phi = -q_c\mathbf{E}\cdot\mathbf{R}$) and vector potential gauge ($\mathbf{A} = -q_c T\mathbf{E}$) [22], respectively, can be represented as

$$i\left[\hbar\partial_T + q_c\mathbf{E}\cdot\left(\nabla_{\mathbf{k}} + \frac{\hbar\mathbf{k}}{m^*}\partial_\omega\right)\right]\tilde{G}^{\langle}_{ss^{(\prime)}}(\mathbf{k},\omega,T)$$
$$-2i\tilde{G}^{\langle}_{s's^{(\prime)}}(\mathbf{k},\omega,T)\begin{cases}\mathrm{Im}\ \Delta_{12}, s'=\downarrow\\ \mathrm{Im}\ \Delta_{21}, s'=\uparrow\end{cases}$$
$$=\int d\tau d\tau' \exp(i\omega\tau)\sum_{s''=\uparrow,\downarrow}\hat{\mathbf{P}}_{ss''}(\mathbf{k}_{1,2},\tau_{1,2},T_{1,2})$$
$$\times\hat{\mathbf{Q}}_{s''s^{(\prime)}}(\mathbf{k}_{1,2},\tau_{1,2},T_{1,2}), \quad (5)$$

where m^* is the effective mass. Additionally,

$$\hat{\mathbf{P}}_{ss''}(\mathbf{k}_{1,2},\tau_{1,2},T_{1,2})\hat{\mathbf{Q}}_{s''s}(\mathbf{k}_{1,2},\tau_{1,2},T_{1,2})$$
$$\equiv[\hat{\Sigma}^r_{ss''}(\mathbf{k}_1,\tau_1,T_1)\hat{G}^{\langle}_{s''s}(\mathbf{k}_2,\tau_2,T_2)+\hat{\Sigma}^{\langle}_{ss''}(\mathbf{k}_1,\tau_1,T_1)$$
$$\times\hat{G}^a_{s''s}(\mathbf{k}_2,\tau_2,T_2)-\hat{G}^r_{s''s}(\mathbf{k}_1,\tau_1,T_1)\hat{\Sigma}^{\langle}_{ss''}(\mathbf{k}_2,\tau_2,T_2)$$
$$-\hat{G}^{\langle}_{s''s}(\mathbf{k}_1,\tau_1,T_1)\hat{\Sigma}^a_{ss''}(\mathbf{k}_2,\tau_2,T_2)],$$
$$\mathbf{k}_{1,2}\equiv\mathbf{k}+\frac{q}{2\hbar}\mathbf{E}(\tau'\pm\frac{\tau}{2}),\tau_{1,2}\equiv\frac{\tau}{2}\mp\tau',\tau'\equiv t_2-T$$

and $T_{1,2}\equiv T\pm\tau_{2,1}$. Average-space ($\mathbf{R}$) dependence is omitted and considered in the next section. By solving the Dyson equation, the retarded Green function can be obtained.

$$\tilde{G}^r_{ss}(\mathbf{k},\omega)\approx\frac{1}{\hbar\omega-e_{\mathbf{k}}-\mathrm{Re}\ \sigma^r_{ss}-i\mathrm{Im}\ \sigma^r_{ss}}-\frac{q_c^2\hbar^2E^2}{4m^*\left(\hbar\omega-e_{\mathbf{k}}-\mathrm{Re}\ \sigma^r_{ss}-i\mathrm{Im}\ \sigma^r_{ss}\right)^4}, \quad (6a)$$

$$\tilde{G}^r_{ss'}(\mathbf{k},\omega)\approx-\frac{q_c^2\hbar^2E^2}{m^*\left(\hbar\omega-e_{\mathbf{k}}-\mathrm{Re}\ \sigma^r_{s's'}-i\mathrm{Im}\ \sigma^r_{s's'}\right)^5}\times\begin{cases}\mathrm{Re}\ \Delta_{21}, & s'=\uparrow\\ \mathrm{Re}\ \Delta_{12}, & s'=\downarrow,\end{cases} \quad (6b)$$

where E is $|\mathbf{E}|$ and σ^r_{ss} denotes the equilibrium retarded self energy. The detailed derivation is shown in Appendix C.

By applying Taylor expansion to the electric field in equation (5) and approximating $\tilde{G}^{\langle}(\mathbf{k},\omega)$ as $\tilde{g}^{\langle}(\mathbf{k},\omega)+E\tilde{G}^{\langle(1)}(\mathbf{k},\omega)+E^2\tilde{G}^{\langle(2)}(\mathbf{k},\omega)$, the first-order KBE for the SHE can then be obtained based on the perturbation method.

$$\frac{q_c}{2\hbar}\tilde{a}^2_{ss}\partial_\omega n_F\hat{\boldsymbol{\varepsilon}}\cdot\left(\tilde{\xi}_{ss}\nabla_{\mathbf{k}}\tilde{\gamma}_{ss}-\tilde{\gamma}_{ss}\nabla_{\mathbf{k}}\tilde{\xi}_{ss}\right)+2i\tilde{G}^{\langle(1)}_{s's}(\mathbf{k},\omega)$$
$$\times\begin{cases}\mathrm{Im}\ \Delta_{12}, s'=\downarrow\\ \mathrm{Im}\ \Delta_{21}, s'=\uparrow\end{cases}=i[\tilde{\gamma}_{ss}\tilde{G}^{\langle(1)}_{ss}(\mathbf{k},\omega)-\tilde{a}_{ss}\tilde{\Sigma}^{\langle(1)}_{ss}(\mathbf{k},\omega)], \quad (7a)$$

$$\tilde{G}^{\langle(1)}_{s's'}(\mathbf{k},\omega)\begin{cases}2\mathrm{Im}\ \Delta_{12}, s'=\downarrow\\ 2\mathrm{Im}\ \Delta_{21}, s'=\uparrow\end{cases}=\tilde{\gamma}_{ss}\tilde{G}^{\langle(1)}_{ss'}(\mathbf{k},\omega)-\tilde{a}_{s's'}\tilde{\Sigma}^{\langle(1)}_{ss'}(\mathbf{k},\omega), \quad (7b)$$

where n_F is Fermi-Dirac distribution. Additionally, $\hat{\boldsymbol{\varepsilon}}\equiv\mathbf{E}/E$, $\tilde{\xi}_{ss}(\mathbf{k},\omega)\equiv\hbar\omega-e_{\mathbf{k}}-\tilde{\sigma}_{ss}(\mathbf{k},\omega)$, $\tilde{\sigma}_{ss}(\mathbf{k},\omega)\equiv[\tilde{\sigma}^r_{ss}(\mathbf{k},\omega)+\tilde{\sigma}^a_{ss}(\mathbf{k},\omega)]/2$, $\tilde{a}_{ss}(\mathbf{k},\omega)\equiv i[\tilde{g}^r_{ss}(\mathbf{k},\omega)-\tilde{g}^a_{ss}(\mathbf{k},\omega)]$, and $\tilde{\gamma}_{ss}(\mathbf{k},\omega)\equiv i[\tilde{\sigma}^r_{ss}(\mathbf{k},\omega)-\tilde{\sigma}^a_{ss}(\mathbf{k},\omega)]$. Note that (non-)equilibrium Green function or (non-) equilibrium self energy is written in a (capital) lower letter in this paper. The second-order KBE for the SHE is shown in Appendix D, where an algorithm for solving the KBE up to the 2nd order solution is provided. In this report, only the 1st order solution is demonstrated.

For an impurity interaction, the lowest-order self energy is given by $N_i\Omega\sum_{\mathbf{q}}[|V(\mathbf{q})|^2\tilde{G}^{\langle,r}_{s_1s_2}(\mathbf{k}-\mathbf{q},\omega)]$, where N_i is the impurity density, Ω is sample volume and $V(\mathbf{q})$ is the Coulomb interaction. Thus $\tilde{\sigma}^r_{ss(e-imp)}(\mathbf{k},\omega)$ can be rearranged as $\sum_{\mathbf{q}}\frac{N_i\Omega|V(\mathbf{q})|^2}{\hbar\omega-e_{\mathbf{k-q}}+i\hbar\tau^{-1}_{ss(e-imp)}}$, where $\tau_{ss(e-imp)}$ is the spin-electron impurity scattering time.

For an electron-phonon interaction, the lowest-order lesser (retarded) self energy is given by

$$\frac{i\hbar}{2\pi}\sum_{\mathbf{q}}\int d\omega'\,|M_{\mathbf{q}}|^2[G^{\langle(\langle)}_{s_1s_2}(\mathbf{k}-\mathbf{q},\omega-\omega')D^{\langle(r)}_{s_1s_2}(\mathbf{q},\omega')$$
$$+G^{(r)}_{s_1s_2}(\mathbf{k}-\mathbf{q},\omega-\omega')D^{(\langle)}_{s_1s_2}(\mathbf{q},\omega')$$
$$+G^{(r)}_{s_1s_2}(\mathbf{k}-\mathbf{q},\omega-\omega')D^{(r)}_{s_1s_2}(\mathbf{q},\omega')],$$

where $|M_{\mathbf{q}}|^2$ and $D_{s_1s_2}(\mathbf{q},\omega')$ are the electron-phonon interaction strength and phonon's propagator, respectively. Then, $\tilde{\Sigma}^{\langle}_{s_1s_2(e-ph)}(\mathbf{k},\omega)$ can be rearranged as

$$\sum_{\mathbf{q},\pm}|M_{\mathbf{q}}|^2\left\{\left[N_{ph}(\mathbf{q})+\frac{1}{2}\pm\frac{1}{2}\right]\tilde{G}^{\langle}_{s_1s_2}(\mathbf{k}-\mathbf{q},\omega\pm\omega_{\mathbf{q}})\right\},$$

where $N_{ph}(\mathbf{q})$ is the phonon number, and $\tilde{\sigma}^r_{ss(e-ph)}(\mathbf{k},\omega)$ is rearranged as $\sum_{\mathbf{q},\pm}|M_{\mathbf{q}}|^2\ \frac{N_{ph}(\mathbf{q})\pm 1+n_F(e_{\mathbf{k-q}})}{\hbar(\omega\pm\omega_{\mathbf{q}})-e_{\mathbf{k-q}}+i\hbar\tau^{-1}_{ss(e-ph)}}$, where $\tau_{ss(e-ph)}$ is the spin-electron phonon scattering time.

To simplify the calculation of self energy, the interaction strength in momentum space, $|\widehat{M}_{\mathbf{q}}|^2$, is assumed as $\widehat{M}_0^2\delta(\mathbf{q})$ called a delta interaction approximation later, where $\left|\widehat{M}_{\mathbf{q}}\right|^2$ for the electron-phonon and impurity interactions is $|M_{\mathbf{q}}|^2$ and $|V(\mathbf{q})|^2$, respectively. Under the approximation, equations (7a) and (7b) can be rewritten as

$$-\frac{q_c}{2\hbar}\tilde{a}^2_{\uparrow\uparrow}\partial_\omega n_F\left(\tilde{\xi}_{\uparrow\uparrow}\partial_{k_y}\tilde{\gamma}_{\uparrow\uparrow}-\tilde{\gamma}_{\uparrow\uparrow}\partial_{k_y}\tilde{\xi}_{\uparrow\uparrow}\right)$$
$$+2\mathrm{Im}\ \Delta_{12}(\mathbf{k})\mathrm{Im}\ \tilde{G}^{\langle(1)}_{\downarrow\uparrow}(\mathbf{k})=\tilde{\gamma}_{\uparrow\uparrow}\mathrm{Im}\ \tilde{G}^{\langle(1)}_{\uparrow\uparrow}(\mathbf{k})$$
$$-\frac{A\widehat{M}_0^2N_\alpha}{4\pi^2}\tilde{a}_{\uparrow\uparrow}\mathrm{Im}\ \tilde{G}^{\langle(1)}_{\uparrow\uparrow}(\mathbf{k}), \quad (8a)$$

$$2\mathrm{Im}\ \Delta_{21}(\mathbf{k})\mathrm{Im}\ \tilde{G}^{\langle(1)}_{\uparrow\uparrow}(\mathbf{k})=$$
$$\tilde{\gamma}_{\downarrow\downarrow}\mathrm{Im}\ \tilde{G}^{\langle(1)}_{\downarrow\uparrow}(\mathbf{k})-\frac{A\widehat{M}_0^2N_\alpha}{4\pi^2}\tilde{a}_{\uparrow\uparrow}\mathrm{Im}\ \tilde{G}^{\langle(1)}_{\downarrow\uparrow}(\mathbf{k}), \quad (8b)$$

$$\mathrm{Im}\ \tilde{G}^{\langle(1)}_{\uparrow\uparrow}(\mathbf{k}) = \frac{-\frac{q_c}{2\hbar}\tilde{a}^2_{\uparrow\uparrow}\partial_\omega n_F\left(\tilde{\xi}_{\uparrow\uparrow}\partial_{k_y}\tilde{\gamma}_{\uparrow\uparrow} - \tilde{\gamma}_{\uparrow\uparrow}\partial_{k_y}\tilde{\xi}_{\uparrow\uparrow}\right)\left[\tilde{\gamma}_{\downarrow\downarrow} - \frac{A\widehat{M}_0^2 N_\alpha}{4\pi^2}\tilde{a}_{\uparrow\uparrow}\right]}{\left[\tilde{\gamma}_{\uparrow\uparrow} - \frac{A\widehat{M}_0^2 N_\alpha}{4\pi^2}\tilde{a}_{\uparrow\uparrow}\right]\left[\tilde{\gamma}_{\downarrow\downarrow} - \frac{A\widehat{M}_0^2 N_\alpha}{4\pi^2}\tilde{a}_{\uparrow\uparrow}\right] - 4\mathrm{Im}\ \Delta_{12}(\mathbf{k})\mathrm{Im}\ \Delta_{21}(\mathbf{k})}. \tag{9}$$

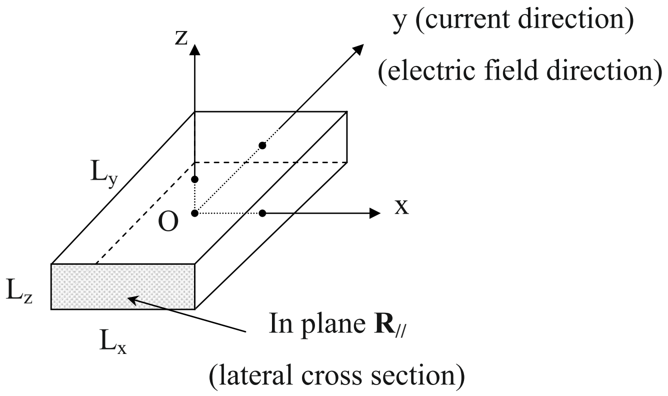


Fig. 1. Electrically-biased sample's coordinate and geometry for the SHE.

where the sample's coordinate is defined in Figure 1, and A denotes sample's lateral area ($L_x L_z$) with respect to the electric field. Notably, N_α is equal to $N_i\Omega$ and $2N_{ph}(0)+1$ for impurity interactions and electron-phonon interactions, respectively.

Equations (8a) and (8b) can easily derive Im $\tilde{G}^{\langle(1)}_{\uparrow\uparrow}(\mathbf{k})$, which is shown as

See equation (9) above.

Via equation (9), one can understand why and when the SHE occurs. A conventional definition of the SC as $-\sum_{\mathbf{k}}\frac{\hbar k_x}{m^*}\mathrm{Im}\ \tilde{G}^{\langle}_{\uparrow\uparrow}(\mathbf{k})E_y$ is used, where the SC arises unless the Fermi sphere of spin species shifts toward the direction perpendicular to the applied field, i.e. Im $\tilde{G}^{\langle(1)}_{\uparrow\uparrow}(\mathbf{k})$ is an asymmetric function of k_x. Since all equilibrium functions and their derivatives with respect to k_y in equation (9) are k_x symmetric, SOC term $\Delta_{ij}(\mathbf{k})$ becomes the unique factor in generating the SC. From Appendix A, Im $\tilde{G}^{\langle(1)}_{\uparrow\uparrow}(\mathbf{k})$ remains k_x symmetric when either BIA or SIA is included in the SOC term except the fact that both BIA and SIA are considered. Accordingly, equation (9) indicates that the SC can only appear in a QW and vanishes in bulk. The prediction is against the observation of the SHE in bulk [39]. This is probably due to the delta interaction approximation used in our calculation. If the interaction strength has a slight broadening in momentum space, a non-zero SC may appear in bulk. Otherwise, an earlier theoretical report based on the Keldysh formalism [40] shows a vanishing SC in bulk, which is the same as our prediction. As for the QW, the SC has a nonzero value even under the delta interaction approximation. The SHE of two-dimensional (2D) electrons [41] and 2D holes [42] in GaAs has been experimentally demonstrated.

In addition to the SOC requirement, the SC between two spin species must be distinct either in direction or magnitude, or their spin fluxes cannot be distinguished experimentally. Thus the differential SC (DSC), which is defined as $J^s_{x,\uparrow\uparrow} - J^s_{x,\downarrow\downarrow}$, is useful. Whether the DSC exists depends on the electron-impurity (-phonon) scattering rate. If the scattering rate varies with the spin species, the DSC exists; otherwise, the DSC goes to zero. In the latter case, although the DSC vanishes, the sum of the SC as $J^s_{x,\uparrow\uparrow} + J^s_{x,\downarrow\downarrow}$ survives, implying that a transverse electric current likely exists. According to equation (9), the flux of electrons is in the direction opposite that of holes and the electrical current caused by the opposite charges will not cancel each other. Therefore, the derived result reveals the existence of Hall current without any external magnetic field except for the DP field, which is self-induced due to the BIA and SIA. In contrast, using the Keldysh formalism in the quasiclassical approximation [26], the Hall current due to the SOC can occur only when the ferromagnetic contact is present.

The work now calculates the SC in a 10 nm-wide GaAs/$Al_{0.3}Ga_{0.7}$ As QW at sheet densities (n_{2D}) of 10^{10} cm^{-2} and 10^{11} cm^{-2} at room temperature, and compares the SC of the impurity and Fröhlich interactions. The extrinsic ($\widehat{M}_0 \neq 0$) and intrinsic ($\widehat{M}_0 = 0$) SCs are evaluated for each case. The extrinsic SC stands for the calculation with both non-equilibrium and equilibrium self energies present (diffusive regime), whereas the intrinsic SC stands for the calculation with equilibrium self energy only (ballistic regime). For impurity interactions, complete ionization (i.e., $n_{2D} = N_i L$) and Brooks- Herring approximation for calculating the electron-impurity scattering time are assumed, where

$$\tau_{ss(e-imp)} = \frac{2^{4.5}\pi\varepsilon^2_{\mathrm{GaAs}}\sqrt{m^*}e^{1.5}_{\mathbf{k}}}{q_c^4 Z N_i\left\{\ln\left[1+8m^*e_k/(\hbar^2 q_s^2)\right] - \left[1+(\hbar^2 q_s^2/(8m^*e_{\mathbf{k}}))\right]^{-1}\right\}}$$

with q_s being $2m^* q_c^2 n_F(\mathbf{k}=0)/\left(\varepsilon_{\mathrm{GaAs}}\hbar^2\right)$, $\varepsilon_{\mathrm{GaAs}}$ being the GaAs dielectric constant, $Z = 14$ for silicon impurity, and L being the width of QW. For Fröhlich interaction (polar optical phonon abbreviated as POP), two symmetric interface phonon modes and the lowest confined phonon mode based on the dielectric continuum model [43–45] are considered. The average electron-POP scattering time $\tau_{ss(e-ph)}$ of 165 fs was used. The SC shown in following figures is multiplied by an electron's charge.

Figures 2a and 2b show the extrinsic and intrinsic SCs caused by impurity, respectively. The extrinsic and intrinsic SCs are in the reverse direction and the intrinsic SC is significantly higher than the extrinsic SC. At a carrier density of 10^{11} (10^{10}) cm^{-2}, the intrinsic SC is even ten-(several) thousand times higher than the extrinsic SC. In

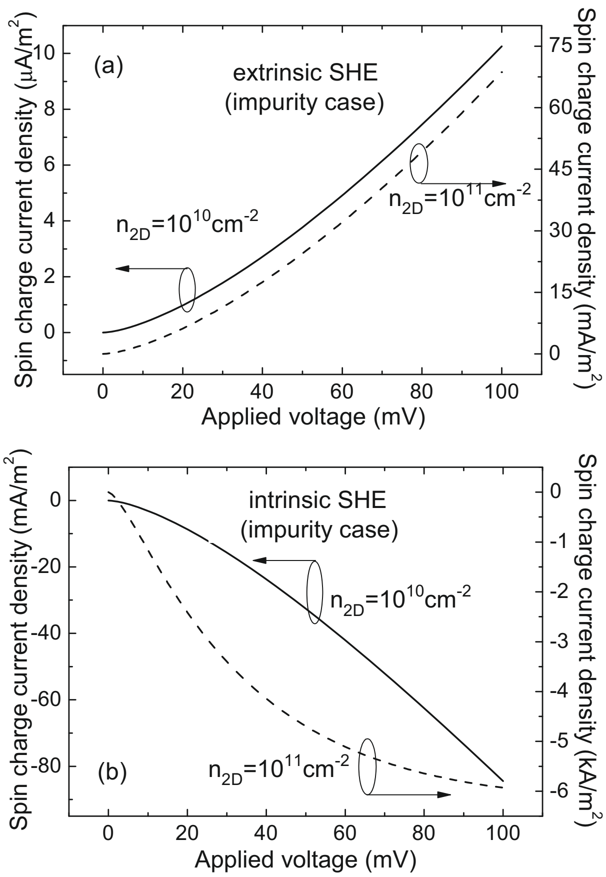


Fig. 2. The first-order impurity-induced spin charge current density for (a) the extrinsic SHE and (b) the intrinsic SHE in a 10 nm-wide GaAs/$Al_{0.3}Ga_{0.7}$As QW at room temperature within the complete ionization and Brooks-Herring approximations.

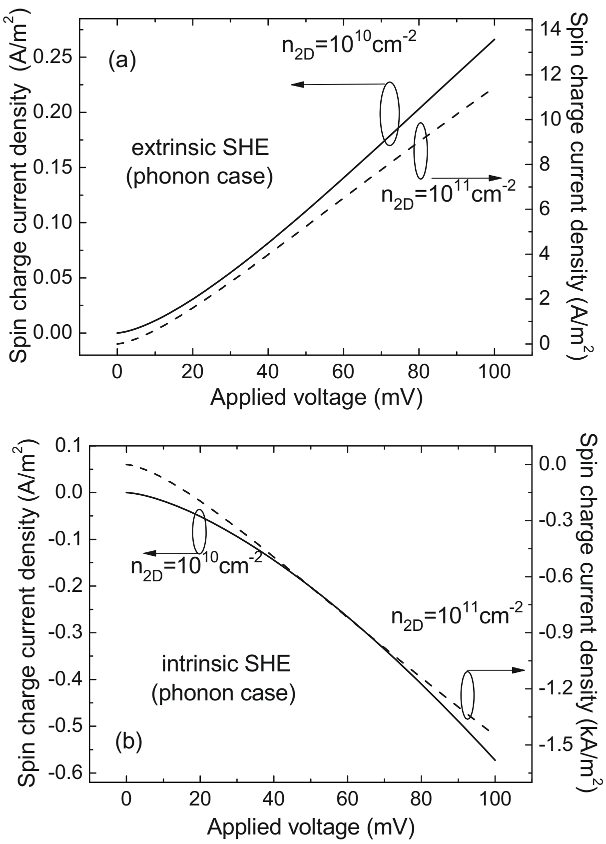


Fig. 3. The first-order POP-induced spin charge current density for (a) the extrinsic SHE and (b) the intrinsic SHE in a 10 nm-wide GaAs/$Al_{0.3}Ga_{0.7}$As QW at room temperature based on the dielectric continuum model, where the two symmetric interface phonon mode and the lowest confined phonon mode are included.

clean limit the spin Hall conductance is due to the intrinsic SC. Once the impurity exists, the intrinsic SC replaced with the extrinsic SC therefore results in a significant reduction of the SC by defect scattering as described in the Inoue's report [24]. Furthermore, the SC, especially the intrinsic SC, is extremely sensitive to the carrier density. While the extrinsic SC at a carrier density of 10^{11} cm^{-2} is 6000 times higher than the extrinsic SC at 10^{10} cm^{-2}, the intrinsic SC at a carrier density of 10^{11} cm^{-2} is several 10 000 times higher than the intrinsic SC at a density of 10^{10} cm^{-2}. The considerable density dependence results from Im $\tilde{\sigma}^r_{ss(e-imp)}$, which is proportional to $N_i \tau^{-1}_{ss(e-imp)}$ when $\hbar\omega - e_{\mathbf{k}} \gg \hbar\tau^{-1}_{ss(e-imp)}$. Due to $\tau^{-1}_{ss(e-imp)} \propto N_i$, Im $\tilde{\sigma}^r_{ss(e-imp)}$ has the N_i^2 dependence. $\tilde{\gamma}_{ss}$ and $\tilde{a}_{ss}$ relates to Im $\tilde{\sigma}^r_{ss}$. $\tilde{\gamma}_{ss} = -2\text{Im}\ \tilde{\sigma}^r_{ss}$, and $\tilde{a}_{ss} \propto \text{Im}\ \tilde{\sigma}^r_{ss}$ when $\hbar\omega - e_{\mathbf{k}} - \text{Re}\ \sigma^r_{ss} \gg \text{Im}\ \sigma^r_{ss}$. Thus $\tilde{\gamma}_{ss}$ and $\tilde{a}_{ss}$ also has the N_i^2 dependence. By canceling N_i factors between the numerator and denominator in equation (9), Im $\tilde{G}^{\langle(1)}_{\uparrow\uparrow}(\mathbf{k})$ for the intrinsic and extrinsic cases can be shown to depend on $\tilde{a}^2_{\uparrow\uparrow}\partial_\omega n_F \propto N_i^5$ and $\tilde{a}^2_{\uparrow\uparrow} \propto N_i^4$, respectively. (Note that $n_{2D} = N_i$.) Thus the max. ratio of the SC between 10^{10} and 10^{11} cm^{-2} can reach to several 10 000 times.

Figures 3a and 3b show the extrinsic and intrinsic SCs caused by the POP, respectively. Like the impurity case, the direction of the extrinsic SC is opposite that of the intrinsic SC, and the intrinsic SC is higher than the extrinsic SC; however, the ratio between the intrinsic SC and extrinsic SC is not so considerable. At a carrier density of 10^{11} (10^{10}) cm^{-2}, the intrinsic SC is approximately 130 (2) times higher than the extrinsic SC. The density dependence of SC is strong, but also not as strong as that of impurity interaction. While the extrinsic SC at a carrier density of 10^{11} cm^{-2} is roughly 40 times higher than the extrinsic SC at 10^{10} cm^{-2}, the intrinsic SC at a carrier density of 10^{11} cm^{-2} is almost 2500 times higher than the intrinsic SC at 10^{10} cm^{-2}. This is because $\tau_{ss(e-ph)}$ does not have the n_{2D} dependence. Thus Im $\tilde{\sigma}^r_{ss(e-ph)}$ as well as $\tilde{\gamma}_{ss}$ and $\tilde{a}_{ss}$ is only proportional to n_{2D}. Furthermore, unlike the impurity case, N_α is independent on the n_{2D}. Therefore, by canceling n_{2D} factors in equation (9), Im $\tilde{G}^{\langle(1)}_{\uparrow\uparrow}(\mathbf{k})$ for both intrinsic and extrinsic cases shows the dependence of $\tilde{a}^2_{\uparrow\uparrow}\partial_\omega n_F \propto n^3_{2D}$. As a result, the maximum ratio of the SC between 10^{10} and 10^{11} cm^{-2} is about 1000 times.

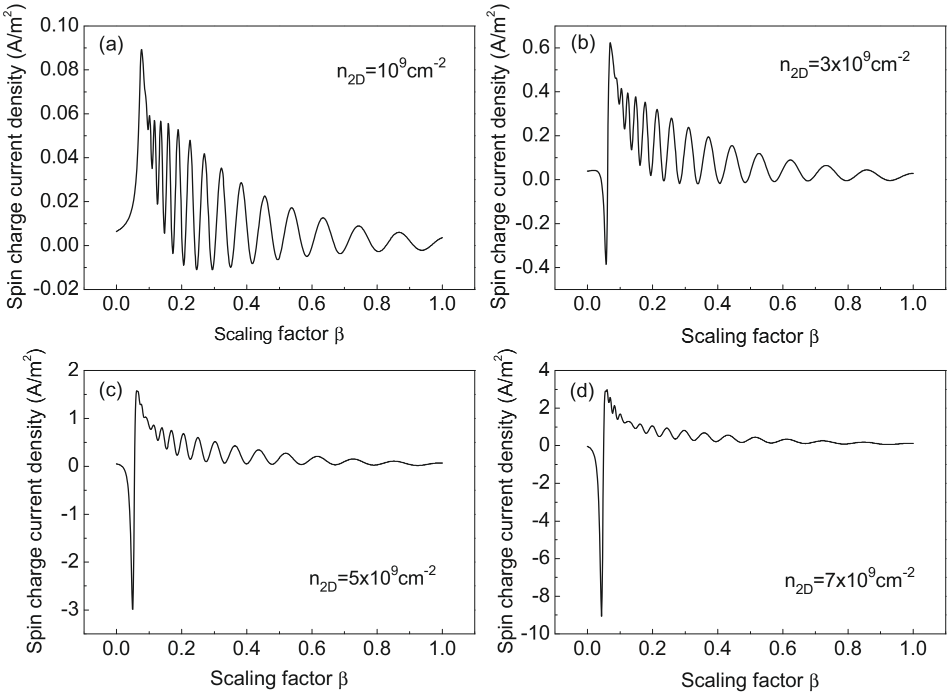


Fig. 4. Spin charge current density caused by Fröhlich interaction as a function of scaling factor at sheet carrier densities of (a) 10^9 (b) 3×10^9 (c) 5×10^9 (d) 7×10^9 cm^{-2}.

Generally, the SC caused by the POP is much higher than the SC caused by impurity. For example, the ratio of the extrinsic POP-induced SC to the extrinsic impurity-induced SC is close to 25 000(160) at a carrier density of 10^{10} (10^{11}) cm^{-2}. The exception is that the intrinsic POP-induced SC becomes one-fourth of the intrinsic impurity-induced SC when the density is at 10^{11} cm^{-2}. Excluding the case, the POP- induced SC is always dominant; however, the importance of Fröhlich interaction on the SHE has not been described. If the spin effect on the electron-POP scattering rate $\tau^{-1}_{ss(e-ph)}$ is also strong, a considerable POP-induced DSC exists, further increasing the importance of Fröhlich interaction on the application of devices based on the SHE.

Additionally, Fröhlich interaction leads to a unique oscillatory behaviour in the SC that does not appear with the impurity-induced SC. By introducing the scaling factor β, i.e., $\widehat{M}_0'^2 \equiv \beta\widehat{M}_0^2$ (0, intrinsic SC; 1, extrinsic SC), Figures 4a–4d show the POP-induced SC as a function of the scaling factor at carrier densities of 10^9, 3×10^9, 5×10^9 and 7×10^9 cm^{-2}, respectively, where oscillation as a function of the scaling factor is shown. The scaling factor can result from Coulomb screening effect. As the carrier density increases, the oscillation weakens and eventually disappears. Notably, the oscillatory behaviour is dependent on the scaling factor. While the oscillatory amplitude shows a modulation as a quantum beat, oscillatory frequency gets lower as the scaling factor increases. Interestingly, the phonon number (N_α) has the same mathematical role in equation (9) as the scaling factor. Therefore, the oscillatory behaviour can also appear in the SC-voltage curve because of the temperature dependence of phonon excitation number, which is associated with an applied voltage.

5 KBE for spin accumulation

This section is to derive the spatially dependent KBE for spin accumulation, where the in-plane ($\mathbf{R}_\parallel$) dependence is considered. With the Neumann boundary condition, the lesser Green function $\tilde{G}^\langle(\mathbf{k},\omega,\mathbf{R}_\parallel)$ can be expressed as $\frac{1}{\sqrt{L_x L_z}}\sum_{\mathbf{K}_\parallel}\tilde{G}^\langle(\mathbf{k},\omega,\mathbf{K}_\parallel)\exp[i\mathbf{K}_\parallel\cdot(\mathbf{R}_\parallel-\mathbf{L}_\parallel/2)]$, where $\mathbf{K}_\parallel=\pi\left(\frac{n_x}{L_x},\frac{n_z}{L_z}\right)$ with $n_{x,z}$ being an integer and $\mathbf{L}_\parallel$ being (L_x,L_z). With the Fourier expansion, the LHS of equation (2b), i.e., the driving term (DT) of spatially-dependent KBE, after Wigner transformation and Fourier transform under the scalar gauge can be written as

$$DT_{ss^{(\prime)}}=\frac{1}{\sqrt{L_xL_z}}\sum_{\mathbf{K}_\parallel}\exp[i\mathbf{K}_\parallel\cdot(\mathbf{R}_\parallel-\mathbf{L}_\parallel/2)] \times\left\{\left[i\hbar\partial_T-\frac{\hbar^2}{m^*}\mathbf{k}\cdot\mathbf{K}_\parallel+iq_c\mathbf{E}\cdot\left(\nabla_\mathbf{k}+\frac{\hbar\mathbf{k}}{m^*}\partial_\omega\right)\right]\right. \times\tilde{G}^\langle_{ss^{(\prime)}}(\mathbf{k},\omega,\mathbf{K}_\parallel,T)-2i\tilde{G}^\langle_{s's^{(\prime)}}(\mathbf{k},\omega,\mathbf{K}_\parallel,T) \times\left.\begin{Bmatrix}\mathrm{Im}\,\Delta_{12},\ s'=\downarrow\\ \mathrm{Im}\,\Delta_{21},\ s'=\uparrow\end{Bmatrix}\right\}. \tag{10}$$

The RHS of equation (2b), i.e. the collision term (CT), becomes difficult to transform to the $(\mathbf{k},\omega)$ domain when the average space $\mathbf{R}$ dependence involves due to the spatial entanglement in $\int d\mathbf{r}_2dt_2\{\Sigma[\mathbf{r}_1-\mathbf{r}_2,\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2),t_1,t_2]$

$G[\mathbf{r}_2 - \mathbf{r}_{1'}, \frac{1}{2}(\mathbf{r}_2 + \mathbf{r}_{1'}), t_2, t_{1'}]\}$. However, the Fourier expansion for the average space $\mathbf{R}$ is efficient to eliminate the entanglement. Hence, the CT can be successfully transformed to the $(\mathbf{k}, \omega)$ domain.

$$CT_{ss^{(\prime)}}(\mathbf{k},\omega,\mathbf{R}_\parallel,T) = \frac{1}{L_x L_z}\sum_{\mathbf{K}_\parallel}\sum_{\mathbf{K}'_\parallel}\exp[i\mathbf{K}_\parallel\cdot(\mathbf{R}_\parallel - \mathbf{L}_\parallel/2)] \times \int d\tau d\tau' \exp(i\omega\tau) \sum_{s''=\uparrow,\downarrow} \hat{\mathbf{P}}_{ss''}\left(\mathbf{k}'_{1,2}, \tau_{1,2}, \mathbf{K}_{\parallel 1,2}, T_{1,2}\right) \times \hat{\mathbf{Q}}_{s''s^{(\prime)}}\left(\mathbf{k}'_{1,2}, \tau_{1,2}, \mathbf{K}_{\parallel 1,2}, T_{1,2}\right), \quad (11)$$

where $\mathbf{k}'_{1,2} = \mathbf{k}_{1,2} \pm \frac{1}{2}\mathbf{K}_{\parallel 2,1}$. $\mathbf{K}_{\parallel 1,2} = \frac{1}{2}(\mathbf{K}_\parallel \pm \mathbf{K}'_\parallel)$. The detailed derivation is shown in Appendix E.

Hence, the first and the second order $\mathbf{R}_\parallel$-dependent KBEs can be determined and shown as, respectively,

$$q_c\hat{\boldsymbol{\varepsilon}}\cdot\left(\nabla_\mathbf{k} + \frac{\hbar\mathbf{k}}{m^*}\partial_\omega\right)\tilde{g}^{\langle}_{ss}\delta_{ss^{(\prime)}} + \frac{i\hbar^2}{m^*}\mathbf{k}\cdot\mathbf{K}_\parallel\tilde{G}^{\langle(1)}_{ss^{(\prime)}}(\mathbf{k},\omega,\mathbf{K}_\parallel) - 2\tilde{G}^{\langle(1)}_{s's^{(\prime)}}(\mathbf{k},\omega,\mathbf{K}_\parallel)\begin{cases}\mathrm{Im}\,\Delta_{12},\ s'=\downarrow\\ \mathrm{Im}\,\Delta_{21},\ s'=\uparrow\end{cases} - \frac{q_c}{2\hbar}\hat{\boldsymbol{\varepsilon}}\cdot\frac{\delta_{ss^{(\prime)}}}{\sqrt{L_x L_z}} \times\sum_{\mathbf{K}'_\parallel}\left[\partial_\omega\tilde{\mathbf{P}}^{eq}_{ss}\nabla_\mathbf{k}\tilde{\mathbf{Q}}^{eq}_{ss} - \nabla_\mathbf{k}\tilde{\mathbf{P}}^{eq}_{ss}\partial_\omega\tilde{\mathbf{Q}}^{eq}_{ss}\right] = \frac{-i}{\sqrt{L_x L_z}} \times\sum_{\mathbf{K}'_\parallel}\sum_{s''=\uparrow,\downarrow}\left[\tilde{\mathbf{P}}_{ss''}(\mathbf{k}'_{1,2},\omega,\mathbf{K}_{\parallel 1,2}) \times\tilde{\mathbf{Q}}_{s''s^{(\prime)}}(\mathbf{k}'_{1,2},\omega,\mathbf{K}_{\parallel 1,2})\right]_1, \quad (12a)$$

$$q_c\hat{\boldsymbol{\varepsilon}}\cdot\left(\nabla_\mathbf{k} + \frac{\hbar\mathbf{k}}{m^*}\partial_\omega\right)\tilde{G}^{\langle(1)}_{ss^{(\prime)}}(\mathbf{k},\omega,\mathbf{K}_\parallel) + \frac{i\hbar^2}{m^*}\mathbf{k}\cdot\mathbf{K}_\parallel\tilde{G}^{\langle(2)}_{ss^{(\prime)}}(\mathbf{k},\omega,\mathbf{K}_\parallel) - 2\tilde{G}^{\langle(2)}_{s's^{(\prime)}}(\mathbf{k},\omega,\mathbf{K}_\parallel)\begin{cases}\mathrm{Im}\,\Delta_{12},\ s'=\downarrow\\ \mathrm{Im}\,\Delta_{21},\ s'=\uparrow\end{cases} - \frac{iq_c^2}{8\hbar^2}\frac{\delta_{ss^{(\prime)}}\hat{\boldsymbol{\varepsilon}}\cdot}{\sqrt{L_x L_z}} \times\sum_{\mathbf{K}'_\parallel}\left[\partial^2_\omega\tilde{\mathbf{P}}^{eq}_{ss}\partial^2_\mathbf{k}\tilde{\mathbf{Q}}^{eq}_{ss} - \partial^2_\mathbf{k}\tilde{\mathbf{P}}^{eq}_{ss}\partial^2_\omega\tilde{\mathbf{Q}}^{eq}_{ss}\right] = \frac{-i}{\sqrt{L_x L_z}}\sum_{\mathbf{K}'_\parallel}\sum_{s''=\uparrow,\downarrow}\left[\tilde{\mathbf{P}}_{ss''}(\mathbf{k}'_{1,2},\omega,\mathbf{K}_{\parallel 1,2}) \times\tilde{\mathbf{Q}}_{s''s^{(\prime)}}(\mathbf{k}'_{1,2},\omega,\mathbf{K}_{\parallel 1,2})\right]_2, \quad (12b)$$

where the subscript (1,2) in the middle bracket denotes the order expansion with respect to the electric field. The retarded Green function (retarded self energy) shown in equations (C.8a) and (C.8b) can be input into the CT on the RHS of $\mathbf{R}_\parallel$-dependent KBEs.

The $\mathbf{R}_\parallel$-dependent KBE is important to the study of the SHE because detection of spin flux density is still a major restriction for current measurements; however, spin accumulation can be verified experimentally using Kerr spectroscopy [12]. Since the KBE is a very general approach, the issue of spin accumulation in a ballistic regime [18,19] or a nanometer scale where Boltzmann theory no longer fits can be governed by the KBE. In addition to spin applications, the analytic KBE is especially important for spatial quantum kinetic effects, which have been still less understood because the analytic spin-independent, spatially-dependent KBE has not been derived before this work. Based on equations (12a) and (12b), spatial quantum kinetic effects such as momentum non-conservation and spatial coherence [46,47] can be studied and compared with temporal quantum kinetic effects involving the energy non-conservation [6] and memory effect [7,8].

6 Conclusion

This work presents a general quantum kinetic theory of spin dynamics, in which the KBE is applied to spin relaxation and the SHE. First, the equation governing the time evolution of spin relaxation via the DP magnetic field among non-equilibrium CCS was constructed. Quantum kinetic oscillation between distinct spin-polarized states within the quantum coherence time, as well as SOC-induced oscillation, was identified. Second, the quantum transport equation of the SHE in the presence of impurity and Fröhlich interactions was formulated. The equation can interpret why the SHE exists and when the SC is no longer zero. Furthermore, the numerical results indicate that the SC is very sensitive to impurity density, while Fröhlich interaction can result in a considerable SC and lead to a unique oscillation in the SC. Finally, the $\mathbf{R}_\parallel$-dependent KBE for spin accumulation was derived, and is especially useful for exploring spatial quantum kinetic effects.

The author Lee would like to thank Prof. S.A. Lyon's enlightenment for the work. The work was financially supported by the project of National Science Council under grant number NSC 95-2112-M-007-013.

Appendix A: SOC terms in DP mechanism [48]

The DP Hamiltonian is given by $H'_{DP} = \hbar\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}(\mathbf{k})$, where $\boldsymbol{\sigma}$ is the Pauli matrix,

$$\boldsymbol{\Omega}_{BIA}(\mathbf{k}) = \frac{\gamma}{\hbar}\begin{pmatrix} k_x(k_y^2 - k_z^2)\\ k_y(k_z^2 - k_x^2)\\ k_z(k_x^2 - k_y^2)\end{pmatrix}$$

and

$$\boldsymbol{\Omega}_{SIA}(\mathbf{k}_\parallel) = \frac{\alpha}{\hbar}\begin{pmatrix} k_y\\ -k_x\\ 0\end{pmatrix}$$

are the effective magnetic field for the bulk inversion asymmetry (BIA) and the surface inversion asymmetry (SIA), respectively. In GaAs, the BIA coefficient γ equals 27 eÅ^3 independent of carrier density and the Rashba coefficient

α decreases linearly from -10^{-4} eV Åat the sheet density of 10^{11} cm^{-2} to -5×10^{-3} eV Åat n_{2D} of 8×10^{11} cm^{-2}.

The SOC term in bulk can therefore be represented as $\Delta_{11}=\Delta_{22}=0$ and $\Delta_{12}=\Delta_{21}^{*}=\gamma k_x(k_y^2-k_z^2)-i\gamma k_y(k_z^2-k_x^2)$. In a QW,

$$\mathbf{\Omega}=\frac{\gamma}{\hbar}\begin{pmatrix}k_x(k_x^2-\langle k_z^2\rangle)\\ k_y(\langle k_z^2\rangle-k_x^2)\\ 0\end{pmatrix}+\mathbf{\Omega}_{SIA}(\mathbf{k}_{\|}),$$

where

$$\langle k_z^2\rangle=\frac{1}{4}\left(\frac{16.5\pi\, m^*\, n_{2D}e^2}{\hbar^2\varepsilon_\infty}\right)^{2/3}$$

and ε is the high-frequency dielectric constant. The SOC diagonal elements in a QW remain zero while the off-diagonal can be represented as $\Delta_{12}=\Delta_{21}^{*}=\gamma k_x(k_y^2-\langle k_z^2\rangle)-i\gamma k_y(\langle k_z^2\rangle-k_x^2)+\alpha k_y+i\alpha k_x$.

Appendix B: Recovery of compact definitions and equations

For clarity, the definitions in Section 2 are presented in an original form.

$$\mathbf{H}\equiv\begin{pmatrix}H_{11}&H_{12}\\H_{21}&H_{22}\end{pmatrix}=\begin{pmatrix}-\frac{\hbar^2}{2m}\nabla^2+U(\mathbf{r},t)+\Delta_{11}&\Delta_{12}\\ \Delta_{21}&-\frac{\hbar^2}{2m}\nabla^2+U(\mathbf{r},t)+\Delta_{22}\end{pmatrix},$$

$$\mathbf{D}(1)\equiv\begin{pmatrix}D_{11}(1)&D_{12}(1)\\D_{21}(1)&D_{22}(1)\end{pmatrix}=\begin{pmatrix}i\hbar\partial_{t_1}-H_{11}(1)&-H_{12}(1)\\-H_{21}(1)&i\hbar\partial_{t_1}-H_{22}(1)\end{pmatrix},$$

$$\mathbf{D}^*(1')\equiv\begin{pmatrix}D_{11}^*(1')&D_{12}^*(1')\\D_{21}^*(1')&D_{22}^*(1')\end{pmatrix}=\begin{pmatrix}-i\hbar\partial_{t_{1'}}-H_{11}^*(1')&-H_{12}^*(1')\\-H_{21}^*(1')&-i\hbar\partial_{t_{1'}}-H_{22}^*(1')\end{pmatrix},$$

$$\mathbf{G}(1,1')\equiv\begin{pmatrix}G_{\uparrow\uparrow}(1,1')&G_{\uparrow\downarrow}(1,1')\\G_{\downarrow\uparrow}(1,1')&G_{\downarrow\downarrow}(1,1')\end{pmatrix},$$
$$\mathbf{\Sigma}(1,1')\equiv\begin{pmatrix}\Sigma_{\uparrow\uparrow}(1,1')&\Sigma_{\uparrow\downarrow}(1,1')\\\Sigma_{\downarrow\uparrow}(1,1')&\Sigma_{\downarrow\downarrow}(1,1')\end{pmatrix}. \quad \text{(B.1)}$$

where 1 and 1′ present $(\mathbf{r}_1,t_1)$ and $(\mathbf{r}_{1'},t_{1'})$, respectively.

The original form of Dyson equation shown in equations (1a) and (1b) is given by

$$\begin{cases}D_{11}(1)G_{\uparrow\uparrow}(1,1')+D_{12}(1)G_{\downarrow\uparrow}(1,1')=\delta_C(1-1')\\ \quad+\int_C d2\,[\Sigma_{\uparrow\uparrow}(1,2)G_{\uparrow\uparrow}(2,1')+\Sigma_{\uparrow\downarrow}(1,2)G_{\downarrow\uparrow}(2,1')], \quad \text{(B.2a)}\\ D_{11}^*(1')G_{\uparrow\uparrow}(1,1')+D_{12}^*(1')G_{\downarrow\uparrow}(1,1')=\delta_C(1-1')\\ \quad+\int_C d2\,[G_{\uparrow\uparrow}(1,2)\Sigma_{\uparrow\uparrow}(2,1')+G_{\downarrow\uparrow}(1,2)\Sigma_{\uparrow\downarrow}(2,1')], \quad \text{(B.2a)'}\end{cases}$$

$$\begin{cases}D_{11}(1)G_{\uparrow\downarrow}(1,1')+D_{12}(1)G_{\downarrow\downarrow}(1,1')=\\ \int_C d2\,[\Sigma_{\uparrow\uparrow}(1,2)G_{\uparrow\downarrow}(2,1')+\Sigma_{\uparrow\downarrow}(1,2)G_{\downarrow\downarrow}(2,1')], \quad \text{(B.2b)}\\ D_{11}^*(1')G_{\uparrow\downarrow}(1,1')+D_{12}^*(1')G_{\downarrow\downarrow}(1,1')=\\ \int_C d2\,[G_{\uparrow\downarrow}(1,2)\Sigma_{\uparrow\uparrow}(2,1')+G_{\downarrow\downarrow}(1,2)\Sigma_{\uparrow\downarrow}(2,1')], \quad \text{(B.2b)'}\end{cases}$$

$$\begin{cases}D_{21}(1)G_{\uparrow\uparrow}(1,1')+D_{22}(1)G_{\downarrow\uparrow}(1,1')=\\ \int_C d2\,[\Sigma_{\downarrow\uparrow}(1,2)G_{\uparrow\uparrow}(2,1')+\Sigma_{\downarrow\downarrow}(1,2)G_{\downarrow\uparrow}(2,1')], \quad \text{(B.2c)}\\ D_{21}^*(1')G_{\uparrow\uparrow}(1,1')+D_{22}^*(1')G_{\downarrow\uparrow}(1,1')=\\ \int_C d2\,[G_{\uparrow\uparrow}(1,2)\Sigma_{\downarrow\uparrow}(2,1')+G_{\downarrow\uparrow}(1,2)\Sigma_{\downarrow\downarrow}(2,1')], \quad \text{(B.2c)'}\end{cases}$$

$$\begin{cases}D_{21}(1)G_{\uparrow\downarrow}(1,1')+D_{22}(1)G_{\downarrow\downarrow}(1,1')=\delta_C(1-1')\\ \quad+\int_C d2\,[\Sigma_{\downarrow\uparrow}(1,2)G_{\uparrow\downarrow}(2,1')+\Sigma_{\downarrow\downarrow}(1,2)G_{\downarrow\downarrow}(2,1')], \quad \text{(B.2d)}\\ D_{21}^*(1')G_{\uparrow\downarrow}(1,1')+D_{22}^*(1')G_{\downarrow\downarrow}(1,1')=\delta_C(1-1')\\ \quad+\int_C d2\,[G_{\uparrow\downarrow}(1,2)\Sigma_{\downarrow\uparrow}(2,1')+G_{\downarrow\downarrow}(1,2)\Sigma_{\downarrow\downarrow}(2,1')], \quad \text{(B.2d)'}\end{cases}$$

Given the Langreth theorem [34,35], if $Z(t_1,t_{1'})=\int_C d\tau\, X(t_1,\tau)Y(\tau,t_{1'})$, then

$$Z^{\langle}(t_1,t_{1'})=\int_C d\tau[X^r(t_1,\tau)Y^{\langle}(\tau,t_{1'})+X^{\langle}(t_1,\tau)Y^a(\tau,t_{1'})]. \quad \text{(B.3)}$$

Applying (B.3) to (B.2), two kinds of KBE shown in equations (2a) and (2b) can then be obtained. To clarify the spin notation, the original form of the two kinds of KBE is shown. For the first kind of KBE in equation (2a), the detailed expression is given by

$$[D_{11}(1)-D_{11}^*(1')]G_{\uparrow\uparrow}^{\langle}(1,1')+[D_{12}(1)-D_{12}^*(1')]G_{\downarrow\uparrow}^{\langle}(1,1') -[\Sigma_{\uparrow\uparrow},G_{\uparrow\uparrow}^{\langle}]-[\Sigma_{\uparrow\uparrow}^{\langle},G_{\uparrow\uparrow}]-[\Sigma_{\uparrow\downarrow},G_{\downarrow\uparrow}^{\langle}]-[\Sigma_{\uparrow\downarrow}^{\langle},G_{\downarrow\uparrow}]= \frac{1}{2}\{\Sigma_{\uparrow\uparrow}^{\rangle},G_{\uparrow\uparrow}^{\langle}\}-\frac{1}{2}\{G_{\uparrow\uparrow}^{\rangle},\Sigma_{\uparrow\uparrow}^{\langle}\}+\frac{1}{2}\{\Sigma_{\uparrow\downarrow}^{\rangle},G_{\downarrow\uparrow}^{\langle}\}-\frac{1}{2}\{G_{\downarrow\uparrow}^{\rangle},\Sigma_{\uparrow\downarrow}^{\langle}\}, \tag{B.4a}$$

$$[D_{11}(1)-D_{11}^*(1')]G_{\uparrow\downarrow}^{\langle}(1,1')+[D_{12}(1)-D_{12}^*(1')]G_{\downarrow\downarrow}^{\langle}(1,1') -[\Sigma_{\uparrow\uparrow},G_{\uparrow\downarrow}^{\langle}]-[\Sigma_{\uparrow\uparrow}^{\langle},G_{\uparrow\downarrow}]-[\Sigma_{\uparrow\downarrow},G_{\downarrow\downarrow}^{\langle}]-[\Sigma_{\uparrow\downarrow}^{\langle},G_{\downarrow\downarrow}]= \frac{1}{2}\{\Sigma_{\uparrow\uparrow}^{\rangle},G_{\uparrow\downarrow}^{\langle}\}-\frac{1}{2}\{G_{\uparrow\downarrow}^{\rangle},\Sigma_{\uparrow\uparrow}^{\langle}\}+\frac{1}{2}\{\Sigma_{\uparrow\downarrow}^{\rangle},G_{\downarrow\downarrow}^{\langle}\}-\frac{1}{2}\{G_{\downarrow\downarrow}^{\rangle},\Sigma_{\uparrow\downarrow}^{\langle}\}, \tag{B.4b}$$

$$[D_{21}(1)-D_{21}^*(1')]G_{\uparrow\uparrow}^{\langle}(1,1')+[D_{22}(1)-D_{22}^*(1')]G_{\downarrow\uparrow}^{\langle}(1,1') -[\Sigma_{\downarrow\uparrow},G_{\uparrow\uparrow}^{\langle}]-[\Sigma_{\downarrow\uparrow}^{\langle},G_{\uparrow\uparrow}]-[\Sigma_{\downarrow\downarrow},G_{\downarrow\uparrow}^{\langle}]-[\Sigma_{\downarrow\downarrow}^{\langle},G_{\downarrow\uparrow}]= \frac{1}{2}\{\Sigma_{\downarrow\uparrow}^{\rangle},G_{\uparrow\uparrow}^{\langle}\}-\frac{1}{2}\{G_{\uparrow\uparrow}^{\rangle},\Sigma_{\downarrow\uparrow}^{\langle}\}+\frac{1}{2}\{\Sigma_{\downarrow\downarrow}^{\rangle},G_{\downarrow\uparrow}^{\langle}\}-\frac{1}{2}\{G_{\downarrow\uparrow}^{\rangle},\Sigma_{\downarrow\downarrow}^{\langle}\}, \tag{B.4c}$$

$$[D_{21}(1)-D_{21}^*(1')]G_{\uparrow\downarrow}^{\langle}(1,1')+[D_{22}(1)-D_{22}^*(1')]G_{\downarrow\downarrow}^{\langle}(1,1') -[\Sigma_{\downarrow\uparrow},G_{\uparrow\downarrow}^{\langle}]-[\Sigma_{\downarrow\uparrow}^{\langle},G_{\uparrow\downarrow}]-[\Sigma_{\downarrow\downarrow},G_{\downarrow\downarrow}^{\langle}]-[\Sigma_{\downarrow\downarrow}^{\langle},G_{\downarrow\downarrow}]= \frac{1}{2}\{\Sigma_{\downarrow\uparrow}^{\rangle},G_{\uparrow\downarrow}^{\langle}\}-\frac{1}{2}\{G_{\uparrow\downarrow}^{\rangle},\Sigma_{\downarrow\uparrow}^{\langle}\}+\frac{1}{2}\{\Sigma_{\downarrow\downarrow}^{\rangle},G_{\downarrow\downarrow}^{\langle}\}-\frac{1}{2}\{G_{\downarrow\downarrow}^{\rangle},\Sigma_{\downarrow\downarrow}^{\langle}\}, \tag{B.4d}$$

where reminding that ΣG and $G\Sigma$ are abbreviated forms of $\int_C d2\Sigma(1,2)G(2,1')$ and $\int_C d2G(1,2)\Sigma(2,1')$, respectively. Additionally, [,] and { , } stand for the commutator and anti-commutator, respectively.

For the second kind of KBE in equation (2b), the detailed expression is given by

$$D_{11}(1)G_{\uparrow\uparrow}^{\langle}(1,1')-D_{11}^*(1')G_{\uparrow\uparrow}^{\langle}(1,1')+D_{12}(1)G_{\downarrow\uparrow}^{\langle}(1,1') -D_{12}^*(1')G_{\downarrow\uparrow}^{\langle}(1,1')=\Sigma_{\uparrow\uparrow}^{r}G_{\uparrow\uparrow}^{\langle}+\Sigma_{\uparrow\uparrow}^{\langle}G_{\uparrow\uparrow}^{a}-G_{\uparrow\uparrow}^{r}\Sigma_{\uparrow\uparrow}^{\langle} -G_{\uparrow\uparrow}^{\langle}\Sigma_{\uparrow\uparrow}^{a}+\Sigma_{\uparrow\downarrow}^{r}G_{\downarrow\uparrow}^{\langle}+\Sigma_{\uparrow\downarrow}^{\langle}G_{\downarrow\uparrow}^{a}-G_{\downarrow\uparrow}^{r}\Sigma_{\uparrow\downarrow}^{\langle}-G_{\downarrow\uparrow}^{\langle}\Sigma_{\uparrow\downarrow}^{a}, \tag{B.5a}$$

$$D_{11}(1)G_{\uparrow\downarrow}^{\langle}(1,1')-D_{11}^*(1')G_{\uparrow\downarrow}^{\langle}(1,1')+D_{12}(1)G_{\downarrow\downarrow}^{\langle}(1,1') -D_{12}^*(1')G_{\downarrow\downarrow}^{\langle}(1,1')=\Sigma_{\uparrow\uparrow}^{r}G_{\uparrow\downarrow}^{\langle}+\Sigma_{\uparrow\uparrow}^{\langle}G_{\uparrow\downarrow}^{a}-G_{\uparrow\downarrow}^{r}\Sigma_{\uparrow\uparrow}^{\langle} -G_{\uparrow\downarrow}^{\langle}\Sigma_{\uparrow\uparrow}^{a}+\Sigma_{\uparrow\downarrow}^{r}G_{\downarrow\downarrow}^{\langle}+\Sigma_{\uparrow\downarrow}^{\langle}G_{\downarrow\downarrow}^{a}-G_{\downarrow\downarrow}^{r}\Sigma_{\uparrow\downarrow}^{\langle}-G_{\downarrow\downarrow}^{\langle}\Sigma_{\uparrow\downarrow}^{a}, \tag{B.5b}$$

$$D_{21}(1)G_{\uparrow\uparrow}^{\langle}(1,1')-D_{21}^*(1')G_{\uparrow\uparrow}^{\langle}(1,1')+D_{22}(1)G_{\downarrow\uparrow}^{\langle}(1,1') -D_{22}^*(1')G_{\downarrow\uparrow}^{\langle}(1,1')=\Sigma_{\downarrow\uparrow}^{r}G_{\uparrow\uparrow}^{\langle}+\Sigma_{\downarrow\uparrow}^{\langle}G_{\uparrow\uparrow}^{a}-G_{\uparrow\uparrow}^{r}\Sigma_{\downarrow\uparrow}^{\langle} -G_{\uparrow\uparrow}^{\langle}\Sigma_{\downarrow\uparrow}^{a}+\Sigma_{\downarrow\downarrow}^{r}G_{\downarrow\uparrow}^{\langle}+\Sigma_{\downarrow\downarrow}^{\langle}G_{\downarrow\uparrow}^{a}-G_{\downarrow\uparrow}^{r}\Sigma_{\downarrow\downarrow}^{\langle}-G_{\downarrow\uparrow}^{\langle}\Sigma_{\downarrow\downarrow}^{a}, \tag{B.5c}$$

$$D_{21}(1)G_{\uparrow\downarrow}^{\langle}(1,1')-D_{21}^*(1')G_{\uparrow\downarrow}^{\langle}(1,1')+D_{22}(1)G_{\downarrow\downarrow}^{\langle}(1,1') -D_{22}^*(1')G_{\downarrow\downarrow}^{\langle}(1,1')=\Sigma_{\downarrow\uparrow}^{r}G_{\uparrow\downarrow}^{\langle}+\Sigma_{\downarrow\uparrow}^{\langle}G_{\uparrow\downarrow}^{a}-G_{\uparrow\downarrow}^{r}\Sigma_{\downarrow\uparrow}^{\langle} -G_{\uparrow\downarrow}^{\langle}\Sigma_{\downarrow\uparrow}^{a}+\Sigma_{\downarrow\downarrow}^{r}G_{\downarrow\downarrow}^{\langle}+\Sigma_{\downarrow\downarrow}^{\langle}G_{\downarrow\downarrow}^{a}-G_{\downarrow\downarrow}^{r}\Sigma_{\downarrow\downarrow}^{\langle}-G_{\downarrow\downarrow}^{\langle}\Sigma_{\downarrow\downarrow}^{a}. \tag{B.5d}$$

Appendix C: Seeking retarded Green function for the spin Hall effect

According to Langreth theorem, $Z^r(t_1,t_{1'}) = \int_C d\tau X^r(t_1,\tau)Y^r(\tau,t_{1'})$. With the relation, (B2$a$) + (B2$a$)' and (B2$c$) + (B2$c$)' can be presented as, respectively

$$[D_{11}(1)+D_{11}^*(1')]G_{\uparrow\uparrow}^{r}(1,1')+[D_{12}(1)+D_{12}^*(1')]G_{\downarrow\uparrow}^{r}(1,1')= 2\delta(1-1')+\Sigma_{\uparrow\uparrow}^{r}G_{\uparrow\uparrow}^{r}+G_{\uparrow\uparrow}^{r}\Sigma_{\uparrow\uparrow}^{r}+\Sigma_{\uparrow\downarrow}^{r}G_{\downarrow\uparrow}^{r}+G_{\downarrow\uparrow}^{r}\Sigma_{\uparrow\downarrow}^{r}, \tag{C.1a}$$

$$[D_{21}(1)+D_{21}^*(1')]G_{\uparrow\uparrow}^{r}(1,1')+[D_{22}(1)+D_{22}^*(1')]G_{\downarrow\uparrow}^{r}(1,1')= \Sigma_{\downarrow\uparrow}^{r}G_{\uparrow\uparrow}^{r}+G_{\uparrow\uparrow}^{r}\Sigma_{\downarrow\uparrow}^{r}+\Sigma_{\downarrow\downarrow}^{r}G_{\downarrow\uparrow}^{r}+G_{\downarrow\uparrow}^{r}\Sigma_{\downarrow\downarrow}^{r}, \tag{C.1b}$$

which is another form of spin-dependent Dyson equation and where $G_{\uparrow\uparrow}^r(1,1')$ and $G_{\downarrow\uparrow}^r(1,1')$ can be found out to input into the KBE for the SHE shown in equation (5).

(C.1a) and (C.1b) after applying the Wigner transformation and Fourier transform under the scalar gauge, i.e. $\int d\tau d\mathbf{r}\exp[i(\omega-q_c\mathbf{E}\cdot\mathbf{R}/\hbar)\tau-i\mathbf{k}\cdot\mathbf{r}]G(\mathbf{r},\tau,\mathbf{R},T)$, become

$$\left(\hbar\omega-e_{\mathbf{k}}+\frac{q_c^2E^2}{8m^*}\partial_\omega^2\right)\tilde{G}_{\uparrow\uparrow}^r(\mathbf{k},\omega)-\mathrm{Re}\ \Delta_{12}\tilde{G}_{\downarrow\uparrow}^r(\mathbf{k},\omega)=1, \tag{C.2a}$$

$$-\mathrm{Re}\ \Delta_{21}\tilde{G}_{\uparrow\uparrow}^r(\mathbf{k},\omega)+\left(\hbar\omega-e_{\mathbf{k}}+\frac{q_c^2E^2}{8m^*}\partial_\omega^2\right)\tilde{G}_{\downarrow\uparrow}^r(\mathbf{k},\omega)=0, \tag{C.2b}$$

where the retarded Green function is assumed to be independent of $\mathbf{R}$ (spatially homogeneous) and T (stationary); the collision term of (C.1) and (C.2) is set to be zero first and considered later. The retarded Green function can be obtained in the τ domain. Applying the transform $\hat{G}^r(\mathbf{k},\tau)\equiv\int_{-\infty}^{\infty}d\omega\exp(-i\omega\tau)\tilde{G}^r(\mathbf{k},\omega)$, note that this is under neither the scalar nor the vector potential gauge, (C.2a) and (C.2b) become

$$\left(i\hbar\partial_\tau-e_{\mathbf{k}}-\frac{q_c^2E^2}{8m^*}\tau^2\right)\hat{G}_{\uparrow\uparrow}^r(\mathbf{k},\tau) -\mathrm{Re}\ \Delta_{12}\hat{G}_{\downarrow\uparrow}^r(\mathbf{k},\tau)=\delta(\tau), \tag{C.3a}$$

$$-\mathrm{Re}\ \Delta_{21}\hat{G}_{\uparrow\uparrow}^r(\mathbf{k},\tau)+\left(i\hbar\partial_\tau-e_{\mathbf{k}}-\frac{q_c^2E^2}{8m^*}\tau^2\right)\hat{G}_{\downarrow\uparrow}^r(\mathbf{k},\tau)=0, \tag{C.3b}$$

where the two retarded Green functions can be found out by using an iterative method. Set $\hat{G}^r_{\downarrow\uparrow}(\mathbf{k},\tau)=0$ in (C.3a), then

$$\hat{G}^r_{\uparrow\uparrow}(\mathbf{k},\tau)=-\frac{i}{\hbar}\theta(\tau)\exp\left[-i\left(\frac{e_{\mathbf{k}}}{\hbar}\tau+\frac{q_c^2E^2}{24m^*\hbar}\tau^3\right)\right],$$

where $\theta(\tau)$ is a step function. Hence, $\tilde{G}^r_{\uparrow\uparrow}(\mathbf{k},\omega)\approx\frac{1}{\hbar\omega-e_{\mathbf{k}}}-\frac{q_c^2\hbar^2E^2}{4m^*(\hbar\omega-e_{\mathbf{k}})^4}$, where the first order Taylor expansion for the exponential function was made. Considering the equilibrium self energy σ^r_{ss} as the collision term in (C.1a) then yields

$$\tilde{G}^r_{\uparrow\uparrow}(\mathbf{k},\omega)\approx\frac{1}{\hbar\omega-e_{\mathbf{k}}-\mathrm{Re}\ \sigma^r_{\uparrow\uparrow}-i\mathrm{Im}\ \sigma^r_{\uparrow\uparrow}}-\frac{q_c^2\hbar^2E^2}{4m^*\left(\hbar\omega-e_{\mathbf{k}}-\mathrm{Re}\ \sigma^r_{\uparrow\uparrow}-i\mathrm{Im}\ \sigma^r_{\uparrow\uparrow}\right)^4}.\quad\text{(C.4a)}$$

Inputting $\hat{G}^r_{\uparrow\uparrow}(\mathbf{k},\tau)$ into (C.3b) can find out $\hat{G}^r_{\downarrow\uparrow}(\mathbf{k},\tau)$ equal to $-\frac{\mathrm{Re}\Delta_{21}}{\hbar^2}\tau\theta(\tau)\exp\left[-i\left(\frac{e_{\mathbf{k}}}{\hbar}\tau+\frac{q_c^2E^2}{24m^*\hbar}\tau^3\right)\right]$, Changing it to the frequency domain then yields

$$\tilde{G}^r_{\downarrow\uparrow}(\mathbf{k},\omega)\approx-\frac{q_c^2\hbar^2\mathrm{Re}\ \Delta_{21}E^2}{m^*\left(\hbar\omega-e_{\mathbf{k}}-\mathrm{Re}\ \sigma^r_{\uparrow\uparrow}-i\mathrm{Im}\ \sigma^r_{\uparrow\uparrow}\right)^5},\quad\text{(C.4b)}$$

where the equilibrium spin-flip term is set zero.

Starting from (B2*b*) + (B2*b*)' and (B2*d*) + (B2*d*)', $\tilde{G}^r_{\uparrow\downarrow}(\mathbf{k},\omega)$ and $\tilde{G}^r_{\downarrow\downarrow}(\mathbf{k},\omega)$ can be found out with using the same iterative method.

$$\tilde{G}^r_{\downarrow\downarrow}(\mathbf{k},\omega)\approx\frac{1}{\hbar\omega-e_{\mathbf{k}}-\mathrm{Re}\ \sigma^r_{\downarrow\downarrow}-i\mathrm{Im}\ \sigma^r_{\downarrow\downarrow}}-\frac{q_c^2\hbar^2E^2}{4m^*\left(\hbar\omega-e_{\mathbf{k}}-\mathrm{Re}\ \sigma^r_{\downarrow\downarrow}-i\mathrm{Im}\ \sigma^r_{\downarrow\downarrow}\right)^4},\quad\text{(C.4a)'}$$

$$\tilde{G}^r_{\uparrow\downarrow}(\mathbf{k},\omega)\approx-\frac{q_c^2\hbar^2E^2\mathrm{Re}\ \Delta_{12}}{m^*\left(\hbar\omega-e_{\mathbf{k}}-\mathrm{Re}\ \sigma^r_{\downarrow\downarrow}-i\mathrm{Im}\ \sigma^r_{\downarrow\downarrow}\right)^5}.\quad\text{(C.4b)'}$$

When considering the spatially inhomogeneous case, (C.2a) and (C.2b) becomes

$$\left(\hbar\omega-e_{\mathbf{k}}+\frac{q_c^2E^2}{8m^*}\partial^2_\omega\right)\tilde{G}^r_{\uparrow\uparrow}(\mathbf{k},\omega,\mathbf{R}_\parallel)+\frac{\hbar^2}{8m^*}\times\partial^2_{\mathbf{R}_\parallel}\tilde{G}^r_{\uparrow\uparrow}(\mathbf{k},\omega,\mathbf{R}_\parallel)-\mathrm{Re}\ \Delta_{12}\tilde{G}^r_{\downarrow\uparrow}(\mathbf{k},\omega,\mathbf{R}_\parallel)=1\quad\text{(C.5a)}$$

$$-\mathrm{Re}\ \Delta_{21}\tilde{G}^r_{\uparrow\uparrow}(\mathbf{k},\omega,\mathbf{R}_\parallel)+\left(\hbar\omega-e_{\mathbf{k}}+\frac{q_c^2E^2}{8m^*}\partial^2_\omega\right)\times\tilde{G}^r_{\downarrow\uparrow}(\mathbf{k},\omega,\mathbf{R}_\parallel)+\frac{\hbar^2}{8m^*}\partial^2_{\mathbf{R}_\parallel}\tilde{G}^r_{\downarrow\uparrow}(\mathbf{k},\omega,\mathbf{R}_\parallel)=0\quad\text{(C.5b)}$$

With the Neumann boundary condition, the Green function can be written as

$$\tilde{G}^r(\mathbf{k},\omega,\mathbf{R}_\parallel)=\frac{1}{\sqrt{L_xL_z}}\sum_{\mathbf{K}_\parallel}\tilde{G}^r(\mathbf{k},\omega,\mathbf{K}_\parallel)\times\exp[iK_x(x-L_x/2)]\exp[iK_z(z-L_z/2)],\quad\text{(C.6)}$$

where $K_{x(z)}=n_{x(z)}\pi/L_{x(z)}$ and $n_{x(z)}$ is an integer. The coordinate refers to Figure 1.

Taking (C.6) into (C.5a) and (C.5b), and changing them to the τ domain yields

$$\left(i\hbar\partial_\tau-e_{\mathbf{k}}-\frac{1}{4}e_{\mathbf{K}_\parallel}-\frac{q_c^2E^2\tau^2}{8m^*}\right)\hat{G}^r_{\uparrow\uparrow}(\mathbf{k},\tau,\mathbf{K}_\parallel)-\mathrm{Re}\ \Delta_{12}\hat{G}^r_{\downarrow\uparrow}(\mathbf{k},\tau,\mathbf{K}_\parallel)=-\eta\delta(\tau),\quad\text{(C.7a)}$$

$$-\mathrm{Re}\ \Delta_{21}\hat{G}^r_{\uparrow\uparrow}(\mathbf{k},\tau,\mathbf{K}_\parallel)+\left(i\hbar\partial_\tau-e_{\mathbf{k}}-\frac{1}{4}e_{\mathbf{K}_\parallel}-\frac{q_c^2E^2\tau^2}{8m^*}\right)\times\hat{G}^r_{\downarrow\uparrow}(\mathbf{k},\tau,\mathbf{K}_\parallel)=0,\quad\text{(C.7b)}$$

where $\eta=[1-\cos(n_x\pi)][1-\cos(n_z\pi)]/(\sqrt{L_xL_z}K_xK_z)$.

(C.7a) and (C.7b) can then be solved with using the same iterative method as that in the spatially homogeneous case. Starting from (B2b) + (B2b)' and (B2d) + (B2d)', $\tilde{G}^r_{\uparrow\downarrow}(\mathbf{k},\omega,\mathbf{K}_\parallel)$ and $\tilde{G}^r_{\downarrow\downarrow}(\mathbf{k},\omega,\mathbf{K}_\parallel)$ can also be found out. As a result, the $\mathbf{K}_\parallel$-dependent retarded Green function can be expressed as

$$\tilde{G}^r_{ss}(\mathbf{k},\omega,\mathbf{K}_\parallel)\approx\frac{-\eta}{\hbar\omega-e_{\mathbf{k}}-e_{\mathbf{K}_\parallel/2}-\mathrm{Re}\ \sigma^r_{ss}-i\mathrm{Im}\ \sigma^r_{ss}}+\frac{\eta q_c^2\hbar^2E^2}{4m^*(\hbar\omega-e_{\mathbf{k}}-e_{\mathbf{K}_\parallel/2}-\mathrm{Re}\ \sigma^r_{ss}-i\mathrm{Im}\ \sigma^r_{ss})^4},\quad\text{(C.8a)}$$

$$\tilde{G}^r_{ss'}(\mathbf{k},\omega,\mathbf{K}_\parallel)\approx\frac{\eta q_c^2\hbar^2E^2}{m^*(\hbar\omega-e_{\mathbf{k}}-e_{\mathbf{K}_\parallel/2}-\mathrm{Re}\ \sigma^r_{s's'}-i\mathrm{Im}\ \sigma^r_{s's'})^5}\times\begin{cases}\mathrm{Re}\ \Delta_{21},\ s'=\uparrow\\ \mathrm{Re}\ \Delta_{12},\ s'=\downarrow\ .\end{cases}\quad\text{(C.8b)}$$

Appendix D: The 2nd KBE for the SHE with an algorithm for seeking solutions

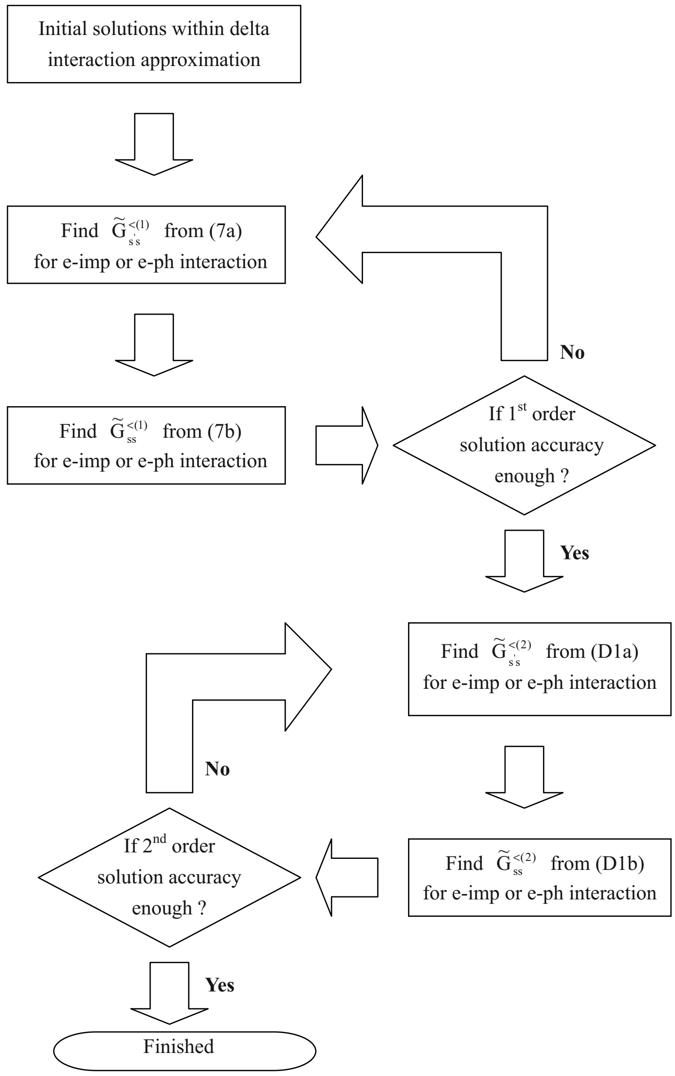
Applying Taylor expansion to the electric field in equation (5) and approximating $\tilde{G}^{\langle}(\mathbf{k},\omega)$ as $\tilde{g}^{\langle}(\mathbf{k},\omega)+E\tilde{G}^{\langle(1)}(\mathbf{k},\omega)+E^2\tilde{G}^{\langle(2)}(\mathbf{k},\omega)$, the second order KBE for SHE based on the perturbation method can be expressed as

$$q_c\hat{\varepsilon}\cdot\left(\nabla_{\mathbf{k}}+\frac{\hbar\mathbf{k}}{m^*}\partial_\omega\right)\tilde{G}^{\langle(1)}_{ss}(\mathbf{k},\omega)-i\left(\frac{q_c}{2\hbar}\right)^2 \times\hat{\varepsilon}\cdot\left(\partial^2_{\mathbf{k}}\tilde{g}^{\langle}_{ss}\partial^2_\omega\tilde{\sigma}_{ss}+\partial^2_\omega\tilde{g}^{\langle}_{ss}\partial^2_{\mathbf{k}}\tilde{\sigma}_{ss}+\partial^2_\omega\tilde{\sigma}^{\langle}_{ss}\partial^2_{\mathbf{k}}\tilde{g}_{ss}+\partial^2_{\mathbf{k}}\tilde{\sigma}^{\langle}_{ss}\partial^2_\omega\tilde{g}_{ss}\right) - \tilde{G}^{\langle(2)}_{s's}(\mathbf{k},\omega)\cdot\begin{cases}2\mathrm{Im}\ \Delta_{12},\ s'=\downarrow\\ 2\mathrm{Im}\ \Delta_{21},\ s'=\uparrow\end{cases} = -[\tilde{\gamma}_{ss}\tilde{G}^{\langle(2)}_{ss}(\mathbf{k},\omega) - \tilde{a}_{ss}\tilde{\Sigma}^{\langle(2)}_{ss}(\mathbf{k},\omega)] - \tilde{g}^{\langle}_{ss}\tilde{\Gamma}^{(2)}_{ss(e-ph)}, \quad \text{(D.1a)}$$

$$q_c\hat{\varepsilon}\cdot\left(\nabla_{\mathbf{k}}+\frac{\hbar\mathbf{k}}{m^*}\partial_\omega\right)\tilde{G}^{\langle(1)}_{ss'}(\mathbf{k},\omega)-2\tilde{G}^{\langle(2)}_{s's'}(\mathbf{k},\omega) \times\begin{cases}\mathrm{Im}\ \Delta_{12},\ s'=\downarrow\\ \mathrm{Im}\ \Delta_{21},\ s'=\uparrow\end{cases} = -\left[\tilde{\gamma}_{ss}\tilde{G}^{\langle(2)}_{ss'}(\mathbf{k},\omega)-\tilde{\sigma}^{\langle}_{ss}\tilde{A}^{(2)}_{ss'}(\mathbf{k},\omega) +\tilde{g}^{\langle}_{s's'}\tilde{\Gamma}^{(2)}_{ss'}(\mathbf{k},\omega)-\tilde{a}_{s's'}\tilde{\Sigma}^{\langle(2)}_{ss'}(\mathbf{k},\omega)\right], \quad \text{(D.1b)}$$

where $\tilde{g}_{ss}(\mathbf{k},\omega) \equiv \frac{1}{2}[\tilde{g}^r_{ss}(\mathbf{k},\omega)+\tilde{g}^a_{ss}(\mathbf{k},\omega)]$. The subscript in $\tilde{g}^{\langle}_{ss}\tilde{\Gamma}^{(2)}_{ss(e-ph)}$ denotes that the term only exists in the case of an e-ph interaction due to the Langreth theorem [34,35]. $\tilde{A}^{(2)}_{ss^{(\prime)}}(\mathbf{k},\omega)\equiv i[\tilde{G}^{r(2)}_{ss^{(\prime)}}\mathbf{k},\omega)-\tilde{G}^{a(2)}_{ss^{(\prime)}}(\mathbf{k},\omega)]$ and $\tilde{\Gamma}^{(2)}_{ss^{(\prime)}}(\mathbf{k},\omega)\equiv i[\tilde{\Sigma}^{r(2)}_{ss^{(\prime)}}(\mathbf{k},\omega)-\tilde{\Sigma}^{a(2)}_{ss^{(\prime)}}(\mathbf{k},\omega)]$ can be derived using retarded Green functions in equation (6b).

The KBE for the SHE can be solved by using an iterative algorithm shown below, where the sufficiently accurate first-order solutions are iteratively found and then input into the second-order equation to find solutions iteratively until accuracy is acceptable.



Appendix E: Derivation for collision term in spatially-dependent KBE

For clarity, we use a simplified notation first. The collision term $C(\mathbf{r}_1,\mathbf{r}_{1'},t_1,t_{1'}) = \int d\mathbf{r}_2dt_2A(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)\times B(\mathbf{r}_2,\mathbf{r}_{1'},t_2,t_{1'})$ after the Wigner transformation becomes

$$C(\mathbf{r},\tau,\mathbf{R},T)=\int d\mathbf{r}_2dt_2A\left(\mathbf{r}_1-\mathbf{r}_2,t_1-t_2,\frac{\mathbf{r}_1+\mathbf{r}_2}{2},\frac{t_1+t_2}{2}\right) \times B\left(\mathbf{r}_2-\mathbf{r}_{1'},t_2-t_{1'},\frac{\mathbf{r}_2+\mathbf{r}_{1'}}{2},\frac{t_2+t_{1'}}{2}\right), \quad \text{(E.1)}$$

where reminding that $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_{1'}$, $\tau=t_1-t_{1'}$, $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_{1'})/2$ and $T=(t_1+t_{1'})/2$.

With the spatial inverse Fourier transform under the vector potential gauge, i.e.,

$$f(\mathbf{r},\tau,\mathbf{R},T)=\int\frac{d\mathbf{k}}{(2\pi)^3}\exp\left[i\left(\mathbf{k}-\frac{q_c\mathbf{E}T}{\hbar}\right)\cdot\mathbf{r}\right] \times\hat{F}(\mathbf{k},\tau,\mathbf{R},T),$$

(E.1) becomes

$$C(\mathbf{r},\tau,\mathbf{R},T)=\int d\mathbf{r}_2dt_2\int\frac{d\mathbf{k}_1}{(2\pi)^3}\frac{d\mathbf{k}_{1'}}{(2\pi)^3} \times\exp\left\{i\left[\mathbf{k}_1-\frac{q_c\mathbf{E}}{2\hbar}(t_1+t_2)\right](\mathbf{r}_1-\mathbf{r}_2)\right\} \times\exp\left\{i\left[\mathbf{k}_{1'}-\frac{q_c\mathbf{E}}{2\hbar}(t_2+t_{1'})\right](\mathbf{r}_2-\mathbf{r}_{1'})\right\} \times A\left(\mathbf{k}_1,\frac{\mathbf{r}_1+\mathbf{r}_2}{2},\frac{t_1+t_2}{2}\right) \times B\left(\mathbf{k}_{1'},\frac{\mathbf{r}_2+\mathbf{r}_{1'}}{2},\frac{t_2+t_{1'}}{2}\right), \quad \text{(E.2)}$$

where the difference time in A and B functions is not marked for simplicity.

Substituting the Fourier expansion, i.e.,

$$F(\mathbf{k},\tau,\mathbf{R}_\parallel,T)=\frac{1}{\sqrt{L_xL_z}}\times\sum_{\mathbf{K}_\parallel}\hat{F}(\mathbf{k},\tau,\mathbf{K}_\parallel,T)\exp[i\mathbf{K}_\parallel\cdot(\mathbf{R}_\parallel-\mathbf{L}_\parallel/2)],$$

into (E.2) yields

$$C(\mathbf{r},\tau,\mathbf{R}_\parallel,T)=\frac{1}{L_xL_z}\sum_{\mathbf{K}_{\parallel 1}}\sum_{\mathbf{K}_{\parallel 2}}\exp\left[i\mathbf{K}_{\parallel 1}\cdot\left(\mathbf{R}_\parallel-\frac{\mathbf{L}_\parallel}{2}\right)\right]\times\exp\left[i\mathbf{K}_{\parallel 2}\cdot\left(\mathbf{R}_\parallel-\frac{\mathbf{r}}{2}-\frac{\mathbf{L}_\parallel}{2}\right)\right]\int dt_2\int\frac{d\mathbf{k}_1}{(2\pi)^3}\times\exp\left\{i\left[\mathbf{k}_1-\frac{q_c\mathbf{E}}{2\hbar}(t_1+t_2)\right]\cdot\mathbf{r}\right\}A\left(\mathbf{k}_1,\mathbf{K}_{\parallel 1}\right)\times B\left[\mathbf{k}_1-\frac{q_c\mathbf{E}\tau}{2\hbar}-\frac{1}{2}(\mathbf{K}_{\parallel 1}+\mathbf{K}_{\parallel 2}),\mathbf{K}_{\parallel 2}\right],\quad\text{(E.3)}$$

where the average time in A and B functions is not marked for simplicity.

(E.3) after the Fourier transform under the vector gauge, i.e.,

$$F(\mathbf{k},\omega,\mathbf{R},T)=\int d\tau d\mathbf{r}\exp\left[i\omega\tau-i\left(\mathbf{k}-\frac{q_c\mathbf{E}T}{\hbar}\right)\cdot\mathbf{r}\right]f(\mathbf{r},\tau,\mathbf{R},T),$$

becomes

$$C(\mathbf{k},\omega,\mathbf{R}_\parallel,T)=\frac{1}{L_xL_z}\sum_{\mathbf{K}_\parallel}\sum_{\mathbf{K}'_\parallel}\exp[i\mathbf{K}_\parallel\cdot(\mathbf{R}_\parallel-\mathbf{L}_\parallel/2)]\times\int d\tau d\tau'\exp(i\omega\tau)A\left(\mathbf{k}'_1,\tau_1,\mathbf{K}_{\parallel 1},T_1\right)B\left(\mathbf{k}'_2,\tau_2,\mathbf{K}_{\parallel 2},T_2\right),\quad\text{(E.4)}$$

where $\mathbf{k}'_{1,2}=\mathbf{k}_{1,2}\pm\frac{1}{2}\mathbf{K}_{\parallel 2,1}$, $\mathbf{k}_{1,2}=\mathbf{k}+\frac{q}{2\hbar}\mathbf{E}(\tau'\pm\frac{\tau}{2})$, $\mathbf{K}_{\parallel 1,2}=\frac{1}{2}\left(\mathbf{K}_\parallel\pm\mathbf{K}'_\parallel\right)$, $\tau'=t_2-T$, $T_{1,2}\equiv T\pm\tau_{2,1}$, and $\tau_{1,2}\equiv\frac{\tau}{2}\mp\tau'$.

Considering the spin-dependent KBE, the collision term can be written as

$$CT_{ss^{(\prime)}}(\mathbf{k},\omega,\mathbf{R}_\parallel,T)=\frac{1}{L_xL_z}\sum_{\mathbf{K}_\parallel}\sum_{\mathbf{K}'_\parallel}\exp[i\mathbf{K}_\parallel\cdot(\mathbf{R}_\parallel-\mathbf{L}_\parallel/2)]\times\int d\tau d\tau'\exp(i\omega\tau)\sum_{s''=\uparrow,\downarrow}\hat{\mathbf{P}}_{ss''}\left(\mathbf{k}'_{1,2},\tau_{1,2},\mathbf{K}_{\parallel 1,2},T_{1,2}\right)\times\hat{\mathbf{Q}}_{s''s^{(\prime)}}\left(\mathbf{k}'_{1,2},\tau_{1,2},\mathbf{K}_{\parallel 1,2},T_{1,2}\right),\quad\text{(E.5)}$$

where

$$\hat{\mathbf{P}}_{ss''}(\mathbf{k}_{1,2},\tau_{1,2},\mathbf{K}_{\parallel 1,2},T_{1,2})\hat{\mathbf{Q}}_{s''s}(\mathbf{k}_{1,2},\tau_{1,2},\mathbf{K}_{\parallel 1,2},T_{1,2})\equiv[\hat{\Sigma}^r_{ss''}(\mathbf{k}_1,\tau_1,\mathbf{K}_{\parallel 1},T_1)\hat{G}^{\langle}_{s''s}(\mathbf{k}_2,\tau_2,\mathbf{K}_{\parallel 2},T_2)+\hat{\Sigma}^{\langle}_{ss''}(\mathbf{k}_1,\tau_1,\mathbf{K}_{\parallel 1},T_1)\hat{G}^a_{s''s}(\mathbf{k}_2,\tau_2,\mathbf{K}_{\parallel 2},T_2)-\hat{G}^r_{s''s}(\mathbf{k}_1,\tau_1,\mathbf{K}_{\parallel 1},T_1)\hat{\Sigma}^{\langle}_{ss''}(\mathbf{k}_2,\tau_2,\mathbf{K}_{\parallel 2},T_2)-\hat{G}^{\langle}_{s''s}(\mathbf{k}_1,\tau_1,\mathbf{K}_{\parallel 1},T_1)\hat{\Sigma}^a_{ss''}(\mathbf{k}_2,\tau_2,\mathbf{K}_{\parallel 2},T_2)].$$

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