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# On the extremal number of edges in hamiltonian connected graphs[☆]

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ABSTRACT

Assume that $n$ and $\delta$ are positive integers with $3 \leq \delta < n$. Let $hc(n, \delta)$ be the minimum number of edges required to guarantee an $n$-vertex graph $G$ with minimum degree $\delta(G) \geq \delta$ to be hamiltonian connected. Any $n$-vertex graph $G$ with $\delta(G) \geq \delta$ is hamiltonian connected if $|E(G)| \geq hc(n, \delta)$. We prove that $hc(n, \delta) = C(n - \delta + 1, 2) + \delta^2 - \delta + 1$ if $\delta \leq \lfloor \frac{n+3\times(n \textbf{ mod } 2)}{6} \rfloor + 1$, $hc(n, \delta) = C(n - \lfloor \frac{n}{2} \rfloor + 1, 2) + \lfloor \frac{n}{2} \rfloor^2 - \lfloor \frac{n}{2} \rfloor + 1$ if $\lfloor \frac{n+3\times(n \textbf{ mod } 2)}{6} \rfloor + 1 < \delta \leq \lfloor \frac{n}{2} \rfloor$, and $hc(n, \delta) = \lceil \frac{n\delta}{2} \rceil$ if $\delta > \lfloor \frac{n}{2} \rfloor$.



## 1. Introduction

In this paper, we use $C(a, b)$ to denote the combination of "$a$" numbers taking "$b$" numbers at a time, where $a, b$ are positive integers and $a \geq b$. For the graph definitions and notations, we follow [1]. Let $G = (V, E)$ be a graph if $V$ is a finite set and $E$ is a subset of $\{(u, v) \mid (u, v)$ is an unordered pair of $V\}$. We say that $V$ is the *vertex set* and $E$ is the *edge set*. Two vertices $u$ and $v$ are *adjacent* if $(u, v) \in E$. The *complete graph* $K_n$ is the graph with $n$ vertices such that any two distinct vertices are adjacent. The *degree* of a vertex $u$ in $G$, denoted by $\deg_G(u)$, is the number of vertices adjacent to $u$. We use $\delta(G)$ to denote $\min\{\deg_G(u) \mid u \in V(G)\}$. A *path* of length $m - 1$, $\langle v_0, v_1, \ldots, v_{m-1}\rangle$, is an ordered list of distinct vertices such that $v_i$ and $v_{i+1}$ are adjacent for $0 \leq i \leq m - 2$. A *cycle* is a path with at least three vertices such that the first vertex is the same as the last one. A *hamiltonian cycle* of $G$ is a cycle that traverses every vertex of $G$ exactly once. A graph is *hamiltonian* if it has a hamiltonian cycle. A *hamiltonian path* is a path of length $|V(G)| - 1$. A graph $G$ is *hamiltonian connected* if there exists a hamiltonian path between any two distinct vertices of $G$. It is easy to see that a hamiltonian connected graph with at least three vertices is hamiltonian.

It is proved by Moon [2] that the degree of any vertex in a hamiltonian connected graph with at least four vertices is at least 3. Therefore, it is natural to consider the $n$-vertex graph $G$ with $n \geq 4$ and $\delta(G) \geq 3$. Assume that $n$ and $\delta$ are positive integers with $3 \leq \delta < n$. Let $hc(n, \delta)$ be the minimum number of edges required to guarantee an $n$-vertex graph with minimum degree $\delta(G) \geq \delta$ to be hamiltonian connected. Any $n$-vertex graph $G$ with $\delta(G) \geq \delta$ is hamiltonian connected if $|E(G)| \geq hc(n, \delta)$. We will prove the following main theorem.

**Theorem A.** *Assume that $n$ and $\delta$ are positive integers with $3 \leq \delta < n$. Then*

$$hc(n, \delta) = \begin{cases} C(n - \delta + 1, 2) + \delta^2 - \delta + 1 & \text{if } \delta \leq \left\lfloor \dfrac{n + 3 \times (n \textbf{ mod } 2)}{6} \right\rfloor + 1, \\ C\left(n - \left\lfloor \dfrac{n}{2} \right\rfloor + 1, 2\right) + \left\lfloor \dfrac{n}{2} \right\rfloor^2 - \left\lfloor \dfrac{n}{2} \right\rfloor + 1 & \text{if } \left\lfloor \dfrac{n + 3 \times (n \textbf{ mod } 2)}{6} \right\rfloor + 1 < \delta \leq \left\lfloor \dfrac{n}{2} \right\rfloor, \\ \lceil n\delta/2 \rceil & \text{if } \delta > \left\lfloor \dfrac{n}{2} \right\rfloor. \end{cases}$$

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We will defer the proof of Theorem A to Section 4. In Section 2, we describe an application of Theorem A, which is the original motivation of this paper. In particular, we establish the relationship between $hc(n, g)$ and $g$-conditional edge-fault tolerant hamiltonian connectivity of the complete graph $K_n$. In Section 3, we present some preliminary results. Section 4 gives the proof of Theorem A.

## 2. An application

A hamiltonian graph $G$ is *k edge-fault tolerant hamiltonian* if $G - F$ remains hamiltonian for every $F \subset E(G)$ with $|F| \le k$. The *edge-fault tolerant hamiltonicity*, $\mathcal{H}_e(G)$, is defined as the maximum integer $k$ such that $G$ is $k$ edge-fault hamiltonian if $G$ is hamiltonian and is undefined otherwise. It is proved by Ore [3] that any $n$-vertex graph with at least $C(n, 2) - (n - 3)$ edges is hamiltonian. Moreover, there exists an $n$-vertex non-hamiltonian graph with $C(n, 2) - (n - 2)$ edges. In other words, $\mathcal{H}_e(K_n) = n - 3$ for $n \ge 3$. In Latifi et al. [4], it is proved that $\mathcal{H}_e(Q_n) = n - 2$ for $n \ge 2$ where $Q_n$ is the $n$-dimensional hypercube. In Li et al. [5], it is proved that $\mathcal{H}_e(S_n) = n - 3$ for $n \ge 3$ where $S_n$ is the $n$-dimensional star graph.

Chan and Lee [6] began the study of the existence of a hamiltonian cycle in a graph such that each vertex is incident with at least a number of nonfaulty edges. In particular, they have obtained results on hypercubes. A graph $G$ is *g-conditional k edge-fault tolerant hamiltonian* if $G - F$ is hamiltonian for every $F \subset E(G)$ with $|F| \le k$ and $\delta(G - F) \ge g$. The *g-conditional edge-fault tolerant hamiltonicity*, $\mathcal{H}_e^g(G)$, is defined as the maximum integer $k$ such that $G$ is $g$-conditional $k$ edge-fault tolerant hamiltonian if $G$ is hamiltonian and is undefined otherwise. Chan and Lee [6] proved that $\mathcal{H}_e^g(Q_n) \le 2^{g-1}(n - g) - 1$ for $n > g \ge 2$ and the equality holds for $g = 2$.

Recently, Fu [7] study the 2-conditional edge-fault tolerant hamiltonicity of the complete graph. In the paper by the authors, Ho et al. [8] extend Fu's result by studying the $g$-conditional edge-fault tolerant hamiltonicity of the complete graph for $g \ge 2$.

Several results (Lick [9], Moon [2], and Ore [10]) have studied hamiltonian connected graphs and some good sufficient conditions for a graph to be hamiltonian connected. Fault tolerant hamiltonian connectivity is another important parameter for graphs as indicated in [11]. A graph $G$ is *k edge-fault tolerant hamiltonian connected* if $G-F$ remains hamiltonian connected for any $F \subset E(G)$ with $|F| \le k$. The *edge-fault tolerant hamiltonian connectivity* of a graph $G$, $\mathcal{HC}_e(G)$, is defined as the maximum integer $k$ such that $G$ is $k$ edge-fault tolerant hamiltonian connected if $G$ is hamiltonian connected and is undefined otherwise. Again, Ore [10] proved that $\mathcal{HC}_e(K_n) = n - 4$ for $n \ge 4$.

Similarly, a graph $G$ is *g-conditional k edge-fault tolerant hamiltonian connected* if $G-F$ is hamiltonian connected for every $F \subset E(G)$ with $|F| \le k$ and $\delta(G - F) \ge g$. The *g-conditional edge-fault tolerant hamiltonian connectivity*, $\mathcal{HC}_e^g(G)$, is defined to be the maximum integer $k$ such that $G$ is $g$-conditional $k$ edge-fault tolerant hamiltonian connected if $G$ is hamiltonian connected and is undefined otherwise.

With the inspiration of the work by Fu [7] in the study of 2-conditional edge-fault tolerant hamiltonicity of the complete graph, Ho et al. [12] begin the study on 3-conditional edge-fault tolerant hamiltonian connectivity of the complete graph. The following result was obtained in [12]:

Let $n \ge 4$ and $F \subset E(K_n)$ with $\delta(K_n - F) \ge 3$. Then $K_n - F$ is hamiltonian connected if $|F| \le 2n - 10$ for $n \notin \{4, 5, 8, 10\}$, $|F| = 0$ for $n = 4$, $|F| \le 2$ for $n = 5$, and $|F| \le 2n - 11$ for $n \in \{8, 10\}$.

We restate this result using our terminology.

**Theorem 1.** *$\mathcal{HC}_e^3(K_n) = 2n - 10$ for $n \notin \{4, 5, 8, 10\}$ and $n \ge 5$, $\mathcal{HC}_e^3(K_4) = 0$, $\mathcal{HC}_e^3(K_5) = 2$, $\mathcal{HC}_e^3(K_8) = 5$, and $\mathcal{HC}_e^3(K_{10}) = 9$.*

Now, we extend the result in [12] and use our main result Theorem A to compute $\mathcal{HC}_e^g(K_n)$ for $3 \le g < n$.

**Theorem 2.** *$\mathcal{HC}_e^g(K_n) = C(n, 2) - hc(n, g)$ for $3 \le g < n$.*

**Proof.** Let $F$ be any faulty edge set of $K_n$ with $|F| \le C(n, 2) - hc(n, g)$ such that $\delta(K_n - F) \ge g$. Obviously, $|E(K_n - F)| \ge hc(n, g)$. By Theorem A, $K_n - F$ is hamiltonian connected. Thus, $\mathcal{HC}_e^g(K_n) \ge C(n, 2) - hc(n, g)$.

Now, we prove that $\mathcal{HC}_e^g(K_n) \le C(n, 2) - hc(n, g)$. Assume that $\mathcal{HC}_e^g(K_n) \ge C(n, 2) - hc(n, g) + 1$. Let $G$ be any graph with $hc(n, g) - 1$ edges such that $\delta(G) \ge g$. Let $F = E(K_n) \setminus E(G)$. In other words, $G = K_n - F$. Obviously, $|F| = C(n, 2) - hc(n, g) + 1$. Since $\mathcal{HC}_e^g(K_n) \ge C(n, 2) - hc(n, g) + 1$, $G$ is hamiltonian connected. This contradicts to the definition of $hc(n, g)$. Thus, $\mathcal{HC}_e^g(K_n) \le C(n, 2) - hc(n, g)$.

Therefore, $\mathcal{HC}_e^g(K_n) = C(n, 2) - hc(n, g)$ for $3 \le g < n$. □

## 3. Preliminary results

The following theorem is proved by Ore [10].

**Theorem 3** ([10]). *Let $G$ be an $n$-vertex graph with $\delta(G) > \lfloor \frac{n}{2} \rfloor$. Then $G$ is hamiltonian connected.*

The following theorem is given by Lick [9].

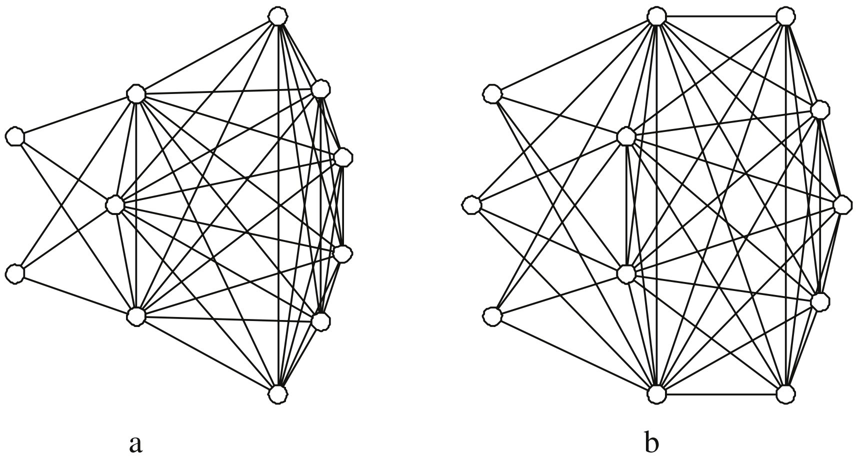


**Fig. 1.** The graphs (a) $H_{3,11}$ and (b) $H_{4,12}$.

**Theorem 4** ([9]). *Let $G$ be an $n$-vertex graph. Assume that the degree $d_i$ of $G$ satisfy $d_1 \le d_2 \le \dots \le d_n$. If $d_{j-1} \le j \le n/2 \Rightarrow d_{n-j} \ge n - j + 1$, then $G$ is hamiltonian connected.*

To our knowledge, no one has ever discussed the sharpness of the above theorem. In the following, we give a logically equivalent theorem.

**Theorem 5.** *Let $G$ be an $n$-vertex graph. Assume that the degree $d_i$ of $G$ satisfy $d_1 \le d_2 \le \dots \le d_n$. If $G$ is non-hamiltonian connected, then there exist at least one integer $2 \le m \le n/2$ such that $d_{m-1} \le m \le n/2$ and $d_{n-m} \le n - m$.*

To discuss the sharpness of Theorem 5, we introduce the following family of graphs. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The *union* of $G_1$ and $G_2$, written $G_1 + G_2$, has edge set $E_1 \cup E_2$ and vertex set $V_1 \cup V_2$ with $V_1 \cap V_2 = \emptyset$. The *join* of $G_1$ and $G_2$, written $G_1 \vee G_2$, obtained from $G_1 + G_2$ by joining each vertex of $G_1$ to each vertex of $G_2$.

The *degree sequence* of an $n$-vertex graph is the list of vertices degree, in nondecreasing order, as $d_1 \le d_2 \le \dots \le d_n$. For $2 \le m \le n/2$, let $H_{m,n}$ denote the graph $(\bar{K}_{m-1} + K_{n-2m+1}) \vee K_m$. The graphs $H_{3,11}$ and $H_{4,12}$ are shown in Fig. 1. Obviously, the degree sequence of $H_{m,n}$ is

$$(\underbrace{m, m, \dots, m}_{m-1}, \underbrace{n-m, n-m, \dots, n-m}_{n-2m+1}, \underbrace{n-1, n-1, \dots, n-1}_{m})$$

A sequence of real numbers $(p_1, p_2, \dots, p_n)$ is said to be *majorised* by another sequence $(q_1, q_2, \dots, q_n)$ if $p_i \le q_i$ for $1 \le i \le n$. A graph $G$ is *degree-majorised* by a graph $H$ if $|V(G)| = |V(H)|$ and the nondecreasing degree sequence of $G$ is majorised by that of $H$. For instance, the 5-cycle is degree-majorised by the complete bipartite graph $K_{2,3}$ because $(2, 2, 2, 2, 2)$ is majorised by $(2, 2, 2, 3, 3)$.

**Lemma 1.** *Let $G = (V, E)$ be a graph, $X$ be a subset of $V$, and $u$, $v$ be any two distinct vertices in $X$. Suppose that there exists a hamiltonian path between $u$ and $v$. Then there are at most $|X| - 1$ connected components of $G - X$.*

Let $S$ be the subset of $V(H_{m,n})$ corresponding to the vertex of $K_m$. Since $2 \le m \le n/2$, $|S| \ge 2$. Let $u$ and $v$ be any two distinct vertices in $S$. Obviously, there are $m$ connected components of $H_{m,n} - S$. By Lemma 1, $H_{m,n}$ does not have a hamiltonian path between $u$ and $v$. Thus, $H_{m,n}$ is not hamiltonian connected. In other words, the result in Theorem 5 is sharp.

So we have the following corollary.

**Corollary 1.** *The graph $H_{m,n}$ is not hamiltonian connected where $n$ and $m$ are integers with $2 \le m \le n/2$.*

Thus, the following theorem is equivalent to Theorem 5.

**Theorem 6.** *If $G$ is an $n$-vertex non-hamiltonian connected graph, then $G$ is degree-majorised by some $H_{m,n}$ with $2 \le m \le n/2$.*

**Corollary 2.** *Let $n \ge 6$. Assume that $G$ is an $n$-vertex non-hamiltonian connected graph. Then $\delta(G) \le \lfloor \frac{n}{2} \rfloor$ and $|E(G)| \le \max\{|E(H_{\delta(G),n})|, |E(H_{\lfloor \frac{n}{2} \rfloor,n})|\}$.*

**Proof.** Let $G$ be any $n$-vertex non-hamiltonian connected graph. With Theorem 3, $\delta(G) \le \lfloor \frac{n}{2} \rfloor$. By Theorem 6, $G$ is degree-majorised by some $H_{m,n}$. Since $\delta(H_{m,n}) = m$, $\delta(G) \le m \le \lfloor \frac{n}{2} \rfloor$. Therefore $|E(G)| \le \max\{|E(H_{m,n})| \mid \delta(G) \le m \le \lfloor \frac{n}{2} \rfloor\}$. Since $|E(H_{m,n})| = \frac{1}{2}(m(m-1) + (n-2m+1)(n-m) + m(n-1))$ is a quadratics function with respect to $m$ and the maximum value of it occurs at the boundary $m = \delta(G)$ or $m = \lfloor \frac{n}{2} \rfloor$, $|E(G)| \le \max\{|E(H_{\delta(G),n})|, |E(H_{\lfloor \frac{n}{2} \rfloor,n})|\}$. $\square$

By Corollary 2, we have the following corollary.

**Corollary 3.** *Let G be an n-vertex graph with* $n \geq 6$. *If* $|E(G)| \geq \max\{|E(H_{\delta(G),n})|, |E(H_{\lfloor \frac{n}{2} \rfloor,n})|\} + 1$, *then G is hamiltonian connected.*

**Lemma 2.** *Let n and k be integers with* $n \geq 6$ *and* $3 \leq k \leq \lfloor \frac{n}{2} \rfloor$. *Then* $|E(H_{k,n})| \geq |E(H_{\lfloor \frac{n}{2} \rfloor,n})|$ *if and only if* $3 \leq k \leq \lfloor \frac{n+3\times(n \bmod 2)}{6} \rfloor + 1$ *or* $k = \lfloor \frac{n}{2} \rfloor$.

**Proof.** We first prove the case that $n$ is even. We claim that $|E(H_{k,n})| \geq |E(H_{\frac{n}{2},n})|$ if and only if $3 \leq k \leq \lfloor \frac{n}{6} \rfloor + 1$ or $k = \frac{n}{2}$. Suppose that $|E(H_{k,n})| < |E(H_{\frac{n}{2},n})|$. Then $|E(H_{k,n})| = \frac{1}{2}(k(k-1) + (n-2k+1)(n-k) + k(n-1)) < |E(H_{\frac{n}{2},n})| = \frac{1}{2}((\frac{n}{2}-1)(\frac{n}{2})) + (\frac{n}{2})(n-1) + (\frac{n}{2})$. This implies $3k^2 - (2n+3)k + (\frac{1}{4}n^2 + \frac{3}{2}n) < 0$, which means $(k - \frac{n}{2})(3k - \frac{n}{2} - 3) < 0$. Thus $|E(H_{k,n})| < |E(H_{\frac{n}{2},n})|$ if and only if $\frac{n}{6} + 1 < k < \frac{n}{2}$. Note that $n$ and $k$ are integers with $n$ is even, $n \geq 6$, and $3 \leq k \leq \frac{n}{2}$. Therefore, $|E(H_{k,n})| \geq |E(H_{\frac{n}{2},n})|$ if and only if $3 \leq k \leq \lfloor \frac{n}{6} \rfloor + 1$ or $k = \frac{n}{2}$.

For odd integer $n$, using the same method, we can prove that $|E(H_{k,n})| < |E(H_{\frac{n-1}{2},n})|$ if and only if $\frac{n+3}{6} + 1 < k < \frac{n-1}{2}$. Given that $n \geq 7$, and $3 \leq k \leq \frac{n-1}{2}$, then $|E(H_{k,n})| \geq |E(H_{\frac{n-1}{2},n})|$ if and only if $3 \leq k \leq \lfloor \frac{n+3}{6} \rfloor + 1$ or $k = \frac{n-1}{2}$. Therefore, the result follows. □

## 4. Proof of Theorem A

By brute force, we can check that $hc(4,3) = 6$, $hc(5,3) = 8$, and $hc(5,4) = 10$. Therefore, the theorem holds for $n = 4, 5$. Next, we consider the cases that $3 \leq \delta \leq \lfloor \frac{n}{2} \rfloor$ and $n \geq 6$.

Suppose that $3 \leq \delta \leq \lfloor \frac{n+3\times(n \bmod 2)}{6} \rfloor + 1$ or $\delta = \lfloor \frac{n}{2} \rfloor$. By Lemma 2, $|E(H_{\delta,n})| \geq |E(H_{\lfloor \frac{n}{2} \rfloor,n})|$. Let $G$ be any $n$-vertex graph with $\delta(G) \geq \delta$ and $|E(G)| \geq |E(H_{\delta,n})| + 1$. By Corollary 3, $G$ is hamiltonian connected. We note that $|E(H_{\delta,n})| + 1 = C(n-\delta+1, 2) + \delta^2 - \delta + 1$. Therefore, $hc(n,\delta) \leq C(n-\delta+1,2) + \delta^2 - \delta + 1$. By Corollary 1, $H_{\delta,n}$ is not hamiltonian connected. Thus, $hc(n,\delta) > |E(H_{\delta,n})| = C(n-\delta+1,2) + \delta^2 - \delta$. Hence, $hc(n,\delta) = C(n-\delta+1,2) + \delta^2 - \delta + 1$.

Suppose that $\lfloor \frac{n+3\times(n \bmod 2)}{6} \rfloor + 1 < \delta < \lfloor \frac{n}{2} \rfloor$. By Lemma 2, $|E(H_{\delta,n})| < |E(H_{\lfloor \frac{n}{2} \rfloor,n})|$. Let $G$ be any $n$-vertex graph with $\delta(G) \geq \delta$ and $|E(G)| \geq |E(H_{\lfloor \frac{n}{2} \rfloor,n})| + 1$. By Corollary 3, $G$ is hamiltonian connected. We note that $|E(H_{\lfloor \frac{n}{2} \rfloor,n})| + 1 = C(n - \lfloor \frac{n}{2} \rfloor + 1, 2) + \lfloor \frac{n}{2} \rfloor^2 - \lfloor \frac{n}{2} \rfloor + 1$. Therefore, $hc(n,\delta) \leq C(n - \lfloor \frac{n}{2} \rfloor + 1, 2) + \lfloor \frac{n}{2} \rfloor^2 - \lfloor \frac{n}{2} \rfloor + 1$. By Corollary 1, $H_{\lfloor \frac{n}{2} \rfloor,n}$ is not hamiltonian connected. Thus, $hc(n,\delta) > |E(H_{\lfloor \frac{n}{2} \rfloor,n})| = C(n - \lfloor \frac{n}{2} \rfloor + 1, 2) + \lfloor \frac{n}{2} \rfloor^2 - \lfloor \frac{n}{2} \rfloor$. Hence, $hc(n,\delta) = C(n - \lfloor \frac{n}{2} \rfloor + 1, 2) + \lfloor \frac{n}{2} \rfloor^2 - \lfloor \frac{n}{2} \rfloor + 1$.

Finally, we consider the case that $\delta > \lfloor \frac{n}{2} \rfloor$ and $n \geq 6$. Let $G$ be any graph with $\delta(G) \geq \delta > \lfloor \frac{n}{2} \rfloor$. By Theorem 3, $G$ is hamiltonian connected. Obviously, $|E(G)| \geq \lceil \frac{n\delta}{2} \rceil$. Thus, $hc(n,\delta) = \lceil \frac{n\delta}{2} \rceil$.

The proof of our main result, Theorem A, is complete. □

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