Optimization of Transceivers with Bit Allocation to Maximize Bit Rate for MIMO Transmission

Chien-Chang Li, Yuan-Pei Lin, Shang-Ho Tsai, and P. P. Vaidyanathan

Abstract—There have been many results on designing transceivers for MIMO channels. In early results, the transceiver is designed for a given bit allocation. In this paper we will *jointly* design the transceiver and bit allocation for maximizing bit rate. By using a high bit rate assumption, we will see that the optimal transceiver and bit allocation can be obtained in a closed form using simple Hadamard inequality and the Poincaré separation theorem. In the simulation, we will demonstrate the usefulness of the joint design. Simulation results, in which a high bit rate assumption is not used in allocating bits, show that a higher bit rate can be achieved compared to previously reported methods.

Index Terms—MIMO systems, transceivers, transmitters, receivers, communication systems.

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) channels arise in applications such as wireless communication systems that use multiple antennas and also telephone cables that consist of many twisted wire pairs. The information capacity for MIMO transmissions was analyzed in [1]-[2]. A lower bound on the bit error rate for nonuniform bit loading was derived in [3]. Optimal transceivers of different design criteria for MIMO channels have been considered in the literature earlier, e.g., [4]-[12]. Optimal transceivers for two design criteria: maximum signal to noise ratio under zero-forcing constraint and minimum mean-square error (MMSE), are developed in [4] for a given bit allocation. The optimal zero-forcing transceiver that minimizes the bit error rate (BER) is derived in [5]. Assuming the bit allocation is given, the system is optimized for a given transmit power. A minimum BER design with a channel independent transmitter is considered in [6]. Zero-forcing solutions with the aim of minimizing the total transmit power for a given BER are developed in [7]. Transceivers with two design criteria: minimum mean-squared error and minimum error rate for a given power constraint, are proposed in [8]. To incorporate quality of service criterion in the design, a weighted minimum mean-squared error criterion subject to a transmit power constraint is proposed in [9]. A unified framework for designing MIMO systems with an MMSE receiver is proposed in [10]. A number of useful objective functions can be considered in

C.-C. Li, Y.-P. Lin, and S.-H. Tsai are with Electrical and Control Engineering, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu, Taiwan 300, ROC (e-mail: halliday.ece92g@nctu.edu.tw, {ypl, shanghot}@mail.nctu.edu.tw).

P. P. Vaidyanathan is with the Dept. Electrical Engineering, California Institute of Technology, Pasadena, CA 91125, USA (e-mail: ppv-nath@systems.caltech.edu).
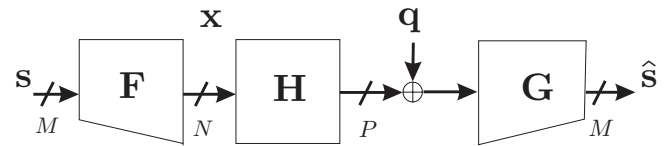
Fig. 1.   MIMO communication system.

this framework. For example, for a given bit allocation, the optimal MMSE power minimizing system can be designed using this unified approach. As an extension of [10], the transmit power is minimized in [11] with different quality of service requirements such as mean-squared error, signal to interference ratio, and bit error rate. In these works [4]-[11], the constellations of the input symbols are assumed to be given. In [12], bit allocation is considered based on a suboptimal transceiver structure that had been obtained using a power minimization criterion in [11].

In this paper, we consider the design of zero-forcing transceivers over a MIMO channel for a given error rate. Unlike earlier designs that optimize the system for a given bit allocation or design the bit allocation and transceiver separately, we *jointly* optimize the transceiver and bit allocation to maximize the transmission rate. Using the high bit rate assumption, we can simplify the optimization problem. The solutions are obtained in two steps. Firstly, we design the optimal bit and power allocation for a given transceiver and a given power constraint. Secondly, we design the optimal transceiver that maximizes the bit rate based on the optimal bit and power allocation. In the second step, the optimal transceiver can be easily found by the *Hadamard inequality* and the *Poincaré separation theorem*. The proposed design does not assume a given bit allocation as in earlier works. Rather, the transceiver and bit allocation are jointly designed. Although the high bit rate assumption is used to derive the optimal transceiver, the assumption is not used in allocating bits in the simulations. Simulations show that a higher bit rate can be achieved due to the incorporation of bit allocation in the design.

II. SYSTEM MODEL

A generic MIMO communication system is shown in Fig. 1. The MIMO channel is modeled by a $P \times N$ memoryless matrix $\mathbf{H}$. The $P \times 1$ channel noise $\mathbf{q}$ is additive white gaussian noise with variance N_0. The transmitter matrix $\mathbf{F}$ is of size $N \times M$. The receiver matrix $\mathbf{G}$ is of size $M \times P$. The input of the transmitter is $\mathbf{s}$, an $M \times 1$ vector of modulation symbols. The symbols are assumed to be zero mean and uncorrelated; hence the autocorrelation matrix $\mathbf{\Lambda}_s = \mathrm{E}[\mathbf{s}\mathbf{s}^\dagger]$, where $\dagger$ denotes the

transpose conjugate, is a diagonal matrix with $[\mathbf{\Lambda}_s]_{kk} = \sigma_{s_k}^2$. The total transmit power is $\mathrm{E}\{\mathbf{x}^\dagger\mathbf{x}\} = \mathrm{Tr}(\mathbf{F}\mathbf{\Lambda}_s\mathbf{F}^\dagger)$, where $\mathbf{x}$ is the transmitter output. To achieve the zero-forcing condition, the transceiver needs to satisfy $\mathbf{GHF} = \mathbf{I}_M$, where $\mathbf{I}_M$ is $M \times M$ identity matrix. The output noise vector is denoted by $\mathbf{e} = \mathbf{Gq}$ with $\sigma_{e_k}^2 = N_0[\mathbf{GG}^\dagger]_{kk}$, where the notation $[\mathbf{X}]_{kk}$ denotes the k-th diagonal element of $\mathbf{X}$. Assume the symbol error rates (SER) are the same for all the subchannels. Let b_k be the number of bits carried by the k-th symbol. For QAM modulation,

$$b_k = \log_2\left(1 + \frac{\sigma_{s_k}^2}{\sigma_{e_k}^2 \Gamma}\right), \tag{1}$$

where $\Gamma = \frac{1}{3}[Q^{-1}(\mathrm{SER}/4)]^2$ is determined by the prescribed symbol error rate [13]. The function $Q(x)$ is the area under a Gaussian tail, i.e., $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-u^2/2}du$. The derivation in this paper is given for the QAM case. The results for the PAM case can be obtained in a similar way (For the PAM case, there is an additional scalar of $\frac{1}{2}$). In this paper, we will use the high bit rate assumption, i.e., $2^{b_k} \gg 1$, and

$$b_k = \log_2\left(\frac{\sigma_{s_k}^2}{\sigma_{e_k}^2 \Gamma}\right). \tag{2}$$

We will see in section 3 that such an assumption facilitates the derivation of the optimal transceiver. The problem of maximizing bit rate subject to a zero-forcing constraint and a total transmit power constraint P_0 can be formulated as

$$\begin{array}{ll} \underset{\mathbf{G},\, \mathbf{F},\, \{\sigma_{s_k}^2\}}{\text{maximize}} & b = \sum_{k=0}^{M-1} \log_2\left(\frac{\sigma_{s_k}^2}{\sigma_{e_k}^2 \Gamma}\right),\ \sigma_{e_k}^2 = N_0[\mathbf{GG}^\dagger]_{kk} \\[2mm] \text{subject to} & \left\{ \begin{array}{l} \mathrm{Tr}(\mathbf{F}\mathbf{\Lambda}_s\mathbf{F}^\dagger) \le \mathrm{P}_0 \\ \mathbf{GHF} = \mathbf{I}_M \\ \sigma_{s_k}^2 \ge 0, k = 0, 1, \cdots, M-1. \end{array} \right. \end{array} \tag{3}$$

III. OPTIMAL ZERO-FORCING TRANSCEIVER

First, we will find the power allocation that maximizes the bit rate for a given zero-forcing transceiver. To this end, we use the Karush-Kuhn-Tucker (KKT) condition [14]. Let $\sigma_{s_k}^{2*}$ be a local maximum for the optimization problem in (3). Then there exists constants α and β_k, for $k = 0, 1, \cdots, M-1$ such that:

1) $\alpha \le 0$.
2) $\beta_k \le 0$, for $k = 0, 1, \cdots, M-1$.
3) $\left.\frac{\partial}{\partial \sigma_{s_k}^2}\left(\sum_{k=0}^{M-1} \log_2\left(\frac{\sigma_{s_k}^2}{\sigma_{e_k}^2 \Gamma}\right) + \alpha(\mathrm{Tr}(\mathbf{F}\mathbf{\Lambda}_s\mathbf{F}^\dagger) - \mathrm{P}_0) + \sum_{k=0}^{M-1} \beta_k(-\sigma_{s_k}^2)\right)\right|_{\sigma_{s_k}^2 = \sigma_{s_k}^{2*}} = 0$.
4) $\left.\alpha(\mathrm{Tr}(\mathbf{F}\mathbf{\Lambda}_s\mathbf{F}^\dagger) - \mathrm{P}_0)\right|_{\sigma_{s_k}^2 = \sigma_{s_k}^{2*}} = 0$.
5) $\left.\beta_k(-\sigma_{s_k}^2)\right|_{\sigma_{s_k}^2 = \sigma_{s_k}^{2*}} = 0$, for $k = 0, 1, \cdots, M-1$.

By solving the above conditions, the optimal power allocation is given by

$$\sigma_{s_k}^2 = \frac{P_0}{M[\mathbf{F}^\dagger\mathbf{F}]_{kk}}. \tag{4}$$

From (4), we can see that the power allocation depends only on the transmitter for the given P_0 and M. Using (4), the bit rate is given by

$$b = \sum_{k=0}^{M-1} \log_2\left(\frac{P_0}{M\Gamma[\mathbf{F}^\dagger\mathbf{F}]_{kk}\sigma_{e_k}^2}\right) \tag{5}$$

$$= \log_2\left(\prod_{k=0}^{M-1} \frac{P_0}{M\Gamma[\mathbf{F}^\dagger\mathbf{F}]_{kk}\sigma_{e_k}^2}\right). \tag{6}$$

Next, we will design the optimal zero-forcing transceiver that maximizes the bit rate in (6). Suppose the $P \times N$ channel matrix $\mathbf{H}$ has rank K. Let the singular value decomposition of $\mathbf{H}$ be $\mathbf{H} = \mathbf{U}\begin{bmatrix} \mathbf{\Lambda} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\mathbf{V}^\dagger$, where the $K \times K$ diagonal matrix $\mathbf{\Lambda}$ contains the nonzero singular values of $\mathbf{H}$. The $P \times P$ matrix $\mathbf{U}$ and the $N \times N$ matrix $\mathbf{V}$ are unitary. We assume that the elements of $\mathbf{\Lambda}$ are in nonincreasing order and $K \ge M$ so that solutions of zero-forcing transceivers exist.

Lemma 1: Without loss of generality, we can express $\mathbf{F}$ to be of the following form:

$$\mathbf{F} = \mathbf{V}\begin{bmatrix} \mathbf{A} \\ \mathbf{0} \end{bmatrix}, \tag{7}$$

for appropriate $K \times M$ matrix $\mathbf{A}$ of rank M.

Proof: Suppose $(\mathbf{G}, \mathbf{F})$ is a transceiver pair that satisfies the zero-forcing condition. As $\mathbf{V}$ is unitary, $\mathbf{F}$ can always be represented as $\mathbf{F} = \mathbf{V}[\ \mathbf{A}^T\quad \mathbf{A}_1^T\ ]^T$, where $\mathbf{A}$ is a $K \times M$ matrix, $\mathbf{A}_1$ is an $(N-K) \times M$ matrix, and the notation T denotes the transpose. Define a new transceiver $\mathbf{F}'$ as $\mathbf{F}' = \mathbf{V}[\ \mathbf{A}^T\quad \mathbf{0}\ ]^T$. Then we have $\mathbf{GHF}' = \mathbf{GHF}$. Therefore, when we replace the transmitter by $\mathbf{F}'$, the new system still satisfies the zero-forcing condition $\mathbf{GHF} = \mathbf{I}_M$. As the receiver is not changed, the new system has the same subchannel noise variances and hence the same bit rate performance. Now, let us compare the transmit power of $\mathbf{F}$ and $\mathbf{F}'$ for the same $\mathbf{\Lambda}_s$. The transmit power when we use $\mathbf{F}$ is $\mathrm{Tr}(\mathbf{F}\mathbf{\Lambda}_s\mathbf{F}^\dagger) = \mathrm{Tr}(\mathbf{A}\mathbf{\Lambda}_s\mathbf{A}^\dagger) + \mathrm{Tr}(\mathbf{A}_1\mathbf{\Lambda}_s\mathbf{A}_1^\dagger)$. Note that the transmit power with $\mathbf{F}'$ is $\mathrm{Tr}(\mathbf{F}'\mathbf{\Lambda}_s\mathbf{F}'^\dagger) = \mathrm{Tr}(\mathbf{A}\mathbf{\Lambda}_s\mathbf{A}^\dagger) \le \mathrm{Tr}(\mathbf{F}\mathbf{\Lambda}_s\mathbf{F}^\dagger)$. This means a transmitter of the form in (7) is no loss of generality. △△△

Lemma 2: It is no loss of generality to choose $\mathbf{G}$ as the pseudo inverse of $\mathbf{HF}$. That is,

$$\mathbf{G} = (\mathbf{A}^\dagger\mathbf{\Lambda}^2\mathbf{A})^{-1}[\ \mathbf{A}^\dagger\mathbf{\Lambda}\quad \mathbf{0}\ ]\mathbf{U}^\dagger, \tag{8}$$

where $\mathbf{A}$ is the matrix given in *(7)*. In this case, the noise variance at the k-th subchannel is given by

$$\sigma_{e_k}^2 = N_0[\mathbf{GG}^\dagger]_{kk} = N_0[(\mathbf{A}^\dagger\mathbf{\Lambda}^2\mathbf{A})^{-1}]_{kk}. \tag{9}$$

Proof: Suppose $(\mathbf{G}, \mathbf{F})$ is a transceiver pair that satisfies zero-forcing condition, and $\mathbf{F}$ is of the form in (7). Let $\mathbf{G}'$ be the pseudo inverse of $\mathbf{HF}$, i.e.,

$$\mathbf{G}' = (\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{HF})^{-1}\mathbf{F}^\dagger\mathbf{H}^\dagger \tag{10}$$

$$= (\mathbf{A}^\dagger\mathbf{\Lambda}^2\mathbf{A})^{-1}[\ \mathbf{A}^\dagger\mathbf{\Lambda}\quad \mathbf{0}\ ]\mathbf{U}^\dagger. \tag{11}$$

Define $\mathbf{\Delta} = \mathbf{G} - \mathbf{G}'$. Since $(\mathbf{G}, \mathbf{F})$ and $(\mathbf{G}', \mathbf{F})$ are both zero-forcing, we have $\mathbf{\Delta HF} = \mathbf{0}$. It follows that $\mathbf{\Delta G}'^\dagger = \mathbf{0}$. When we use $\mathbf{G}$, the noise variance at k-th subchannel is given by $N_0[(\mathbf{G}' + \mathbf{\Delta})(\mathbf{G}' + \mathbf{\Delta})^\dagger]_{kk} \ge N_0[\mathbf{G}'\mathbf{G}'^\dagger]_{kk}$, where we have

used $\Delta \mathbf{G}'^{\dagger} = \mathbf{0}$ and $[\Delta \Delta^{\dagger}]_{kk} > 0$. Therefore, we will have smaller subchannel noise variances when we replace $\mathbf{G}$ with $\mathbf{G}'$ and hence a higher bit rate can be achieved. Using (8), we have the subchannel noise variance $\sigma_{e_k}^2$ as in (9).   △△△

Note that the transmitter and receiver in Lemma 1 and Lemma 2 have the same form as those in [3] and [7]. The transceivers in [3] and [7] are designed for minimizing the transmit power for a given bit allocation, while we jointly design the optimal transceiver and bit allocation for maximizing the transmission rate. Lemma 1 and Lemma 2 lead us to conclude that the matrix $\mathbf{A}$ in (7) is the only part of the transceiver left to be designed. Using the expression of $\mathbf{F}$ in Lemma 1 and the expression of $\sigma_{e_k}^2$ in Lemma 2, the bit rate in (6) becomes

$$ b = \log_2 \left[(\frac{P_0}{MN_0\Gamma})^M \frac{1}{\Phi} \right], \tag{12} $$

where $\Phi = \prod_{k=0}^{M-1} [\mathbf{A}^{\dagger}\mathbf{A}]_{kk} [(\mathbf{A}^{\dagger}\mathbf{\Lambda}^2\mathbf{A})^{-1}]_{kk}$. To maximize b, we need to find $\mathbf{A}$ that minimizes Φ.

Optimal structure of A: Applying the Hadamard inequality [15], we have

$$ \Phi = \prod_{k=0}^{M-1} [\mathbf{A}^{\dagger}\mathbf{A}]_{kk} [(\mathbf{A}^{\dagger}\mathbf{\Lambda}^2\mathbf{A})^{-1}]_{kk} \tag{13} $$

$$ \geq \det[\mathbf{A}^{\dagger}\mathbf{A}] \det[(\mathbf{A}^{\dagger}\mathbf{\Lambda}^2\mathbf{A})^{-1}]. \tag{14} $$

The equality holds if and only if the matrix $\mathbf{A}$ satisfies the following two conditions:
1). $\mathbf{A}^{\dagger}\mathbf{A}$ is diagonal, and 2). $\mathbf{A}^{\dagger}\mathbf{\Lambda}^2\mathbf{A}$ is diagonal. The first condition means that the columns of $\mathbf{A}$ are orthogonal, while the second means that the columns of $\mathbf{\Lambda}\mathbf{A}$ are orthogonal. As $\mathbf{\Lambda}\mathbf{A}$ is orthogonal, we can express it as $\mathbf{\Lambda}\mathbf{A} = \mathbf{Q}\mathbf{D}$, for some $K \times M$ unitary matrix $\mathbf{Q}$ such that $\mathbf{Q}^{\dagger}\mathbf{Q} = \mathbf{I}_M$, and some $M \times M$ nonsingular diagonal matrix $\mathbf{D}$. As $\mathbf{\Lambda}$ is nonsingular, we can write $\mathbf{A} = \mathbf{\Lambda}^{-1}\mathbf{Q}\mathbf{D}$. Then the product of the two determinant quantities in (14) becomes $\det[\mathbf{A}^{\dagger}\mathbf{A}]\det[(\mathbf{A}^{\dagger}\mathbf{\Lambda}^2\mathbf{A})^{-1}] = \det(\mathbf{Q}^{\dagger}\mathbf{\Lambda}^{-2}\mathbf{Q})$. Hence the bit rate in (12) is simplified to

$$ b = \log_2 \left[(\frac{P_0}{MN_0\Gamma})^M \frac{1}{\det(\mathbf{Q}^{\dagger}\mathbf{\Lambda}^{-2}\mathbf{Q})} \right]. \tag{15} $$

Note that the bit rate in (15) is independent of $\mathbf{D}$. Without loss of generality, we can choose $\mathbf{D}$ to be any $M \times M$ nonsingular diagonal matrix. For example, we can choose $\mathbf{D} = \mathbf{I_M}$. To achieve the maximal bit rate, we need to find $\mathbf{Q}$ that minimizes $\det(\mathbf{Q}^{\dagger}\mathbf{\Lambda}^{-2}\mathbf{Q})$.

Optimal Q: We can find $\mathbf{Q}$ with the help of the Poincaré separation theorem [15], which is presented below for convenience.

Poincaré separation theorem [15]*: Let* $\mathbf{B}$ *be an* $n \times n$ *Hermitian matrix and* $\mathbf{C}$ *be an* $n \times r$ *unitary matrix with* $\mathbf{C}^{\dagger}\mathbf{C} = \mathbf{I}_r$*. Then we have* $\rho_i(\mathbf{B}) \leq \rho_i(\mathbf{C}^{\dagger}\mathbf{B}\mathbf{C}) \leq \rho_{n-r+i}(\mathbf{B})$, $i = 0, 1, \cdots, r-1$, *where the notation* $\rho_i(\mathbf{X})$ *denotes the* i-*th smallest eigenvalue of* $\mathbf{X}$.

By the Poincaré separation theorem, we have $[\mathbf{\Lambda}^{-2}]_{ii} \leq \rho_i(\mathbf{Q}^{\dagger}\mathbf{\Lambda}^{-2}\mathbf{Q})$, $i = 0, 1, \cdots, M-1$, where we have used the property that the diagonal elements of $\mathbf{\Lambda}$ are in nonincreasing order. Since the diagonal matrix $\mathbf{\Lambda}$ is nonsingular,

we know that $\mathbf{\Lambda}^{-2}$ is positive definite and $[\mathbf{\Lambda}^{-2}]_{ii} > 0$ for $i = 0, 1, \cdots, K-1$. Therefore, we have

$$ \det(\mathbf{Q}^{\dagger}\mathbf{\Lambda}^{-2}\mathbf{Q}) = \prod_{i=0}^{M-1} \rho_i(\mathbf{Q}^{\dagger}\mathbf{\Lambda}^{-2}\mathbf{Q}) \tag{16} $$

$$ \geq \prod_{i=0}^{M-1} [\mathbf{\Lambda}^{-2}]_{ii} = \det(\mathbf{\Lambda}_{\mathrm{M}}^{-2}), \tag{17} $$

where $\mathbf{\Lambda}_M$ is an $M \times M$ diagonal matrix whose diagonal elements consist of the M largest singular values of $\mathbf{H}$. The inequality in (17) becomes an equality when we choose $\mathbf{Q} = [\ \mathbf{I}_M \ \ \mathbf{0}\ ]^T$. With this choice of $\mathbf{Q}$ and $\mathbf{D} = \mathbf{I}_M$, we have $\mathbf{A} = \mathbf{\Lambda}^{-1}\mathbf{Q}\mathbf{D} = [\ \mathbf{\Lambda}_M^{-1} \ \ \mathbf{0}\ ]^T$. Using Lemma 1 and Lemma 2, the optimal transceiver is given by

$$ \mathbf{F} = \mathbf{V} \left[\begin{array}{c} \mathbf{\Lambda}_M^{-1} \\ \mathbf{0} \end{array} \right], \quad \mathbf{G} = [\ \mathbf{I}_M \ \ \mathbf{0}\ ]\mathbf{U}^{\dagger}. \tag{18} $$

The maximal bit rate in (15) is given by $b = \log_2[(\frac{P_0}{MN_0\Gamma})^M \det(\mathbf{\Lambda}_{\mathrm{M}}^2)]$. Substituting (18) into (4) and (9), we have $\sigma_{s_k}^2 = P_0[\mathbf{\Lambda}_M^2]_{kk}/M$ and $\sigma_{e_k}^2 = N_0$. Using (1), the number of bits allocated to the k-th subchannel becomes

$$ b_k = \log_2 \left[1 + (\frac{P_0}{MN_0\Gamma})[\mathbf{\Lambda}_M^2]_{kk} \right]. \tag{19} $$

We can see that more bits are assigned to subchannels that correspond to larger singular values of the channel.

Remarks:

1) Note that if we choose $\mathbf{D} = \mathbf{\Lambda}_M$, we have $\mathbf{F} = \mathbf{V}[\ \mathbf{I}_M \ \ \mathbf{0}\ ]^T$, $\mathbf{G} = [\ \mathbf{\Lambda}_M^{-1} \ \ \mathbf{0}\ ]\mathbf{U}^{\dagger}$.
   In this case, $\sigma_{s_k}^2 = P_0/M$ and all subchannels are assigned the same power. The bit allocation and bit rate are the same as the case when we choose $\mathbf{D} = \mathbf{I}_M$. This is because the signal to noise ratio $\sigma_{s_k}^2/\sigma_{e_k}^2$ is not affected by $\mathbf{D}$. Therefore, bit assignment and hence bit rate performance will be the same.

2) In (19), the bits are not integers in general. We can use truncation, i.e., $\tilde{b}_k = \lfloor b_k \rfloor$, where the notation $\lfloor z \rfloor$ denotes the largest integer that is less than or equal to z. Zero bits may be assigned to some subchannels ($\tilde{b}_k = 0$ if $P_0[\mathbf{\Lambda}_M^2]_{kk} < MN_0\Gamma$) and the power allocated to these subchannels is wasted. In this case, we will remove the worst subchannel and compute bit and power allocation in the remaining subchannel. We continue like this until all the power is used by subchannels with nonzero bits. The iterative bit allocation algorithm is given below.
   Integer bit allocation algorithm:
   Let M_0 be the number of subchannels that will be assigned nonzero bits. Initially, set $M_0 = M$.
   a) Compute $\xi_k = \frac{P_0[\mathbf{\Lambda}_M^2]_{kk}}{M_0 N_0 \Gamma}$ for $k = 0, 1, \cdots, M_0-1$.
   b) If $\xi_k \geq 1$ for $k = 0, 1, \cdots, M_0-1$, then go to step (c). Else, if $\xi_k < 1$ for some subchannels, set $M_0 = M_0 - 1$ and go to step (a).
   c) Compute the bit allocation by $b_k = \lfloor \log_2(1 + \xi_k) \rfloor$ for $0 \leq k < M_0$. For $M_0 \leq k < M$, we set $b_k = 0$.

IV. SIMULATIONS

In the simulation, we evaluate the performance of the proposed method. The number of subchannels M is 4. The

channel used is a 4×4 MIMO channel ($P = N = 4$). The elements of $\mathbf{H}$ are complex Gaussian random variables whose real and imaginary parts are independent with zero mean and variance $1/2$. The noise vector $\mathbf{q}$ is assumed to be complex Gaussian with $\mathrm{E}[\mathbf{q}\mathbf{q}^\dagger] = \mathbf{I}_4$. QAM modulation is used for the input symbols. Optimal zero-forcing transceiver in (18) is used for the proposed method. Although the high bit rate assumption ($b_k \gg 1$) is used in the derivation of the optimal transceivers, the assumption is not used in the computation of transmission bit rate in the simulations. We will use the integer bit allocation in remark 2 instead. Fig. 2 shows the transmission rates for different transmit power to noise ratio (P_0/N_0). The symbol error rates are 10^{-2} for all the subchannels. The transmission rates are averaged over 10^4 random channel realizations. For comparison, we have also shown the results of three zero-forcing systems: the maximum signal to noise ratio (MSNR) transceiver in [4], the unit noise variance (UNV) transceiver in [7], and the SVD-waterfilling solution in [2], and also the results of two optimal transceivers in [8] that are designed using a minimum mean square error (MMSE) criterion and a maximum mutual information (MMI) criterion. Both of the MMSE [8] and MMI [8] systems use MMSE reception. In the UNV [7] and MSNR [4] systems, as all the subchannels have the same signal to noise ratios, the same bits are assigned for all subchannels. For the MMSE and MMI systems, we use the bit loading method mentioned in equation (46) of [8]. Fig. 2 shows that the proposed method can achieve a bit rate considerably higher than MMSE [8], UNV [7], and MSNR [4], and slightly better than MMI [8] and SVD-waterfilling [2]. We should note that, although the proposed system is zero-forcing, it is still better than the two MMSE systems in [8], in which the noise statistics is also taken into consideration. In Fig. 3, we plot the bit error rates averaged over 10^4 random channel realizations when the total number of bits per block is fixed to eight for the same six systems. For the proposed method, we compute the bit allocation that is obtained when $P_0/N_0 = 12$ dB (the corresponding bits per block is eight in Fig. 2) and the same bit allocation is used in generating the plot in Fig. 3. Similarly, we allocate bits for the other five system such that the number of total bits is eight. In Fig. 3, we can see that the proposed method has the smallest bit error rate. The bit error rate of the proposed zero-forcing system is even smaller than the MMI [8], which use a MMSE receiver.

V. CONCLUSION

In this paper, we have designed the transceiver over an MIMO channel for maximizing transmission rate. The bit allocation and transceiver were jointly optimized subject to a total power constraint for a fixed error rate. Using a high bit rate assumption, we showed that we can simultaneously obtain the optimal bit allocation and transceiver easily We have demonstrated through simulations that the proposed method can indeed achieve a higher transmission rate although the high bit rate assumption is not used in the computation of bit allocation.
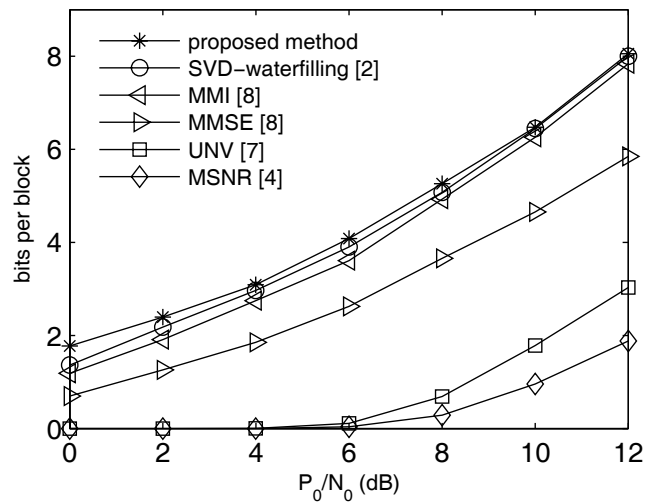

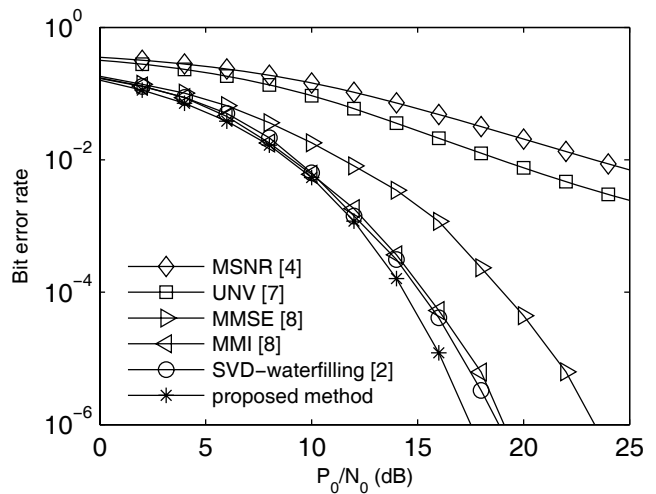Fig. 2. Transmission bit rates for a fixed error rate.

Fig. 3. Bit error rate performance.

REFERENCES

[1] G. G. Raleigh and J. M. Cioffi, "Spatio-temporal coding for wireless communication," *IEEE Trans. Commun.*, vol. 46, pp. 357-366, Mar. 1998.
[2] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecommun.*, vol. 10, pp. 585-595, Nov./Dec. 1999.
[3] A. Yasotharan, "Lower bound on the BER of a pre- and postequalized vector communication system," *IEEE Trans. Commun.*, vol. 53, pp. 453-462, Mar. 2005.
[4] A. Scaglione, G. B. Giannakis, and S. Barbarossa, "Redundant filterbank precoders and equalizers, part I: unification and optimal designs," *IEEE Trans. Signal Process.*, vol. 47, pp. 1988-2006, July 1999.
[5] Y. Ding, T. N. Davidson, Z.-Q. Luo, and K.-M. Wong, "Minimum BER block precoders for zero-forcing equalization," *IEEE Trans. Signal Process.*, vol. 51, pp. 2410-2423, Sep. 2003.
[6] Y.-P. Lin and S.-M. Phoong, "BER minimized OFDM systems with channel independent precoders," *IEEE Trans. Signal Process.*, vol. 51, pp. 2369-2380, Sep. 2003.
[7] A. Yasotharan, "Multirate zero-forcing Tx-Rx design for MIMO channel under BER constraints," *IEEE Trans. Signal Process.*, vol. 54, pp. 2288-2301, June 2006.
[8] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Trans. Signal Process.*, vol. 50, pp. 1051-1064, May 2002.

[9] H. Sampath, P. Stoica, and A. Paulraj, "Generalized linear precoder and decoder design for MIMO channelsusing the weighted MMSE criterion," *IEEE Trans. Commun.*, vol. 49, pp. 2198-2206, Dec. 2001.

[10] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization," *IEEE Trans. Signal Process.*, vol. 51, pp. 2381-2401, Sep. 2003.

[11] D. P. Palomar, M. A. Lagunas, and J. M. Cioffi, "Optimum linear joint transmit-receive processing for MIMO channels with QoS constraints," *IEEE Trans. Signal Process.*, vol. 52, pp. 1179-1197, May 2004.

[12] D. P. Palomar and S. Barbarossa, "Design MIMO communication systems: constellation chioce and linear transceiver design," *IEEE Trans. Signal Process.*, vol. 53, pp. 3804-3818, Oct. 2005.

[13] G. D. Dudevoir, J. S. Chow, S. Kasturia, and J. M. Cioffi, "Vector coding for T1 transmission in the ISDN digital subscriber loop," in *Proc. IEEE International Conf. Commun.*, pp. 536-540, June 1989.

[14] E. K. P. Chong and S. H. Zak, *An Introduction to Optimization.* New York: John Wiley & Sons, 1996.

[15] R. A. Horn and C. R. Johnson, *Matrix Analysis.* New York: Cambridge University Press, 1985.