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Regionalization of unit hydrograph parameters: 1. Comparison of regression analysis techniques

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Abstract: Hydrologic regionalization is a useful tool that allows for the transfer of hydrological information from gaged sites to ungaged sites. This study developed regional regression equations that relate the two parameters in Nash's IUH model to the basin characteristics for 42 major watersheds in Taiwan. In the process of developing the regional equations, different regression procedures including the conventional univariate regression, multivariate regression, and seemingly unrelated regression were used. Multivariate regression and seeming unrelated regression were applied because there exists a rather strong correlation between the Nash's IUH parameters. Furthermore, a validation study was conducted to examine the predictability of regional equations derived by different regression procedures. The study indicates that hydrologic regionalization involving several dependent variables should consider their correlations in the process of establishing the regional equations. The consideration of such correlation will enhance the predictability of resulting regional equations as compared with the ones from the conventional univariate regression procedure.

Key words: Hydrologic regionalization, unit hydrograph, regression analysis, multivariate regression, seemingly unrelated regression, validation.

1 Introduction

Rainfall-runoff modeling is an important aspect of hydrological investigation. It provides essential information needed for a variety of problems including watershed management, hydrological engineering design of hydraulic structures, and others. There are numerous hydrological rainfall-runoff models of varying degrees of sophistication. Among them, the unit hydrograph (UH) model, ever since its conception by Sherman (1932), is one of the most widely applied hydrological engineering tools for rainfall-runoff analysis.

Methodologies to determine a discrete unit hydrograph (DUH) of a selected duration from storm events with deduced effective rainfalls and direct runoffs are abundant (Singh, 1988). Recently, Zhao (1992) and Zhao et al. (1994) have conducted systematic investigations on the DUH and instantaneous UH (IUH) determination when rainfall-runoff data from several storms occurring in a given watershed are available.

Furthermore, methodologies to assess the uncertainties associated with the derived UH were proposed (Yeh et al., 1993).

Frequently, rainfall-runoff analysis and modeling have to be performed for watersheds in which data are not available or existing data are too scarce. In such circumstance, hydrological regionalization is needed for the purpose of transferring relevant hydrologic information. Although there are various types of techniques that have been used in hydrological regionalization, this study, in particular, is limited to the commonly used regression-type of regionalization procedure. Furthermore, without being bogged down with deriving regional DUHs of various durations, this study focuses on the derivation of regional relationships for parameters in an IUH model. Specifically, Nash's IUH model was adopted in this study. From the regional equations, the parameters in Nash's IUH model are computed from which the IUH and DUH of any specified duration can be obtained. The primary objective of this paper is to examine the performance of various regression techniques applicable to hydrologic regionalization. When applying a developed regional equation, uncertainties exist in the regionalization of a UH. The accompanying paper (Yeh et al., 1995) deals with the uncertainties involved in regional regression equations for the UH characteristics.

2 Hydrological regionalization by regression analysis

When hydrological data is short in time, scarce in space, or nonexistent, regionalization provides a mechanism for transferring hydrological information from where records are long and/or available. Techniques for hydrological regionalization are many (Cunnane, 1988; Stedinger et al., 1992). In general, techniques can be broadly classified into those (1) substitute space for time, (2) identify model 'structure', or (3) combination of the two.

Regression analysis is one of the most widely used approaches for hydrological regionalization. The approach attempts to identify the 'structure' of a model that describes the functional relationship between the hydrological parameters of interest and physiographical/meteorological characteristics of a watershed as

$$y = g(x_1, x_2, \cdots, x_r | \theta_1, \theta_2, \cdots, \theta_q) + \epsilon_y \tag{1}$$

in which y = the hydrological response of interest; $g(\cdot)$ = a general function relation for y involving r basin physiographical/meteorological characteristics, represented by x_j, j=1,2,$\cdots$,r; θ's are parameters that describe the model behavior; ϵ_y = model error terms due to lack of fit to the observed system response y. Once the functional relation is established, the developed regional regression equations can be applied to estimate hydrological parameters of interest in ungaged watersheds or sites.

Although the selection of regional regression type is subjective, one often incorporates some physical justifications. For example, using logarithmic transform is only applicable to variables that cannot be negative. Many hydrologic and physiographical characteristics of a watershed are nonnegative by nature. Sometimes, logarithmic transform of a variable is used in regression analysis because of statistical reasons, such as variance stabilization.

In general, the function $g(\cdot)$ and its parameter values are not known. The primary task of the regression analysis is to identify the functional relation $g(\cdot)$ and to estimate its parameters θ's that best describes the relation between the available basin physiographical characteristics (x's) and observed hydrological response (y). In other words, regression analysis is like system identification in which one attempts to estimate the system throughput from the observed inputs to the system and output from the system.

2.1 Univariate regression analysis and its drawbacks

In the univariate regression (UVR) analysis, one only deals with a single dependent variable which could be related to several independent variables such as shown in equation (1) Without losing generality, suppose that the regional regression equation $g(\cdot)$ is linear and can be expressed as

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_r x_{ir} + \in_i, i = 1, 2, \cdots, n \tag{2a}$$

with the subscript 'i' representing the i-th observation or, in matrix form, as

$$\mathbf{y} = \mathbf{X}\beta + \in . \tag{2b}$$

The ordinary least squares (OLS) estimation of unknown regression coefficients, β's, for equation (2b) is

$$\mathbf{b} = (\mathbf{X}^t\mathbf{X})^{-1}\mathbf{X}^t\mathbf{y}$$

in which $\mathbf{b}$ is the vector of the OLS estimators of β and the superscript 't' represents the transpose of a matrix or vector. It has been shown (Montgomery and Peck, 1982) that

$$E(\mathbf{b}) = \beta;\ \mathbf{C}(\mathbf{b}) = \sigma^2(\mathbf{X}^t\mathbf{X})^{-1});\ E\mathbf{e} = 0;\ \mathbf{C}(\mathbf{b}, \mathbf{e}) = 0$$

in which $E(\cdot)$ = the expectation operator; $\mathbf{C}(\cdot)$ = the covariance operator yielding a covariance matrix; $\mathbf{e}=\mathbf{y}-\mathbf{X}\mathbf{b}$, the estimated errors vector; σ^2 = the variance associated with the error terms which can be estimated by s^2 as

$$s^2 = \frac{\mathbf{e}^t\mathbf{e}}{n-(r+1)} .$$

If the errors are independent normal random variables with mean 0 and variance σ^2, the following properties hold

$$\mathbf{b} \sim N_{r+1}(\beta, \sigma^2(\mathbf{X}^t\mathbf{X})^{-1});\ \mathbf{e}^t\mathbf{e} \sim \sigma^2\chi^2_{n-r-1} . \tag{3}$$

Equation (3) shows that, under the normality condition, the OLS estimators, $\mathbf{b}$, are multivariate normal random variables and the sum of error squared has a chi-square distribution with (n-r-1) degrees of freedom.

For a given observation on independent variables, $\mathbf{x}_0$, the predicted dependent variable, y_0, has the mean and variance as the following

$$E(y_0|\mathbf{x}_0) = \mathbf{x}^t\beta \tag{4a}$$

$$Var(y_0|\mathbf{x}_0) = \sigma^2\left[1 + \mathbf{x}_0^t(\mathbf{X}^t\mathbf{X})^{-1}\mathbf{x}_0\right] . \tag{4b}$$

After the regression coefficients are estimated, the resulting regression equation can be used for prediction. For a given $\mathbf{x}_0$, the mean and variance of y_0 can be estimated by equations (4a-b) with β replaced by $\mathbf{b}$, and σ^2 by s^2.

In hydrologic regionalization, one frequently deals with several hydrological characteristics simultaneously. For example, in defining the UH on a regional basis, an engineer often relates the peak discharge, time-to-peak, and shape parameters to basin and meteorological characteristics. In fact, parameters describing a UH are often correlated. The conventional practice of univariate regression analysis treats each UH parameter separately and, hence, does not take into account the correlation among the UH parameters.

2.2 Multivariate regression analysis

In the multivariate regression (MVR) framework, one considers several correlated dependent variables simultaneously in establishing the empirical relationships. It is an extension of univariate regression of equation (2b). Suppose that there are m correlated dependent variables $Y_1, Y_2, \cdots, Y_m$ which are functions of r regressors in a form of equation (2b) as

$$\mathbf{Y}_{n\times m} = \mathbf{X}_{n\times(r+1)}\mathbf{B}_{(r+1)\times m} + \mathbf{E}_{n\times m} \tag{5}$$

in which $\mathbf{Y} = (\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_m)$, an n×m matrix containing vectors of m dependent variables each with n observations; $\mathbf{X}$ = an n×(r+1) matrix containing (r+1) values of regressors from n observations as defined previously in equation (2b); $\mathbf{B} = (\beta_0, \beta_1, \beta_2, \cdots, \beta_r)^t$, a (r+1)×m matrix containing (r+1) row vectors of regression coefficients with β_j being the vector of the j-th regression coefficients of the m different regression equations; and $\mathbf{E} = (\in_1, \in_2, \cdots, \in_m)$, an n×m matrix containing vectors of n residuals for the m dependent variables.

As indicated in equation (5), a multivariate regression model requires that all m dependent variables have exactly the same regressors and the functional relationships have the same form. Under the conditions that

$$E(\in_1) = \mathbf{0}; \mathrm{Cov}(\in_i, \in_k) = \sigma_{ik}\mathbf{I}, \ i = 1, 2, \cdots, m$$

with σ_{ik} representing the covariance between dependent variables y_i and y_k. The covariance condition stated above indicates that the random error terms associated with different dependent variables are cross-correlated. However, there is no correlation among residuals for the same dependent variable. The OLS estimators, $\mathbf{B}$, of regression coefficients can be obtained as

$$\mathbf{B} = (\mathbf{X}^t\mathbf{X})^{-1}\mathbf{X}^t\mathbf{Y} \, .$$

Similar to the univariate regression, the following statistical properties for the OLS estimators hold (Johnson and Wichern, 1992)

$$E(\mathbf{B}) = B; \ \mathbf{C}(\mathbf{b}_i, \mathbf{b}_k) = \sigma_{ik}(\mathbf{X}^t\mathbf{X})^{-1} \text{ for } i, k = 1, 2, \cdots, m$$

$$E(\mathbf{E}) = \mathbf{O}; E\left(\frac{\mathbf{E}^t\mathbf{E}}{n-r-1}\right) = S = [\sigma_{ik}]; \mathbf{C}(\mathbf{e}_i, \mathbf{b}_k) = \mathbf{O}, \text{for } i \neq k$$

$$\mathbf{C}(\mathbf{B}) \;=\; \mathbf{S}\otimes(\mathbf{X}^t\mathbf{X})^{-1} \;=\; \begin{bmatrix} \sigma_{11}(\mathbf{X}^t\mathbf{X})^{-1} & \sigma_{12}(\mathbf{X}^t\mathbf{X})^{-1} & \cdots & \sigma_{1m}(\mathbf{X}^t\mathbf{X})^{-1} \\ \sigma_{21}(\mathbf{X}^t\mathbf{X})^{-1} & \sigma_{22}(\mathbf{X}^t\mathbf{X})^{-1} & \cdots & \sigma_{2m}(\mathbf{X}^t\mathbf{X})^{-1} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \sigma_{m1}(\mathbf{X}^t\mathbf{X})^{-1} & \sigma_{m1}(\mathbf{X}^t\mathbf{X})^{-1} & \cdots & \sigma_{mm}(\mathbf{X}^t\mathbf{X})^{-1} \end{bmatrix} \tag{6}$$

in which $\mathbf{S}=[\sigma_{ij}]$ an m×m covariance matrix of error terms associated with the m dependent variables; $\otimes$ = a Kronecker product operator; and $\mathbf{C(B)}$ = covariance matrix of size m(r+1)×m(r+1) of the OLS regression estimators.

Given a set of observed regressors or independent variables, $\mathbf{x}_0$, the m predicted dependent variables, $\mathbf{y}_0$, have the following expected values and the covariance matrix

$$\mathrm{E}(\mathbf{y}_0^t|\mathbf{x}_0) \;=\; \mathbf{x}_0^t\mathbf{B} \tag{7a}$$

$$\mathbf{C}(\mathbf{y}_0|\mathbf{x}_0) \;=\; [\sigma_{ik}(1+\mathbf{x}_0^t(\mathbf{X}^t\mathbf{X})^{-1}\mathbf{x}_0]_{m\times n},\ \text{for } i,k=1,2,\cdots,m \tag{7b}$$

in which $\mathbf{C}(\mathbf{y}_0|\mathbf{x}_0)$ = an m×m covariance matrix for the m predicted dependent variables. From the OLS estimation, the mean vector and covariance matrix of $\mathbf{y}_0$ can be estimated by

$$\hat{\mathrm{E}}(\mathbf{y}_0^t|\mathbf{x}_0) \;=\; \mathbf{x}_0^t\mathbf{B} \tag{8a}$$

$$\hat{\mathrm{C}}(\mathbf{y}_0|\mathbf{x}_0) \;=\; [s_{ik}(1+\mathbf{x}_0^t(\mathbf{X}^t\mathbf{X})^{-1}\mathbf{x}_0]_{m\times n},\ \text{for } i,k=1,2,\cdots,m \tag{8b}$$

in which $s_{ik} = r_{ik}s_is_k$, the estimated covariance between the residuals of y_{0i} and y_{0k} with r_{ik} being the sample correlation between the residuals of y_{0i} and y_{0k}, and s_i and s_k being the standard errors associated with y_{0i} and y_{0k}, respectively.

2.3 Seemingly unrelated regression

The seemingly unrelated regression (SUR) is concerned with a model consisting of m multiple regression equations which are not entirely identical in their functional relationships. It is a generalization of the MVK When the m multiple regression equations have the same functional forms, the SUR and MVR are identical. A comprehensive discussions on the subject of SUR are given by Srivastava and Giles (1987).

In matrix form, the i-th of the m equations under consideration by the SUR are

$$\mathbf{y}_i \;=\; \mathbf{X}_i\beta_i + \eta_i,\ \text{for } i=1,2,\cdots,m \tag{9}$$

in which $\mathbf{y}_i$ = an n×1 vector of the i-th dependent variables: $\mathbf{X}_i$ = an n×r_i matrix containing n observations of r_i regressors; β_i = a $r_i \times 1$ vector of regressors; and η_i = an n×1 vector of errors. Putting equation (9) together for all m equations, one has

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{y}_m \end{bmatrix} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2 & \cdots & \mathbf{0} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{X}_m \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \beta_m \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \cdot \\ \cdot \\ \cdot \\ \eta_m \end{bmatrix}$$

which can be put in a compact form as

$$\mathrm{y} = \mathrm{x}\beta + \eta \tag{10}$$

where $\mathbf{y}$ = an nm×1 vector; $\mathbf{X}$ = an nm×R matrix, β = a R×1 vector; and η = an nm×1 vector with $\mathrm{R} = \sum_i \mathrm{r_i}$.

In the SUR, the following assumptions about the error terms are made

$$\mathrm{E}(\eta_i) = \mathbf{0}; \mathrm{E}(\eta_i \eta_j^t) = \sigma_{ij}\mathbf{I}_n \text{ fori}, j = 1, 2, \cdots, m \tag{11a}$$

where $\mathbf{I}_n$ = an n×n identity matrix. More compactly, equation (11a) can be written as

$$\mathrm{E}(\eta) = \mathbf{0}$$

$$\mathbf{C}(\eta) = \mathrm{E}(\eta\eta^t) = \begin{bmatrix} \sigma_{11}\mathbf{I}_n & \sigma_{12}\mathbf{I}_n & \cdots & \sigma_{1m}\mathbf{I}_n \\ \sigma_{21}\mathbf{I}_n & \sigma_{22}\mathbf{I}_n & \cdots & \sigma_{2m}\mathbf{I}_n \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \sigma_{m1}\mathbf{I}_n & \sigma_{m2}\mathbf{I}_n & \cdots & \sigma_{mm}\mathbf{I}_n \end{bmatrix} = \mathbf{S} \otimes \mathbf{I}_n) = \Psi . \tag{11b}$$

To take into account the correlation among the m dependent variables, the generalized least square (GLS) method can be applied and the resulting estimators are (Srivastava and Giles, 1987)

$$\mathbf{b}_{\mathrm{GLS}} = [\mathbf{X}^t(\mathbf{S}^{-1} \otimes \mathbf{I}_n)\mathbf{X}]^{-1}\mathbf{X}^t(\mathbf{S}^{-1} \otimes \mathbf{I}_n)\mathbf{y} \tag{12}$$

It can be shown that $\mathbf{b}_{\mathrm{GLS}}$ is an unbiased estimators of β and the corresponding covariance matrix is

$$\mathbf{C}(\mathbf{b}_{\mathrm{GLS}}) = [\mathrm{Cov}(\mathbf{b}_i, \mathbf{b}_j)] = [\mathbf{X}^t(\mathbf{S}^{-1} \otimes \mathbf{I}_n)\mathbf{X}]^{-1} . \tag{13}$$

Srivastava and Giles (1987) show that the GLS estimator by equation (12) is better than the OLS estimator because the determinant of the associated covariance matrix is smaller or equal to that associated with the OLS one.

Given a set of values of regressors, the predicted m dependent variables have the means and covariance matrix as

$$E[\mathbf{y}_0|\mathbf{X}_0] = \mathbf{X}_0\beta \tag{14a}$$

$$\mathbf{C}(\mathbf{y}_0) = \mathbf{X}_0^t\mathbf{C}(\mathbf{b}_{GLS})\mathbf{X}_0 + \mathbf{C}(\eta) \tag{14b}$$

3 Regionalization of Nash's IUH parameters

3.1 Descriptions of rainfall-runoff data

To develop a regional UH, representative DUHs of various durations for a total of 42 watersheds in Taiwan were extracted from Huang (1992) and the corresponding basin characteristics that have potential effects on the UH are listed in Table 1. The basin characteristics in Table 1 are basin area (in km^2), basin length (in km) which is the distance between the stream gage and the most remote point on the watershed boundary, basin slope (in m/m), and L_{ca} representing the distance along the main channel between the basin outlet and the centroid of the basin.

3.2 Determination of basin-wide representative Nash's IUH parameters

In this study, the parameters in Nash's IUH are considered to be the dependent variables which are related to the basin characteristics in Table 1. The regional regression equations for Nash's IUH parameters allow estimation of the IUH for ungaged watersheds.

Nash's IUH model (Nash, 1957) is

$$U(t) = \frac{1}{K\Gamma(N)}\left(\frac{t}{K}\right)^{N-1} e^{-t/K}, t \geq 0 \tag{15}$$

in which U(t) = the IUH ordinate at time t; K = model parameter representing the storage coefficient; N = model parameter representing the number of hypothetical reservoirs; $\Gamma(\cdot)$ = gamma function. Once the parameters N and K are determined, Δt-hour DUH can be approximated by

$$U_k \approx \frac{U(k\Delta t|N,K) - U((k-1)\Delta t|N,K)}{2} \tag{16}$$

in which U_k = the k-th ordinate in a Δt-hour DUH, k=1,2,$\cdots$; $U(k\Delta t)$ = Nash's IUH ordinate at $t=k\Delta t$.

Based on the representative DUH of known duration for each watershed, the two parameters N and K for the watershed can be determined by the method of moment (Bras, 1987) or some types of optimization techniques. Based on the previous study (Yang et al., 1992), an optimization technique would yield a better fit between the computed DUH and the given DUH. In this study, the downhill simplex search algorithm developed by Nelder and Mead (1965) was used to determine the optimal N and K for each watershed with the following objective function

$$\underset{N,K}{\text{Minimize}} \sum_{k=1}^{M} [U_{k,comp}(N,K) - U_{k,repr}]^2 \tag{17}$$

Table 1. Basin characteristics and Nash's IUH parameters in Taiwan

ID1	STATION NAME	AREA (SQ.KM)	LENGTH (KM)	Lca (KM)	SLOPE (M/M)	N	K
1	SHIN CHI	146.46	25.35	6.33	0.001836	2.812	3.460
2	JAW SHIN	489.00	24.30	11.00	0.0372	2.040	3.804
3	SWAN TOU	282.89	51.00	25.50	0.0278	4.752	2.364
4	TUNGTOU	259.20	33.20	18.70	0.02905	2.245	1.973
5	KWAN IN	338.00	34.38	12.75	0.0471	1.766	4.865
6	DARLUKUN	247.28	27.00	8.80	0.0347	2.246	2.298
7	LI SHAN	249.40	31.00	16.00	0.0592	4.529	1.391
8	SIN BEI	309.86	34.10	14.10	0.0183	4.702	1.231
9	YEIH MEY	539.52	91.55	48.30	0.01420	4.193	1.684
10	TZEN WEN	1157.46	123.50	60.80	0.0044	3.714	2.450
11	SIN YIN	226.66	39.45	25.30	0.00263	7.091	1.081
12	NUO CHOU	149.68	35.40	14.10	0.0130	4.016	0.733
13	BE GARN	597.46	52.00	22.00	0.0010	3.074	4.943
14	CHAN PAN	101.09	18.65	10.75	0.0217	2.453	0.962
15	SHIH GUN	676.50	96.70	47.90	0.0017	3.807	2.383
16	DAGIN	360.20	63.45	27.09	0.01404	2.585	2.198
17	SIN JUN	90.50	28.00	14.50	0.0022	4.577	1.418
18	MAR YUAN	85.49	17.22	9.70	0.07967	1.235	6.448
19	JEI SHOU	94.75	23.00	14.50	0.0865	2.960	1.459
20	INPANCO	262.18	25.00	11.00	0.0526	3.676	1.239
21	LIYITAN	53.45	23.00	14.50	0.0296	9.467	0.172
22	SIN WU LU	638.78	49.70	15.51	0.0249	2.359	4.896
23	WAN LON	232.61	33.10	15.91	0.0528	3.424	1.229
24	SIN HAU	321.70	31.50	18.50	0.0065	4.710	1.178
25	SANDIMON	408.51	57.23	25.45	0.01720	3.137	1.609
26	TSO ZAN	121.31	24.50	13.50	0.0036	6.063	0.821
27	CHU KO	83.15	10.65	7.45	0.01764	2.833	1.708
28	SHILOW	2988.00	162.30	59.50	0.00827	4.505	1.701
29	GANZILIN	954.24	64.90	22.00	0.018288	1.931	4.041
30	SWENCHI	549.17	57.00	27.90	0.0243	2.073	1.149
31	YEN PING	476.16	60.00	28.30	0.025	2.536	2.878
32	HAU LAIN	1500.11	55.58	19.00	0.0092	2.366	5.133
33	JOW CHU	3076.66	150.70	63.60	0.00780	4.690	1.987
34	CHUN TE	139.62	37.00	15.20	0.0027	6.792	0.846
35	YU TAIN	160.53	48.00	30.30	0.0042	2.043	2.233
36	PUZI	288.94	63.55	31.70	0.0034	4.248	2.119
37	GIGI	2298.00	129.10	35.30	0.01014	4.320	1.414
38	NAN PEI	408.00	50.50	24.25	0.0616	1.76	98.351
39	ERCUNPU	485.48	51.39	22.00	0.0175	2.995	1.296
40	TAI DONG	1584.29	94.40	35.08	0.00824	2.006	5.728
41	LAN YANG	820.69	65.44	30.00	0.01777	1.765	3.132
42	LAU LONG	812.03	91.70	35.30	0.02097	3.317	1.930

in which $U_{k,comp}(N,K)$ and $U_{k,repr}$ are the k-th ordinates of the computed and representative DUHs, respectively. The optimal values of N and K in Nash's IUH model for the 42 watersheds are listed in the last two columns of Table 1.

3.3 Analysis of Nash's IUH parameters

The summary statistics of N, K, and their logarithmic transform for the 42 watersheds are listed in Table 2. The means of N and K are both significantly larger than their medians indicating that the distributions for N and K are positively skewed. On the other hand, the mean and median for N and K in the log-space are very close, indicating that they are approximately symmetric. Normal plots for N and K shown in Figures 1(a)-(b) indicate that both parameters in their log-space are close being normal random variables. Therefore, the two parameters N and K were treated as log-normal random variables.

Table 3 shows the correlations between the two IUH parameters and the four basin characteristics. Note that correlation coefficient indicates the strength of linear relation between two random variables. To detect potential nonlinearity between two variables, the rank correlation coefficient may be useful. In case that rank correlation is significantly larger than the simple correlation coefficient, the existence of nonlinear relationship between the two variables is pronounced. It can be observed that basin length and L_{ca} possess a strong linear relationship. This indicates that one of them would be redundant in regression analysis and, in fact, only one of the length variables is used in the final regression equations presented later. Furthermore, there exists quite large correlation among watershed area and the two length variables. In the log-space, their correlations drop slightly as compared in the original space.

From Table 3, one also observes that.there exists rather significant correlation between N and K in both the original as well as log-spaces. In the log-space, correlation between N and K is even stronger. Therefore, such correlation among' dependent variables should be taken into account explicitly. Comparing the two simple correlation matrices of N and K in Table 3(a) and (b) with the rank correlation matrix in Table 3(c) indicates that, in log-space, a linear relationship will generally suffice to describe the relation between the two IUH parameters and basin characteristics.

3.4 Development of regional equations for parameters in Nash's IUH

In this section, the developed regional equations using the three different regression analysis for the two Nash's IUH parameters are presented. The three regression analysis considered are univariate regression (UVR), multivariate regression (MVR), and seemingly unrelated regression (SUR). Descriptions of the basic theory for regression coefficient estimation for the three methods are given previously. The statistical package, SAS (Statistical Analysis Systems, 1989), was used to conduct the various regression analyses.

Based on the above examination of N and K of the 42 watersheds, ln(N) and ln(K) closely satisfy the normality assumption in regression analysis. The compliance of normality condition facilitates further inferences about the statistical properties of estimated regression parameters. In search of the functions that best describe the relationships between N, K and basin characteristics, the following four functional forms were used

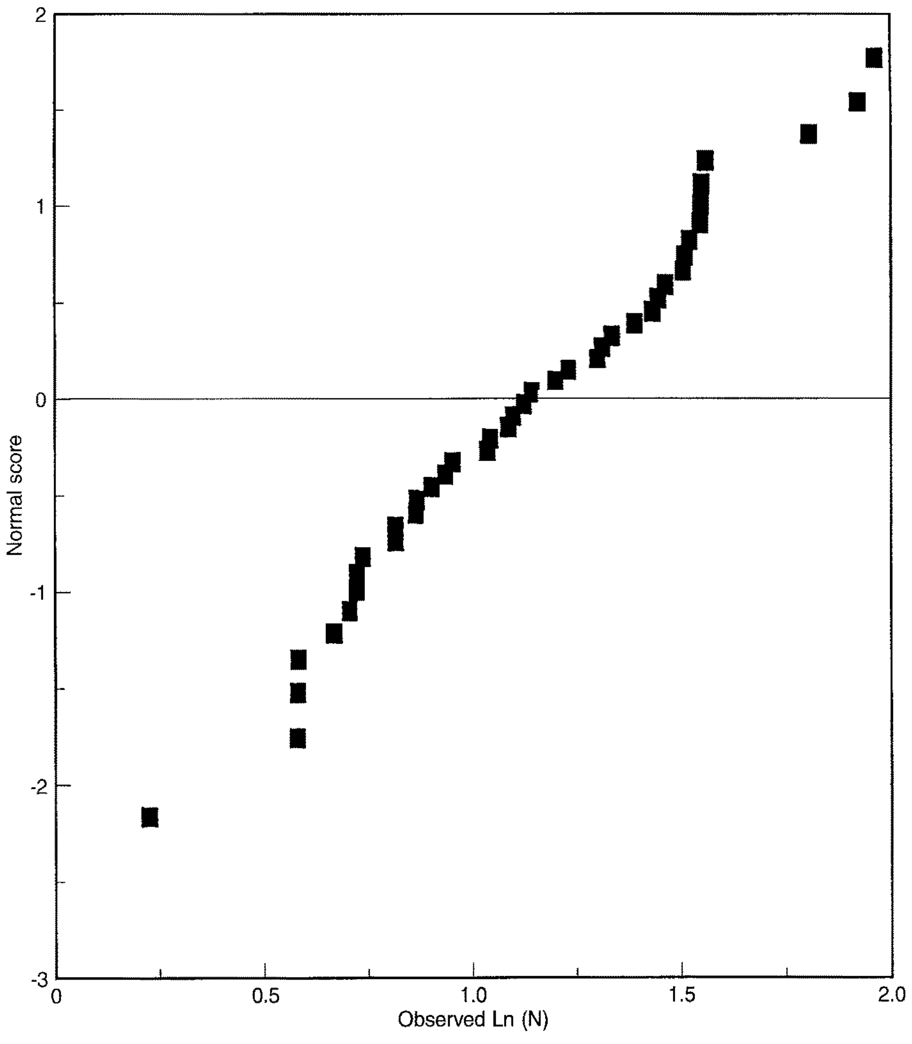

Figure 1(a) Normal Probability Plot for Observed ln(N)

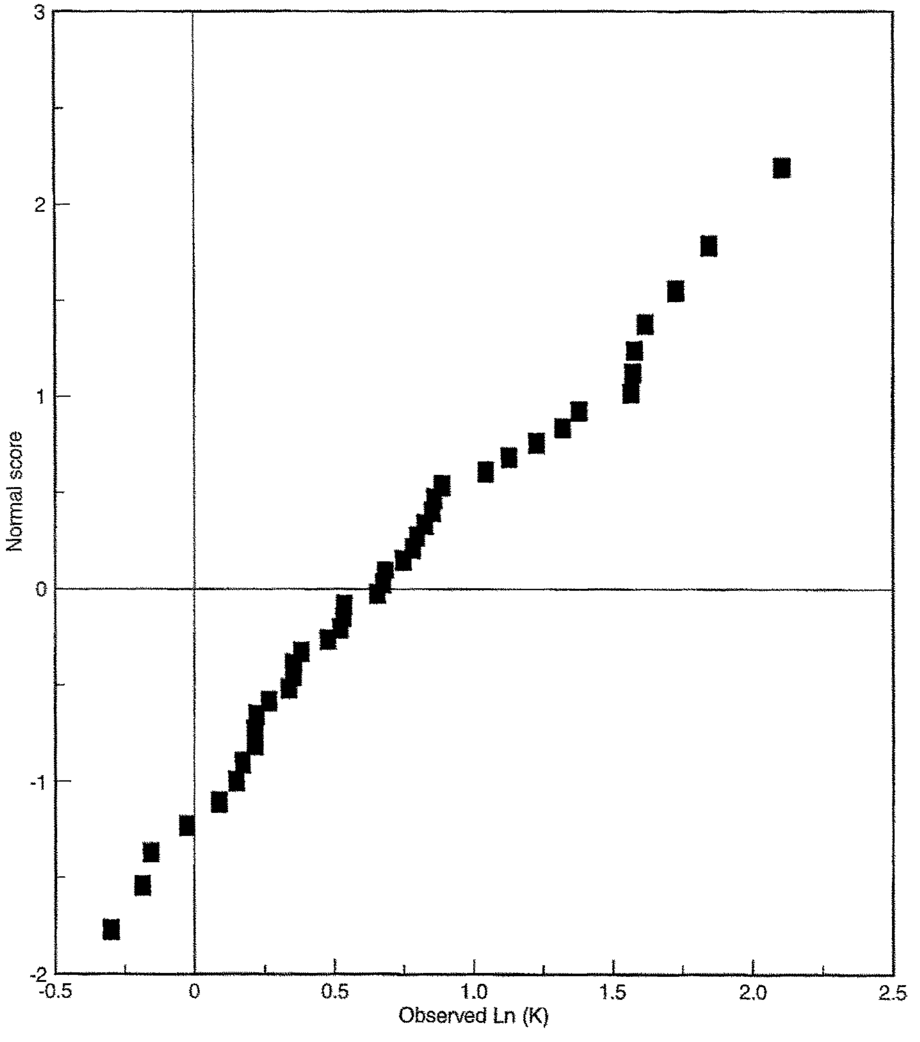

Figure 1(b) Normal Probability Plot for Observed ln(K)

Table 2. Summary statistics of N, K, In(N), and In(K)

	MEAN	MEDIAN	STDEV	MIN	MAX
N	3.520	3.105	1.658	1.235	9.467
K	2.475	1.951	1.760	0.172	8.351
ln(N)	1.163	1.133	0.436	0.211	2.248
ln(K)	0.676	0.669	0.716	-1.760	2.122

Table 3. Correlation matrices for involved variables

(a) Correlation matrix of variables in the original scale

	N	K	Area	Length	Lca
K	-0.606				
Area	-0.018	0.120			
Length	0.031	0.036	0.884		
Lca	0.089	-0.031	0.731	0.936	
Slope	-0.295	0.259	-0.287	-0.389	-0.384

(b) Correlation matrix of log-transformed variables

	ln(N)	ln(K)	ln (Area)	ln (Length)	ln(Lca)
ln (K)	-0.753				
ln (Area)	-0.141	0.416			
ln (Length)	0.070	0.222	0.870		
ln (Lca)	0.155	0.089	0.725	0.926	
ln (Slope)	-0.392	0.048	-0.135	-0.298	-0.283

(c) Rank correlation matrix for the involved variables

	N	K	Area	Length	Lca
K	-0.675				
Area	-0.183	0.436			
Length	0.022	0.280	0.857		
Lca	0.128	0.156	0.718	0.918	
Slope	-0.391	0.072	-0.182	-0.394	-0.351

$$\mathbf{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_r x_r + \in_y \tag{18a}$$

$$y = \beta_0 + \beta_1 \log(x_1) + \beta_2 \log(x_2) + \cdots + \beta_r \log(x_r) + \in_y \tag{18b}$$

$$\log(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_r x_r + \in_{\log y} \tag{18c}$$

$$\log(y) = \beta_0 + \beta_1 \log(x_1) + \beta_2 \log(x_2) + \cdots + \beta_r \log(x_r) + \in_{\log y} \tag{18d}$$

Furthermore, quadratic terms with no interaction was incorporated to account for potential nonlinearity that cannot be clearly detected from comparing simple correlation and rank correlation.

The full model which incorporates all four basin variables and their squared terms would result in nine unknown regression coefficient (including the intercept). Using the full model, the F-statistics and coefficient of determination, R^2, were used to compare the relative significance among the four different functional forms of equations (18a-d). The resulting statistics show that equation (18d) has the highest values of F-statistics and R^2 indicating that it is the best among the four candidate models for these 42 watersheds.

Once the 'best' empirical model is selected, it is simplified by gradually deleting terms that are not statistically significant. The decision of deleting or retaining terms of independent variables is often based on subjective judgement. In this study, the deletion or retention of an independent variable term was based on simultaneous considerations oft-ratios, adjusted R^2, and standard error (SE) of the regression model. The goal of the exercise is to include terms that maximize the value of adjusted R^2 while minimizing the value of SE. With that, the resulting model would retain terms with t-ratios exceeding one. Sometimes, terms with t-ratios slightly lower than one are retained to improve the value of SE. Table 4 summarizes the regression coefficients and the relevant statistical information for the final regional regression equations of N and K for watersheds in Taiwan. The values in brackets are the standard deviations corresponding to the estimated regression coefficients above them. In Table 4, the terms with no regression coefficient given indicate that the corresponding estimated regression coefficient is statistically insignificant.

In a multivariate regression, it is required that all regression equations employ the same independent variable terms. Because the intercept terms for N is significant while it is not for K, two multivariate regression models were developed with one containing intercept terms and the other does not. Detail SAS outputs by the univariate regression (UVR), multivariate regression with intercept VRW), without intercept (MVRWO), and seemingly unrelated regression (SUR) for N and K can be found in Yeh and Tung (1994) and will not be presented herein.

By the UVR analysis, the development of regional regression equations for N and K were made separately. From Table 4, one observes that basin length is not included in the final regional regression equations. This is mainly because the two length variables are highly correlated (see Table 3). Therefore, the use of only one of the two length variables is sufficient. From the UVR, correlation between the residuals of the two IUH parameters is not assessed in any way.

By the MVR analysis, the results are identical to the UVR if the independent variable terms are the same. As shown in Table 4, the measures of goodness-of-fit (such as R^2 and SE) associated with individual regional regression equation by the MVR are slightly worse than those by the UVR. However, the main advantage of conducting the MVR is to allow an assessment of the correlation between residuals associated with different regression equations.

Using the SUR, terms of independent variable in the regional regression equations for N and K are identical to those in the UVR. As can be seen from Table 4, the estimated regression coefficients by the SUR are quite different than those by the UVR. By taking into account the correlation between the two IUH parameters, the standard deviations associated with the regression coefficients by the SUR are smaller than those by the UVR. The value of SE associated with the regional equation for K by the SUR is slightly larger than that by the UVR, whereas the opposite occurs for N.

Table 4. Summary of regression coefficients and relevant statistics by different regression procedures for ln(N) and ln(K) in Nash's IUH model

Regr. Method	UNR		SUR	
Dep. Variable	ln(N)	ln(K)	ln(N)	ln(K)
Intercept	2.912057 [2.065304]	********	1.672693 [1.30959]	********
ln(Area)	-1.156248 [0.603056]	2.402552 [0.751758]	-0.830733 [0.43328]	1.661954 [0.383625]
ln(Lca)	0.286432 [0.159429]	-2.114992 [1.465371]	0.287655 [0.15942]	-0.459775 [0.232095]
ln(Slope)	-0.541320 [0.438287]	1.961785 [0.570128]	-0.669783 [0.40582]	2.113411 [0.556811]
$\ln(\text{Area})^2$	0.077304 [0.049942]	-1.52259 [0.064107]	0.050120 [0.03559]	-0.087751 [0.030613]
$\ln(\text{Lca})^2$	*******	0.279505 [0.244349]	*******	*******
$\ln(\text{Slope})^2$	-0.045117 [0.047114]	0.209849 [0.062073]	-0.058571 [0.04380]	0.227697 [0.060329]
SE	0.37989	0.55079	0.378352	0.553081
R-sq	0.3342	0.7286		
Resid. Correl.	0.000000		-0.773258	

Note: UNR-Univariate regression;
SUR-Seemingly unrelated regression.

Table 4 (continued)

Method	MVRW		MVRWO	
Dep. Variable	ln(N)	ln(K)	ln(N)	ln(K)
Intercept	2.912057 [2.065304]	-2.333498 [3.023451]	********	********
ln(Area)	-1.156248 [0.603056]	2.27483952 [0.882829]	-.391406 [0.266991]	1.661953 [0.383625]
ln(Lca)	0.286432 [0.159429]	-0.457473 [0.233393]	0.289305 [0.161531]	-0.459775 [0.232095]
ln(Slope)	-0.541320 [0.0438287]	1.871538 [0.641620]	-.843161 [0.387523]	2.113410 [0.556811]
$\ln(\text{Area})^2$	0.077304 [0.049942]	-0.138933 [0.073111]	0.013431 [0.021305]	-0.087751 [0.030613]
$\ln(\text{Slope})^2$	0.045117 [0.047114]	0.202366 [0.068971]	-.076728 [0.041987]	0.227696 [0.060329]
SE	0.37989	0.55613	0.3849	0.553081
R-sq	0.3342	0.4709	0.915145	0.718700
Resid. Correl.	-0.779629		-0.781939	

Note: MVRW-Multivariate regression with intercept;
MVRWO-Multivariate regression without intercept.

4 Comparison of predictive performance of different regional regression equations

4.1 Performance evaluation

From Table 4, some information with regard to the relative performance among the different regression analyses can be extracted. However, the predictability of regional regression equations by the different regression analyses is not entirely clear. For this reason, an experiment was conducted to examine the relative predictability of regional regression equations obtained by the different regression procedures.

The investigation was conducted in the following manner. The total of 42 watersheds shown in Table 1 were split into two subsets of equal size. Subset 1 consists of the first 21 watersheds whereas subset 2 contains the remaining ones. To avoid subjectivity involved in finding the 'best' regression model, the experiment used the full model in which all independent variable terms are included for both N and K. Since both N and K have the same independent variable terms, the SUR is identical to the MVR. Furthermore, a linear model with 5 independent variable terms and a quadratic model with 9 terms were used to fit the data in the investigation.

In this investigation, one subset of data was used to develop the regional equations for N and K by both UVR and MVR which, in turn, was used to predict the expected values of N and K as well as their variances in the other data subset. Since the predicted N and K will be used to determine the IUH, the predictability of the resulting regional equations by the UVR and MVR were judged on the basis of the predicted IUH peak discharge and the time-to-peak. Knowing N and K for Nash's IUH model, the peak discharge (U_p) and the time-to-peak (T_p) can be obtained by

$$U_p = \frac{2.78}{K\Gamma(N)} e^{-(N-1)}(N-1)^{(N-1)} \tag{19}$$

$$T_p = (N-1)K \tag{20}$$

Note that, for any given watershed with known basin characteristics, the N and K computed from the regional regression equations are estimated values possessing certain degrees of uncertainty. These uncertainties will be transmitted to the computed U_p and T_p resulting in uncertainty in peak discharge and time-to-peak. Under this circumstance, a proper measure for the relative accuracy of estimated N and K by the different regression procedures can be expressed in terms of the expected losses.

Consider, in general, that a system response W is related to several system inputs Ys through a functional relation, $W(\mathbf{Y})$, in which the inputs Ys are estimated involving certain degrees of uncertainty. For a set of estimated inputs, $\mathbf{Y}=\mathbf{y}$, the corresponding system response, w(y), may potentially deviate from the true response, $w_0(\mathbf{y}_0)$, incurring losses as

$$L(\mathbf{y}) = |w_0(\mathbf{y}_0) - w(\mathbf{y})|^{\alpha} \tag{21}$$

in which $L(\mathbf{y})$ = the loss function corresponding to a specified input $\mathbf{y}$; $\mathbf{y}_0$ = the true system inputs; and $\alpha \geq 1$. Due to the fact that the system inputs Ys are subject to uncertainty, the value of loss function is also random. Therefore, the expected losses can be computed as

$$E(L|\alpha) = E_Y\{|w(\mathbf{y}_0) - W(\mathbf{Y})|^{\alpha}\} = \int |w(\mathbf{y}_0 - w(\mathbf{y})|^{\alpha} f_Y(\mathbf{y})d\mathbf{y} \tag{22}$$

in which $E(L|\alpha)$ = the expected losses over all possible values of system inputs $\mathbf{Y}$; $f_Y(\mathbf{y})$ = a joint probability density function of stochastic inputs. To express the measure of deviation in terms of a metric distance, $E(L|\alpha)$ can be modified into the Minkowski distance as

$$D(\alpha) = \{E_Y[|w(\mathbf{y}_0) - W(\mathbf{Y})|^{\alpha}]\}^{1/\alpha} \tag{23}$$

When α=1 or α=2, $D(\alpha)$ represents the well-known metropolitan distance and Euclidean distance, respectively.

In the problem context of this study, the system inputs $\mathbf{Y}$=(N, K), the system response W could be U_p or T_p, and $f_Y(\mathbf{y})$ in equation (22) is the bivariate probability density function of N and K. Based on equation (23), three error criteria associated with predicting the true U_p and T_p were considered in this validation study and they are

$$BIAS_m(W) = E_{N,K}[W_0(N_0, K_0) - W_m(N, K)] \tag{24}$$

$$\mathrm{MAE_m(W)} = \mathrm{E_{N,K}}\left[|\mathrm{W_0(N_0,K_0)} - \mathrm{W_m(N,K)}|\right] \quad (25)$$

$$\mathrm{RMSE_m(W)} = \left\{\mathrm{E_{N,K}}\left[|\mathrm{W_0(N_0,K_0)} - \mathrm{W_m(N,K)}|^2\right]\right\}^{1/2} \quad (26)$$

in which $\mathrm{BIAS_m(W)}$, $\mathrm{MAE_m(W)}$, and $\mathrm{RMSE_m(W)}$ are bias, mean-absolute- error, and root-mean-squared-error for the system response W by method m, respectively; W could be $\mathrm{U_p}$ or $\mathrm{T_p}$; $\mathrm{W_0(N_0,K_0)}$ is the true values of $\mathrm{U_p}$ or $\mathrm{T_p}$ obtained from the observed N and K; and m represents the type of regression method used to estimate N and K. In addition, the standard deviation of $\mathrm{W_m}$(N,K) was calculated to indicate the degrees of uncertainty associated with the estimated $\mathrm{U_p}$ and $\mathrm{T_p}$, that is,

$$\mathrm{STD_m(W)} = \left\{\mathrm{E_{N,K}}\left[\mathrm{W_m^2(N,K)}\right] - \mathrm{E^2_{N,K}}\left[\mathrm{W_m(N,K)}\right]\right\}^{1/2} \quad (27)$$

in which $\mathrm{STD_m(W)}$ = standard deviation of $\mathrm{W_m}$(N,K).

In the validation study, the values of N and K in the subset used for validation purpose were treated as the true values from which the true values of $\mathrm{U_p}$ and $\mathrm{T_p}$ by equations (19) and (20) were calculated. Furthermore, from the previous analysis of N and K values for the 42 watersheds (Figures 1a and 1b), they are treated as bivariate log-normal random variables with the means and covariance computed by the regional regression equations under consideration.

Along with the MVR, two considerations were given to the results of UVR: one, denoted as UVR0, considers that the estimated N and K by the UVR are uncorrelated and the other, denoted as UVR1, treats them as bivariate lognormal random variables having the sample correlation of N and K of the 21 watersheds used for estimation.

4.2 Results

Sample results of validation study based on the first half data set are shown in Tables 5-6 each corresponding to the use of linear model and quadratic model, respectively. To shorten the presentation, results of validation study based on the second half data set are not presented herein (see Yeh and Tung, 1994) but only discussed. Part (a) of Tables 5-6 contains the observed and estimated mean and standard deviation of N and K in the log-space for the 21 watersheds in the validation subset. In computing the values of error criteria and the standard deviation associated with $\mathrm{U_p}$ and $\mathrm{T_p}$ of Nash's IUH, one recognizes that, from equations (19) and (20), the value of N must be greater than or equal to one to ensure the existence of $\mathrm{U_p}$ and $\mathrm{T_p}$. Consequently, in this validation study, the values of $\mathrm{U_p}$ and $\mathrm{T_p}$ were computed only when $\mathrm{N} \geq 1$. The column (2) of parts (b)-(d) of Tables 5-6 contains the probability that $\mathrm{N} \geq 1$. It should be pointed out that the values of error criteria and the standard deviation presented in parts (b)-(d) of Tables 5-6, in fact, are the conditional BIAS, MAE, and RMSE for $\mathrm{N} \geq 1$. Based on equations (24)-(27), the conditional error criteria and standard deviation of $\mathrm{U_p}$ and $\mathrm{T_p}$ can be expressed as

$$\mathrm{BIAS_m(W|N \geq 1)} = \mathrm{E_{N,K}}\left[\mathrm{W_0(N_0,K_0)} - \mathrm{W_m(N \geq 1,K)}\right]/\mathrm{P(N \geq 1)} \quad (28)$$

$$\mathrm{MAE_m(W|N \geq 1)} = \mathrm{E_{N,K}}\left[|\mathrm{W_0(N_0,K_0)} - \mathrm{W_m(N \geq 1,K)}|\right]/\mathrm{P(N \geq 1)} \quad (29)$$

$$\mathrm{RMSE_m(W|N \geq 1)} = \left\{\mathrm{E_{N,K}}\left[|\mathrm{W_0(N_0,K_0)} - \mathrm{W_m(N \geq 1,K)}|^2\right]\right\}^{1/2}/\mathrm{P(N \geq 1)} \quad (30)$$

$$\mathrm{STD_m}(W|N \geq 1) = \left\{E_{N,K}\left[|W_m^2(N \geq 1, K)\right] - E_{N,K}^2\left[W_m(N \geq 1, K)\right]\right\}^{1/2} / P(N \geq 1) \tag{31}$$

in which P(N≥1) can be computed by integrating the joint log-normal probability density function for N and K over the domain N≥1, K≥0.

As can be seen from Tables 5-6 and those not presented, without considering the correlation between N and K would result in the least desirable predictability, especially on the time-to-peak. Using the results from the UVR, along with the sample correlation yields significant improvement on the prediction accuracy. Interestingly, all methods result in over-prediction on the time-to-peak as indicated by the negative value of BIAS_T_p. Between the MVR and UVR1 for a given model type, except for Table 5, the MVR results in more accurate prediction on U_p and T_p.

It is interesting to note that the use of a quadratic model does not necessarily yield more accurate prediction of U_p and T_p than that of using a simpler linear model. Comparing part (a) of Tables 5 and 6, one notices that, in the great majority of the validation cases, the errors associated with the prediction of ln(N) and In(K) by using a linear model are smaller than that by using a quadratic model. However, this observation is reversed when the second 21 watersheds were used for estimation. More specifically, when the first 21 watersheds were used for estimation set, the values

Table 5. Validation results using the first 21 watersheds for estimation with linear model.

5(a) Observed N and K, their estimated means and standard devitaitons for watersheds in validation subject

		N			K	
Valid. Case	Obs. ln(N)	Est. mn_lnN	Est. std_lnN	Obs. ln(K)	Est. mn_lnK	Est. std_lnK
1	.858E+00	.102E+01	.970E+00	.159E+01	.106E+01	.706E+00
2	.123E+01	.757E+00	.952E+00	.206E+00	.947E+00	.692E+00
3	.155E+01	.122E+01	.954E+00	.164E+00	.532E+00	.694E+00
4	.114E+01	.105E+01	.889E+00	.476E+00	.744E+00	.647E+00
5	.180E+01	.150E+01	.952E+00	.197E+00	.169E+00	.693E+00
6	.104E+01	.989E+00	.105E+01	.535E+00	.574E+00	.766E+00
7	.151E+01	.115E+01	.948E+00	.531E+00	.968E+00	.690E+00
8	.658E+00	.104E+01	.908E+00	.140E+01	.103E+01	.660E+00
9	.729E+00	.894E+00	.899E+00	.139E+00	.897E+00	.654E+00
10	.931E+00	.917E+00	.902E+00	.106E+01	.855E+00	.656E+00
11	.861E+00	.117E+01	.994E+00	.164E+01	.104E+01	.723E+00
12	.155E+01	.111E+01	.990E+00	.687E+00	.955E+00	.721E+00
13	.192E+01	.169E+01	.103E+01	.167E+00	.103E+00	.751E+00
14	.714E+00	.142E+01	.986E+00	.803E+00	.110E+00	.717E+00
15	.145E+01	.150E+01	.951E+00	.751E+00	.216E+00	.692E+00
16	.146E+01	.122E+01	.957E+00	.346E+00	.104E+01	.697E+00
17	.570E+00	.672E+00	.961E+00	.212E+01	.106E+01	.699E+00
18	.110E+01	.104E+01	.877E+00	.259E+00	.831E+00	.638E+00
19	.696E+00	.119E+01	.904E+00	.175E+01	.898E+00	.658E+00
20	.568E+00	.964E+00	.891E+00	.114E+01	.952E+00	.648E+00
21	.120E+01	.996E+00	.913E+00	.658E+00	.915E+00	.655E+00

5(b) Error criteria corresponding to univariate regression (UVR0), $\rho(N,K)=0$

Valid.		IUH Peak Discharge					IUH Time-to-Peak		
Case	P(N>1)	BIAS_Up	MAE_Up	RMSE_Up	STD_Up	BIAS_Tp	MAE_Tp	RMSE_Tp	STD_Tp
1	.853E+00	-.303E+00	.735E+00	.102E+01	.975E+00	-.851E+01	.125E+02	.337E+02	.326E+02
2	.787E+00	.159E+01	.161E+01	.205E+01	.129E+01	-.714E+01	.848E+01	.235E+02	.224E+02
3	.900E+00	.902E+00	.105E+01	.133E+01	.979E+00	-.619E+01	.861E+01	.222E+02	.213E+02
4	.882E+00	.722E+00	.901E+00	.117E+01	.923E+00	-.627E+01	.784E+01	.191E+02	.181E+02
5	.943E+00	.187E+01	.188E+01	.213E+01	.102E+01	-.559E+01	.788E+01	.197E+02	.189E+02
6	.826E+00	.219E+01	.219E+01	.263E+01	.145E+01	-.767E+01	.924E+01	.282E+02	.272E+02
7	.888E+00	-.127E+01	.132E+01	.164E+01	.104E+01	-.906E+01	.123E+02	.318E+02	.304E+02
8	.875E+00	-.389E+00	.744E+00	.995E+00	.916E+00	-.955E+01	.110E+02	.274E+02	.257E+02
9	.840E+00	.115E+01	.122E+01	.155E+01	.104E+01	-.859E+01	.882E+01	.212E+02	.194E+02
10	.845E+00	.189E+00	.688E+00	.935E+00	.916E+00	-.528E+01	.781E+01	.196E+02	.189E+02
11	.881E+00	-.115E+01	.124E+01	.156E+01	.106E+01	-.112E+02	.151E+02	.414E+02	.398E+02
12	.869E+00	-.149E+00	.153E+01	.188E+01	.114E+01	-.820E+01	.129E+02	.351E+02	.341E+02
13	.949E+00	.167E+01	.170E+01	.197E+01	.105E+01	-.786E+01	.106E+02	.293E+02	.282E+02
14	.924E+00	-.122E+01	.130E+01	.158E+00	.101E+00	-.660E+01	.754E+01	.198E+02	.187E+02
15	.943E+00	.308E+00	.750E+00	.964E+00	.913E+00	-.329E+01	.851E+01	.199E+02	.197E+02
16	.898E+00	-.686E+00	.907E+00	.118E+01	.964E+01	-.129E+02	.147E+02	.381E+02	.359E+02
17	.758E+00	-.304E+00	.724E+00	.107E+01	.102E+01	-.422E+01	.900E+01	.245E+02	.241E+02
18	.882E+00	.873E+00	.997E+00	.128E+01	.931E+00	-.760E+01	.847E+01	.201E+02	.187E+02
19	.906E+00	-.134E+01	.138E+01	.167E+01	.987E+00	-.765E+01	.107E+02	.261E+02	.250E+02
20	.860E+00	-.546E+01	.666E+00	.898E+00	.896E+00	-.826E+01	.902E+01	.219E+02	.203E+02
21	.862E+00	-.327E+01	.679E+00	.915E+00	.914E+00	-.692E+01	.922E+01	.234E+02	.223E+02
AVG	.875E+00	.270E+00	.115E+01	.145E+01	.102E+01	-.755E+01	.100E+02	.260E+02	.248E+02

5(c) Error criteria corresponding to univariate regression with sample correlation (UVR1), ρ(N,K)=-.737

Valid.		IUH Peak Discharge				IUH Time-to-Peak			
Case	P(N>1)	BIAS_Up	MAE_Up	RMSE_Up	STD_Up	BIAS_Tp	MAE_Tp	RMSE_Tp	STD_Tp
1	.853E+00	-.443E+00	.544E+00	.724E+00	.573E+00	-.116E+01	.491E+01	.804E+01	.796E+01
2	.787E+00	.140E+01	.140E+01	.168E+01	.921E+00	-.213E+01	.331E+01	.600E+01	.561E+01
3	.900E+00	.803E+00	.825E+00	.991E+00	.581E+00	.-133E+01	.344E+01	.560E+01	.544E+01
4	.882E+00	.615E+00	.657E+00	.819E+00	.541E+00	-.208E+01	.336E+01	.556E+01	.516E+01
5	.943E+00	.180E+01	.180E+01	.192E+01	.667E+00	-.128E+01	.317E+01	.504E+01	.488E+01
6	.826E+00	.201E+01	.201E+01	.229E+01	.110E+01	-.188E+01	.327E+01	.595E+01	.264E+01
7	.888E+00	-.138E+01	.138E+01	.155E+01	.713E+00	-.213E+01	.491E+01	.810E+01	.781E+01
8	.875E+00	-.503E+00	.570E+00	.737E+00	.539E+00	-.364E+01	.478E+01	.801E+01	.714E+01
9	.840E+00	.101E+01	.102E+01	.122E+01	.677E+00	-.416E+01	.431E+01	.696E+01	.558E+01
10	.845E+00	.533E+01	.378E+00	.516E+00	.513E+00	-.904E+00	.326E+01	.529E+01	.521E+01
11	.881E+00	-.127E+01	.127E+01	.145E+01	.716E+00	-.228E+01	.575E+01	.964E+01	.937E+01
12	.869E+00	-.162E+00	.162E+01	.182E+01	.834E+00	-.586E+00	.508E+01	.805E+01	.803E+01
13	.949E+00	.161E+01	.161E+01	.174E+01	.625E+00	-.170E+01	.397E+01	.650E+01	.627E+01
14	.924E+00	-.114E+01	.114E+01	.129E+01	.615E+00	-.242E+01	.308E+01	.515E+01	.455E+01
15	.943E+00	.246E+00	.435E+00	.559E+00	.501E+00	.120E+01	.390E+01	.523E+01	.509E+01
16	.898E+00	-.788E+00	.809E+00	.980E+00	.583E+00	-.474E+01	.613E+01	.103E+02	.916E+01
17	.758E+00	-.516E+00	.593E+00	.827E+00	.647E+00	.118E+01	.398E+01	.599E+01	.588E+01
18	.882E+00	.768E+00	.787E+00	.915E+00	.562E+00	-.327E+01	.390E+01	.640E+01	.550E+01
19	.906E+00	-.143E+01	.143E+01	.158E+01	.673E+00	-.187E+01	.449E+01	.720E+01	.695E+01
20	.860E+00	-.178E+01	.396E+00	.537E+00	.506E+00	-.357E+01	.413E+01	.686E+01	.585E+01
21	.862E+00	-.158E+00	.398E+00	.539E+00	.515E+00	-.178E+01	.379E+01	.632E+01	.607E+01
AVG	.875E+00	.151E+00	.100E+01	.118E+01	.649E+00	-.193E+01	.414E+01	.677E+01	.634E+01

5(d) Error criteria corresponding to multivariate regression (MVR), $\rho(N,K)=-.724$

Valid. Case	P(N>1)	IUH Peak Discharge BIAS_Up	MAE_Up	RMSE_Up	STD_Up	IUH Time-to-Peak BIAS_Tp	MAE_Tp	RMSE_Tp	STD_Tp
1	.853E+00	-.404E+00	.549E+00	.731E+00	.583E+00	-.126E+01	.501E+01	.830E+01	.820E+01
2	.787E+00	.141E+01	.141E+01	.169E+01	.929E+00	-.220E+01	.338E+01	.618E+01	.578E+01
3	.900E+00	.805E+00	.829E+00	.999E+00	.591E+00	-.140E+01	.351E+01	.577E+01	.560E+01
4	.882E+00	.617E+00	.661E+00	.827E+00	.551E+00	-.213E+01	.343E+01	.571E+01	.530E+01
5	.943E+00	.180E+01	.180E+01	.193E+01	.676E+00	-.134E+01	.324E+01	.520E+01	.502E+01
6	.826E+00	.201E+01	.201E+01	.230E+01	.111+01	-.195E+01	.334E+01	.615E+01	.583E+01
7	.888E+00	-.137E+01	.137E+01	.155E+01	.721E+00	-.222E+01	.501E+01	.835E+01	.805E+01
8	.875E+00	-.501E+00	.573E+00	.743E+00	.548E+00	-.371E+01	.486E+01	.822E+01	.734E+01
9	.840E+00	.101E+01	.102E+01	.122E+01	.686E+00	-.421E+01	.437E+01	.711E+01	.573E+01
10	.845E+00	.556E+01	.386E+00	.527E+00	.524E+00	-.961E+00	.333E+01	.544E+01	.535E+01
11	.881E+00	-.126E+01	.127E+01	.146E+01	.724E+00	-.240E+01	.588E+01	.996E+01	.966E+01
12	.869E+00	-.162E+01	.162E+01	.183E+01	.841E+00	-.682E+00	.519E+01	.831E+01	.828E+01
13	.949E+00	.161E+01	.161E+01	.174E+01	.661E+01	-.177E+01	.407E+01	.672E+01	.648E+01
14	.924E+00	.114E+01	.114E+01	.130E+01	.625E+00	-.248E+01	.314E+01	.530E+01	.469E+01
15	.943E+00	.247E+00	.443E+00	.569E+00	.512E+00	.114E+01	.397E+01	.536E+01	.524E+01
16	.898E+00	-.786E+00	.810E+00	.985E+00	.593E+00	-.485E+01	.625E+01	.106E+02	.944E+01
17	.758E+00	-.513E+00	.596E+00	.833E+00	.657E+00	.111E+01	.404E+01	.615E+01	.605E+01
18	.882E+00	.769E+00	.791E+00	.958E+00	.971E+00	-.333E+01	.397E+01	.655E+01	.565E+01
19	.906E+00	-.143E+01	.143E+01	.158E+01	.680E+00	-.195E+01	.458E+01	.741E+01	.715E+01
20	.860E+00	-.176E+00	.176E+00	.403E+00	.546E+00	-.363E+01	.419E+01	.702E+01	.601E+01
21	.826E+00	-.156E+00	.405E+00	.549E+00	.526E+00	-.184E+01	.387E+01	.651E+01	.624E+01
AVG	.875+00	.153E+00	.101E+01	.118E+01	.658E+00	-.200E+01	.422E+01	.697E+01	.653E+01

Table 6. Validation results using the first 21 watersheds for estimation with quadratic model.

6(a) Observed N and K, their estimated means and standard devitaitons for watersheds in validation subject

	N			K		
Valid. Case	Obs. ln(N)	Est. mn_lnN	Est. std_lnN	Obs. ln(K)	Est. mn_lnK	Est. std_lnK
1	.858E+00	.104E+01	.105+01	.159E+01	.117E+01	.824E+00
2	.123E+01	.849E+00	.101E+00	.206E+00	.104E+01	.785E+00
3	.155E+01	.108E+01	.104E+01	.164E+00	.826E+00	.819E+00
4	.114E+01	.873E+00	.938E+00	.476E+00	.576E+00	.736E+00
5	.180E+01	.150E+01	.966E+00	-.197E+00	.153E+00	.759E+00
6	.104E+01	.133E+01	.116E+01	.535E+00	.799E-00	.914E+00
7	.151E+01	.143E+01	.993E+00	.531E+00	.773E+00	.780E+00
8	.658E+00	.937E+00	.906E+00	.140E+01	.104E+01	.711E+00
9	.729E+00	.724E+00	.904E+00	.139E+00	.956E+00	.710E+00
10	.931E+00	.749E+00	.908E+00	.106E+01	.810E+00	.713E+00
11	.861E+00	.947E+00	.105E+01	.164E+01	.133E+01	.825E+00
12	.155E+01	.131E+01	.108E+01	.687E+00	.961E+00	.850E+00
13	.192E+01	.183E+01	.105E+01	-.167E+00	.193E+00	.828E+00
14	.714E+00	.118E+01	.107E+01	.803E+00	-.303E-01	.839E+00
15	.145E+01	.165E+01	.107E+01	.751E+00	.474E+00	.837E+00
16	.146E+01	.131E+01	.112E+01	.346E+00	.636E+00	.880E+00
17	.570E+00	.852E+00	.108E+01	.212E+01	.137E+01	.849E+00
18	.110E+01	.866E+00	.902E+00	.259E+00	.802E+00	.708E+00
19	.696E+00	.115E+01	.910E+00	.175E+01	.943E+00	.714E+00
20	.568E+00	.749E+00	.902E+00	.114E+01	.102E+01	.708E+00
21	.120E+01	.107E+01	.960E+00	.658E+00	.745E+00	.745E+00

6(b) Error criteria corresponding to univariate regression (UVR0), $\rho(N,K)=0$

Valid.		IUH Peak Discharge				IUH Time-to-Peak			
Case	P(N>1)	BIAS_Up	MAE_Up	RMSE_Up	STD_Up	BIAS_Tp	MAE_Tp	RMSE_Tp	STD_Tp
1	.840E+00	-.155E+00	.811E+00	.111E+01	.110E+01	-.148E+02	.184E+02	.580E+02	.560E+02
2	.802E+00	.174E+01	.177E+01	.222E+01	.138E+01	-.111E+02	.123E+02	.366E+02	.349E+02
3	.849E+00	.116E+01	.128E+01	.166E+01	.119E+02	.111E+02	134E+02	415E+02	399E+02
4	.824E+00	.490E+00	.829E+00	.114E+01	.10E+01	-.443E+01	.651E+01	.179E+02	.173E+02
5	.939E+00	.185E+01	.187E+01	.214E+01	108E+01	-.601E+01	.838E+01	.222E+02	.214E+02
6	.873E+00	.188E+01	.192E+01	.233E+01	.138E+01	-.906E+01	.108E+02	.394E+02	.383E+02
7	.925E+00	-.131E+01	.139E+01	.169E+01	.107E+01	-.127E+02	.157E+02	.435E+02	.416E+02
8	.850E+00	-.431E+00	.787E+00	.107E+00	.975E+00	-.869E+01	.103E+02	.270E+02	.256E+02
9	.794E+00	.115E+01	.123E+01	.162E+01	.114E+01	-.819E+01	.846E+01	.219E+02	.203E+02
10	.809E+00	.918E-01	.709E+00	.985E+00	.981E+00	-.420E+01	.709E+01	.187E+02	.182E+02
11	.816E+00	-.943E+01	.112E+01	.152E+01	.119E+01	-.161E+02	.199E+02	.638E+02	.617E+02
12	.886E+00	-.137E+01	.146E+01	.182E+01	.120E+01	-.167E+02	.208E+02	.665E+02	.644E+02
13	.959E+00	.184E+01	.187E+01	.215E+01	.111E+01	-.128E+02	.151E+02	.446E+02	.427E+02
14	.865E+00	.992E+00	.117E+01	.153E+01	.117E+01	-.533E+01	.661E+01	.211E+02	.204E+02
15	.939E+00	.662E+00	.987E+00	.126E+01	.107E+01	-.130E+02	.170E+02	.515E+02	.499E+02
16	.879E+00	-.997E+00	.120E+01	.156E+01	.120E+01	-.142E+02	.167E+02	.565E+02	.547E+02
17	.785E+00	.137E+00	.826E+00	.117E+01	.116E+01	-.172E+02	.206E+02	.700E+02	.679E+02
18	.831E+00	.773E+00	.957E+00	.128E+01	.102E+01	-.648E+01	.756E+01	.199E+02	.188E+02
19	.897E+00	-.132E+01	.137E+01	.168E+01	.104E+00	-.834E+01	.115E+02	.292E+02	.280E+02
20	.811E+00	-.362E+01	.701E+00	.974E+00	.974E+00	-.806E+01	.890E+01	.235E+02	.221E+02
21	.867E+00	-.144E+00	.752E+00	.101E+01	.100E+01	-.743E+01	.990E+01	.273E+02	.263E+02
AVG	.859E+00	.289E+00	.119E+01	.152E+01	.112E+01	-.103E+02	.127E+02	.381E+02	.367E+02

6(c) Error criteria corresponding to univariate regression with sample correlation (UVR1), ρ(N,K)=-.737

Valid.			IUH Peak Discharge				IUH Time-to-Peak		
Case	P(N>1)	BIAS_Up	MAE_Up	RMSE_Up	STD_Up	BIAS_Tp	MAE_Tp	RMSE_Tp	STD_Tp
1	.840E+00	-.331E+00	.529E+00	.720E+00	.640E+00	-.280E+01	.607E+01	.108E+02	.104E+02
2	.802E+00	.154E+01	.154E+01	.183E+01	.981E+00	-.343E+01	.438E+01	.802E+01	.725E+01
3	.849E+00	.995E+00	.101E+01	.124E+01	.745E+00	.-257E+01	447E+01	801E+01	758E+01
4	.824E+00	.320E+00	.490E+00	.669E+00	.588E+00	-.462E+00	.246E+01	.403E+01	.401E+01
5	.939E+00	.178E+01	.178E+01	.191E+01	.703E+00	-.115E+01	.312E+01	.498E+01	.485E+01
6	.873E+00	.172E+01	.172E+01	.196E+01	.939E+00	-.164E+01	.316E+01	.583E+01	.559E+01
7	.925E+00	-.140E+01	.140E+01	.156E+01	.686E+00	-.340E+01	.570E+01	.957E+01	.895E+01
8	.850E+00	-.576E+00	.636E+00	.827E+00	.539E+00	-.273E+01	.408E+01	.698E+01	.624E+01
9	.794E+00	.959E+00	.971E+00	.121E+01	.745E+00	-.352E+01	.371E+01	.630E+01	.523E+01
10	.809E+00	-.850-01	.406E+00	.566E+00	.560E+00	.502E+01	.286E+01	.446E+01	.445E+01
11	.881E+00	-.113E+01	.114E+01	.140E+01	.831E+00	-.303E+01	.651E+01	.118E+02	.114E+02
12	.886E+00	-.150E+01	.151E+01	.171E+01	.823E+00	-.319E+01	.672E+01	.118E+02	.114E+02
13	.959E+00	.178E+01	.178E+01	.191E+01	.686E+00	-.360E+01	.526E+01	.892E+01	.816E+01
14	.865E+00	.836E+00	.876E+00	.109E+01	.706E+00	-.103E+01	.215E+01	.381E+01	.367E+01
15	.939E+00	.583E+00	.676E+00	.844E+00	.611E+00	.-.238+01	.569E+01	.953E+01	.923E+01
16	.879E+00	-.115E+01	.116E+01	.138E+01	.773E+00	-.318E+01	.517E+01	.924E+01	.887E+01
17	.785E+00	-.961E+01	.476E+00	.675E+00	.668E+00	.-.326E+01	.640E+01	.121E+01	.116E+02
18	.831E+00	.615E+00	.669E+00	.868E+00	.613E+00	-.210E+01	.300E+01	.519E+01	.474E+01
19	.897E+00	-.142E+01	.142E+01	.159E+01	.712E+00	-.180E+01	.447E+01	.723E+01	.700E+01
20	.811E+00	-.211E+00	.432E+00	.602E+00	.564E+00	-.294E+01	.361E+01	.632E+01	.560E+01
21	.867E+00	-.282E+00	.477E+00	.640E+00	.575E+00	-.145E+01	.364E+01	.609E+01	.592E+01
AVG	.859E+00	.140E+00	.100E+01	.120E+01	.702E+00	-.236E+01	.441E+01	.767E+01	.725E+01

6(d) Error criteria corresponding to multivariate regression (MVR), ρ(N,K)=-.851

Valid.		IUH Peak Discharge				IUH Time-to-Peak			
Case	P(N>1)	BIAS_Up	MAE_Up	RMSE_Up	STD_Up	BIAS_Tp	MAE_Tp	RMSE_Tp	STD_Tp
1	.840E+00	-.358E+00	.462E+00	.622E+00	.508E+00	-.159E+01	.469E+01	.744E+01	.727E+01
2	.802E+00	.151E+01	.151E+01	.175E+01	.888E+00	-.264E+01	.350E+01	.582E+01	.519E+01
3	.849E+00	.969E+00	.974E+00	.116E+01	.629E+00	-.169E+01	.346E+01	.556E+01	.530E+01
4	.824E+00	.293E+00	.402E+00	.546E+00	.460E+00	-.288E-01	.198E+01	.292E+01	.292E+01
5	.939E+00	.177E+01	.177E+01	.187E+01	.615E+00	-.617E+00	.243E+01	.353E+01	.348E+01
6	.873E+00	.169E+01	.169E+01	.188E+01	.832E+00	-.954E+00	.236E+01	.384E+02	.372E+01
7	.925E+00	-.141E+01	.141E+01	.153E+01	.592E+00	-.239E+01	.446E+01	.680E+01	.636E+01
8	.850E+00	-.398E+00	.618E+00	.772E+00	.489E+00	-.206E+01	.330E+01	.518E+01	.475E+01
9	.794E+00	.930E+00	.933E+00	.113E+01	.643E+00	-.300E+01	.318E+01	.496E+01	.395E+01
10	.809E+00	-.113E+00	.322E+00	.449E+00	.435E+00	.512E+00	.236E+01	.335E+01	.331E+01
11	.816E+00	-.116E+01	.116E+01	.138E+01	.739E+00	-.171E+01	.502E+01	.810E+01	.792E+01
12	.886E+00	-.153E+00	.153E+01	.169E+01	.730E+00	-.184E+01	.513E+01	.801E+01	.780E+01
13	.959E+00	.177E+01	.177E+01	.187E+01	.585E+00	-.265E+01	.407E+01	.623E+01	.564E+01
14	.865E+00	.812E+00	.826E+00	.999E+00	.582E+00	-.596E+01	.165E+01	.260E+01	.254E+01
15	.939E+00	.571E+00	.615E+00	.750E+00	.486E+00	-.129E+01	.434E+01	.649E+01	.636E+01
16	.879E+00	-.117E+01	.117E+01	.134E+01	.664E+00	-.211E+01	.392E+01	.637E+01	.601E+01
17	.785E+00	-.132E+00	.379E+00	.537E+00	.521E+00	-.190E+01	.490E+01	.820E+01	.798E+01
18	.831E+00	.590E+00	.612E+00	.773E+00	.499E+00	-.161E+01	.244E+01	.387E+01	.352E+01
19	.897E+00	-.144E+01	.144E+01	.157E+01	.634E+00	-.106E+01	.355E+01	.526E+01	.515E+01
20	.811E+00	-.238E+00	.363E+00	.504E+00	.444E+00	-.237E+01	.299E+01	.480E+01	.417E+01
21	.867E+00	-.304E+00	.410E+00	.545E+00	.453E+00	-.797E+01	.287E+01	.435E+01	.427E+01
AVG	.859+00	.117E+00	.970E+01	.113E+01	.592E+00	-.1545E+01	.364E+01	.541E+01	.512E+01

of error criteria and standard deviation associated with the prediction of U_p and T_p by the UVR indicate that the linear model outperforms the quadratic model. On the other hand, the quadratic model is superior to the linear model when the second half of the 42 watersheds were used as the estimation set. In both cases, the quadratic model yields a more accurate prediction than the linear model when the MVR was used. This indicates that the regional regression equations developed by the MVR approach, when dependent variables are correlated, would consistently perform better in prediction than that by the UVR.

5 Summary and conclusions

Hydrologic regionalization is a useful tool that allows transferring hydrological information from gaged sites to ungaged sites. This study developed regional regression equations that relate the two parameters in Nash's IUH model, namely, N and K, to the basin characteristics using data from 42 watersheds in Taiwan. In the process of developing the regional equations, various regression procedures were employed. In particular, the conventional univariate regression, multivariate regression, and seemingly unrelated regression were considered. Multivariate regression and seemingly unrelated regression were applied because, based on the previous study by the authors, there exists rather strong correlation between N and K. The conventional regression procedure does not take into account the correlation among the dependent variables which is not theoretically sound. Based on the data from 42 watersheds in Taiwan, a set of regional equations were developed using the various types of regression procedures.

To assess the relative performance of the regional equations derived by three different regression procedures, a numerical experiment was conducted in the study using data splitting validation technique by which the 42 watersheds were divided into two subsets of equal size each of which, in turn, was used for the estimation purpose and validation purpose. The objective of the validation study was to examine the predictability of regional equations derived by different regression procedures. The criteria used in the performance evaluation were the bias, mean-absolute-error, and root mean-squared-error in predicting the peak discharge and time-to-peak of observed IUH in the validation set.

Based on the study, the following conclusions are obtained:

1. Many of the statistical characteristics of a regional equation are readily available from statistical analysis packages which can be used for uncertainty and reliability analysis of hydrologic and hydraulic designs.

2. In hydrologic regionalization involving several dependent variables, their correlations should be considered in the process of establishing the regional equations. Numerical experiment conducted in this study has indicated that the consideration of such correlation will enhance the predictability of resulting regional equations as compared with the ones from the conventional univariate regression procedure.

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