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A new simple unifying approach of finding the state equation model and its practical application

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ABSTRACT

In the control signal process theory, the state equation is an important approach for modelling, analyzing and designing a wide range of systems. Furthermore, the state equation is often the most efficient form from the standpoint of computer simulation for highly complex systems. The existing researches exploit the fact that for networks containing dependent sources, bridging elements and coupling elements, the derivation of the closed-form state equation becomes rather complicated. On the other hand, a simple and valuable unifying approach of finding the state equations for complicated physical systems is successfully presented in the paper. The new unifying approach needs only the substitution theorem and the author's invented simple matrix operations. The main fascinating peculiarity of this approach is exposed in its systematic, graphic and human structure that improves the shortcomings of those traditional approaches. One comparative example is proposed to show the distinguished advantages our method offers over existing methods. In order to demonstrate the practical applicability, the paper has investigated an electronic operational-amplifier circuit that can be easily stabilized by our approach.

1. Introduction

It's universally known that the state equation plays an important role to investigate the significant properties of physical systems, such as controllability, observability and stability in the control and signal process theory [9]. There are five fundamental reasons for representing the physical systems in state equation: (1) this form lends itself most easily to analog or/and digital computer programming, (2) the extension of the analysis to nonlinear and/or time varying systems is quite easy, (3) many system-theoretic concepts are readily applicable to systems, (4) it allows us to handle multi-input multi-output systems within precisely the same notational framework that we will utilize for single-input single-output systems, and (5) it allows us to determine the internal behaviors of the physical systems easily while still giving the input–output information we desire.

The state equation can also be skillfully utilized to design the controller that globally exponentially stabilizes the control system via the famous approaches, such as our recent researches [3–8] of applying singularly perturbed approach [16,17] and feedback linearization approach. However, the existing researches [1,2,12, p. 683,10, p. 521,15] show that for networks containing dependent sources and coupling elements, the derivation of the closed-form state equation becomes rather complicated. Especially, these processes presented by Nise [14, p. 130] and Ziemer et al. [19] are too complicated and troublesome to

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be accepted. In order to solve these shortcomings, the simple unifying approach, as a means of finding the state equations of the control systems, has been derived in this paper. Moreover, the simple unifying approach, proposed in this paper, needs only the substitution theorem and simple algebraic techniques which have been not appeared in any researches. For the sake of convenience, the simple unifying approach of finding the state equations is expressed by Chen's Electric Unifying Approach (C.E.U.A.). In order to emphasize the significant contributions and practical application of C.E.U.A., three comparative examples and one amplifier circuit are proposed to be in comparison with those existing methods.

2. The Chen's Electric Unifying Approach

The main structure of the C.E.U.A. is shown as below:

STEP 1: View the branch capacitor voltages $v_{C_k}(t)$ and the branch inductor currents $i_{L_k}(t)$ as state variables. Utilizing the substitution theorem, the capacitors C_k (inductors L_k) are replaced with the current sources $i_{C_k}(t)$ (the voltage sources $v_{L_k}(t)$), respectively. The use of the substitution theorem prevents the derivative terms $i_{C_k}(t)$ and $C\,dv_C/dt$ from appearing in the system and hence the following modified node voltage method (mesh-current method) could be applied.

STEP 2: Using the transforming technique of the voltage source and the current source changes all voltage (current) sources to be current (voltage) sources. If the voltage (current) source is not in series (parallel) with a resistor, we can put a zero (infinite) resistor R_{CCC} to be in series (parallel) with the voltage (current) source. It's worth noting that the traditional node voltage method (mesh-current method) cannot solve these cases.

STEP 3: According to the following matrix equations, list the node-voltage matrix equations (mesh-current matrix equations):

$$\begin{bmatrix} Y_{11} & -Y_{12} & \cdots & -Y_{1N} \\ -Y_{21} & Y_{22} & \cdots & -Y_{2N} \\ \vdots & \vdots & & \vdots \\ -Y_{N1} & -Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} \pm I_{S11} \pm I_{S12} \pm \cdots \\ \pm I_{S21} \pm I_{S22} \pm \cdots \\ \vdots \\ \pm I_{SN1} \pm I_{SN2} \pm \cdots \end{bmatrix} \tag{2 - 1a}$$

$$\begin{bmatrix} Z_{11} & -Z_{12} & \cdots & -Z_{1N} \\ -Z_{21} & Z_{22} & \cdots & -Z_{2N} \\ \vdots & \vdots & & \vdots \\ -Z_{N1} & -Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_{11} \\ I_{22} \\ \vdots \\ I_{NN} \end{bmatrix} = \begin{bmatrix} \pm V_{S11} \pm V_{S12} \pm \cdots \\ \pm V_{S21} \pm V_{S22} \pm \cdots \\ \vdots \\ \pm V_{SN1} \pm V_{SN2} \pm \cdots \end{bmatrix} \tag{2-1b}$$

where V_i, $i = 1,2,\cdots,N$, and I_{ii}, $i = 1,2,\cdots,N$, are called the node voltages and the mesh currents, respectively. Moreover, I_{ii}, $i = 1,2,\cdots,N$, are in clockwise direction.

STEP 4: If the dependent current (voltage) variables are involved in the node-voltage matrix equations (mesh-current matrix equations), then the dependent variables must be transferred to be node-voltage variables (mesh-current variables) and moving-term operations are immediately executed. In general, an electrical systems with a dependent source will increase complexity in applying network analysis to find the state equations [14]. On the other hand, this problem will be easily solved by the C.E.U.A.

STEP 5: Utilizing the simple matrix operations replaces the node-voltage variables (mesh-current variables) with the input variables $V_{sk}(s)$, the state variables $V_{C_k}(s)$, $I_{L_k}(s)$ and the desired variables $I_{C_k}(s)$, $V_{L_k}(s)$. These simple matrix operations are summarized item by item as follows:

(1)

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} x\,\boxed{-\;k} \\ y \\ z \end{bmatrix} = \left[\begin{matrix} J \\ K \\ L \end{matrix} \boxed{\begin{matrix} -\;Ak \\ -\;Dk \\ -\;Gk \end{matrix}} \right]$$

(2)

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix}$$
$$\Rightarrow \left[\begin{matrix} A & B \\ D & E \\ G & H \end{matrix} \boxed{\begin{matrix} \cdot\,k \\ \cdot\,k \\ \cdot\,k \end{matrix}} \begin{matrix} C \\ F \\ I \end{matrix} \right] \begin{bmatrix} x \\ \cancel{y}\,\boxed{k} \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix}$$

(3)

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A & B \boxed{+A} & C \\ D & E \boxed{+D} & F \\ G & H \boxed{+G} & I \end{bmatrix}\begin{bmatrix} x \boxed{-y} \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix}$$

(4)

$$\begin{bmatrix} \cancel{A} & \cancel{B} & \cancel{C} \\ D & \boxed{\cancel{0}} & F \\ G & \boxed{\cancel{0}} & I \end{bmatrix}\begin{bmatrix} x \\ \cancel{y} \\ z \end{bmatrix} = \begin{bmatrix} \cancel{J} \\ K \\ L \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D & F \\ G & I \end{bmatrix}\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} K \\ L \end{bmatrix}$$

(5)

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}\begin{bmatrix} \boxed{x} \\ \boxed{y} \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \boxed{B} & \boxed{A} & C \\ \boxed{E} & \boxed{D} & F \\ \boxed{H} & \boxed{G} & I \end{bmatrix}\begin{bmatrix} \boxed{y} \\ \boxed{x} \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix}$$

(6)

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \boxed{-J} & B & C \\ \boxed{-K} & E & F \\ \boxed{-L} & H & I \end{bmatrix}\begin{bmatrix} x/\boxed{x^2} \\ y/\boxed{x} \\ z/\boxed{x} \end{bmatrix} = \begin{bmatrix} \boxed{-A} \\ \boxed{-D} \\ \boxed{-G} \end{bmatrix}$$

STEP 6: Move those inputs terms $V_{sk}(s)$ to the left side of the matrix equations. View the variables $I_{C_k}(s)$, $V_{L_k}(s)$ as new node-voltage variables (mesh-current variables) to construct a new node-voltage equations (mesh-current equations). Apply the Cramer's rule and the L'Hospital's rule to obtain the variables $i_{C_k}(v_{L_k})$.

STEP 7: Replace the variables $i_{C_k}(v_{L_k})$ with $C_k\frac{dv_{C_k}}{dt}$ $\left(L_k\frac{di_{L_k}}{dt}\right)$, respectively, to obtain the state equation.

3. Significant contributions of the Chen's Electric Unifying Approach

Be short of the length of the paper, we only put forth those significant contributions in this section.

When we investigate the AC small-signal analysis of the difference amplifier circuits, the symmetric method is adopted for the traditional approaches [13,18]. A mortal wound of the traditional approach is that both difference transistors and collector resistors need to be absolutely equivalent. The constraint is impractical and never to be satisfied now, even in future.

On the other hand, it's easy to be solved neatly via the C.E.U.A. For example, evaluate the voltage gain v_o/v_s of the non-symmetric difference amplifier shown in Fig. 1. The relative small-signal equivalent circuit is immediately derived in Fig. 2. Applying the C.E.U.A. yields

$$\begin{bmatrix} \frac{1}{R_s}+\frac{1}{r_{\pi1}} & 0 & 0 \\ g_{m1} & \frac{1}{r_{\pi2}}+g_{m2} & 0 \\ 0 & 0 & \frac{1}{R_{c2}} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_o \end{bmatrix} = \begin{bmatrix} \frac{v_s}{R_s} \\ 0 \\ 0 \end{bmatrix} \tag{3-1}$$

Hence the voltage gain is facile to be obtained via the Cramer's rule by the following equation

$$\frac{v_o}{v_s} = \frac{-g_{m1}g_{m2}r_{\pi1}r_{\pi2}R_{c2}}{(1+g_{m2}r_{\pi2})(R_s+r_{\pi1})} \tag{3-2}$$

In general, when an F.E.T. (a B.J.T.) is connected in common-source (common-emitter) amplifier structure, the capacitor C_{gd} (C_μ) of its high-frequency hybrid-π equivalent circuit appears in the feedback path from the amplifier output to its input. The bridging element of C_{gd} (C_μ) complicates the analysis of the high-frequency response. The traditional approach [13,18] utilized the approximate method based on the famous Miller's theorem to solve the upper 3-db frequency of the amplifier. Fortunately, in view of the merits of the C.E.U.A., we need not use the Miller's theorem to get the solution. The unprecedented undertakings of terminating the utilization of the Miller's theorem will mark a new epoch in the analysis and design of the high-frequency response of the electronic amplifier. A demonstrated example is shown in Fig. 3. The high-frequency hybrid-π equivalent model is drawn in Fig. 4. Without using the Miller's theorem, we apply the Gray–Searle method [11] based on the C.E.U.A. to find the upper 3-db frequency of Fig. 4. First, obtain the contribution of the capacitor C_μ while reducing the capacitor C_π and the input signal source to zero and determine the equivalent resistance R_{eq-C_μ} seen from C_μ. Adding an individual current source i_t to these two ends of the capacitor C_μ, shown in Fig. 5, we immediately get the node-voltage matrix equations via the C.E.U.A. as follows:

$$\begin{bmatrix} \frac{1}{R_s}+\frac{1}{r_\pi} & 0 \\ g_m & \frac{1}{R_c} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -i_t \\ i_t \end{bmatrix} \tag{3-3}$$

Hence the equivalent resistance R_{eq-C_μ} is evaluated, using the Cramer's rule, by the following equation

$$R_{eq-C_\mu} = \frac{v_t}{i_t} = \frac{v_2 - v_1}{i_t} = \frac{R_c(R_s r_\pi g_m + R_s + r_\pi) + R_s r_\pi}{R_s + r_\pi} \tag{3-4}$$

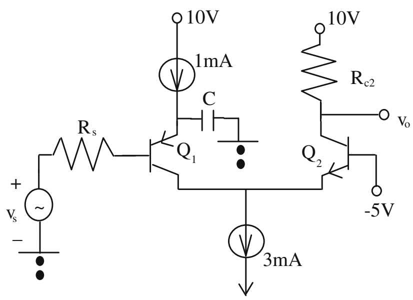


Fig. 1. The non-symmetric difference amplifier.

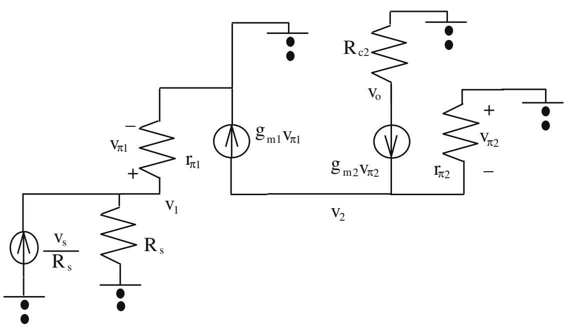


Fig. 2. The small-signal equivalent circuit of Fig. 1.

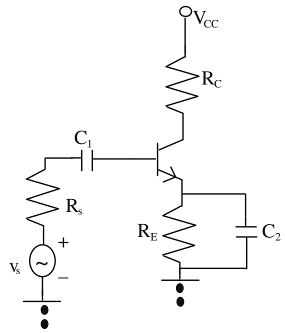


Fig. 3. The common-emitter amplifier.

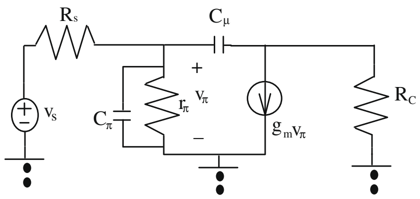


Fig. 4. The high-frequency model of Fig. 3.

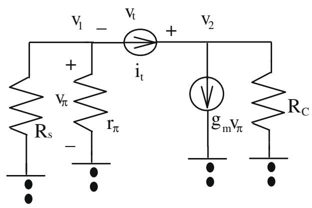


Fig. 5. The circuit of determining equivalent resistance R_{eq-C_μ}.

Secondly, similar procedures are repeated for the capacitor C_π and the equivalent resistance R_{eq-C_π} seen from C_π can be directly derived as follows:

$$R_{eq-C_\pi} = R_s \| r_\pi \tag{3-5}$$

From the Gray–Searle method, we see that the upper 3-db frequency is given by

$$\omega_H = \frac{1}{R_{eq-C_\mu} \cdot C_\mu + R_{eq-C_\pi} \cdot C_\pi} \tag{3-6}$$

4. Demonstrated example

Consider the following complicated electric system with dependent source shown in Fig. 6. Applying the C.E.U.A. to derive the state equations as follows:

STEP 1: View the branch capacitor voltages $v_{C1}(t)$ and the branch inductor currents $i_{L1}(t)$ as state variables. Utilizing the substitution theorem, the capacitor C_1 and inductor L_1 are replaced with the current source $i_{C_1}(t)$ and the voltage source $v_{L_1}(t)$, respectively, as shown in Fig. 7.

STEP 2: To apply the C.E.U.A., we put a zero resistor R_{CCC} to be in series with the voltage source. Using the transforming technique of the voltage source and the current source, Fig. 8 can be redrawn to be in Fig. 9.

STEP 3 and STEP 4: List the node-voltage matrix equations and move the dependent source term to the left side of the matrix equations:

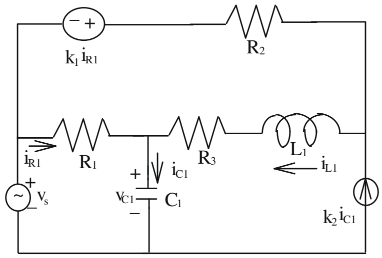


Fig. 6. The complicated electric system.

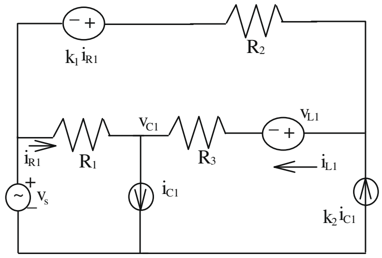


Fig. 7. The equivalent system of Fig. 6 with substitution theorem.

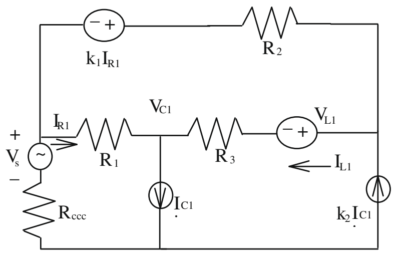


Fig. 8. The s-domain equivalent system of Fig. 7.

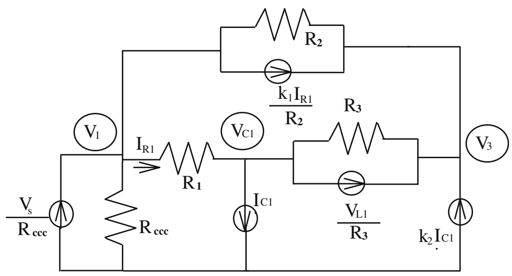


Fig. 9. The equivalent circuit of Fig. 8.

$$\begin{bmatrix} \frac{1}{R_{CCC}}+\frac{1}{R_1}+\frac{1}{R_2}+\frac{k_1}{R_1R_2} & -\frac{1}{R_1}-\frac{k_1}{R_1R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_2}-\frac{k_1}{R_1R_2} & -\frac{1}{R_3}+\frac{k_1}{R_1R_2} & \frac{1}{R_2}+\frac{1}{R_3} \end{bmatrix}\begin{bmatrix} v_1 \\ v_{C1} \\ v_3 \end{bmatrix}=\begin{bmatrix} \frac{v_s}{R_{CCC}} \\ -i_{C1}-\frac{v_{L1}}{R_3} \\ \frac{v_{L1}}{R_3}+k_2i_{C1} \end{bmatrix} \tag{4-1}$$

Multiplying the first row of the matrix equations with R_{CCC}, we obtain

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_2}-\frac{k_1}{R_1R_2} & -\frac{1}{R_3}+\frac{k_1}{R_1R_2} & \frac{1}{R_2}+\frac{1}{R_3} \end{bmatrix}\begin{bmatrix} v_1 \\ v_{C1} \\ v_3 \end{bmatrix}=\begin{bmatrix} v_s \\ -i_{C1}-\frac{v_{L1}}{R_3} \\ \frac{v_{L1}}{R_3}+k_2i_{C1} \end{bmatrix} \tag{4-2}$$

STEP 5: Utilizing the author's invented matrix operations to replace the node-voltage variables (mesh-current variables) with v_s, v_{C1}, i_{L1}, i_{C1}, v_{L1}:

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} & 1 \\ -\frac{1}{R_2}-\frac{k_1}{R_1R_2} & \frac{1}{R_2}+\frac{k_1}{R_1R_2} & -\frac{R_3}{R_2}-1 \end{bmatrix} \begin{bmatrix} v_s \\ v_{C1} \\ \frac{-v_3+v_{L1}+v_{C1}}{R_3}(\text{i.e. } -i_{L1}) \end{bmatrix} = \begin{bmatrix} v_s \\ -i_{C1} \\ -\frac{v_{L1}}{R_2}+k_2 i_{C1} \end{bmatrix} \tag{4-3}$$

STEP 6: Move input term v_s to the left side of the matrix equations. View the variables $i_{C1}(s)$, $v_{L1}(s)$ as new node-voltage variables to construct a new node-voltage equations:

$$\begin{bmatrix} -1 & 0 \\ k_2 & -\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} i_{C1} \\ v_{L1} \end{bmatrix} = \begin{bmatrix} -\frac{v_s}{R_1}+\frac{v_{C1}}{R_1}-i_{L1} \\ \left(-\frac{1}{R_2}-\frac{k_1}{R_1R_2}\right)v_s+\left(\frac{1}{R_2}+\frac{k_1}{R_1R_2}\right)v_{C1}+\left(\frac{R_3}{R_2}+1\right)i_{L1} \end{bmatrix} \tag{4-4}$$

Apply the Cramer's rule to calculate i_{C1}, v_{L1} as follows:

$$\begin{bmatrix} i_{C1} \\ v_{L1} \end{bmatrix} = \begin{bmatrix} \frac{v_s}{R_1}-\frac{v_{C1}}{R_1}+i_{L1} \\ \left(1+\frac{k_1}{R_1}+\frac{k_2R_2}{R_1}\right)v_s+\left(-1-\frac{k_1}{R_1}-\frac{k_2R_2}{R_1}\right)v_{C1}+(-R_3-R_2+k_2R_2)i_{L1} \end{bmatrix} \tag{4-5}$$

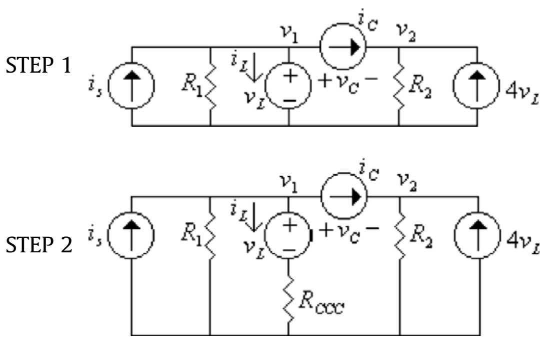
STEP 7: Replace the variables i_{C1} and v_{L1} with $C_1\frac{dv_{C1}}{dt}$ $\left(L_1\frac{di_{L1}}{dt}\right)$, respectively, to obtain the state equations:

$$\begin{bmatrix} \frac{dv_{C1}}{dt} \\ \frac{di_{L1}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1C_1} & \frac{1}{C_1} \\ \frac{-k_1}{R_1L_1}-\frac{k_2}{R_1L_1}R_2-\frac{1}{L_1} & \frac{(k_2-1)R_2-R_3}{L_1} \end{bmatrix} \begin{bmatrix} v_{C1} \\ i_{L1} \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1C_1} \\ \frac{1}{L_1}+\frac{k_1}{R_1L_1}+\frac{k_2R_2}{R_1L_1} \end{bmatrix} v_s \tag{4-6}$$

5. Comparative example

To be in comparison with those traditional existing methods, one example is proposed to exploit the significant contributions for C.E.U.A. [10, p. 521] and [14, p. 130] show that for networks containing dependent sources and coupling elements, the derivation of the closed-form representation will become rather complicated for applying the existing network analysis to find the state equations. The following example is proposed to show that based on the C.E.U.A., these problems are easily overcome via simple matrix algebra. Following the procedures of C.E.U.A. item by item, the state equation can be easily to be list as follows.

Consider the electrical network with dependent source shown in Fig. 10 [14, p. 130].



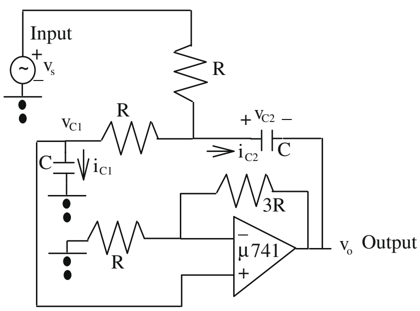


Fig. 10. A unstable electric system.

STEP 3

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_{CCC}} & 0 \\ 0 & \frac{1}{R_2} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_S - i_C + \frac{v_L}{R_{CCC}} \\ i_C + 4v_L \end{bmatrix}$$

STEP 4 and STEP 5

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_{CCC}} & 0 \\ \frac{1}{R_2} & -\frac{1}{R_2} \end{bmatrix}\begin{bmatrix} v_1 \\ -v_2+v_1 \end{bmatrix} = \begin{bmatrix} i_S - i_C + \frac{v_L}{R_{CCC}} \\ i_C + 4v_L \end{bmatrix}$$

$$\begin{bmatrix} \left(\frac{1}{R_1}+\frac{1}{R_{CCC}}\right)\times R_{CCC} & 0 \\ \left(\frac{1}{R_2}\right)\times R_{CCC} & -\frac{1}{R_2} \end{bmatrix}\begin{bmatrix} \frac{v_1-v_L}{R_{CCC}}(\text{i.e. } i_L) \\ v_C \end{bmatrix} = \begin{bmatrix} i_S - i_C - \frac{v_L}{R_1} \\ i_C + v_L\left(4-\frac{1}{R_2}\right) \end{bmatrix}$$

STEP 6

$$\begin{bmatrix} -\frac{1}{R_1} & -1 \\ 4-\frac{1}{R_2} & 1 \end{bmatrix}\begin{bmatrix} v_L \\ i_C \end{bmatrix} = \begin{bmatrix} i_L - i_S \\ -\frac{1}{R_2}v_C \end{bmatrix}$$

STEP 7

$$\begin{cases} v_L = L\frac{di_L}{dt} = \left(\frac{1}{4-\frac{1}{R_1}-\frac{1}{R_2}}\right)i_L + \left(\frac{-\frac{1}{R_2}}{4-\frac{1}{R_1}-\frac{1}{R_2}}\right)v_C + \left(\frac{-1}{4-\frac{1}{R_1}-\frac{1}{R_2}}\right)i_S \\ i_C = C\frac{dv_C}{dt} = \left(\frac{\frac{1}{R_2}-4}{4-\frac{1}{R_1}-\frac{1}{R_2}}\right)i_L + \left(\frac{\frac{1}{R_2}\left(4-\frac{1}{R_2}\right)}{4-\frac{1}{R_1}-\frac{1}{R_2}}-\frac{1}{R_2}\right)v_C + \left(\frac{4-\frac{1}{R_2}}{4-\frac{1}{R_1}-\frac{1}{R_2}}\right)i_S \end{cases}$$

Remark 1. Be due to the effect of the dependent source, the procedure (STEP 3) in [14, p. 131] is too complicated to be manipulated.

6. Practical application

Consider the following unstable system shown in Fig. 10. Our goal is to design a controller such that the overall system is stable. Applying the C.E.U.A. and the virtual grounded property of the operational amplifier, the state equations and the output equation are facile to be obtained immediately as follows:

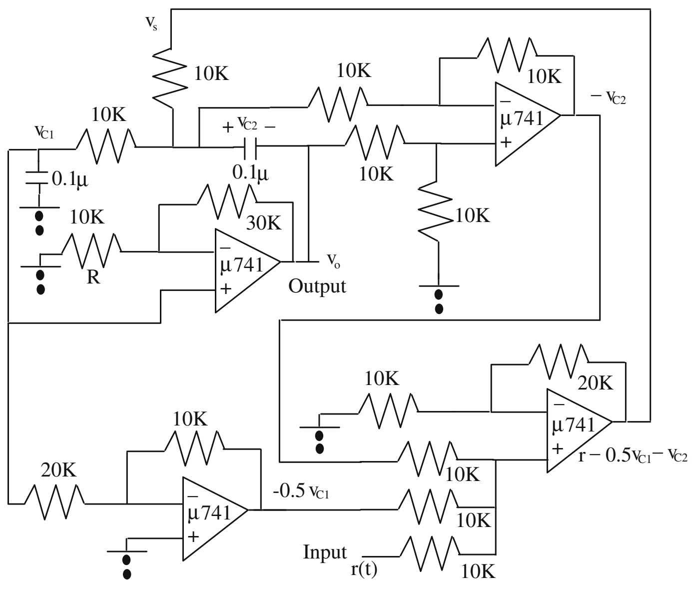


Fig. 11. Feedback-controlled system with stabilizable controller.

$$\begin{bmatrix} \frac{dv_{C1}}{dt} \\ \frac{dv_{C2}}{dt} \end{bmatrix} = \begin{bmatrix} \frac{3}{RC} & \frac{1}{RC} \\ \frac{-7}{RC} & \frac{-2}{RC} \end{bmatrix} \begin{bmatrix} v_{C1} \\ v_{C2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{RC} \end{bmatrix} v_s := AX + Bv_s$$

$$[v_o] = [4 \quad 0] \begin{bmatrix} v_{C1} \\ v_{C2} \end{bmatrix} := CX$$

Based on the state feedback approach, our controller is given by

$$v_s = r - KX = r - [k_1 \quad k_2] \begin{bmatrix} v_{C1} \\ v_{C2} \end{bmatrix}$$

The characteristic equation of the feedback-controlled system is

$$|sI - (A - BK)| = s^2 + s\left(\frac{k_2 - 1}{RC}\right) + \left(\frac{1 + k_1 - 3k_2}{R^2C^2}\right) = 0$$

According to the Routh–Hurwitze criterion, we can choose $k_1 = 1$ and $k_2 = 0.5$ such that the feedback-controlled system is stable. Fig. 11 shows the hardware implementation of the feedback-controlled system with desired controller, where we choose $R = 10\ \text{k}\Omega$ and $C = 0.1\ \mu\text{F}$.

7. Conclusion

The peculiarities of simplifying the investigation of designing and finding the state equations of the complicated physical systems are unprecedented for the C.E.U.A. It's our belief that, from the aforementioned significant contributions of the C.E.U.A., the C.E.U.A. has a situation with great potentialities of taking the place of those traditional approaches in future. Moreover, the C.E.U.A. will be skillfully utilized in the fields of the power electronics, the power system analysis, etc.

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